Sub-wavelength quantum imaging for astronomy

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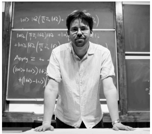
AIP Congress, 2022





Motivation: surpassing Rayleigh's criterion

Quantum-limited estimation for LIDARs

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Experiment: quantum super-resolution imaging and hypothesis testing

Exoplanet atmostpheric spectroscopy

The Rayleigh Criterion

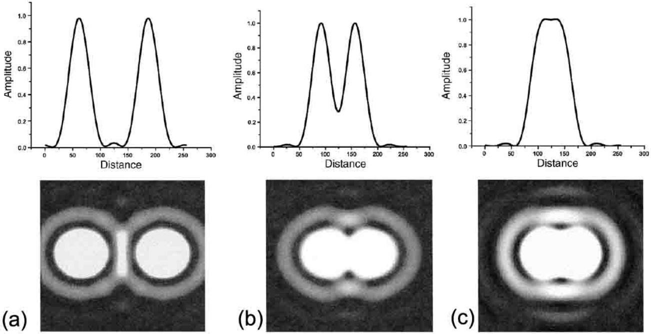


Figure: taken from www.globalsino.com

Minimum resolvable angular separation

$$\theta \approx 1.22\frac{\lambda}{D}$$

Task: estimate θ_2

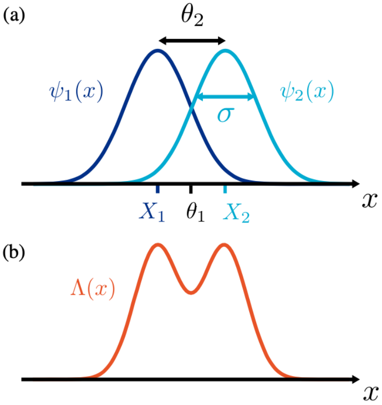


Figure: [1] Tsang et al., Phys. Rev. X 6, 031033 (2016)

Cramer-Rao bound:

$$\Delta^2\varphi = \langle\varphi^2\rangle - \langle\varphi\rangle^2 \geq \frac{1}{\nu I(\varphi)}$$

Fisher information:

$$I(\varphi) = \sum_i p(i|\varphi)\left(\frac{\partial \log[p(i|\varphi)]}{\partial\varphi}\right)^2$$

Quantum state:

$$\rho_\varphi = \sum_j \lambda_j |j\rangle\langle j|$$

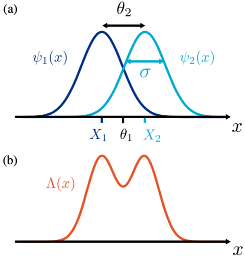
Quantum Fisher information

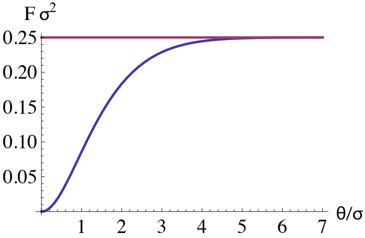
$$F(\rho_\varphi) = \sum_{\lambda_j+\lambda_k\neq 0} 2\frac{|\langle j|\frac{\partial\rho}{\partial\varphi}|k\rangle|^2}{\lambda_j+\lambda_k},$$

Superresolution of two sources

Model: two incoherent, quasi-monochromatic point sources

$$\rho \approx \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|), \qquad |\psi_i\rangle = \int_{-\infty}^{\infty} \psi_i(x)\,|x\rangle$$

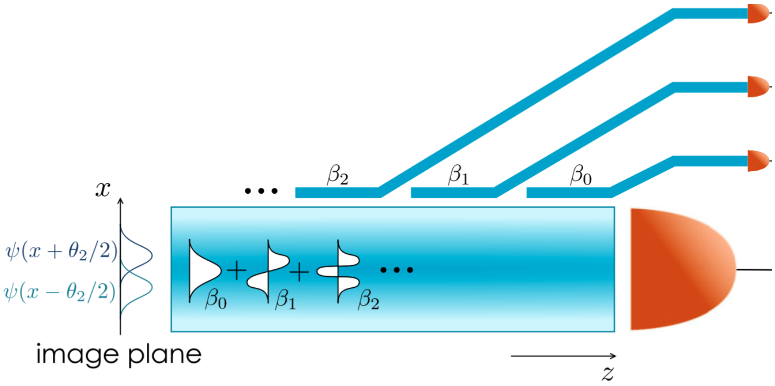




$$F(\theta_2) = \frac{1}{4\sigma^2}$$

[1] Tsang et al., Phys. Rev. X 6, 031033 (2016)

Optimal measurement: FI = QFI



[1] Tsang et al., Phys. Rev. X 6, 031033 (2016)

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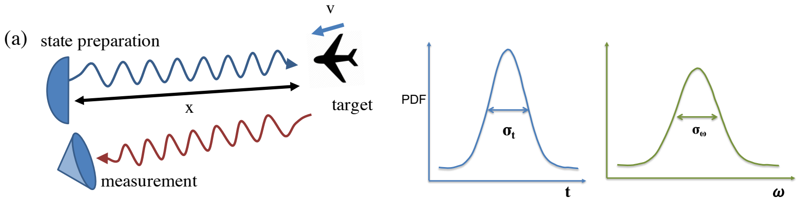
Position and velocity estimation



Range x and velocity v:

$$\bar{t} = \frac{2x}{c}, \qquad \bar{\omega} \approx \omega_0 \left(1 - 2\frac{v}{c}\right)$$

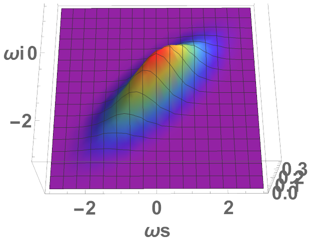
The energy-time uncertainty relation [2]

$$\sigma_t \sigma_\omega \geq 1/2.$$

[2] E. Arthurs and J. Kelly, the Bell System Technical Journal 44, 725 (1965)

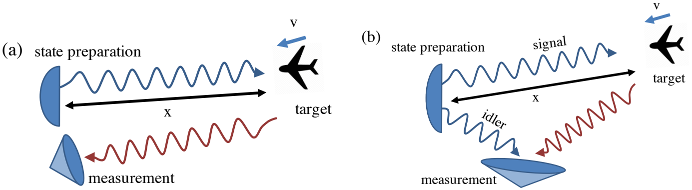
Time-frequency entanglement (SPDC) [3] :

$$|\Psi_0\rangle = \int d\omega \int d\omega_i \; \tilde{\Psi}_0(\omega, \omega_i, \kappa)\, |\omega\rangle\, |\omega_i\rangle\,, \qquad \kappa \in \{0, 1\}$$



[3] MacLean et al., Phys. Rev. Lett. 120, 053601

Position and velocity estimation



$$J(\bar{t},\bar{\omega}) = \begin{pmatrix} 4\sigma_\omega^2 & 0 \\ 0 & 1/\sigma_\omega^2 \end{pmatrix} \qquad J_{\text{ent}}(\bar{t},\bar{\omega}) = \begin{pmatrix} 4\sigma_\omega^2 & 0 \\ 0 & \frac{1}{\sigma_\omega^2}\frac{1}{1-\kappa^2} \end{pmatrix}$$

$$\sigma_t\sigma_\omega = \frac{1}{2} \qquad \sigma_t\sigma_\omega = \frac{1}{2}\sqrt{1-\kappa^2}$$

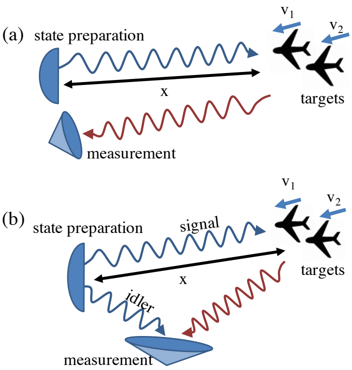
Joint optimal estimation condition [5] not satisfied,

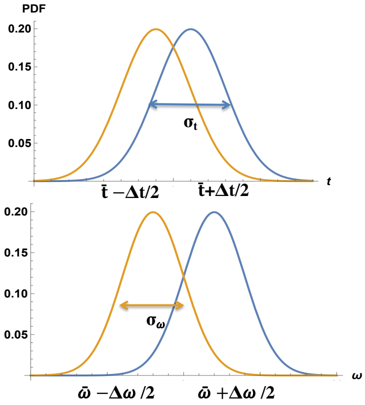
$$\mathrm{Tr}\left(\rho[L_{\bar{t}}, L_{\bar{\omega}}]\right) = -4i$$

[5] S. Ragy, et al. Phys. Rev. A 94, 052108 (2016).
[6] Q. Zhuang, et al., Phys. Rev. A 96, 040304 (2017) .

Position and velocity *separation* estimation

$$\rho = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|)$$





Quantum metrology for superresolution

PRL **117**, 190802 (2016) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2016

Ultimate Precision Bound of Quantum and Subwavelength Imaging

Cosmo Lupo[1] and Stefano Pirandola[1,2]
[1]*York Centre for Quantum Technologies (YCQT), University of York, York YO10 5GH, United Kingdom*
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(Received 6 July 2016; published 4 November 2016)

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PHYSICAL REVIEW LETTERS **122**, 140505 (2019)

**Towards Superresolution Surface Metrology:
Quantum Estimation of Angular and Axial Separations**

Carmine Napoli,[1,2,*] Samanta Piano,[2,†] Richard Leach,[2,‡] Gerardo Adesso,[1,§] and Tommaso Tufarelli[1,∥]
[1]*School of Mathematical Sciences and Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems, University of Nottingham, University Park Campus, Nottingham NG7 2RD, United Kingdom*
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PHYSICAL REVIEW LETTERS **121**, 180504 (2018)

Quantum Limited Superresolution of an Incoherent Source Pair in Three Dimensions

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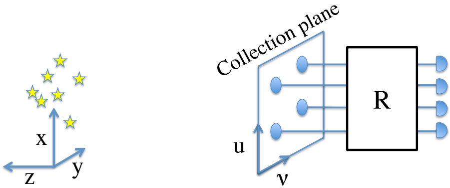
(Received 26 May 2018; published 31 October 2018)

Arbitrary number of sources



N_s incoherent sources, quasi-monochromatic, coordinate $\vec{r_s}$.
N_c collectors, position $\vec{\omega_j}$. Photon impinging $\rightarrow |j\rangle$

$$|\psi(r_s)\rangle = \frac{1}{\sqrt{N_c}} \sum_j^{N_c} e^{i\phi(\vec{r_s},\vec{\omega}_j)} |j\rangle \,,$$

$$\rho = \sum_s^{N_s} p(s) \, |\psi(r_s)\rangle \langle \psi(r_s)|$$

C Lupo, Z Huang, P Kok, Phys. Rev. Lett. 124, 080503 (2020)

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1 or 2 sources?

Two hypotheses H_0, H_0

Classical: $p_0(x), p_1(x)$, quantum: ρ_0, ρ_1

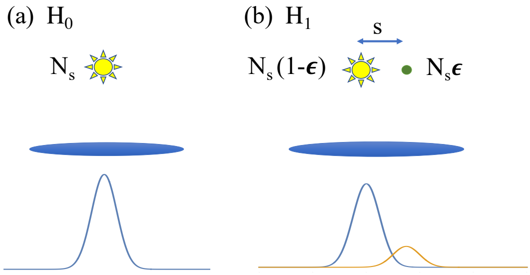


Figure: (a) H_0:there is only 1 source. (b) H_1: two near-by sources present.

Huang & Lupo, Phys. Rev. Lett. 127, 130502 (2021);
Editor's Selection and Featured in Physics

Quantum state discrimination

1. Symmetric discrimination: trace distance - quantum Chernoff bound [6], $P_e \sim \exp[-n\ f(T)]$

$$T_c(p_0, p_1) = 1/2 \int dx\ |p_0(x) - p_1(x)|$$

$$T_Q(\rho_0, \rho_1) = 1/2||\rho_0 - \rho_1||_1$$

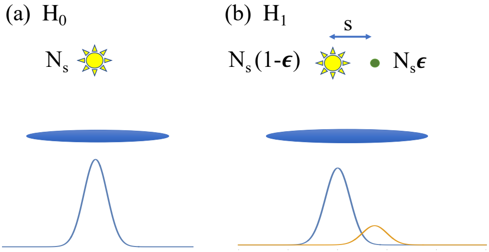
2. Asymmetric: relative entropy - quantum Stein lemma [7],

$$P_e \sim \exp[-nD(\rho_0||\rho_0) + O(\alpha^{-1}, \ln n)]$$

$$D_c(p_0||p_1) = \int\ dx\ p_0(x)(\ln p_0(x) - \ln p_1(x))$$

$$D_Q(\rho_0||\rho_1) = \mathrm{Tr}[\rho_0(\ln \rho_1 - \ln \rho_1)]$$

[6] Audenaert et al., Phys. Rev. Lett. 98, 160501 (2007)
[7] F. Hiai, D. Petz, Commun. Math. Phys. 143, 99 (1991)

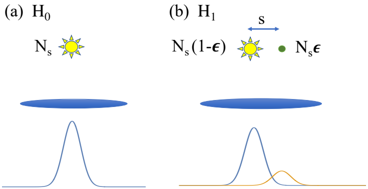


Two hypotheses associated with the density matrices

$$\rho_0 = |\psi_{x_0}\rangle \langle\psi_{x_0}| \ ,$$
$$\rho_1 = (1-\epsilon)\,|\psi_{x_0}\rangle \langle\psi_{x_0}| + \epsilon\,|\psi_{x_0+s}\rangle \langle\psi_{x_0+s}| \ .$$

Huang & Lupo, Phys. Rev. Lett. 127, 130502

Classical probability distributions



On-screen probability distributions:

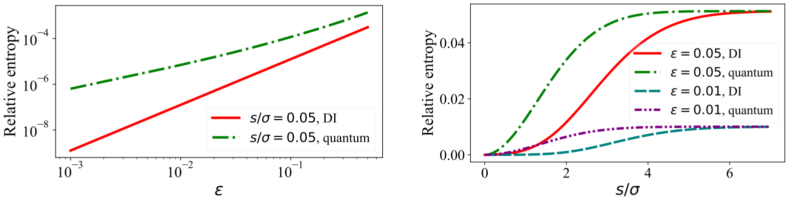
$$p_0(x) = |\psi(x - x_0)|^2,$$
$$p_1(x) = (1-\epsilon)|\psi(x - x_0)|^2 + \epsilon|\psi(x - (x_0 + s))|^2.$$

Classical relative entropy:

$$D(p_0||p_1) \approx \frac{s^2\epsilon^2}{2\sigma^2} + O(\epsilon^3)$$

Huang & Lupo, Phys. Rev. Lett. 127, 130502

Classical vs Quantum



Classical relative entropy:

$$D(p_0||p_1) \approx \frac{s^2\epsilon^2}{2\sigma^2}$$

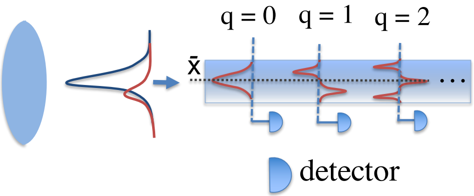
Quantum relative entropy;

$$D(\rho_a||\rho_b) \approx \frac{s^2\epsilon}{4\sigma^2} + O(\epsilon^2 s^2).$$

Huang & Lupo, Phys. Rev. Lett. 127, 130502

Optimal measurement - SPADE

Near-optimal measurement: SPADE



For our case, we point the optical imaging system towards the optical "center of mass".

$$\bar{x} = (1-\epsilon)x_0 + \epsilon(x_0 + s)\,. \tag{1}$$

Huang & Lupo, Phys. Rev. Lett. 127, 130502 (2021)

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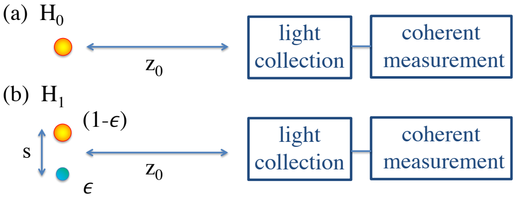
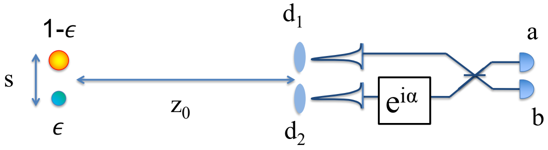


- Sub-Rayleigh quantum state discrimination
- Optimal angular separation estimation (equally bright sources)
- Two tasks achieved with *a single measurement*
- Binary stars, microscopy

Experimental scheme

Instead of a lens, place two collectors at d_1, d_2:



$$|\psi_{\text{star}}\rangle = \frac{1}{\sqrt{2}}(|d_1\rangle + e^{i\phi}|d_2\rangle), \quad |\psi_{\text{planet}}\rangle = \frac{1}{\sqrt{2}}(|d_1\rangle + e^{-i\phi}|d_2\rangle)$$

In the limit of $\epsilon \ll 1$, $\theta = s/z_0$ (angular)

$$D(\rho_0||\rho_1) \approx \frac{\epsilon\theta^2 k^2 d^2}{4}$$

Zanforlin, et al., Nat. Commun. 13, 5373 (2022)

Simplifying the experiment

H_0: the probabilities of photon arriving at detectors a and b are

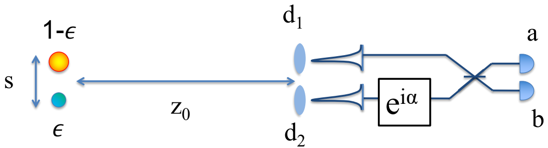
$$p_{H_0}(a) = \frac{1}{2}(1 + \nu\cos(\phi + \alpha))$$
$$p_{H_0}(b) = \frac{1}{2}(1 - \nu\cos(\phi + \alpha))$$

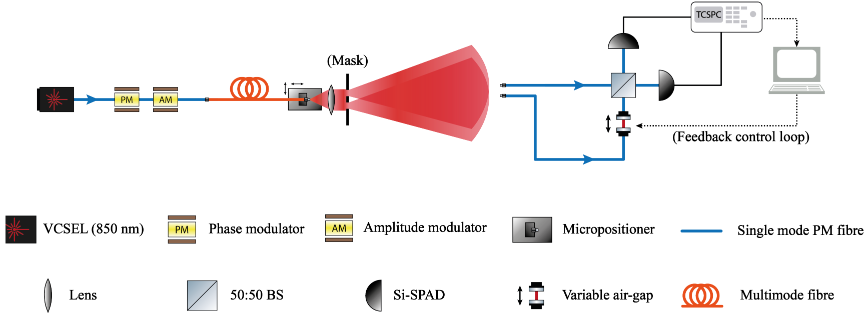
H_1:

$$p_{H_1}(a) = \frac{1}{2}[(1-\epsilon)(1 + \nu\cos(\phi + \alpha)) + \epsilon(1 + \nu\cos(-\phi + \alpha))]$$
$$p_{H_1}(b) = \frac{1}{2}[(1-\epsilon)(1 - \nu\cos(\phi + \alpha)) + \epsilon(1 - \nu\cos(-\phi + \alpha))]$$

$$D_c(p_0||p_1) = \sum_x p_0(x)\left(\log p_0(x) - \log p_1(x)\right), \qquad x = a, b$$

The experimental setup





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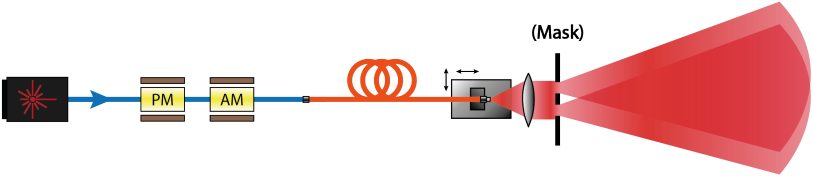
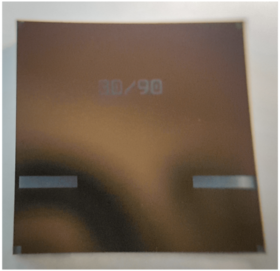




Zanforlin, et al., Nat. Commun. 13, 5373 (2022)

Results - relative entropy

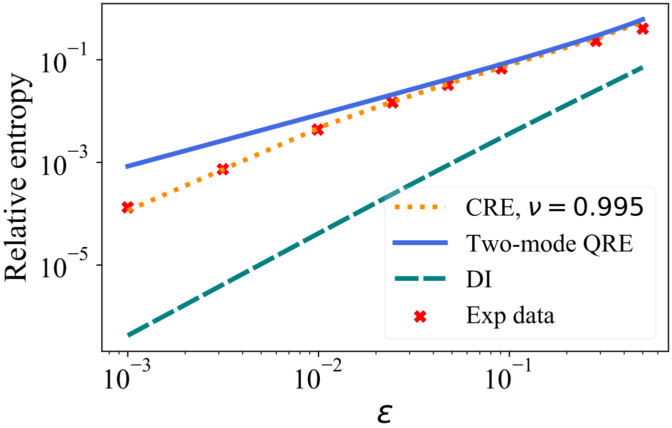


Figure: Relative entropy of the two hypothesises

Zanforlin, et al., Nat. Commun. 13, 5373 (2022)

Angular separation measurement

When the source intensities are equal, the QFI for the angular separation is

$$\text{QFI} = \frac{k^2}{4}(d_1 - d_2)^2$$

The probabilities of detecting the photon at either detector are

$$p_1 = \frac{1}{2}\left(1 + \nu\cos(\alpha)\cos\left[\phi\right]\right), \quad \phi = kd\theta/2$$
$$p_2 = \frac{1}{2}\left(1 - \nu\cos(\alpha)\cos\left[\phi\right]\right)$$

We use Baye's theorem to update the probability distribution.

Zanforlin, et al., Nat. Commun. 13, 5373 (2022)

Sample distribution

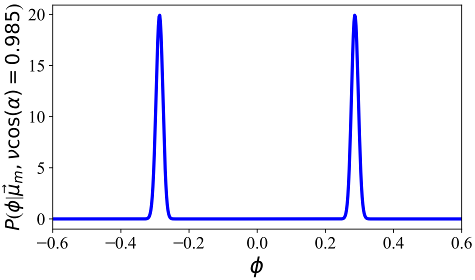


Figure: Sample distribution: after 10,000 detection events, 204 arrived at detector *b*.

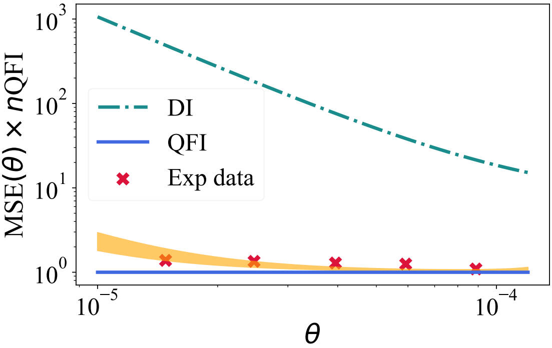
$$\nu \cos(\alpha) \in [-1, 1]$$

Estimator:

$$\hat{\theta}_{\text{est}} = 2|\hat{\phi}_{\text{est}}|/(kd) \qquad (2)$$

Results - MSE

$$\text{MSE}(\theta) = \Delta^2\theta + (\bar{\theta} - \theta_{\text{true}})^2$$



Two sources 15 μ rad apart, we resolve θ to 1.7% accuracy.
Factor 2 within QCR bound.

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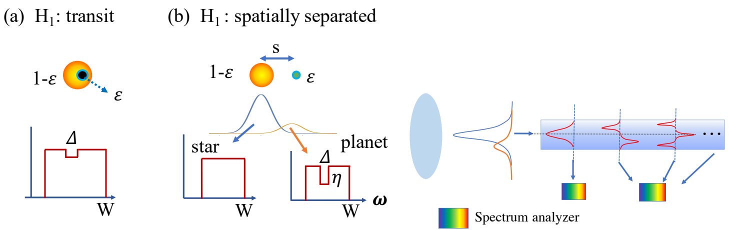
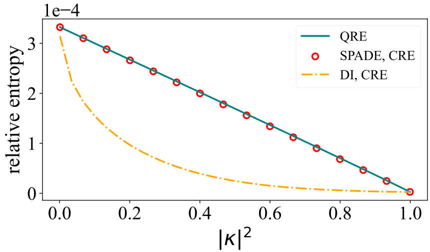
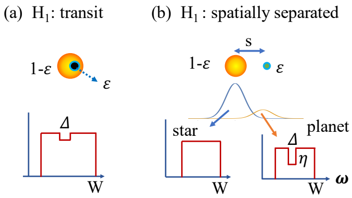
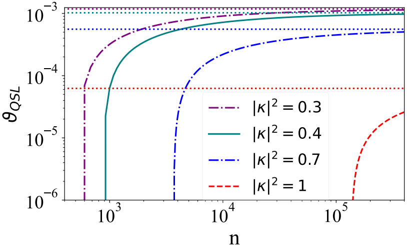




Huang, Schwab, Lupo, arXiv:2211.06050 (2022)

Application to spectroscopy





$$\vartheta_{\mathrm{QSL}} := D(\rho_0 \| \rho_1) + \sqrt{\frac{V(\rho_0 \| \rho_1)}{n}}\, \Phi^{-1}(\alpha) + O\left(\frac{\log n}{n}\right)$$

Huang, Schwab, Lupo, arXiv:2211.06050 (2022)

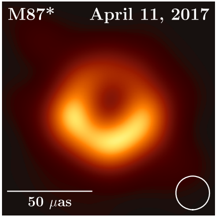
Conclusions

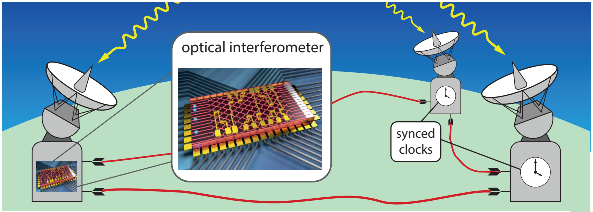
- We compute the type-II error probability exponent of discriminating between 1 or two sources with arbitrary intensity.
- in the limit that $\epsilon \ll 1$, the quantum relative entropy is larger than that of direct imaging by a factor of $1/\epsilon$.
- For a lens: two measurement methods that are optimal in this regime
- We significantly simplified the scheme to a two-mode interferometer, demonstrates sub-Rayleigh scaling and approach the QCRB.

Huang & Lupo, Phys. Rev. Lett. 127, 130502
Zanforlin, et al., Nat. Commun. 13, 5373 (2022)

To the future

£360K (≈AU $650K) grant from the EPSRC
DECRA Fellowship (2023) from the ARC
This work is funded by the Sydney Quantum Academy





- Long-distance optical coherence, entanglement-assisted network
- Quantum error correction to combat to loss to decoherence
- Current collaborations: Bristol, Heriot-Watt, Erlangen, UWA

PhD scholarship available

Questions?

Thank you for your attention.

Figure: (Left) my hamster in the UK; (right) my jenday conure

PhD scholarship available