



Modelling the Experimental Analysis of State Injection for Error-Corrected Quantum Systems

Anthony O'Rourke

Dr. Simon Devitt



G'day!



I'm Anthony O'Rourke a PhD Student at UTS: QSI



THE UNIVERSITY OF
SYDNEY

My undergraduate degree was in Arts / Science
(Advanced) (Honours) in Physics, Music & French



I'm passionate about quantum computing, reading
and increasing diversity in STEM



Lac de Yaté, New Caledonia

Problem:

- Injecting high-fidelity, arbitrary quantum states into an error-corrected system is resource-intensive

Solution:

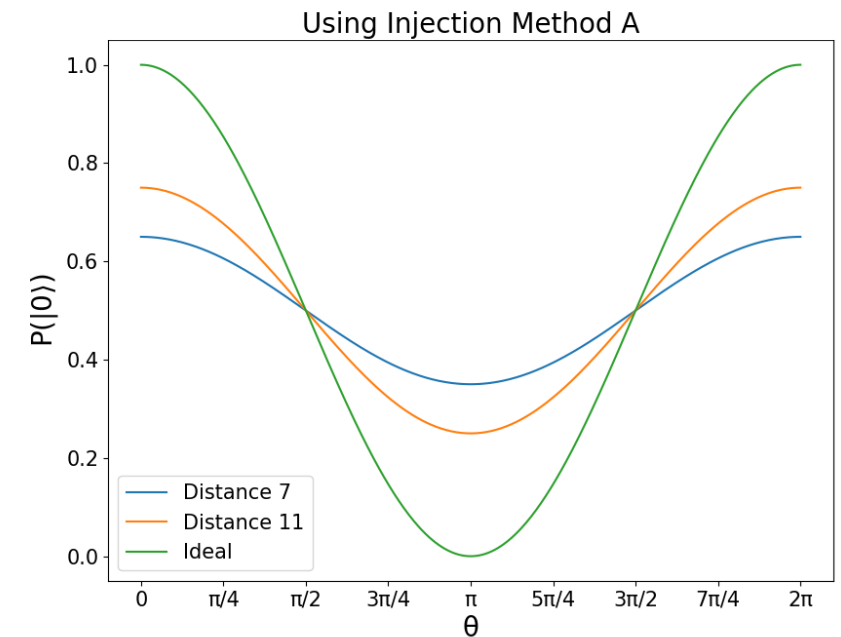
- Model an **experiment** comparing the three methods of state injection and find which takes the lowest resources for the highest fidelity of states

Significance:

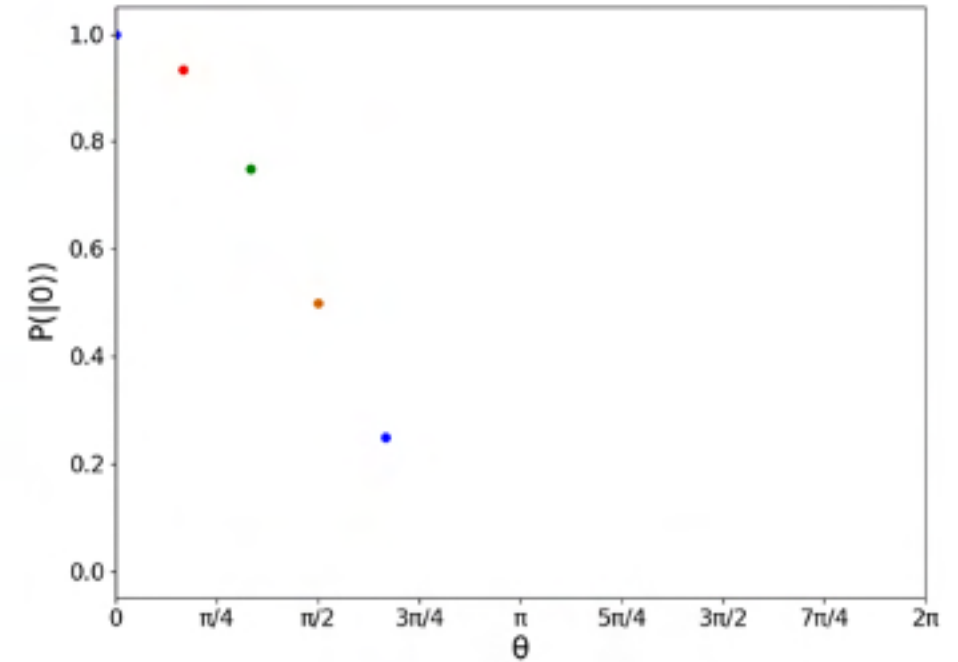
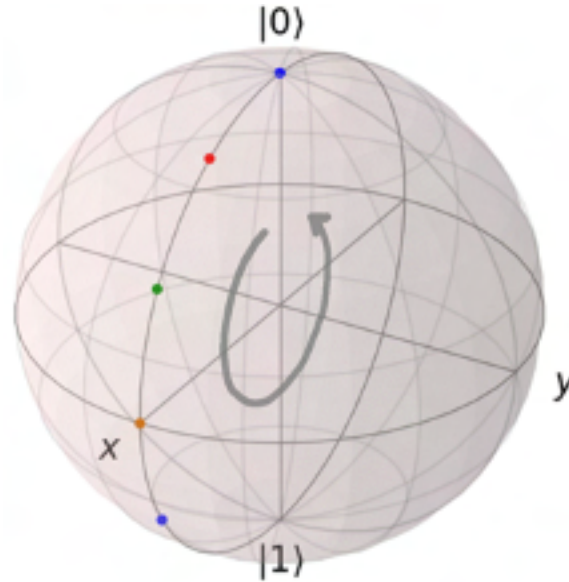
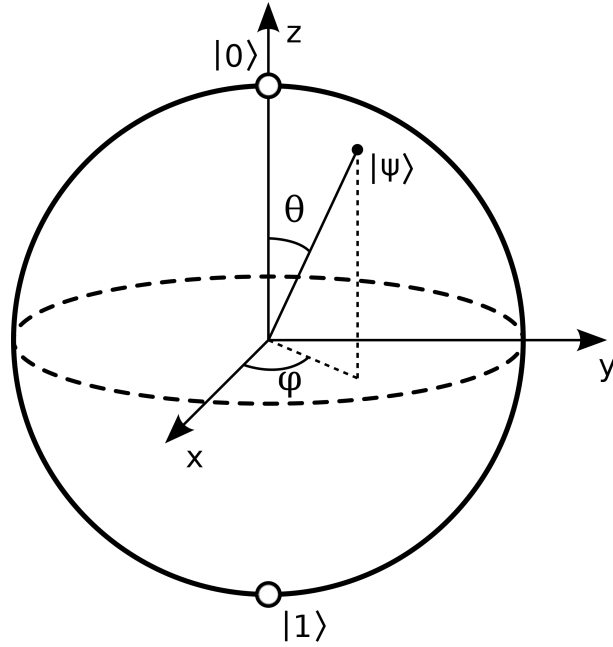
- The state injection method requiring the least resources for the highest fidelity state can be used to make a universal gate set for error-corrected qubits.

Key takeaways from today:

1. A Rabi oscillation experiment can be performed on an error-corrected, logical qubit.
2. The state injection method which maximises state fidelity maximises the Rabi curve's visibility
3. This injection method will be the best for minimising qubit numbers and other hardware requirements for achieving a universal gate set



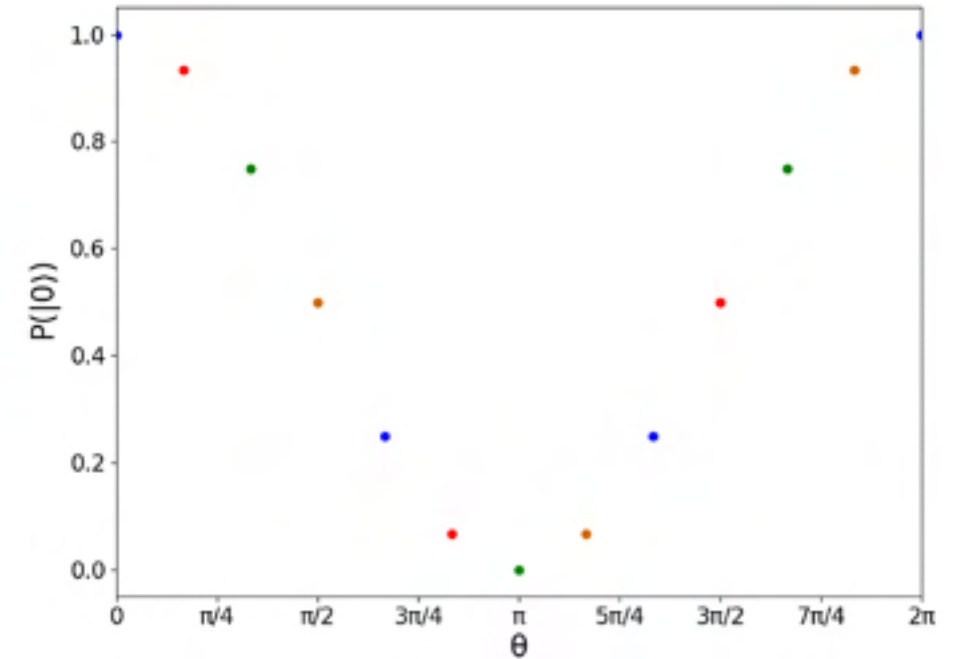
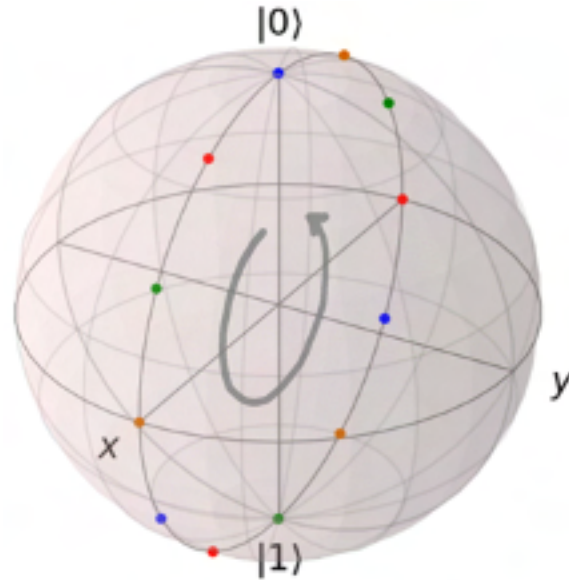
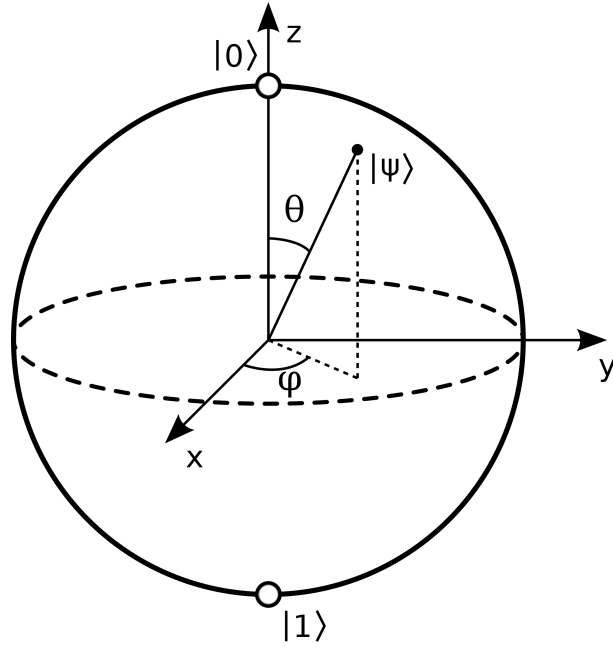
Rabi Oscillation



$$\begin{aligned} |\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \\ &:= \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle \end{aligned}$$

$$P(|0\rangle) = |\alpha|^2 = |\cos\theta/2|^2$$

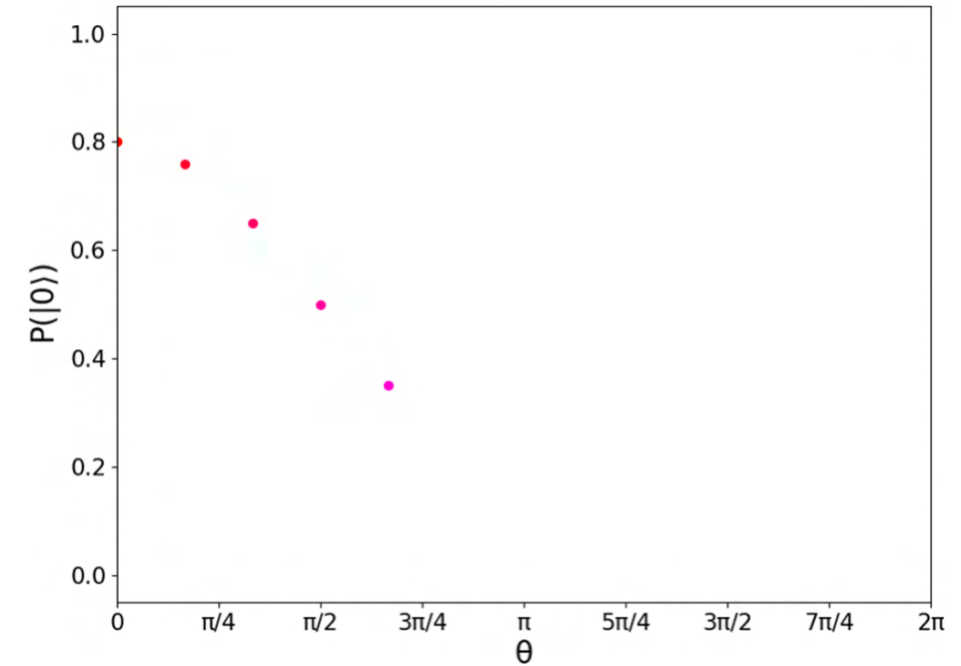
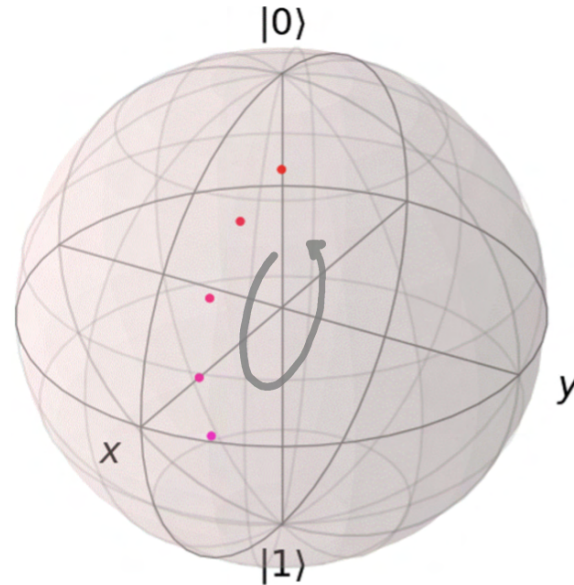
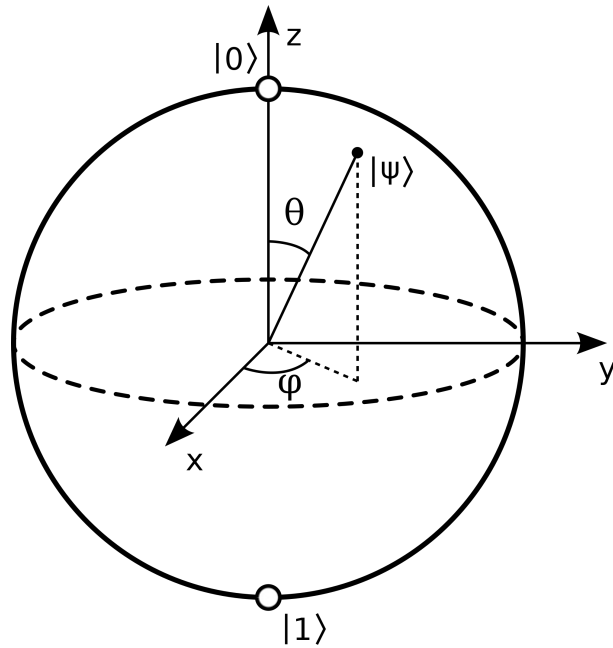
Rabi Oscillation



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$:= \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

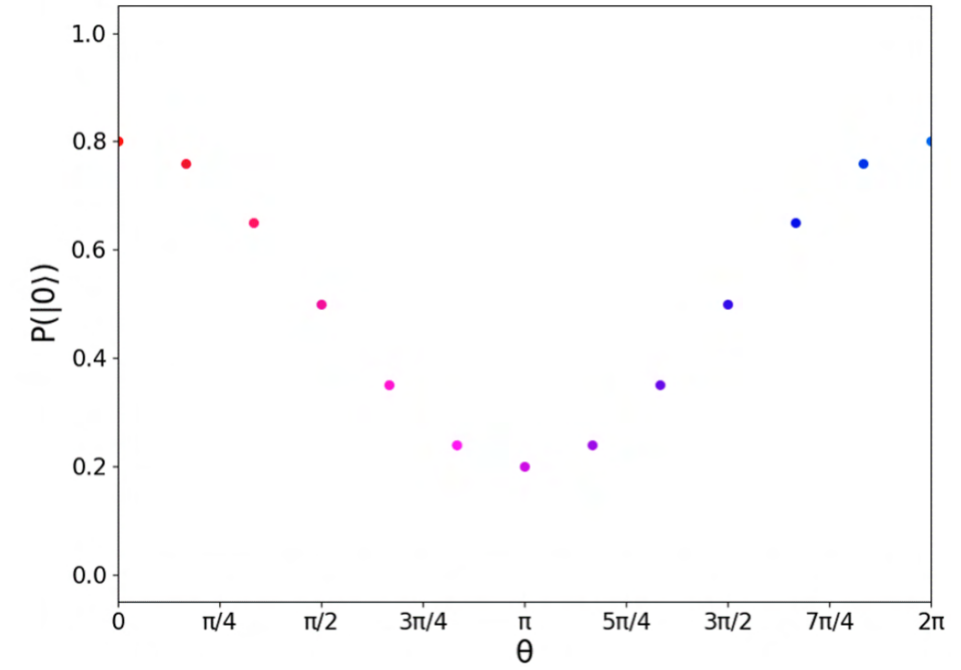
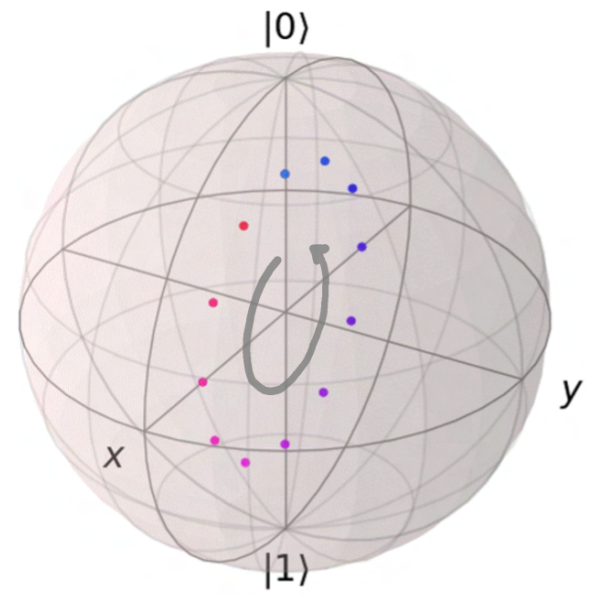
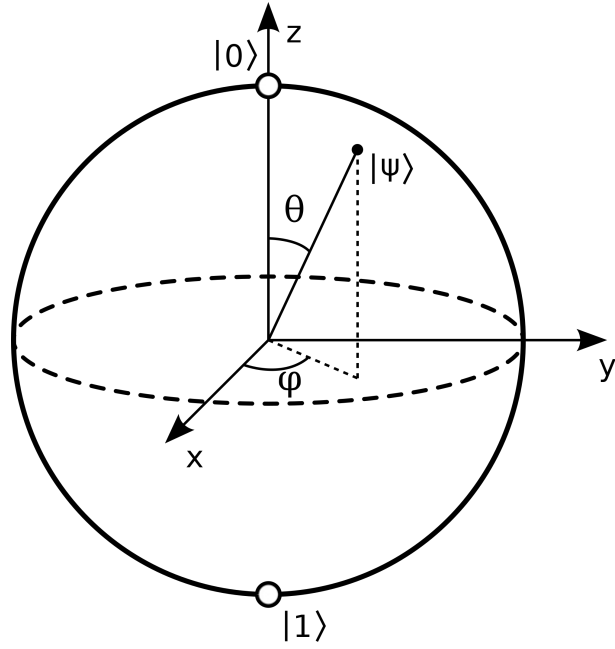
$$P(|0\rangle) = |\alpha|^2 = |\cos\theta/2|^2$$

Rabi Oscillation – with depolarising noise



The curve's *visibility* is reduced

Rabi Oscillation – with depolarising noise



The curve's *visibility* is reduced

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$:= \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

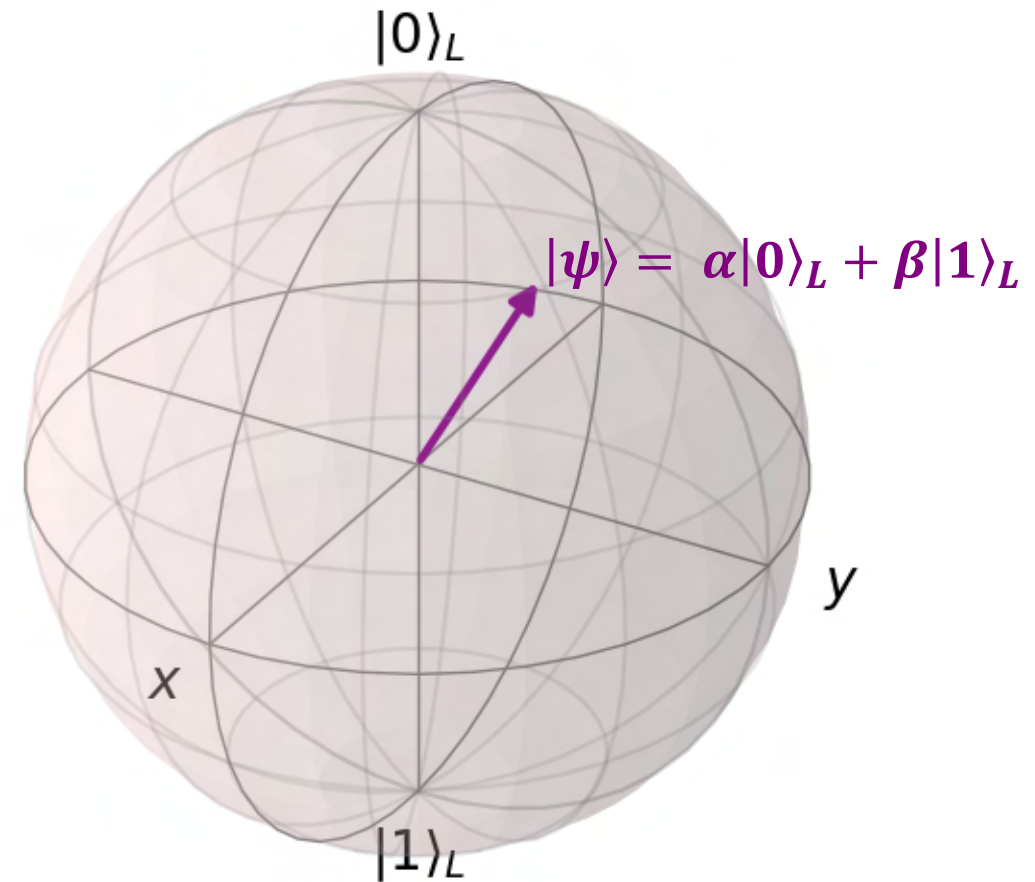
Quantum error correction:

- Encode the state of one qubit into a “logical” qubit made of n qubits

E.g. $\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$

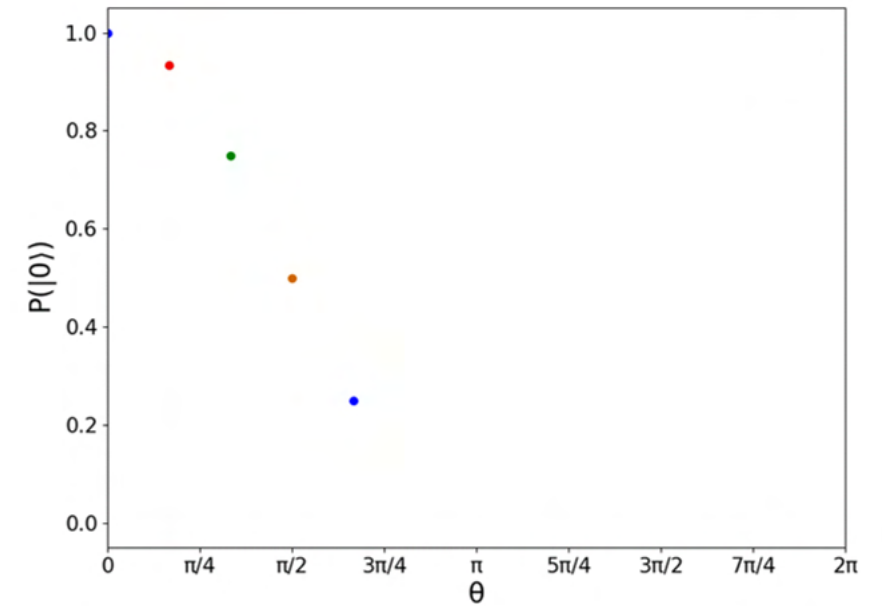
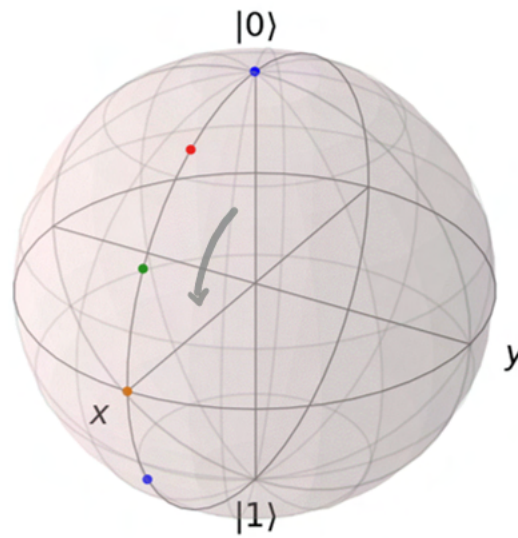
$$:= \alpha|0\rangle_L + \beta|1\rangle_L$$

- The *distance* of an error-correcting code is the number of single qubit operations between $|0\rangle_L$ and $|1\rangle_L$.
- Increasing distance increases the amount of errors that can be corrected



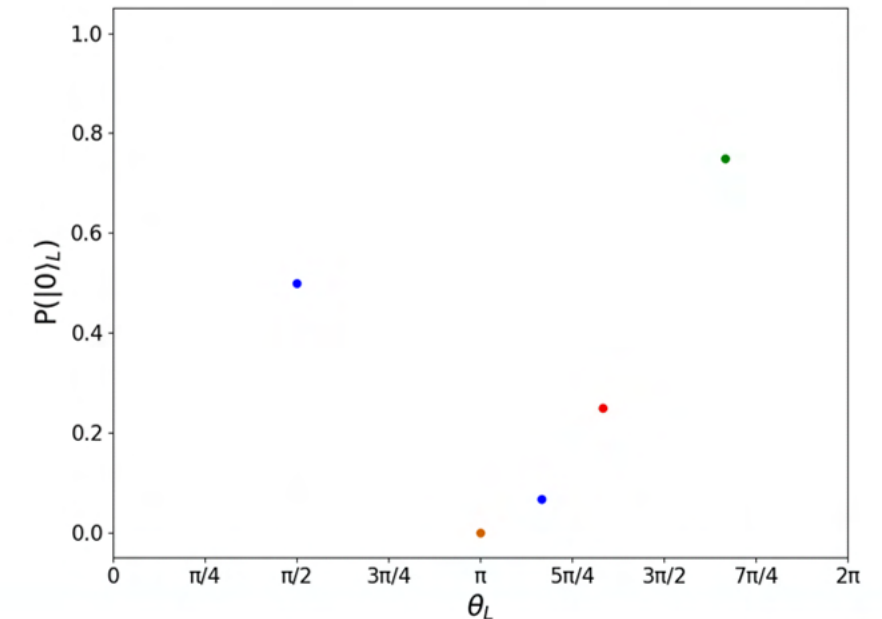
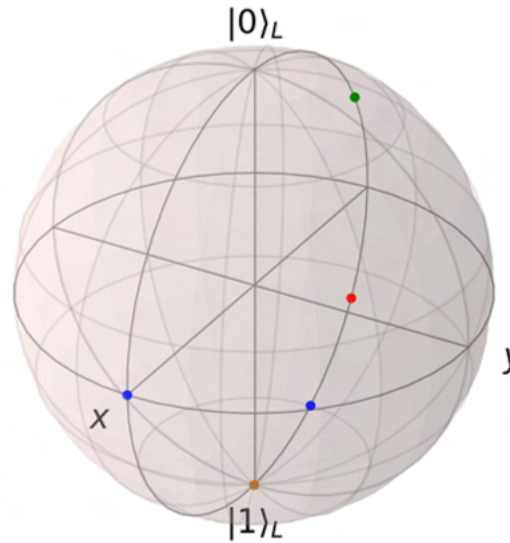
Rabi oscillation:

- Increasing pulse time of microwave on qubit rotates the qubit by θ



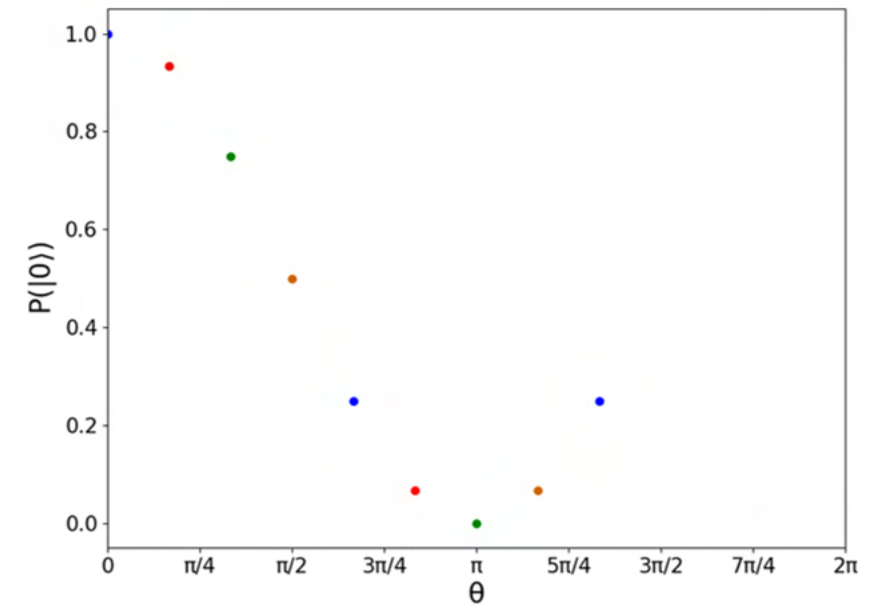
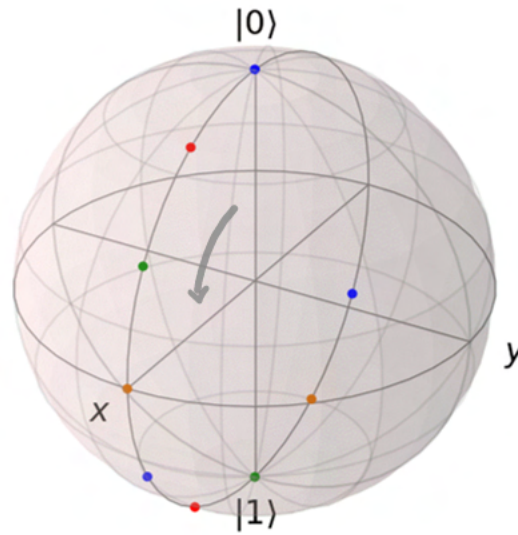
Logical Rabi oscillation:

- To reach arbitrary logical states requires one of three methods of state *injection*
 - Standard method¹
 - Ying Li's method²
 - Transversal Injection³
 - Reduces amount of resource-intensive state distillation
 - Can be done at higher fidelity at lower distances



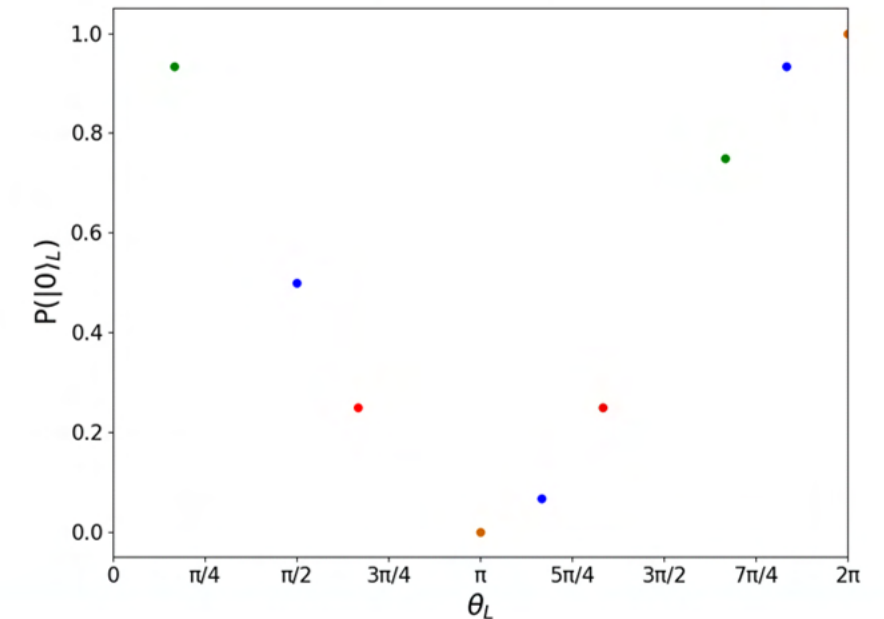
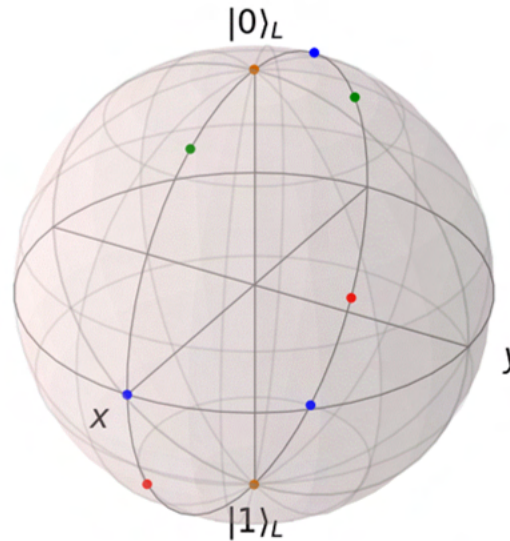
Rabi oscillation:

- Increasing pulse time of microwave on qubit rotates the qubit by θ



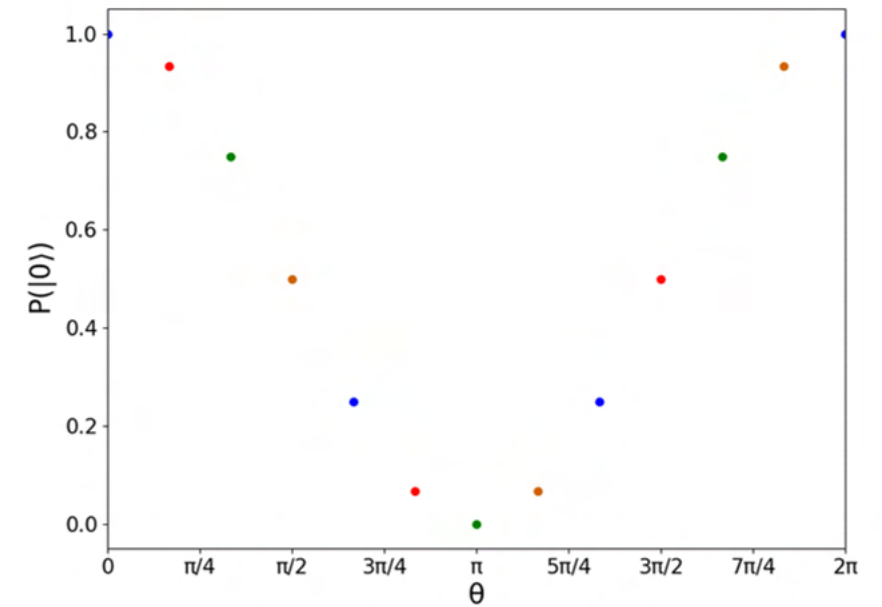
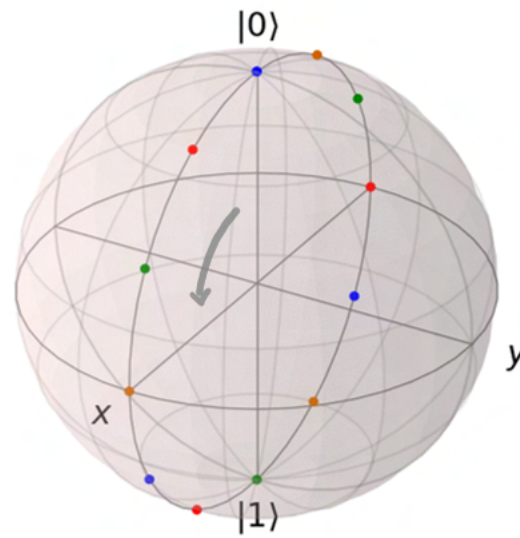
Logical Rabi oscillation:

- To reach arbitrary logical states requires one of three methods of state *injection*
 - Standard method¹
 - Ying Li's method²
 - Transversal Injection³
 - Reduces amount of resource-intensive state distillation
 - Can be done at higher fidelity at lower distances



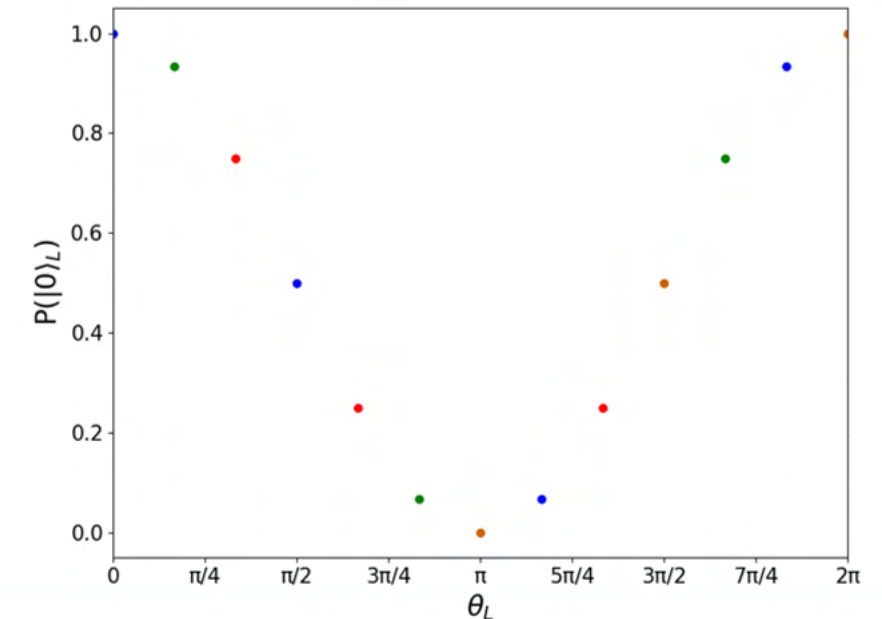
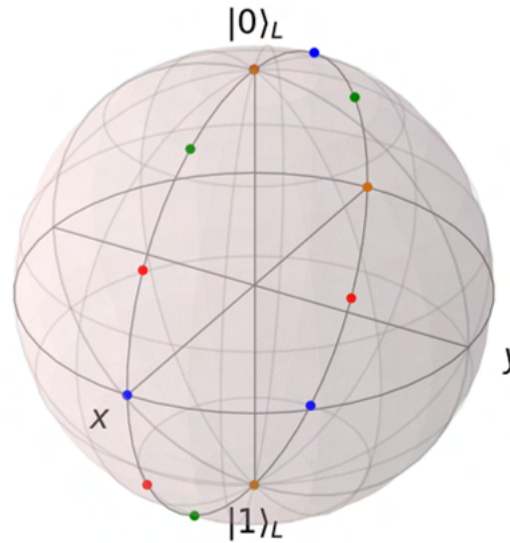
Rabi oscillation:

- Increasing pulse time of microwave on qubit rotates the qubit by θ

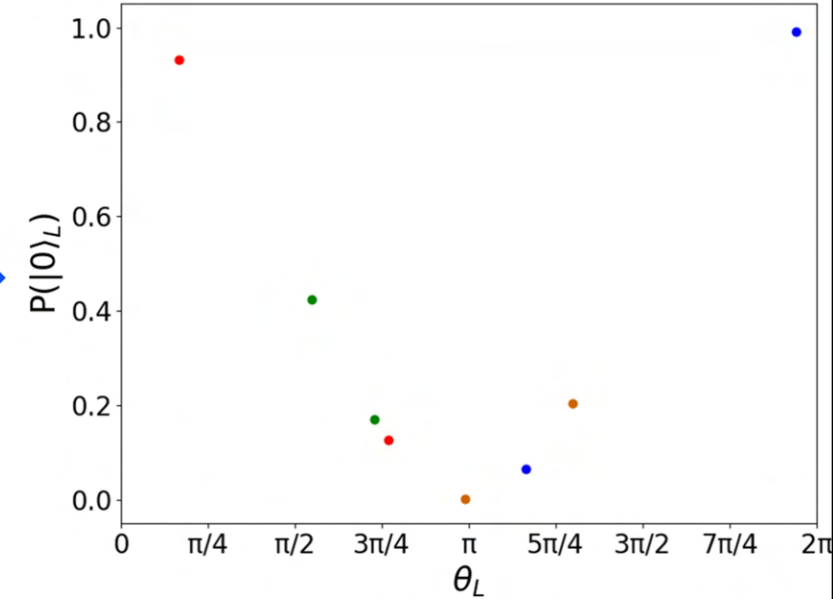
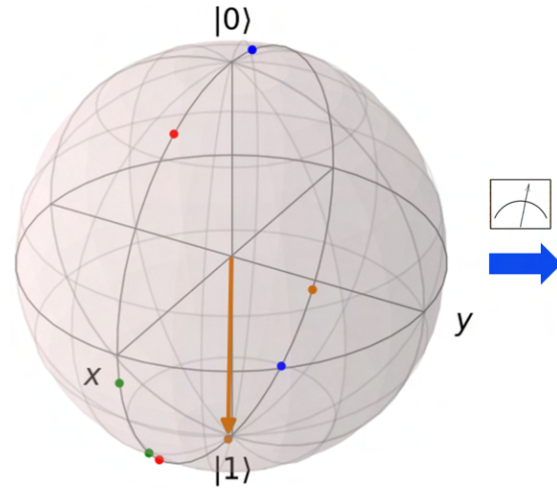
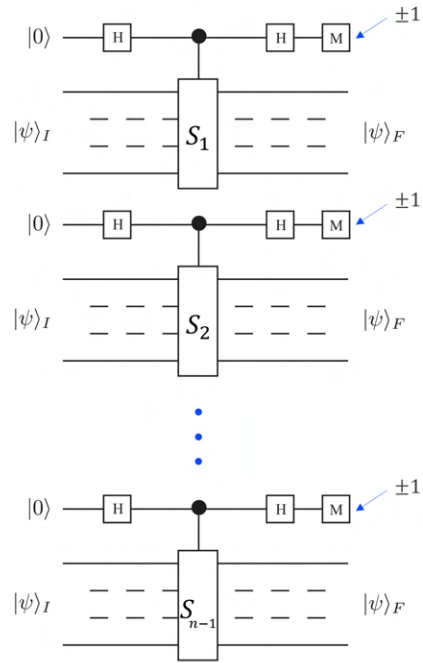
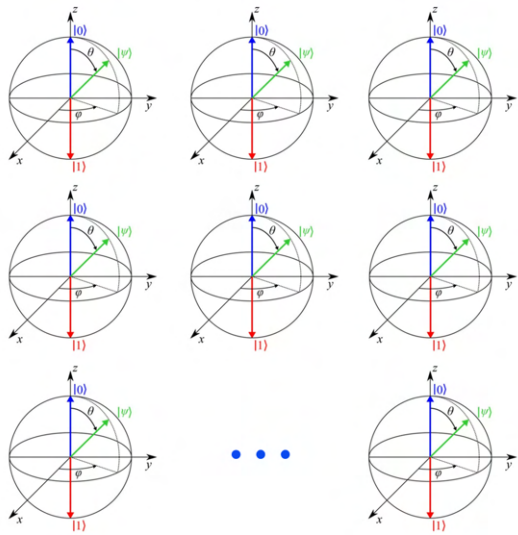


Logical Rabi oscillation:

- To reach arbitrary logical states requires one of three methods of state *injection*
 - Standard method¹
 - Ying Li's method²
 - Transversal Injection³
 - Reduces amount of resource-intensive state distillation
 - Can be done at higher fidelity at lower distances



Logical Rabi using Transversal Injection



Step 1: rotate n data qubits by θ and ϕ to get $|\psi\rangle = (\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle)^{\otimes n}$

Step 2: apply the projective measurements of the $n - 1$ code stabilizers

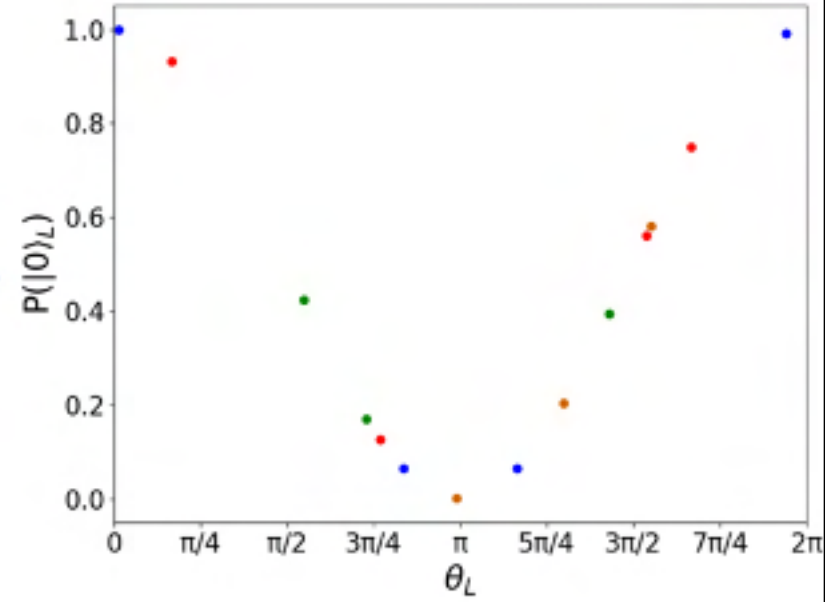
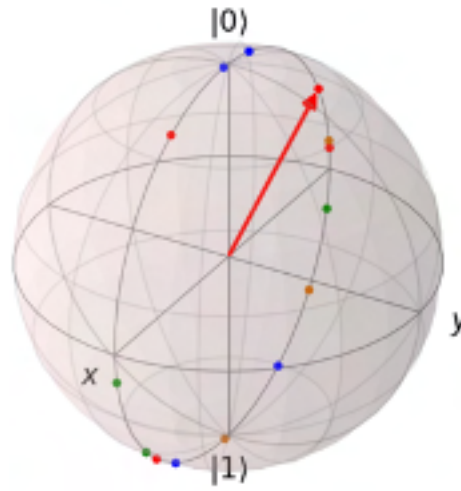
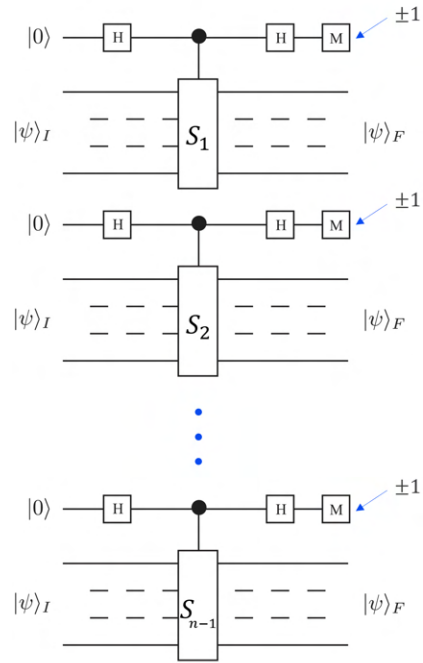
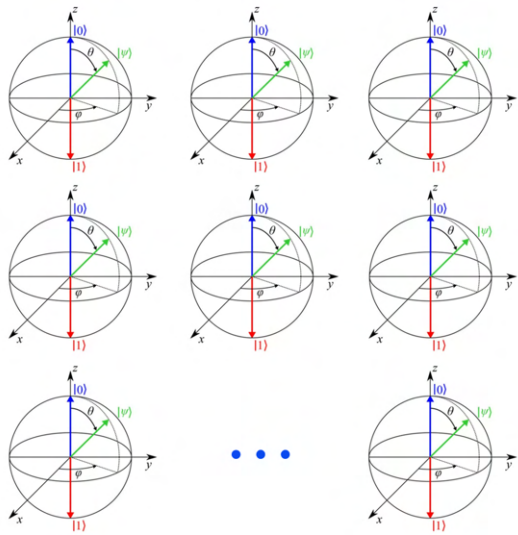
The **logical** state thus projected to can be calculated from:

- The initial choice of θ and ϕ
- The (random) stabilizer measurement results known as the *trajectory*.

E.g. $+1, -1, -1, \dots, +1$
 $\underbrace{\hspace{10em}}_{n - 1 \text{ eigenvalues}}$

Step 3: measure in the Z_L basis then repeat Steps 1-3. This builds up $P(|0\rangle_L)$ for each θ_L

Logical Rabi using Transversal Injection



Step 1: rotate n data qubits by θ and ϕ to get $|\psi\rangle = (\cos\frac{\theta}{2} |0\rangle + e^{i\phi} \sin\frac{\theta}{2} |1\rangle)^{\otimes n}$

Step 2: apply the projective measurements of the $n - 1$ code stabilizers

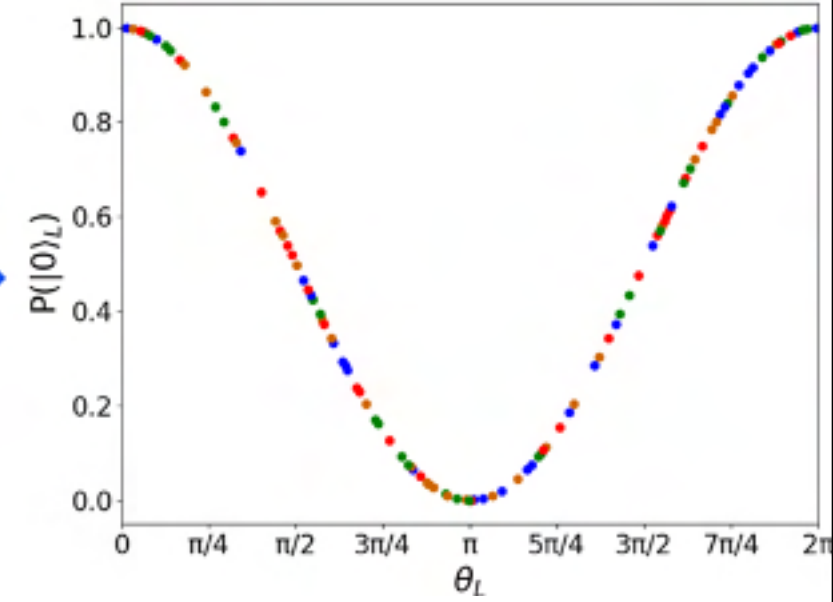
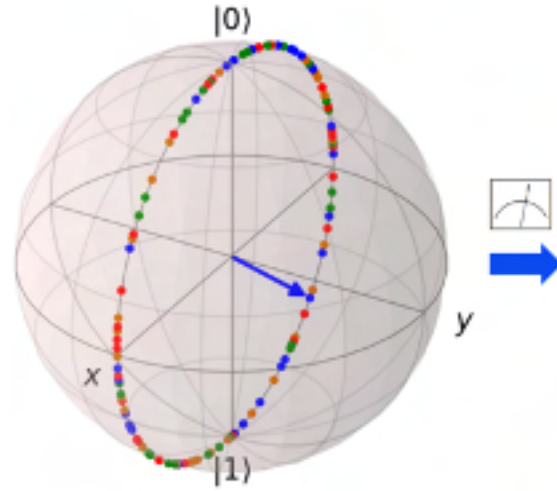
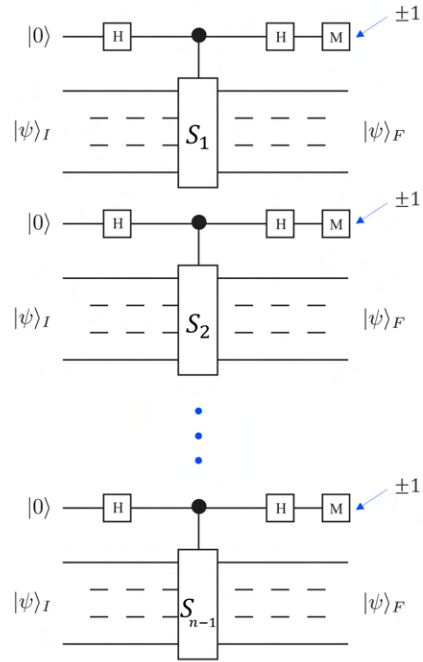
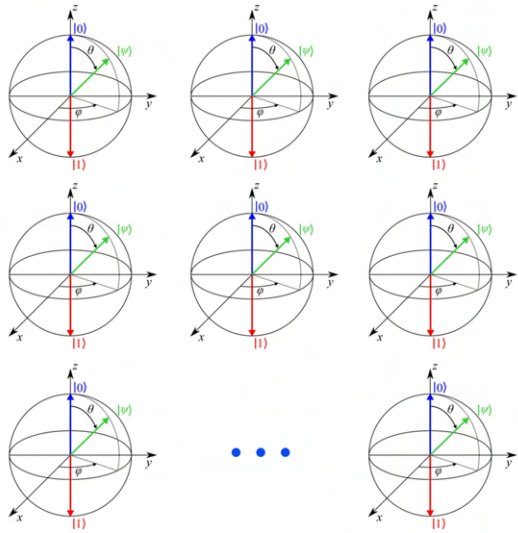
The **logical** state thus projected to can be calculated from:

- The initial choice of θ and ϕ
- The (random) stabilizer measurement results known as the *trajectory*.

E.g. $\underbrace{+1, -1, -1, \dots, +1}_{n - 1 \text{ eigenvalues}}$

Step 3: measure in the Z_L basis then repeat Steps 1-3. This builds up $P(|0\rangle_L)$ for each θ_L

Logical Rabi using Transversal Injection



Step 1: rotate n data qubits by θ and ϕ to get $|\psi\rangle = (\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle)^{\otimes n}$

Step 2: apply the projective measurements of the $n - 1$ code stabilizers

The **logical** state thus projected to can be calculated from:

- The initial choice of θ and ϕ
- The (random) stabilizer measurement results known as the *trajectory*.

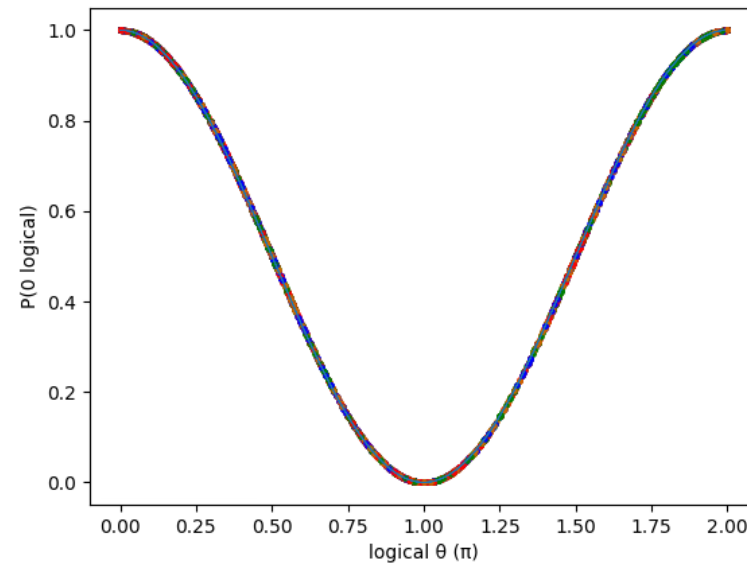
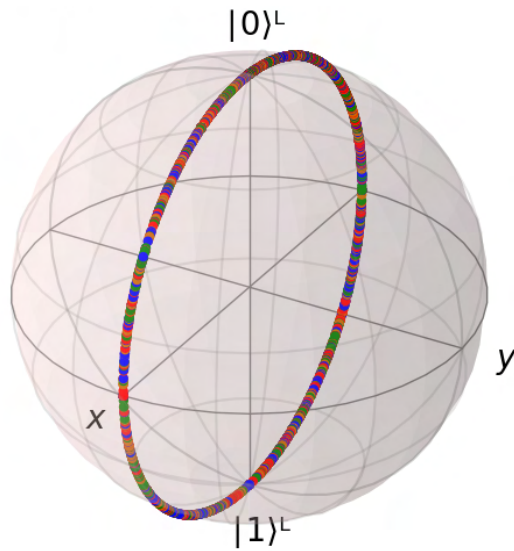
E.g. $+1, -1, -1, \dots, +1$
 $\underbrace{\hspace{10em}}_{n - 1 \text{ eigenvalues}}$

Step 3: measure in the Z_L basis then repeat Steps 1-3. This builds up $P(|0\rangle_L)$ for each θ_L

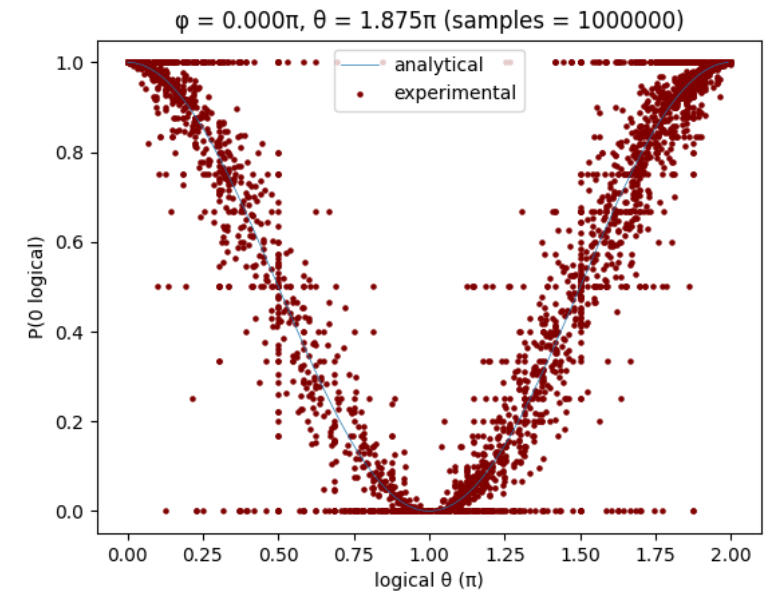
Logical Rabi Oscillation using Transversal Injection

- Distance 3 surface code ($n = 13$).
- E.g. $\phi = 0$, $\theta = 1.875\pi$:

Analytical:

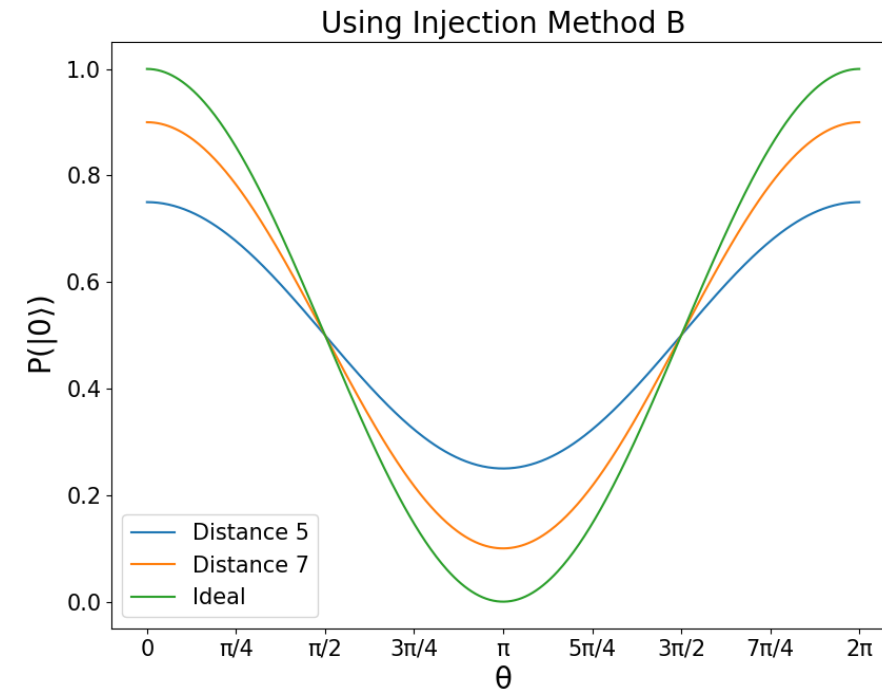
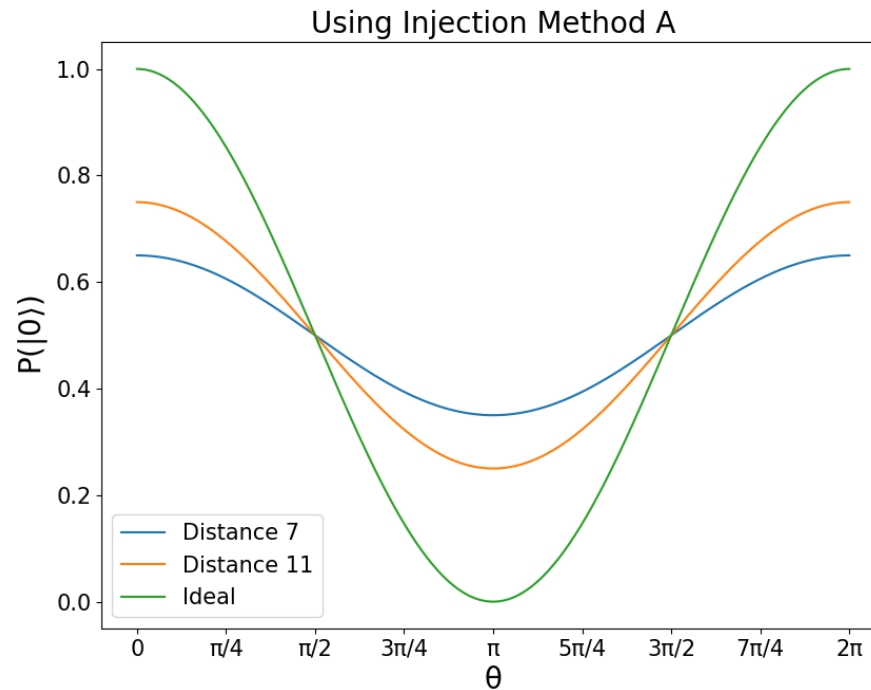


Simulation:



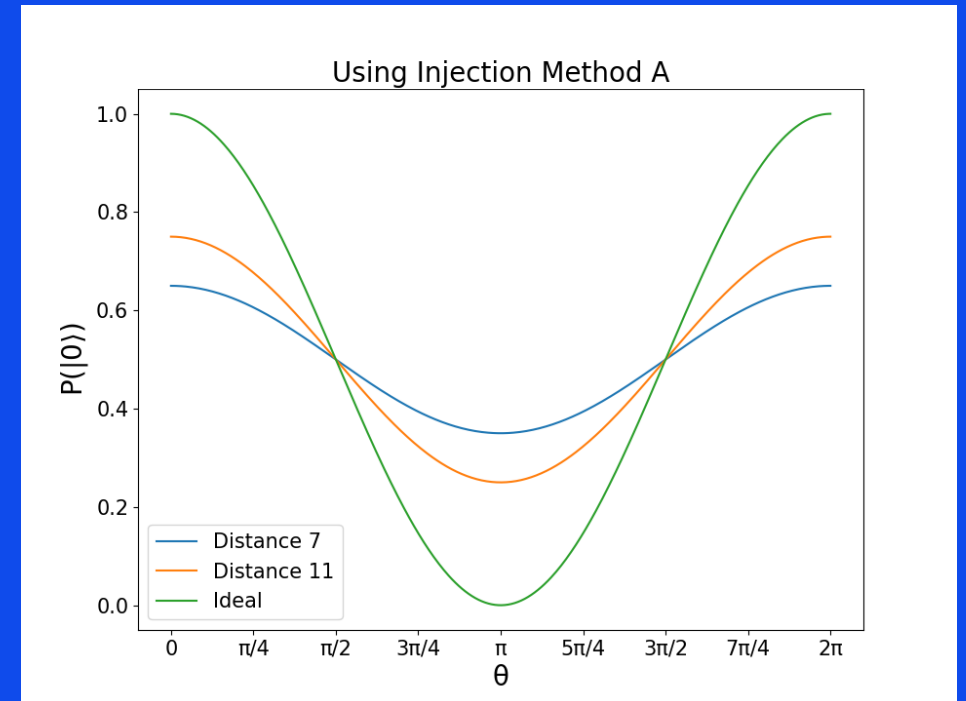
Logical Rabi Oscillation - Expectation

- Expect to see higher visibility of the Rabi curve with
 - Increasing code distance
 - Higher fidelity of injected states



Key takeaway from today:

Transversal injection will maximise the fidelity of injected states and thus maximises the logical Rabi curve's visibility, and this can be used for minimising qubit numbers and state distillation in achieving a universal gate set.



References:

1. A. G. Fowler, M. Mariantoni, J. M. Martinis and A. N. Cleland, Surface codes: Towards practical large-scale quantum computation. *Physical Review A* 86(3), 032324 (2012).
2. Y. Li, A magic state's fidelity can be superior to the operations that created it. *New Journal of Physics* 17(2), 023037 (2015).
3. Gavriel, J., Herr, D., Shaw, A., Bremner, M. J., Paler, A., & Devitt, S. J. (2022). Transversal Injection: A method for direct encoding of ancilla states for non-Clifford gates using stabiliser codes. *arXiv preprint arXiv:2211.10046*.