

# SINGLE STEP PARITY CHECK GATE SET for QUANTUM ERROR CORRECTION

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<sup>1</sup> University of New South Wales, Sydney

<sup>2</sup> Centre for Quantum Computation & Communication Technologies

<sup>3</sup> University of Technology Sydney



UNSW  
SYDNEY

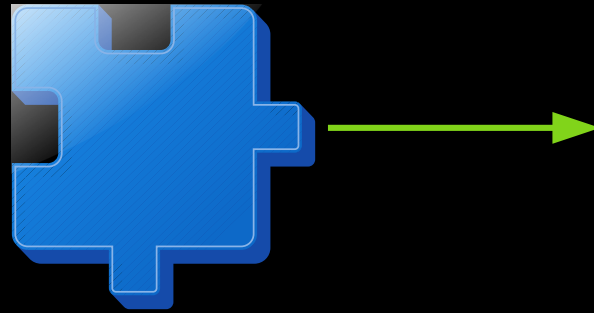


# 1. INTRODUCTION - THRESHOLD

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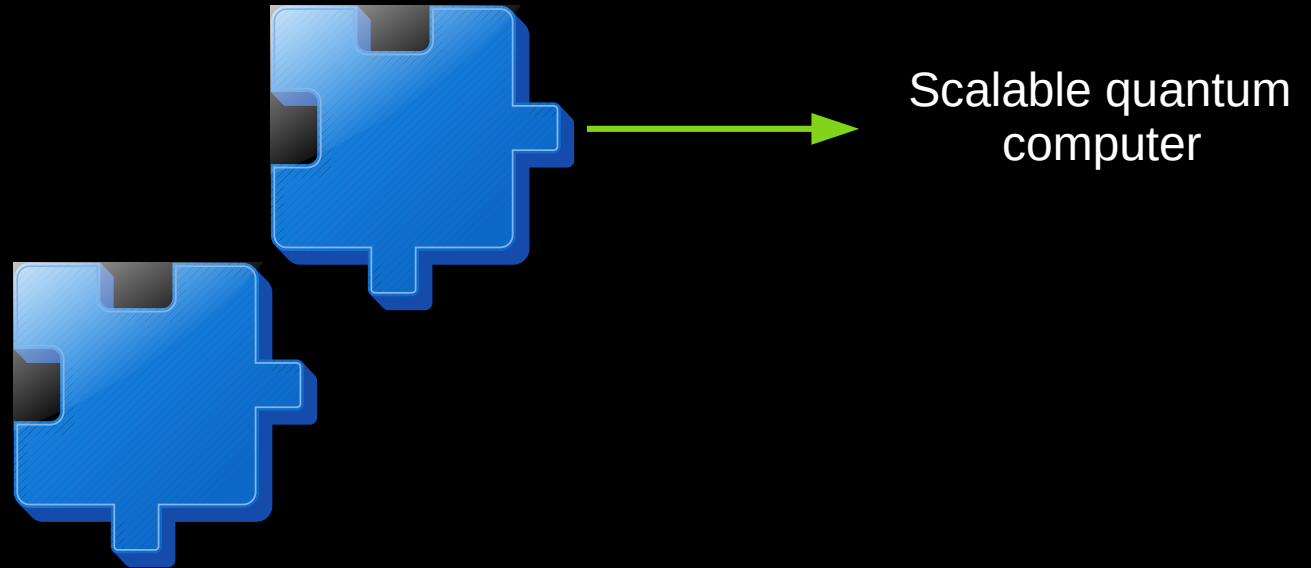


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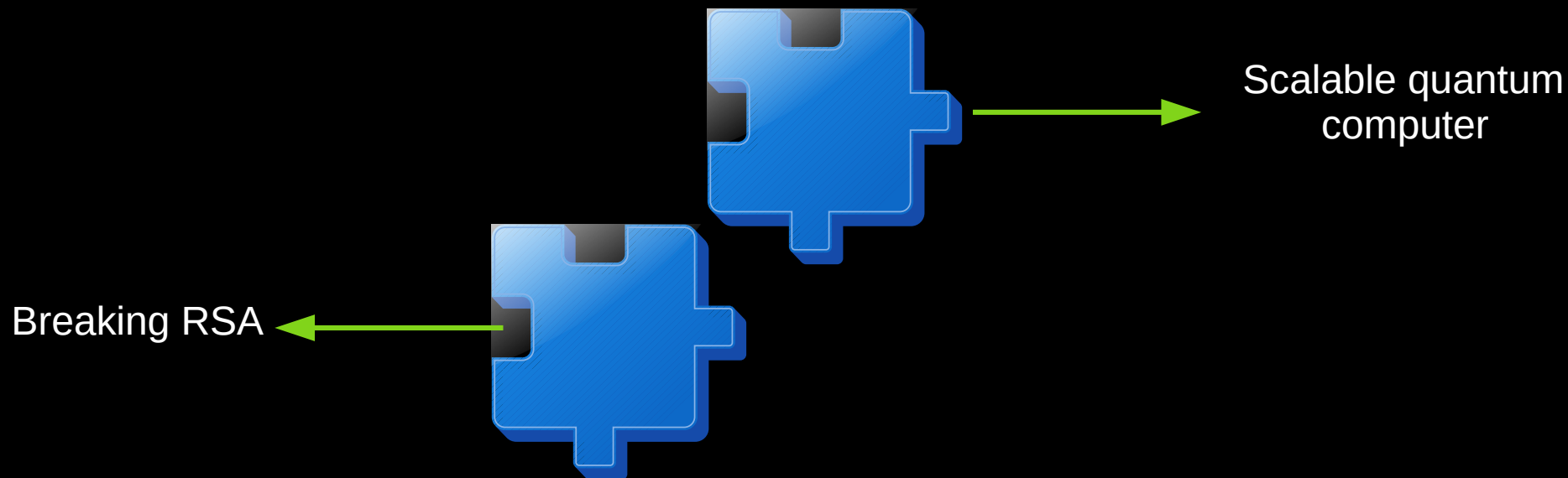


Scalable quantum  
computer

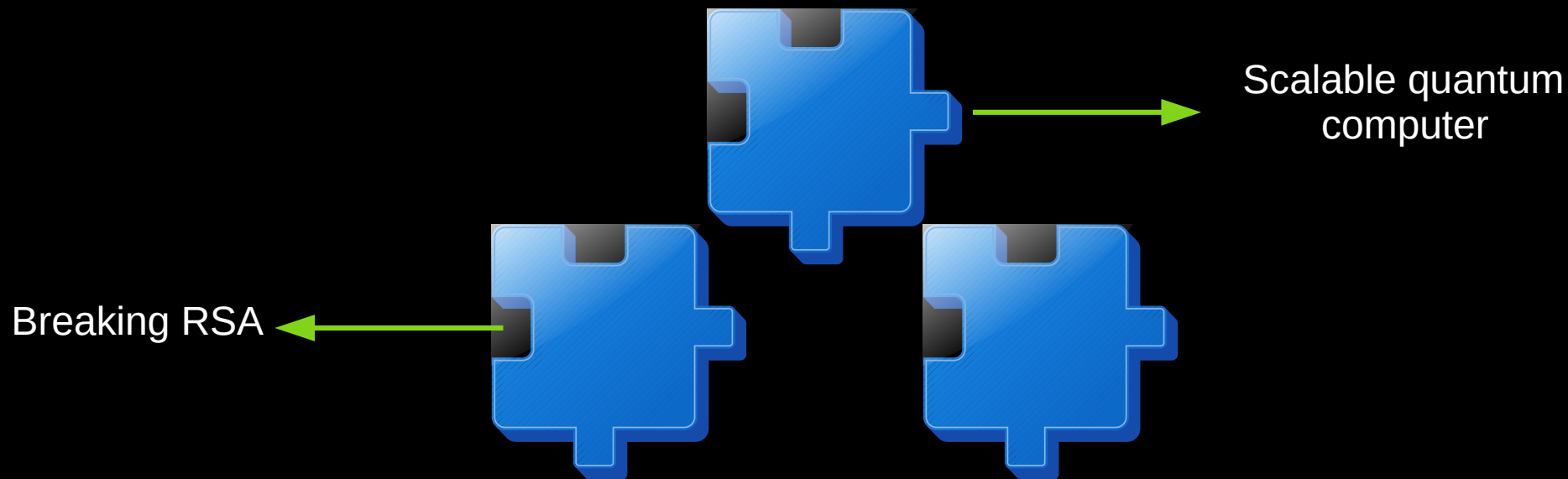
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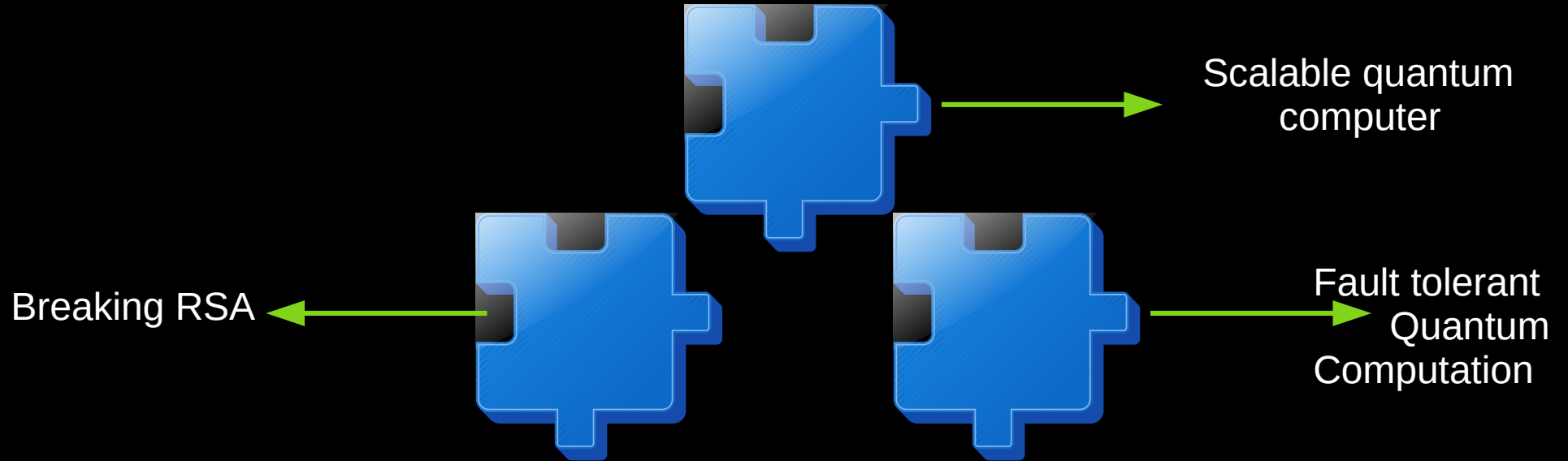
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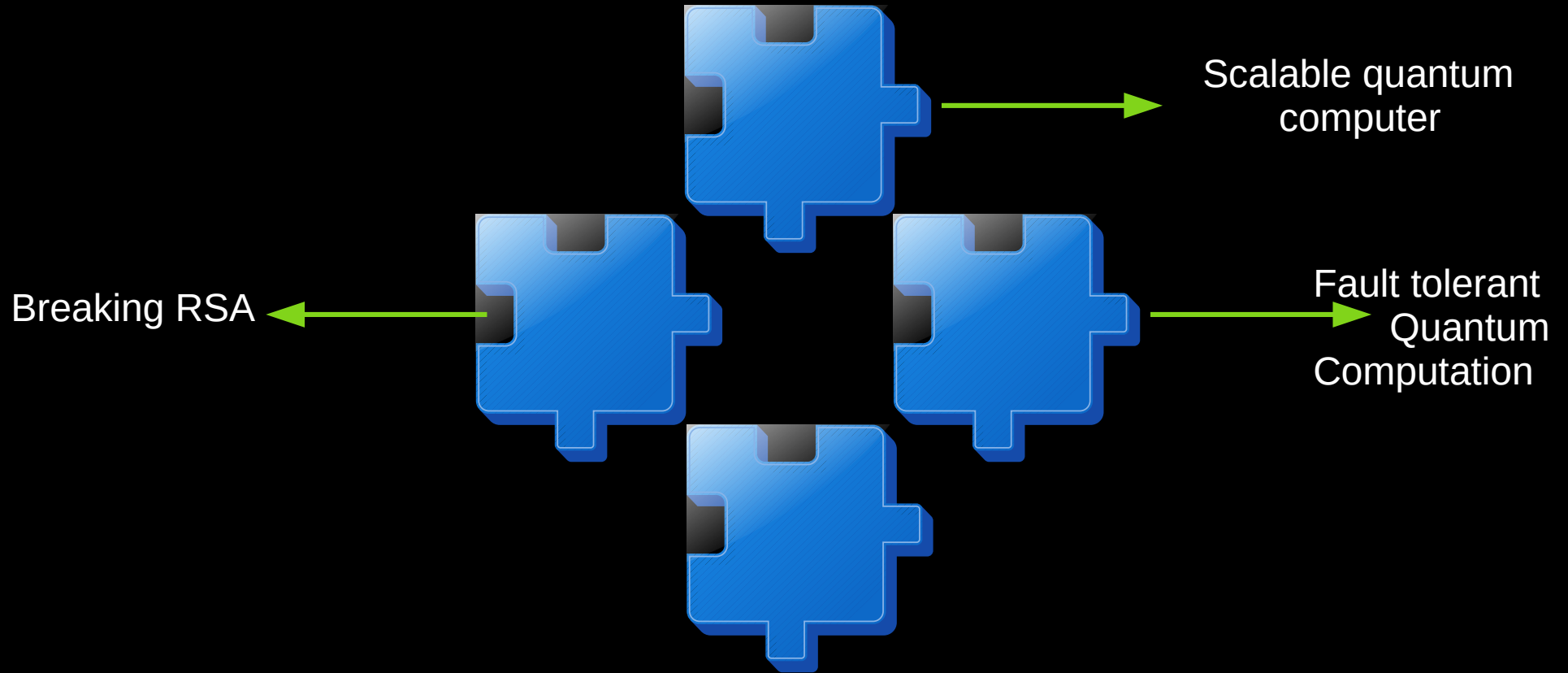


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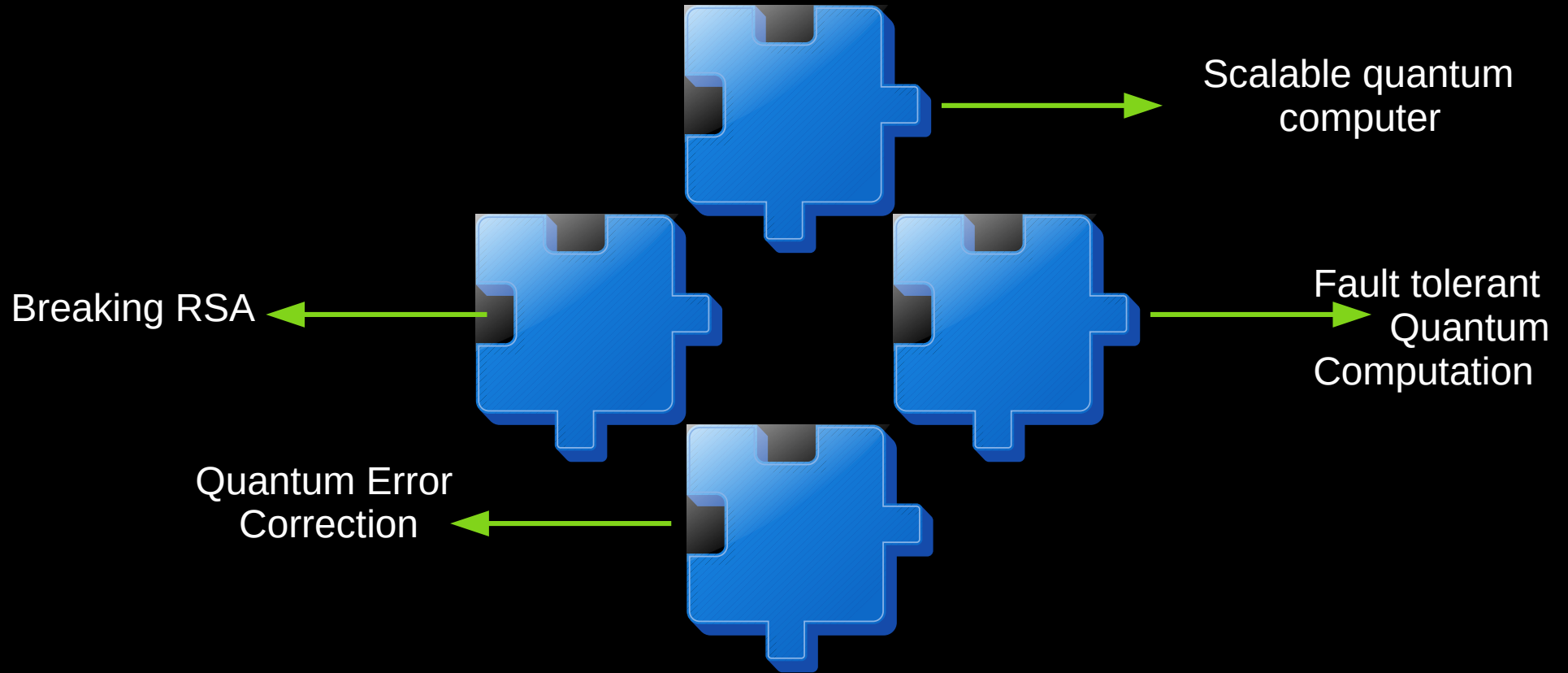




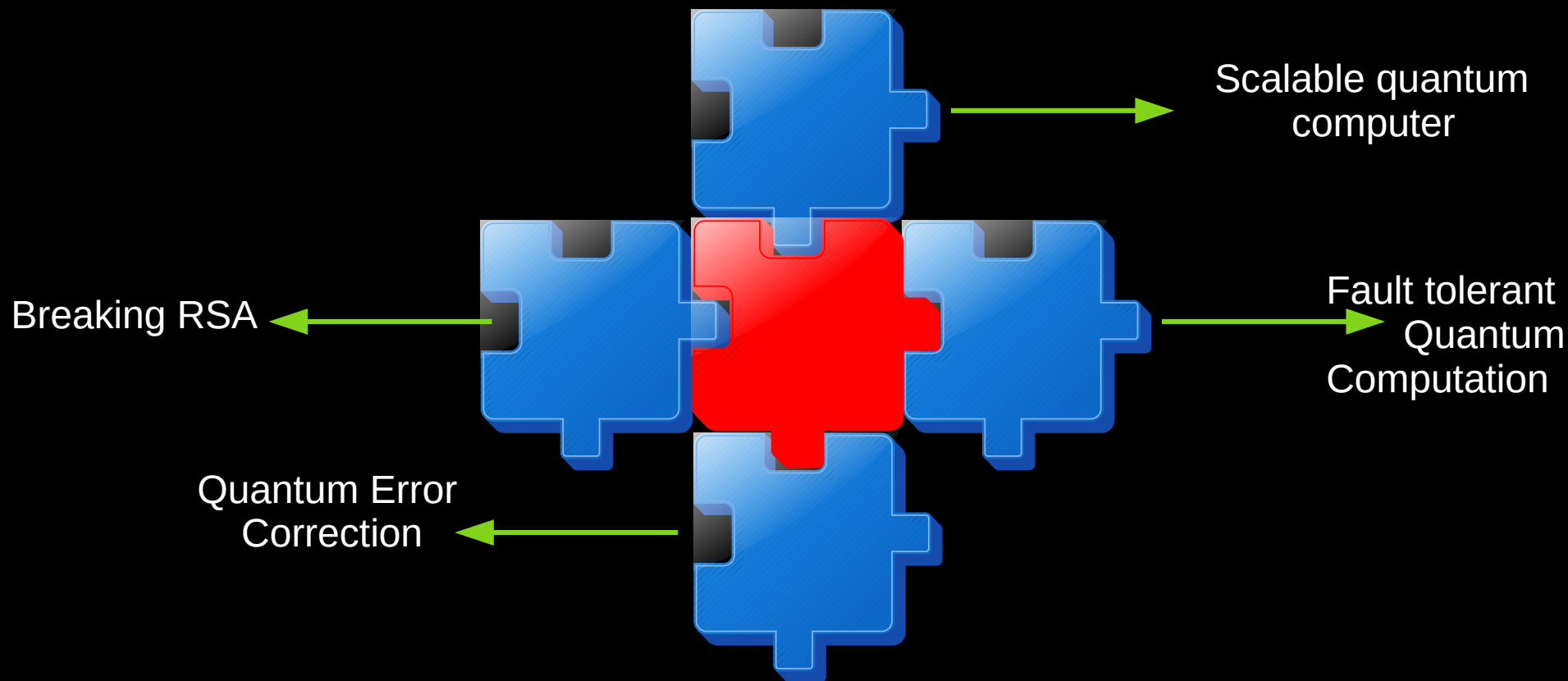
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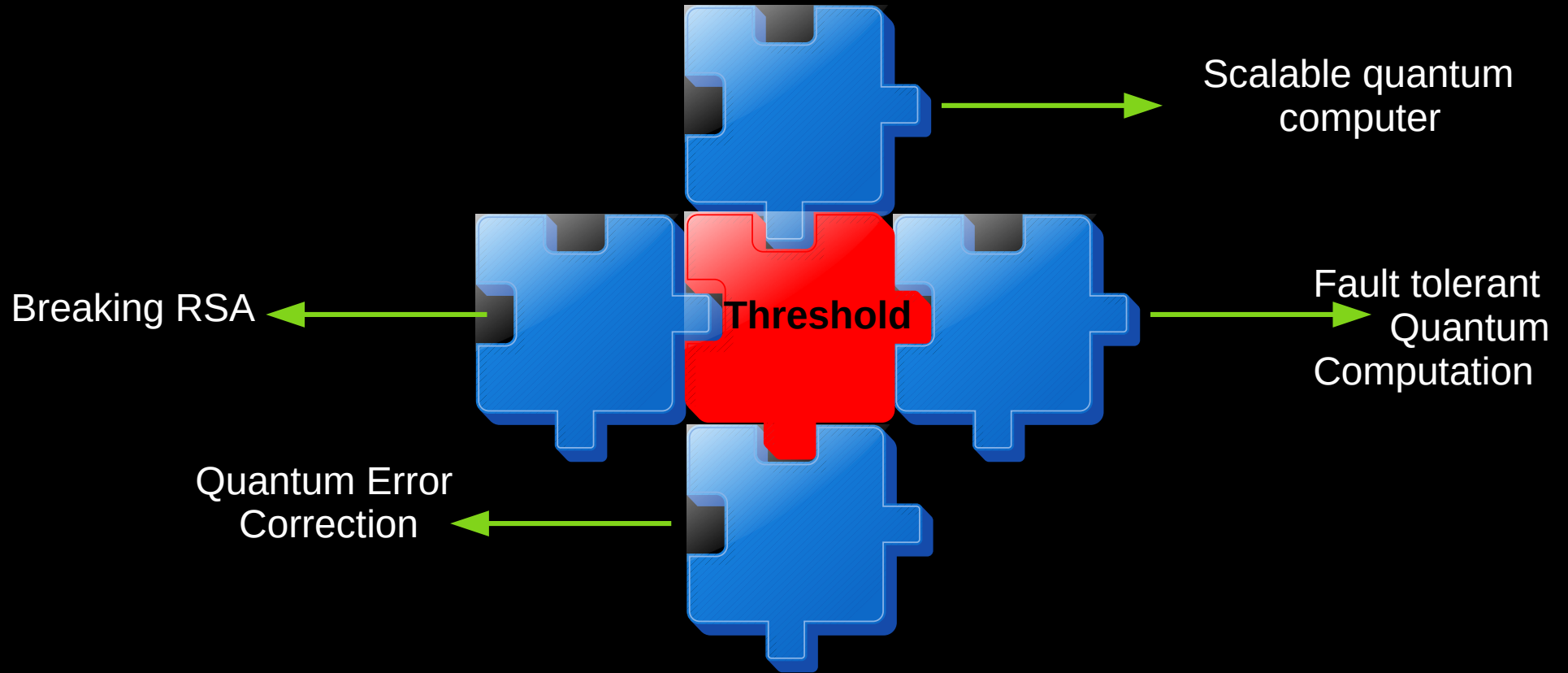
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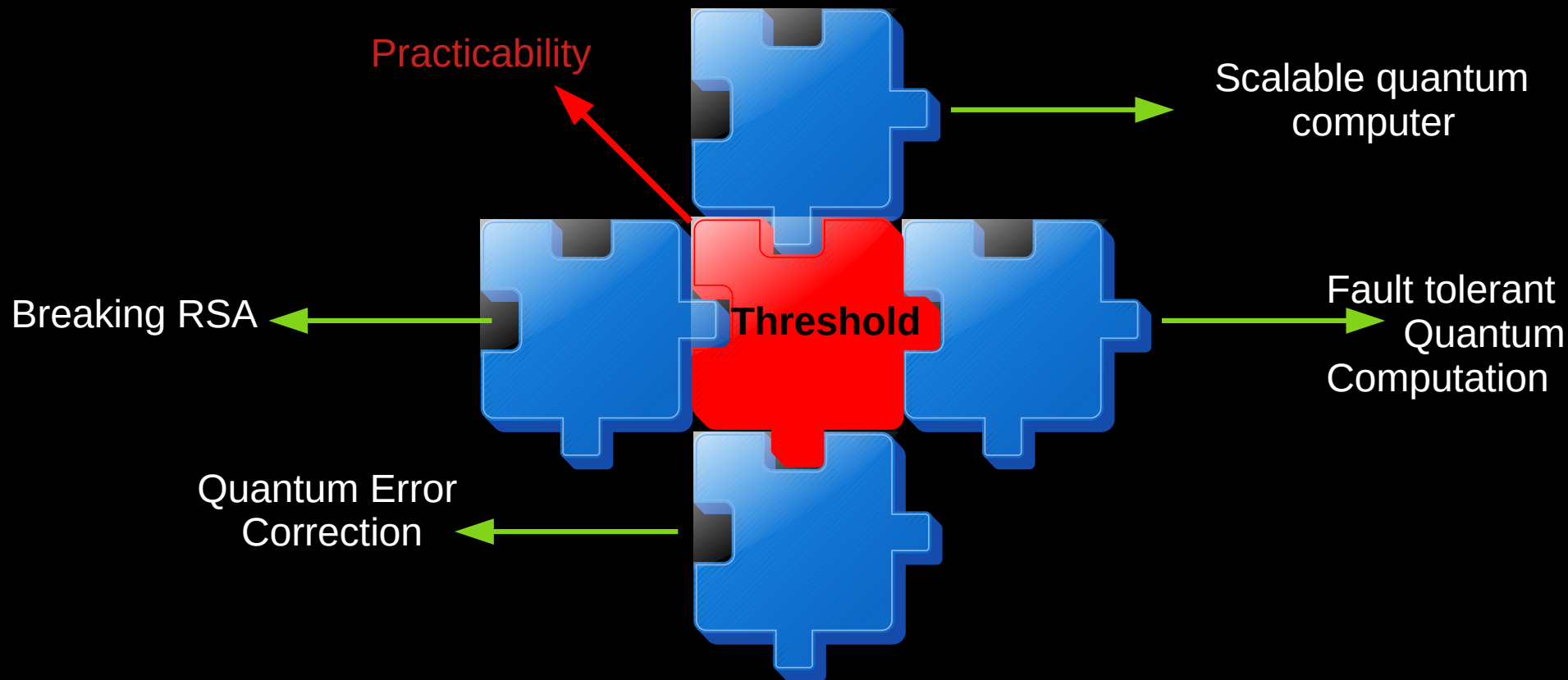
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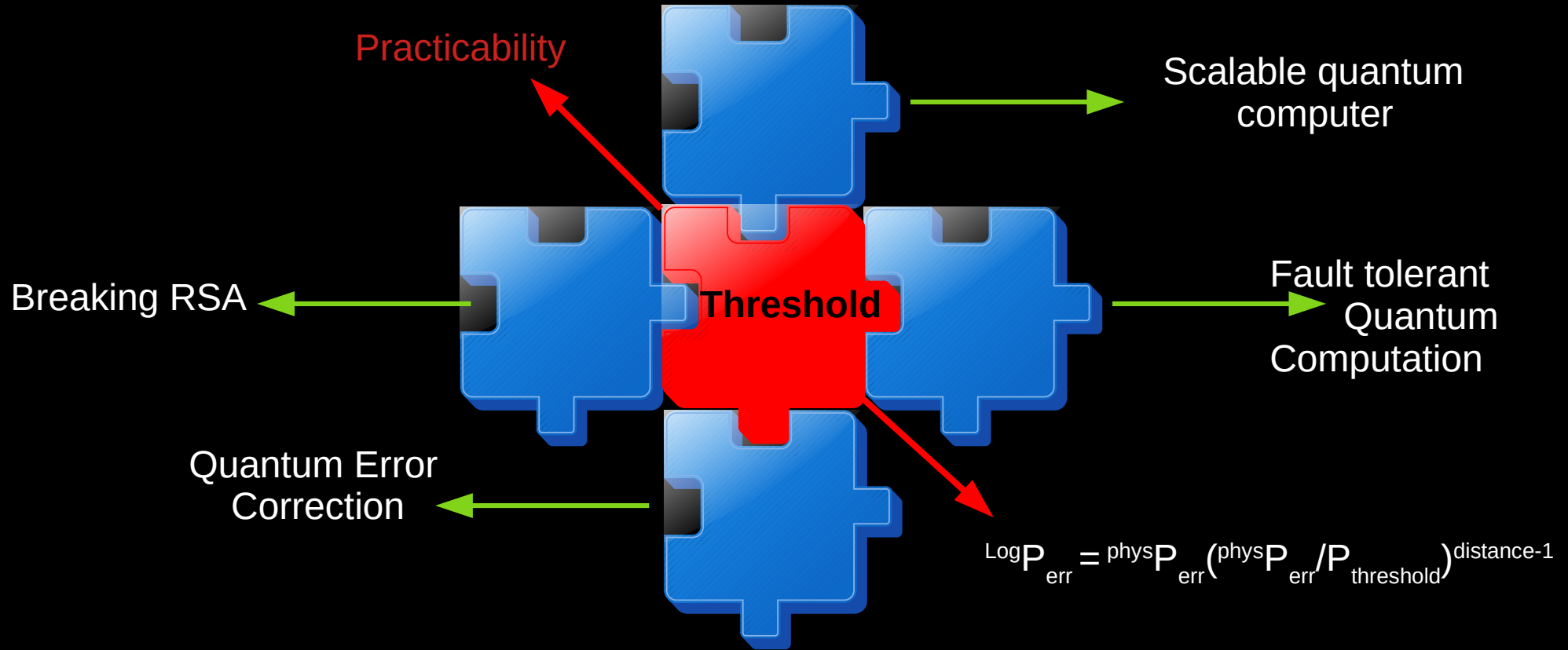
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## 2. WHAT IS THE PROBLEM?

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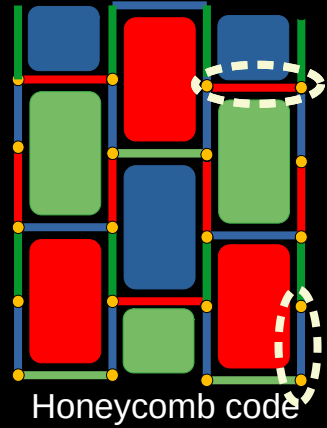


## 2. WHAT IS THE PROBLEM?



Building blocks...

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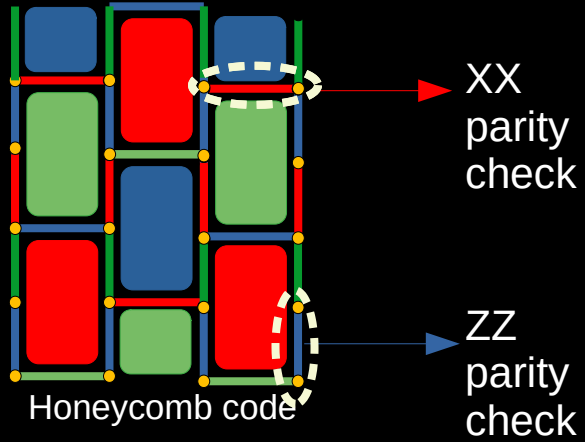


Building blocks...

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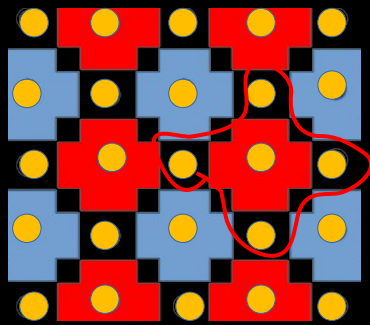
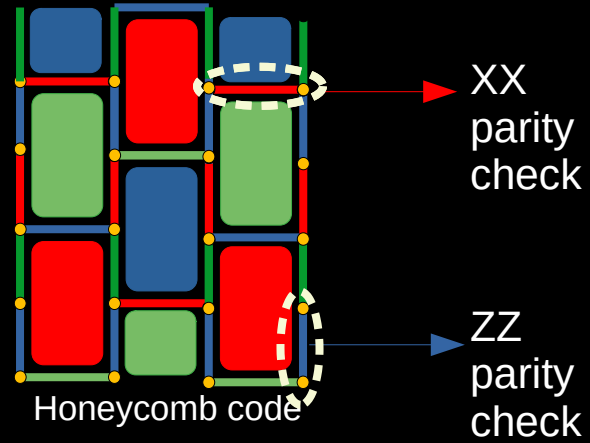
Building blocks...



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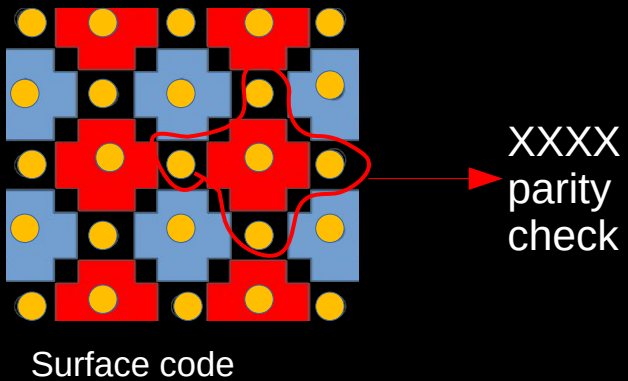
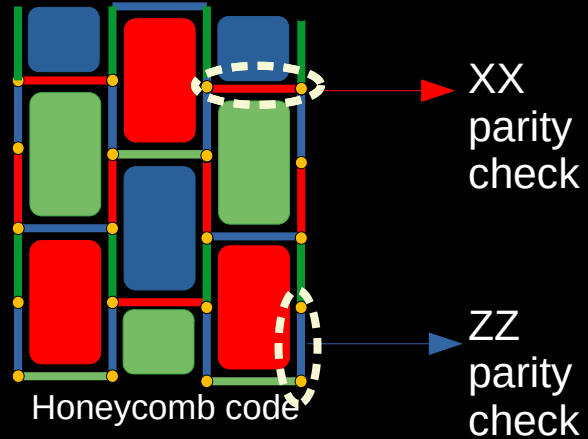
Building blocks...



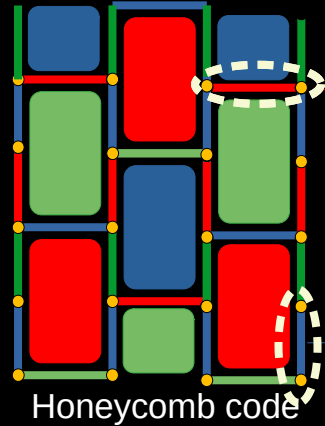
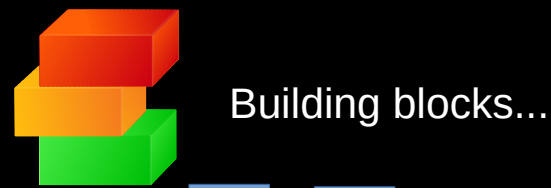
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Building blocks...

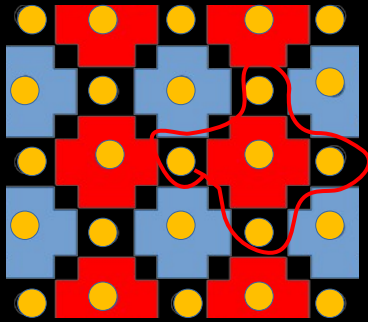


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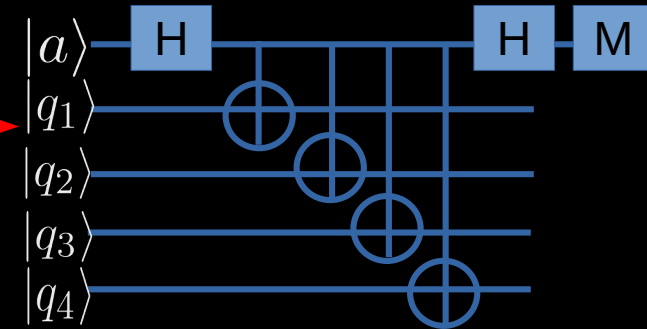
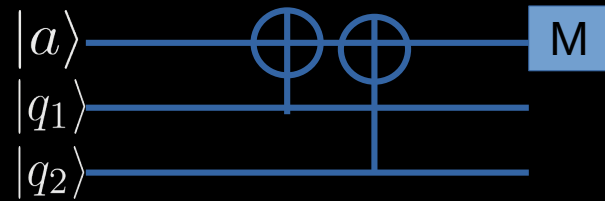
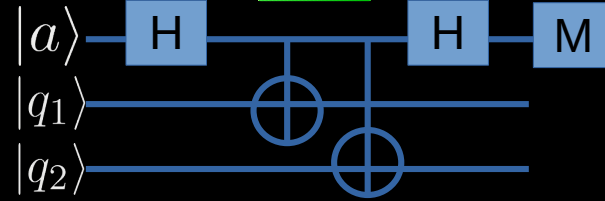


XX  
parity  
check

ZZ  
parity  
check

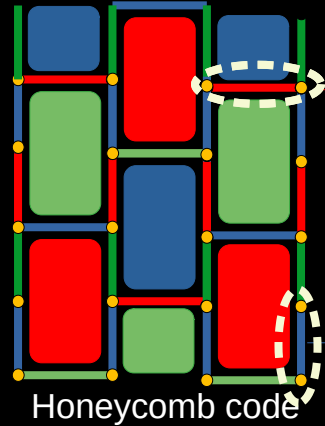
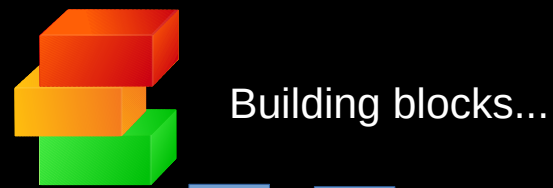


XXXX  
parity  
check



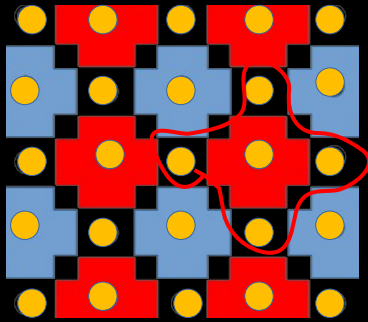


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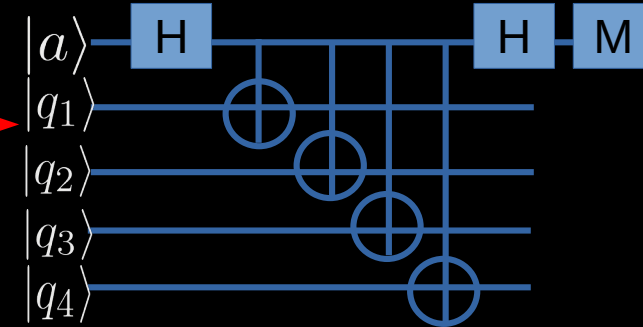
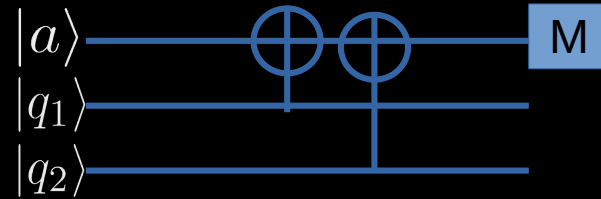
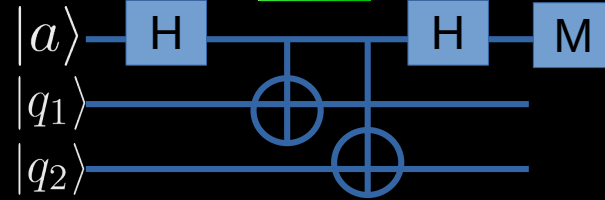


XX  
parity  
check

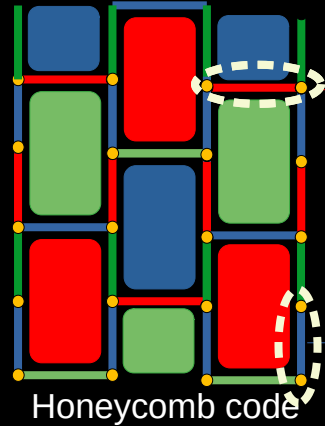
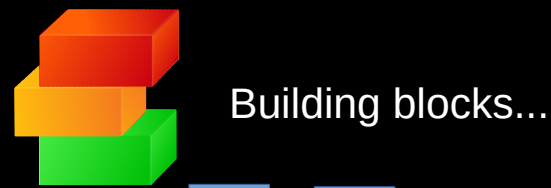
ZZ  
parity  
check



XXXX  
parity  
check

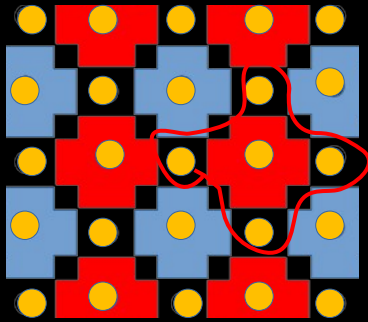


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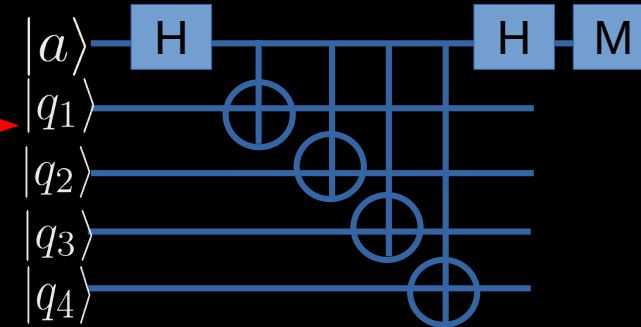
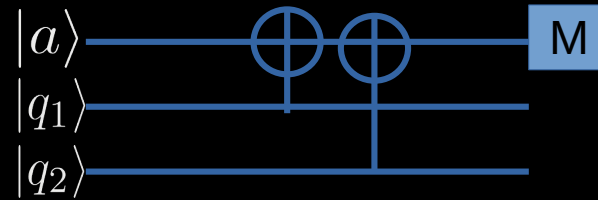
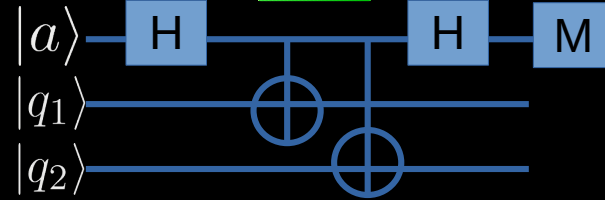


XX  
parity  
check

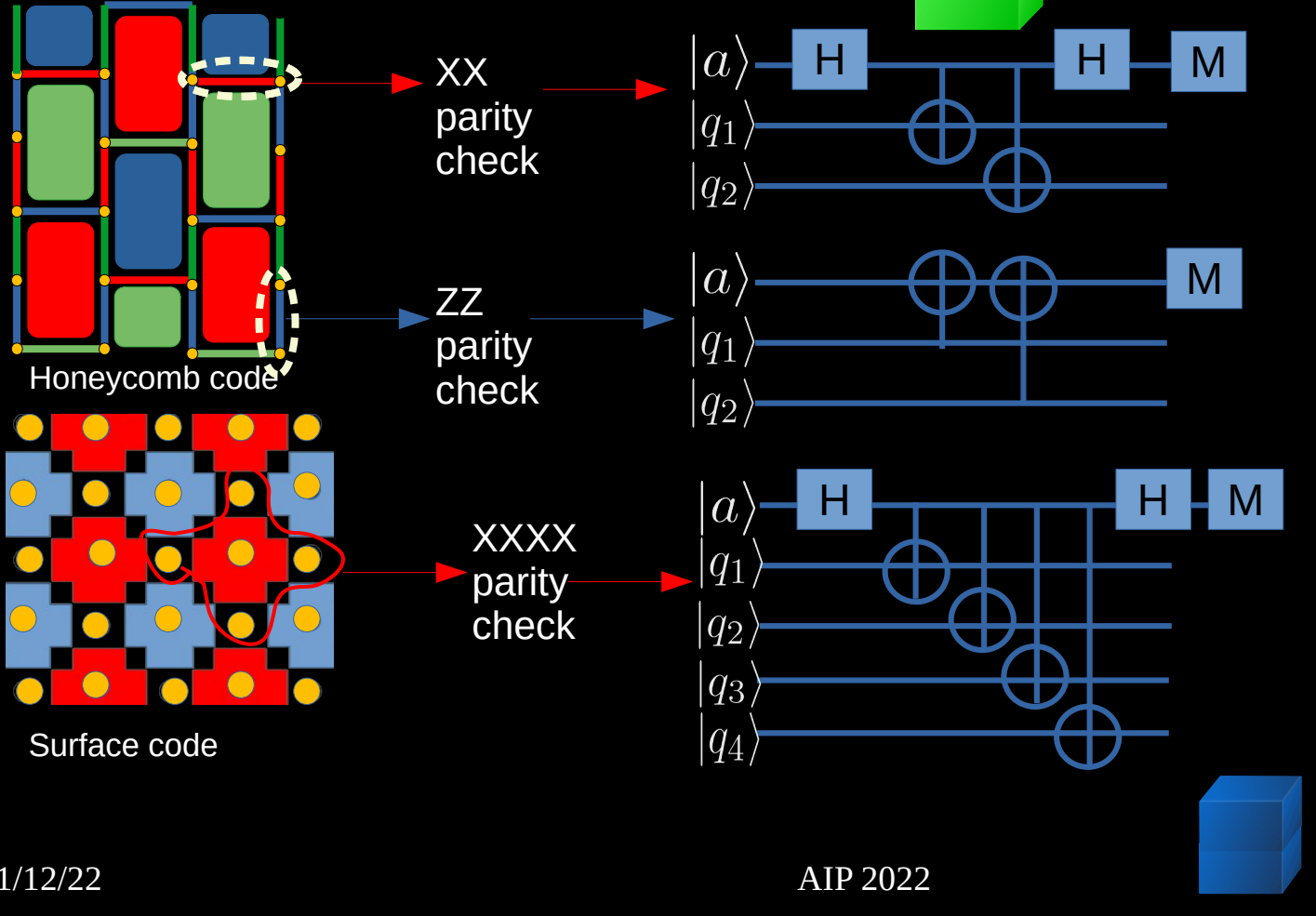
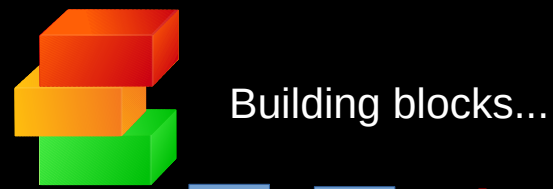
ZZ  
parity  
check



XXXX  
parity  
check



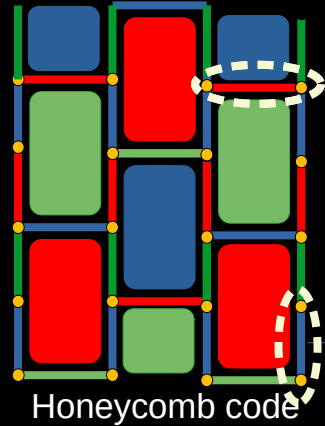
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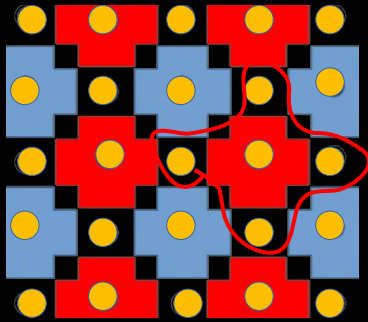


Building blocks...

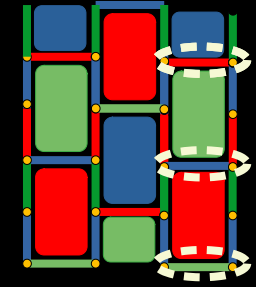
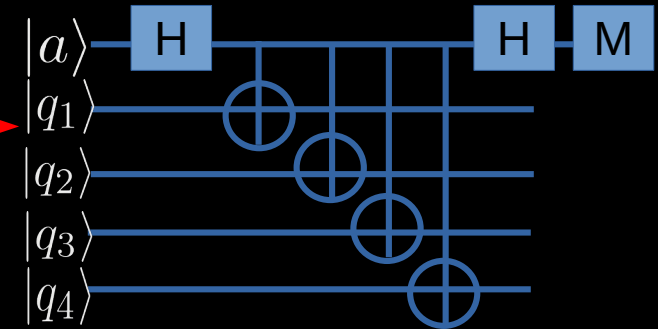
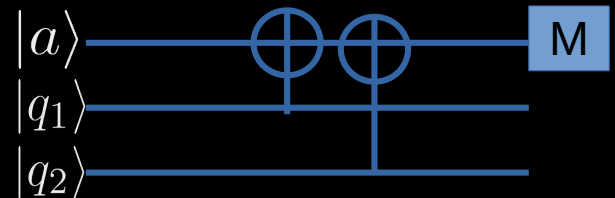
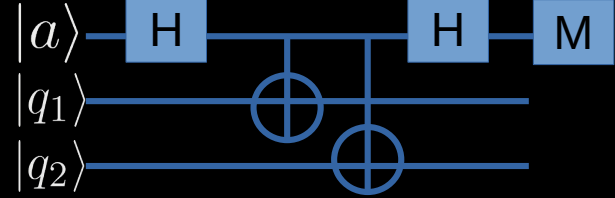


XX  
parity  
check

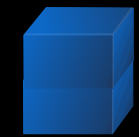
ZZ  
parity  
check



XXXX  
parity  
check



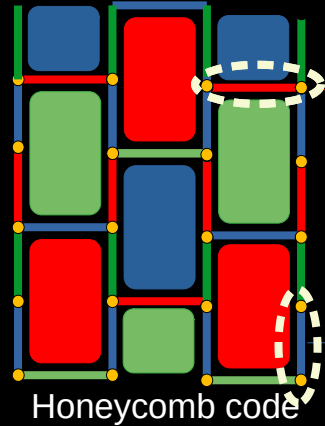
A Fault-Tolerant Honeycomb Memory, Gidney et al, Quantum 5, 605 (2021).



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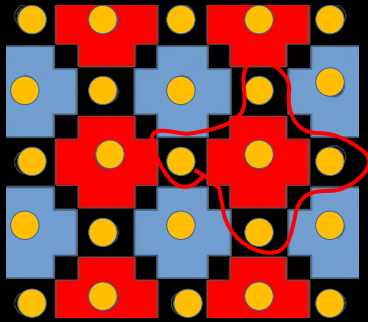


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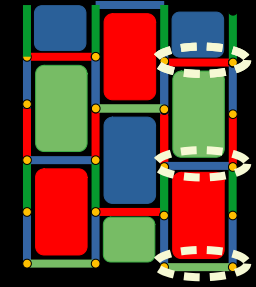
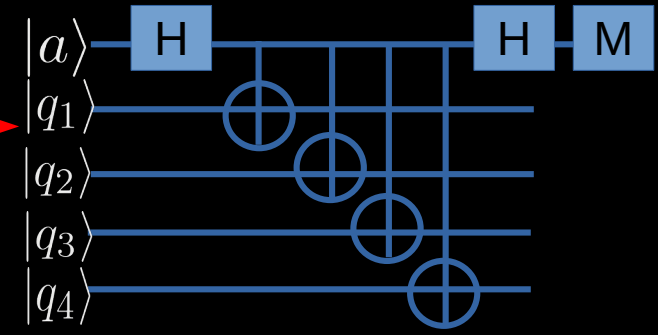
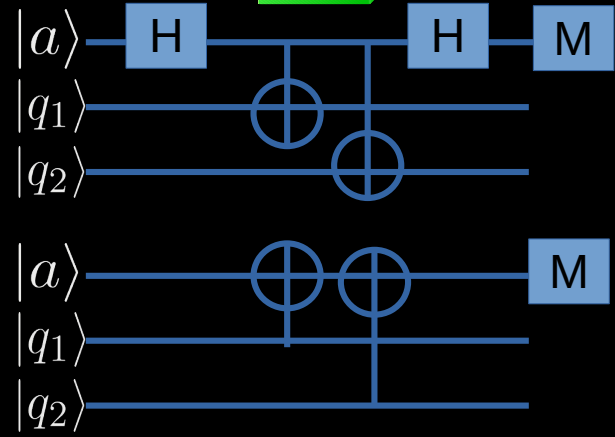


XX  
parity  
check

ZZ  
parity  
check



Surface code

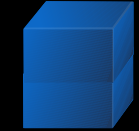


Direct XX parity check

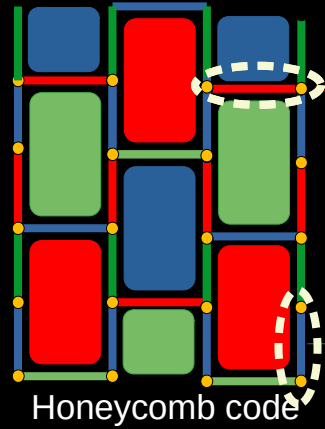
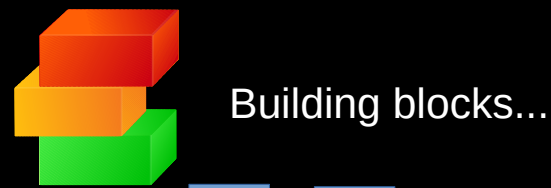
Direct YY parity check

Direct ZZ parity check

A Fault-Tolerant Honeycomb Memory, Gidney et al, Quantum 5, 605 (2021).

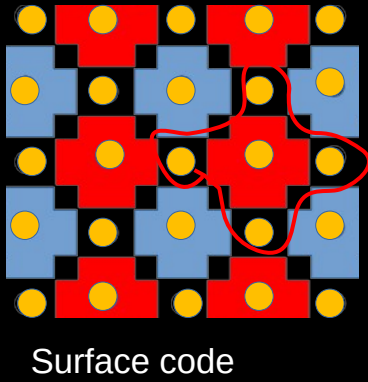


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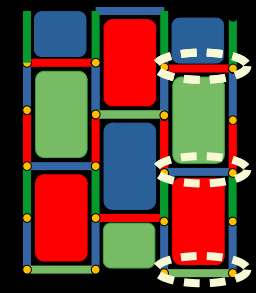
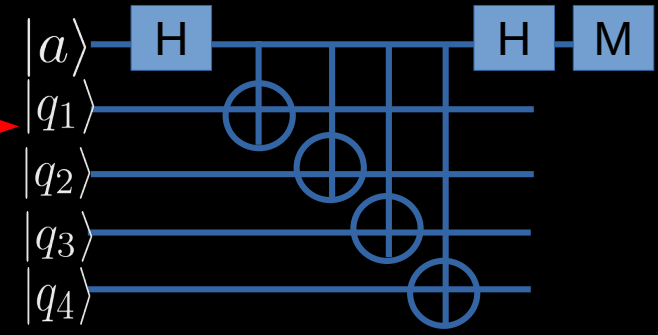
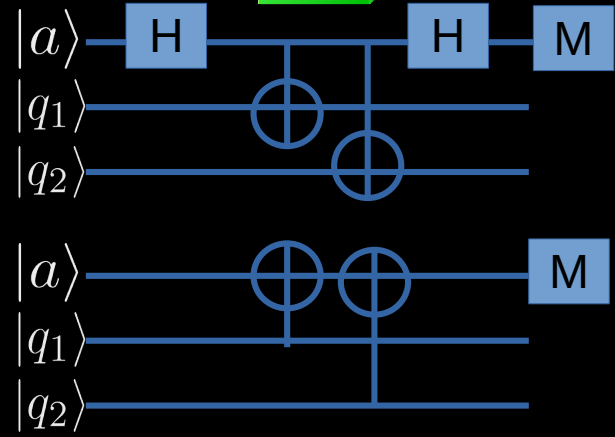


XX  
parity  
check

ZZ  
parity  
check



XXXX  
parity  
check



Direct XX parity check  
Direct YY parity check  
Direct ZZ parity check

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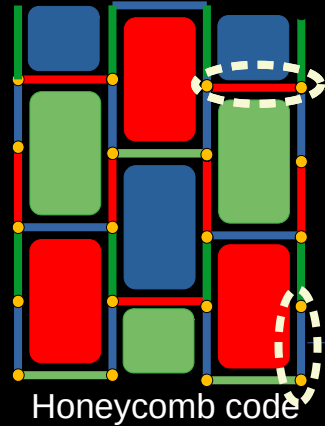
Three times higher  
threshold than  
surface code!



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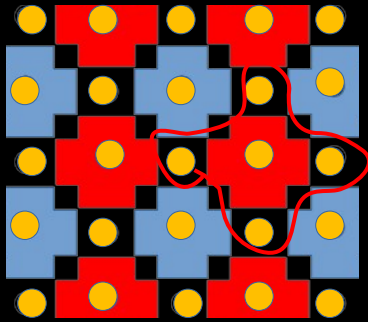


Building blocks...

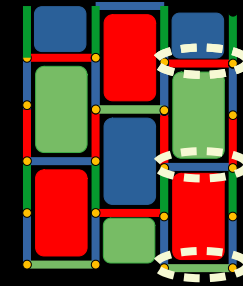
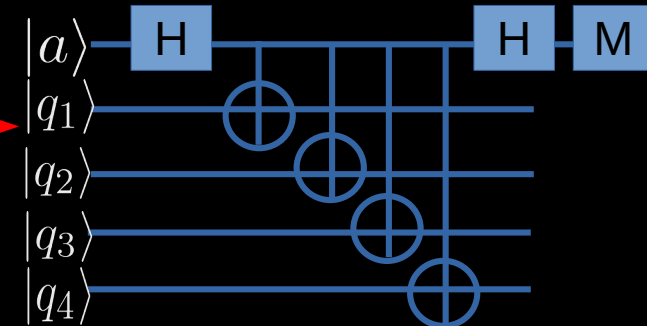
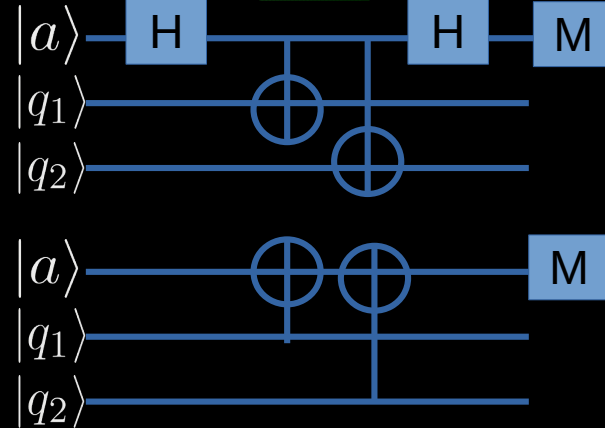


XX  
parity  
check

ZZ  
parity  
check



Surface code



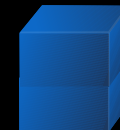
Direct XX parity check

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Three times higher  
threshold than  
surface code!  
If there is  $M_{pp2} \dots ???$



# 3. CURRENT METHODS

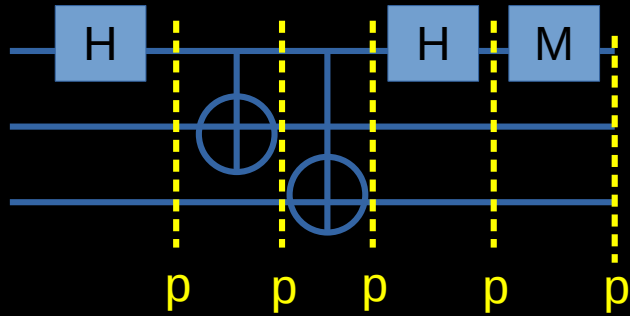


# 3. CURRENT METHODS

From fidelity to  $\rho$

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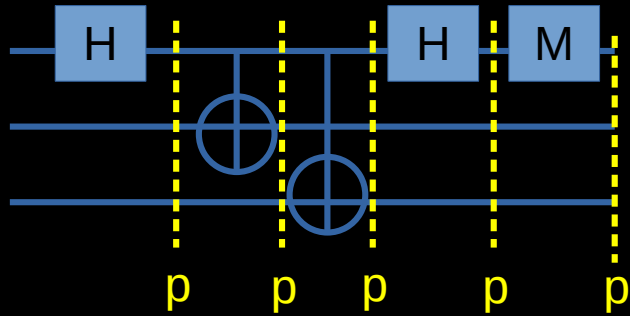
From fidelity to  $p$



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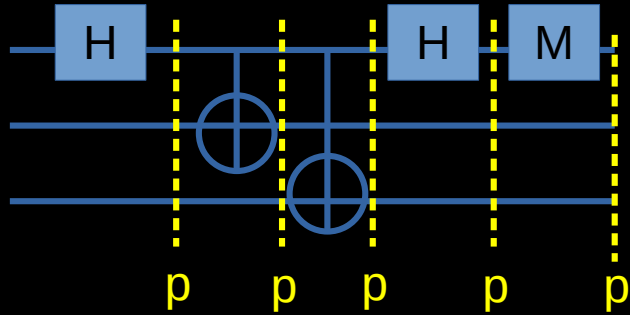
From fidelity to  $p$

1. Take GST matrices



### 3. CURRENT METHODS

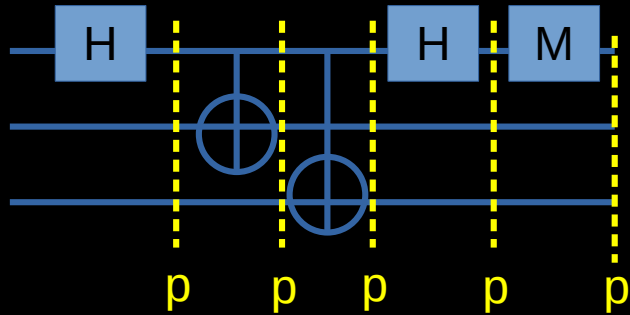
From fidelity to  $p$



1. Take GST matrices
2. They are actually PTM matrices

### 3. CURRENT METHODS

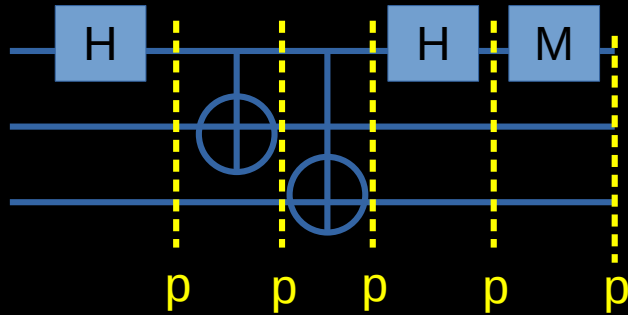
From fidelity to  $p$



1. Take GST matrices
2. They are actually PTM matrices
3. Find Kraus Operators

### 3. CURRENT METHODS

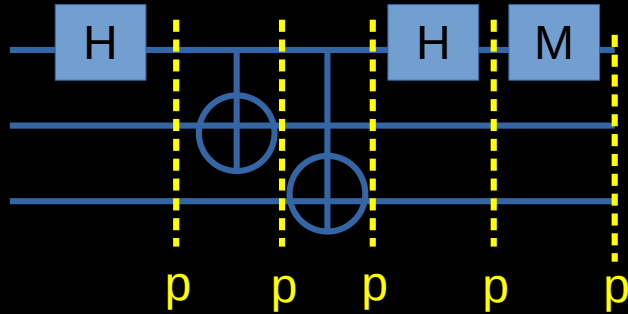
From fidelity to  $p$



1. Take GST matrices
2. They are actually PTM matrices
3. Find Kraus Operators
4. Write Kraus operators in terms of Pauli Matrices

### 3. CURRENT METHODS

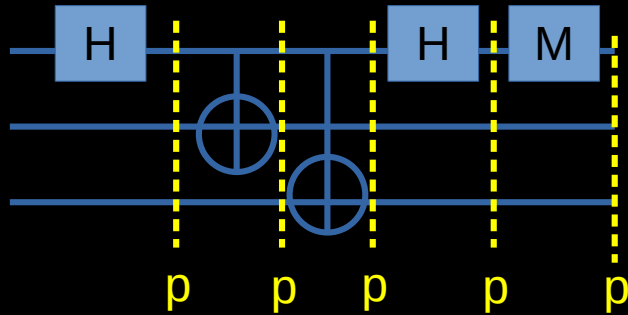
From fidelity to  $\rho$



1. Take GST matrices
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5. Write final density matrices

### 3. CURRENT METHODS

From fidelity to  $p$



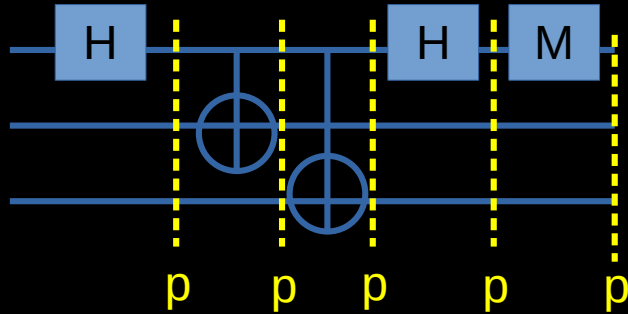
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As a result, we analysed the succession/perfection rate in terms of accumulated error in the stochastic Pauli channels



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From fidelity to  $p$

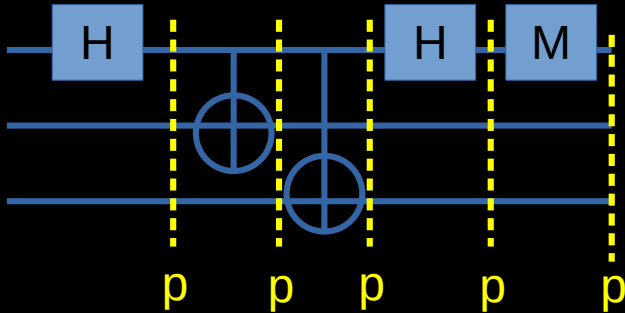


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From fidelity to  $\rho$



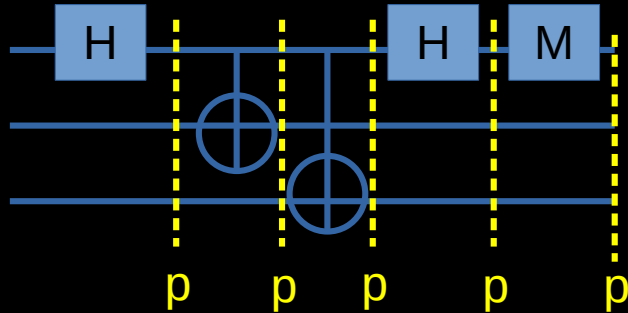
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- Each gate is created with 99.43% fidelity

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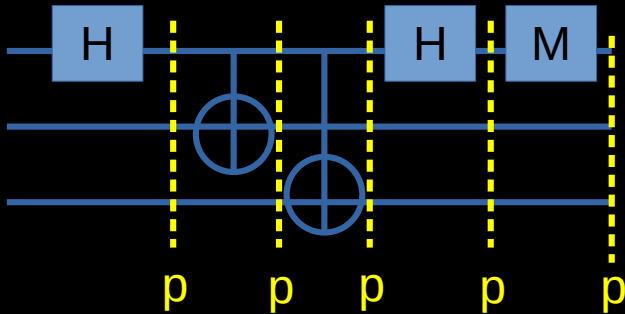
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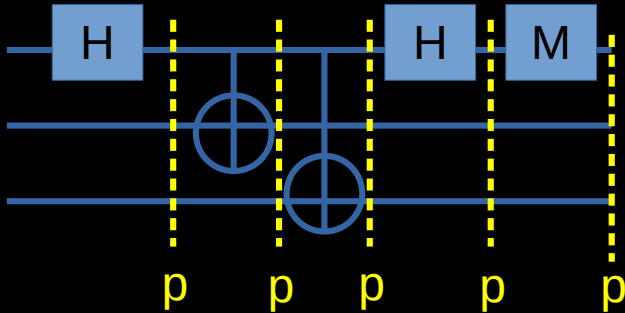
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Surface codes: Towards practical large-scale quantum computation, Fowler et al, Phys. Rev. A 86, 032324

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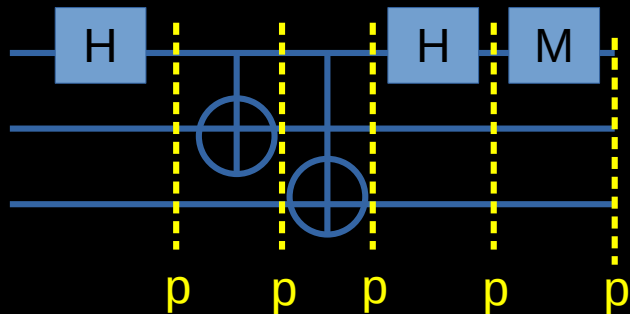
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Surface codes: Towards practical large-scale quantum computation, Fowler et al, Phys. Rev. A 86, 032324

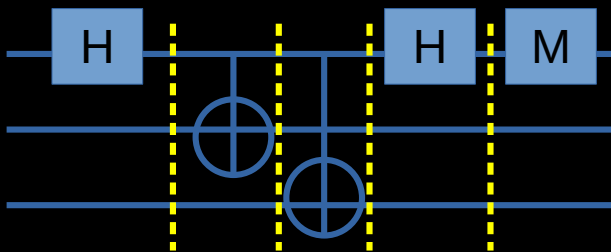
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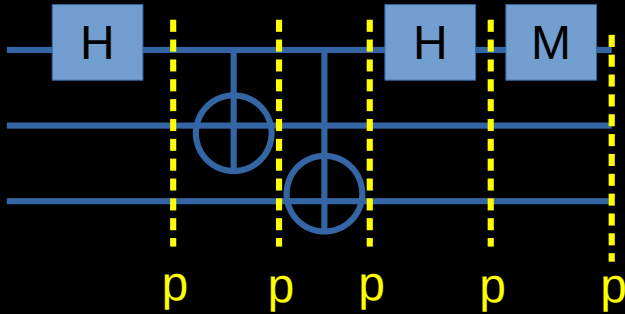


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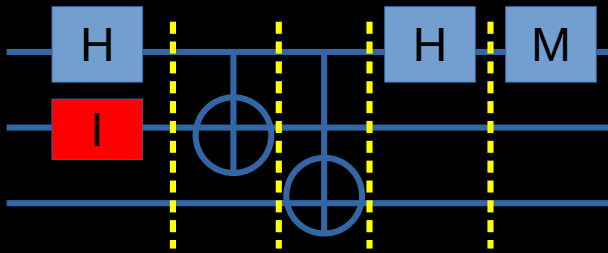
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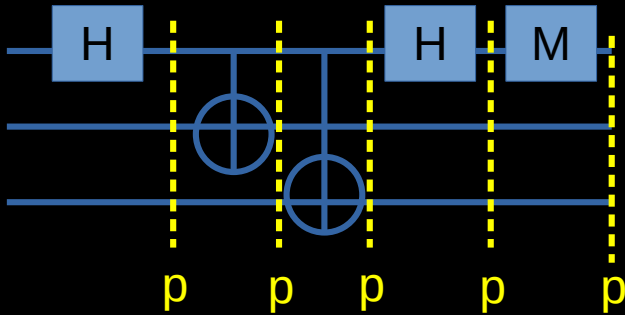


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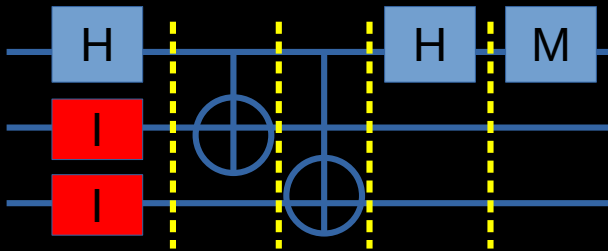
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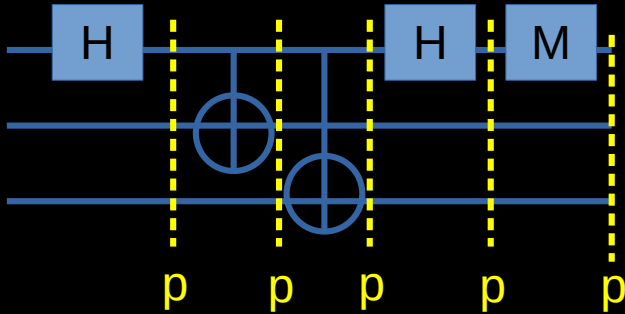
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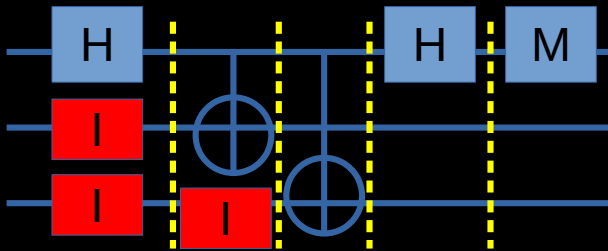
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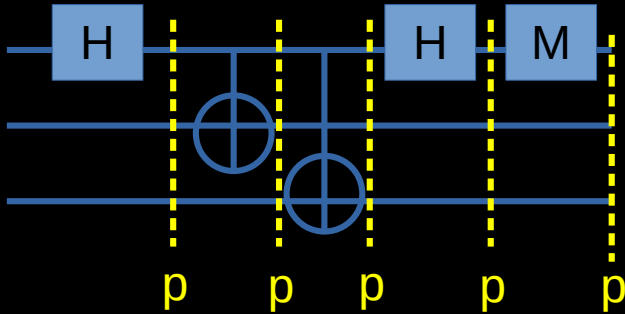


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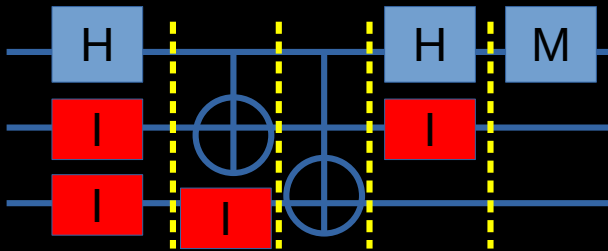
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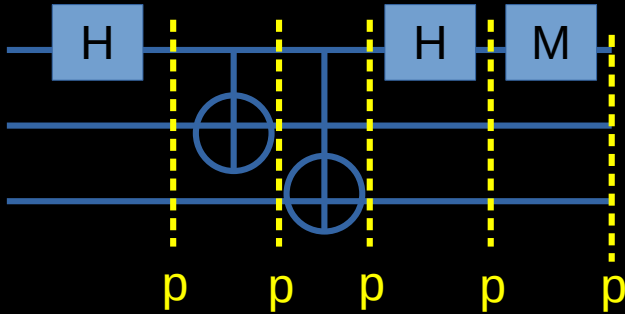


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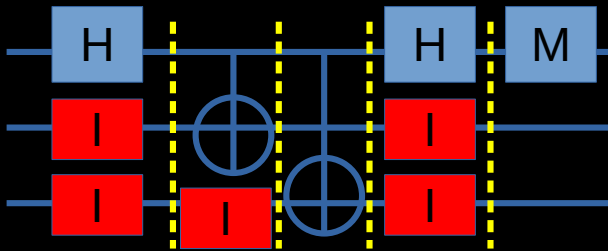
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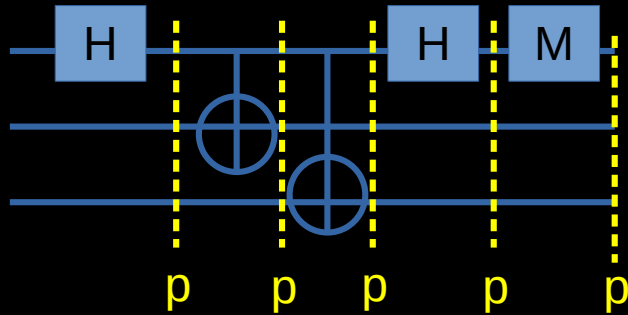


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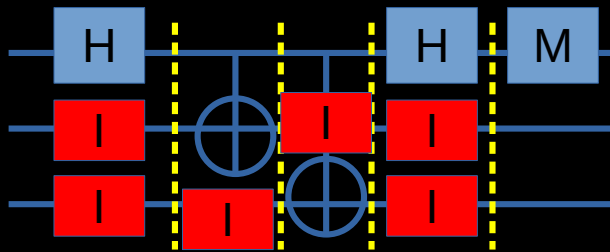
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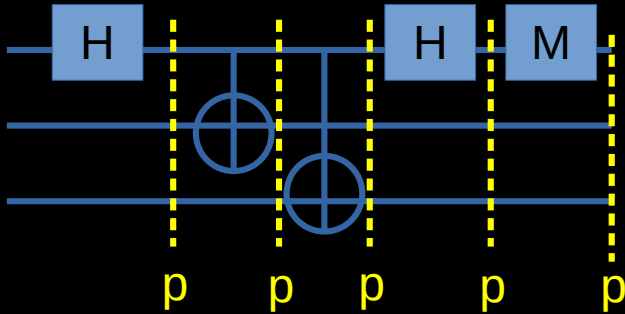


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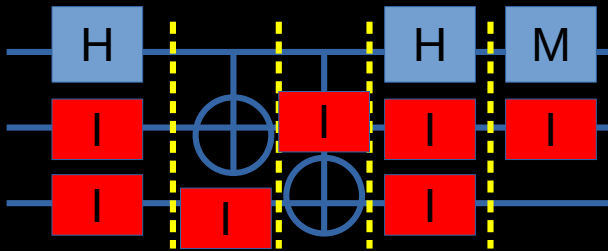
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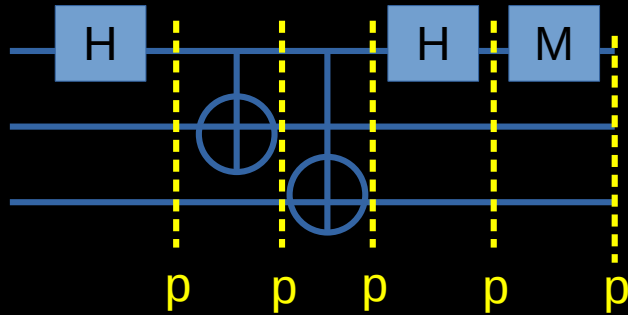


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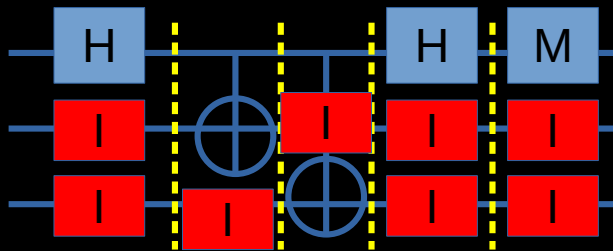
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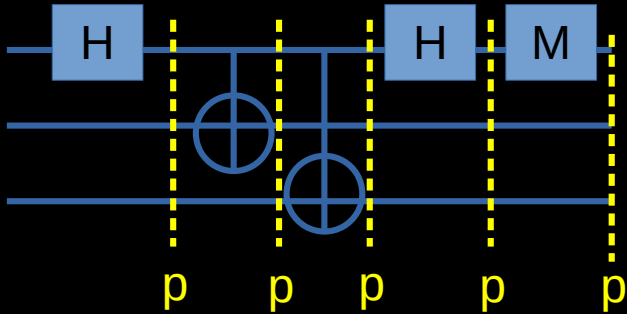


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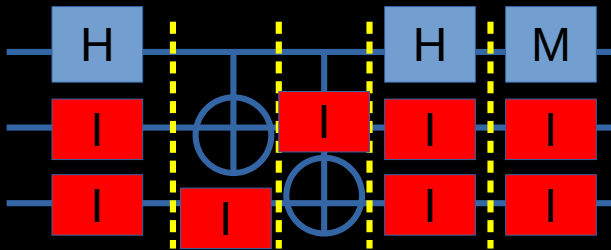
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$P_f = 0.974$

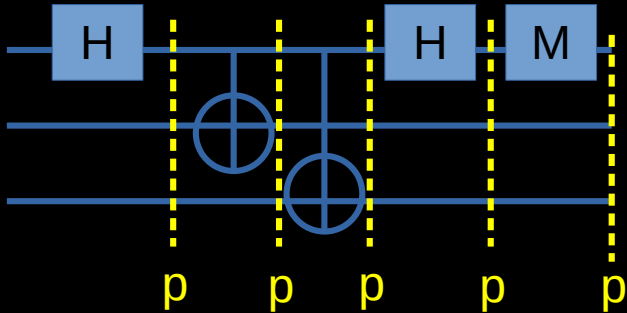
11/12/22

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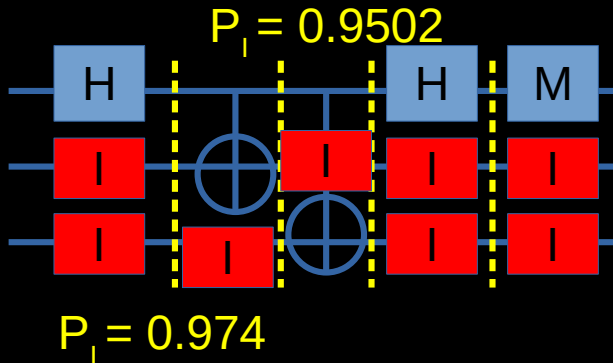
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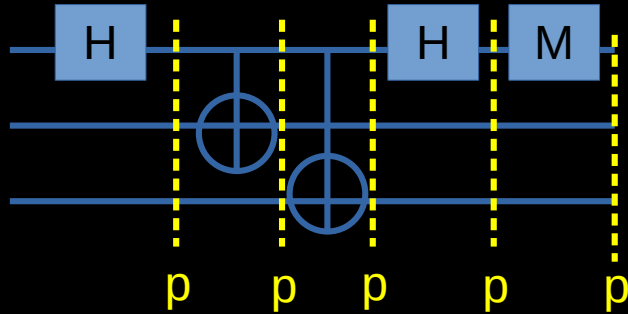
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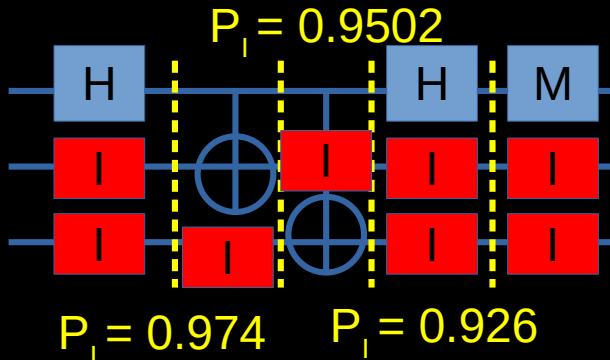
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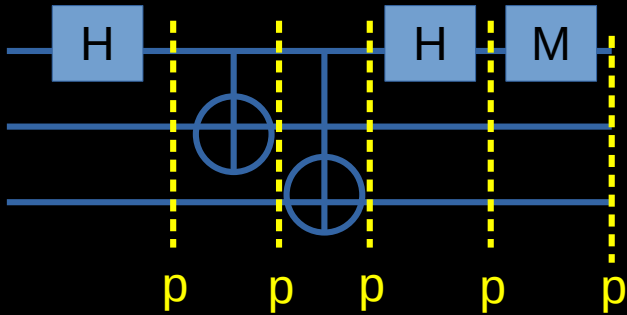


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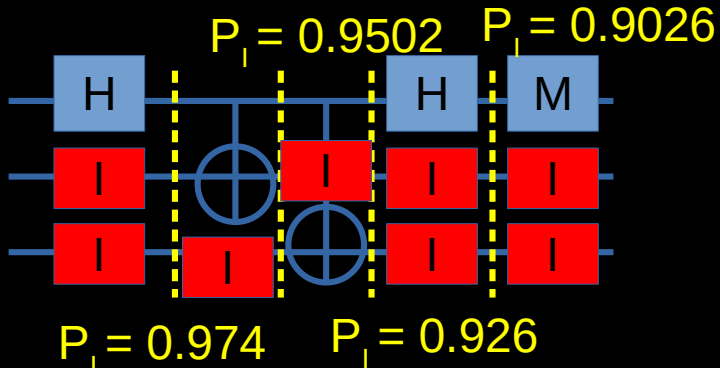
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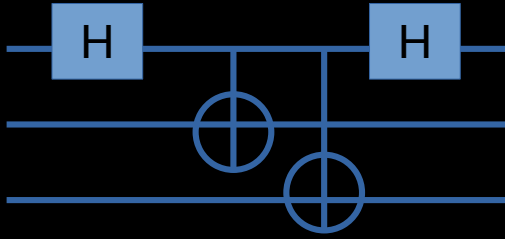


CAN WE FIND A BETTER WAY?



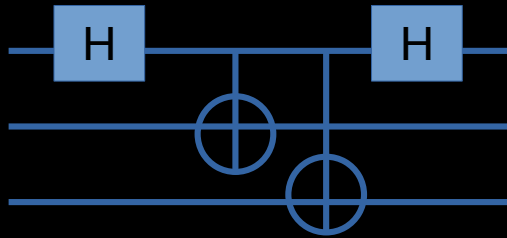
## 4. SINGLE STEP PARITY CHECK GATE SET

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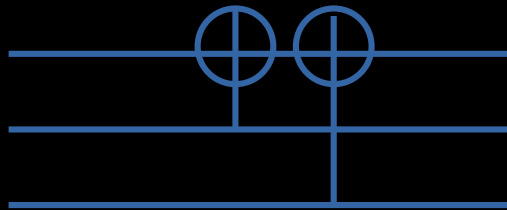


XX parity check circuit

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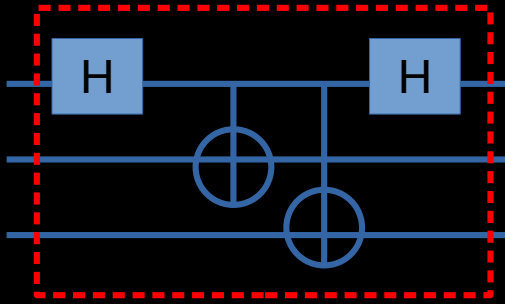
XX parity check circuit



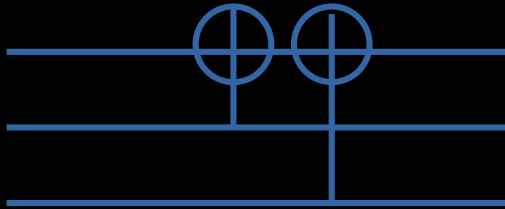
ZZ parity check circuit



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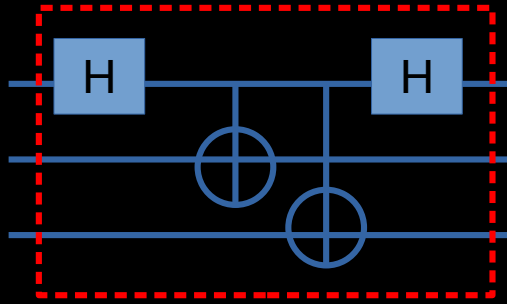


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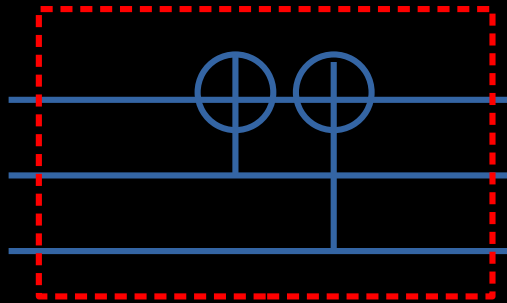


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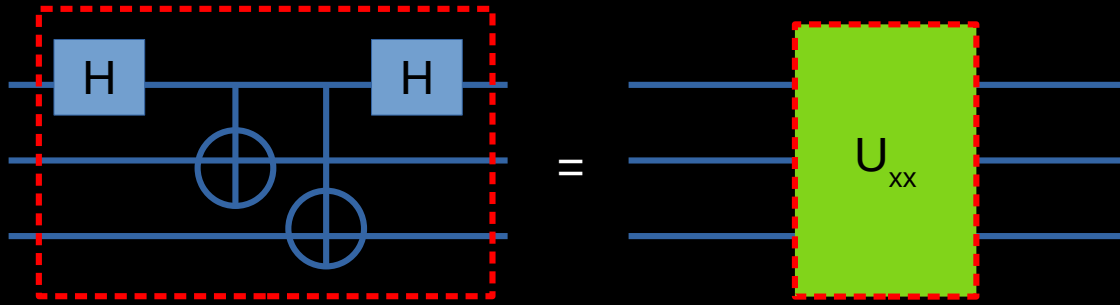


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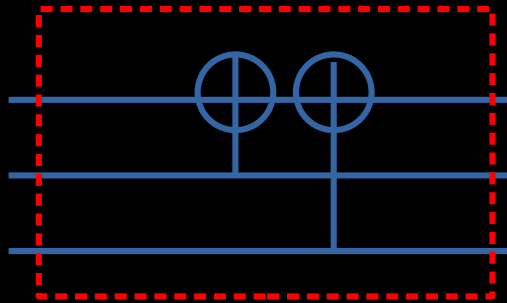


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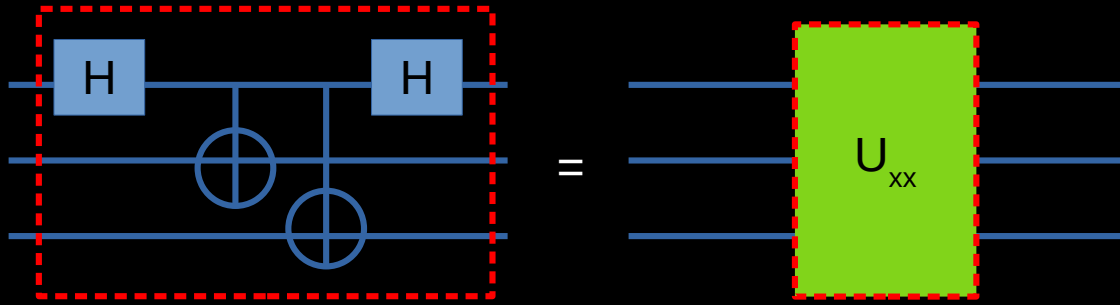


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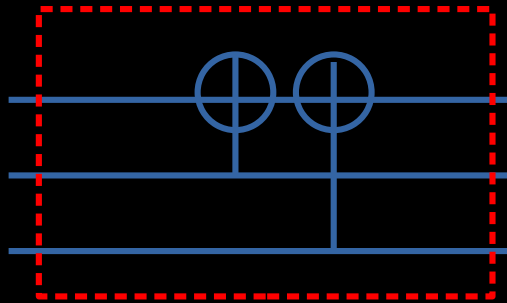


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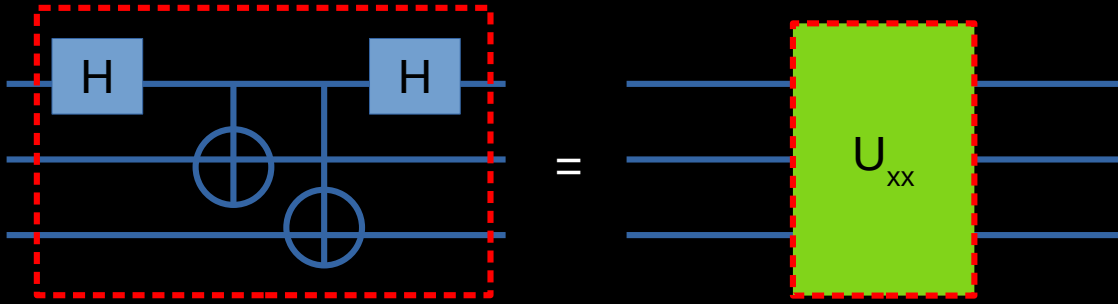


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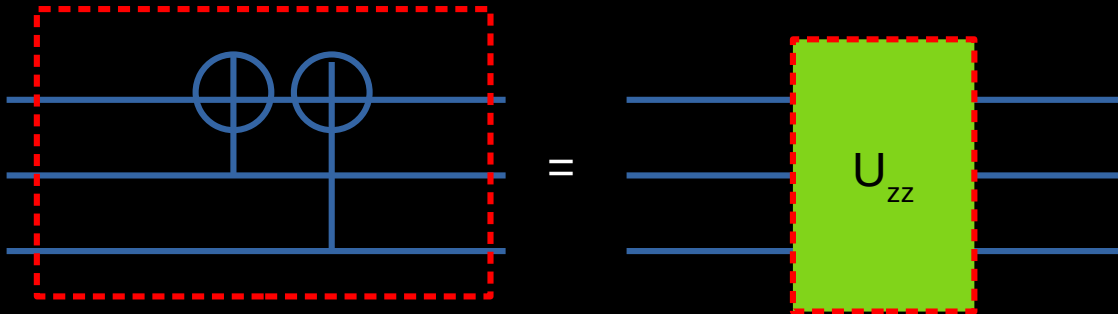


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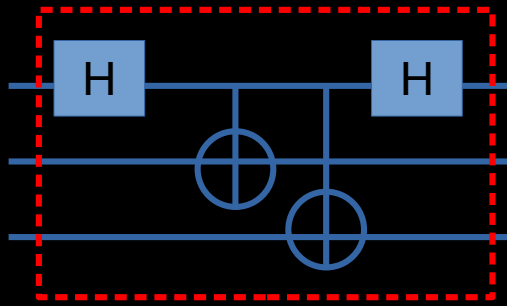


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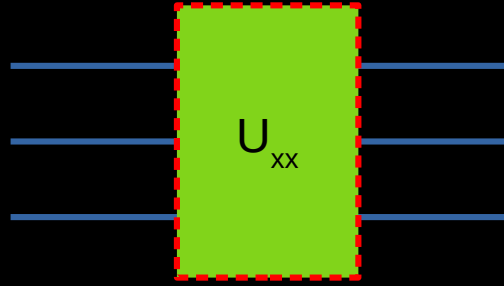
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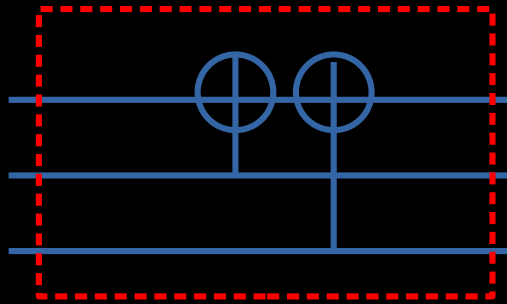
XX parity check circuit

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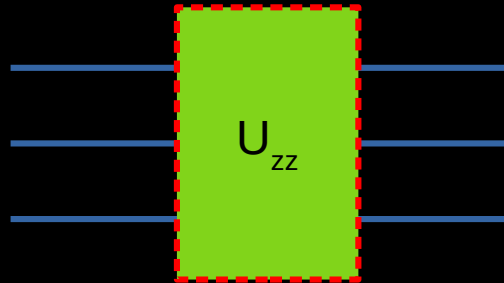
=

$$U_{XX} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

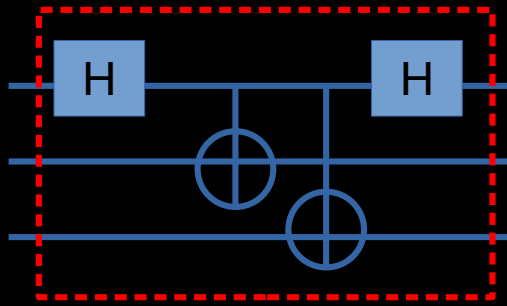


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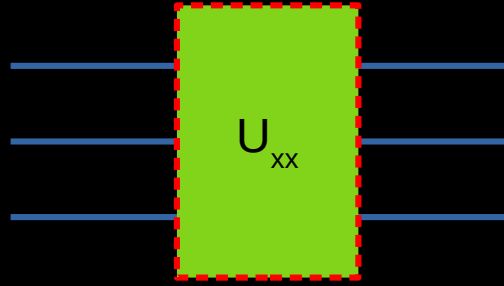


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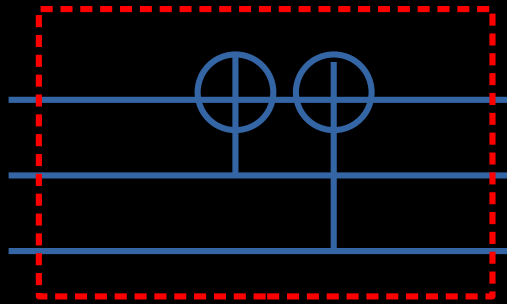
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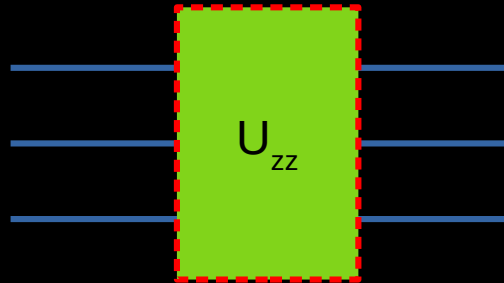
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ZZ parity check circuit

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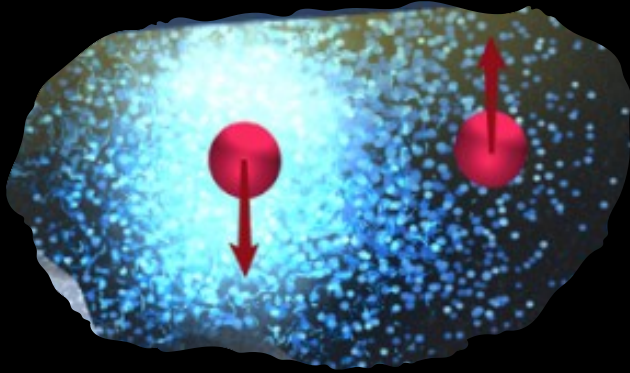
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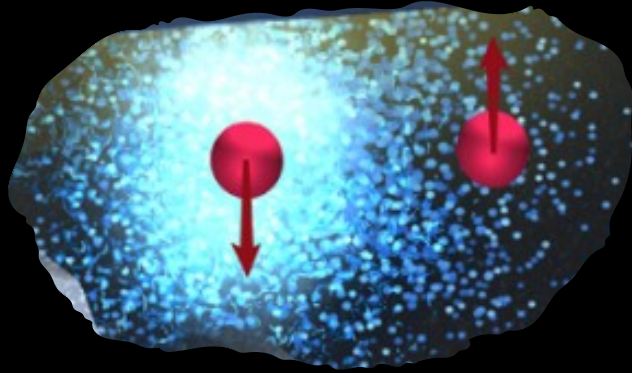


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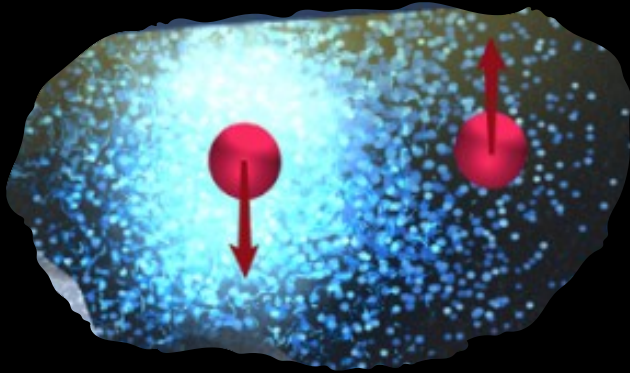


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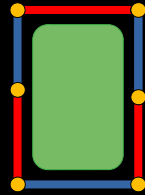


Precision tomography of a three-qubit donor quantum processor in silicon by Madzik & Asaad et al, Nature. 2022 Jan;601(7893):348-353.

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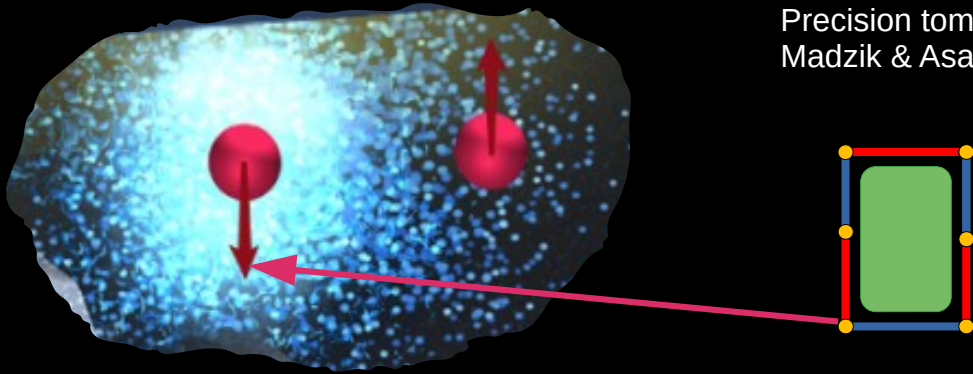


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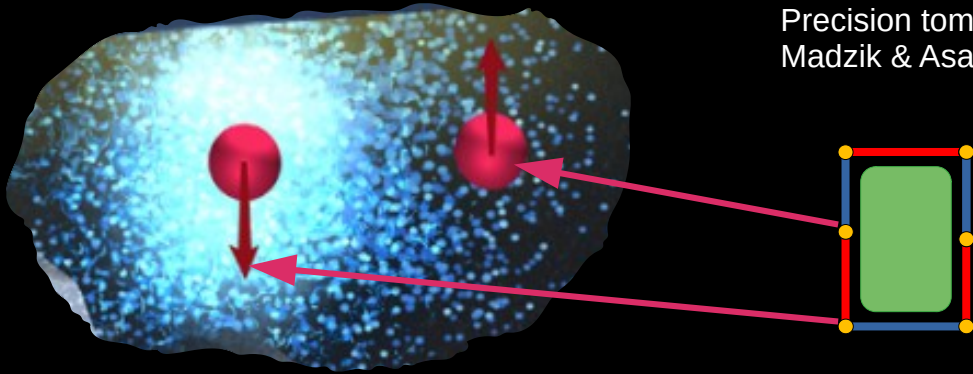
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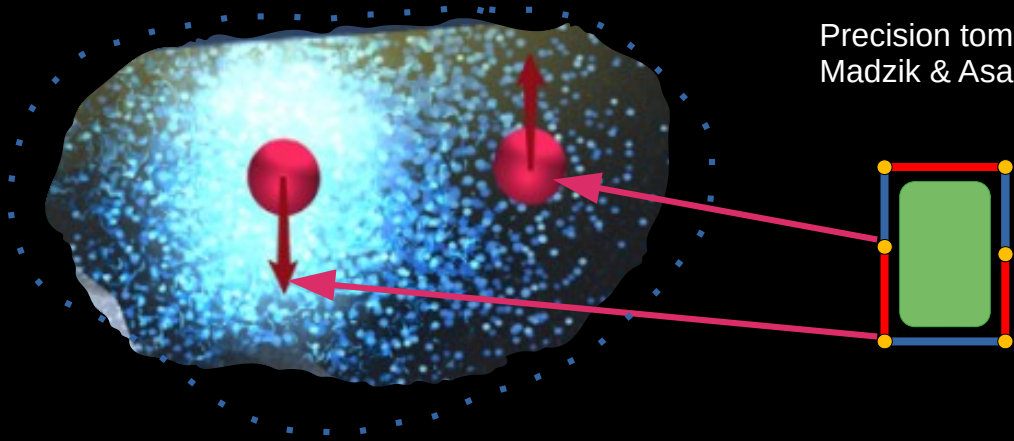
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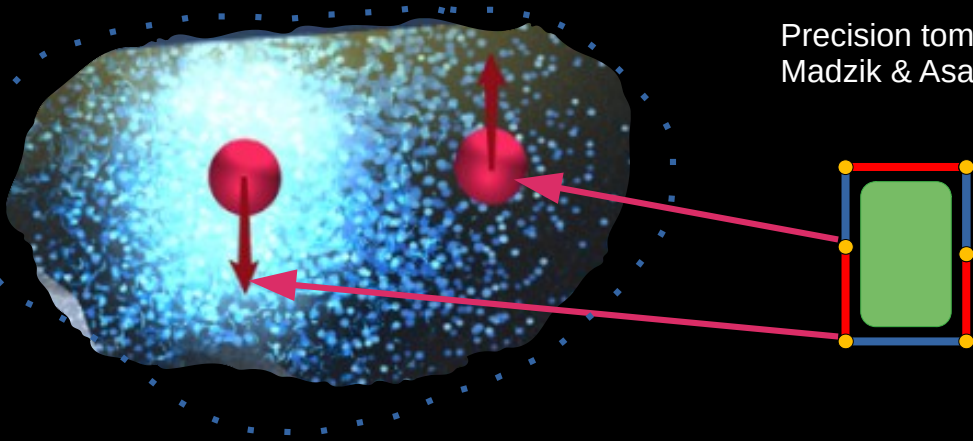
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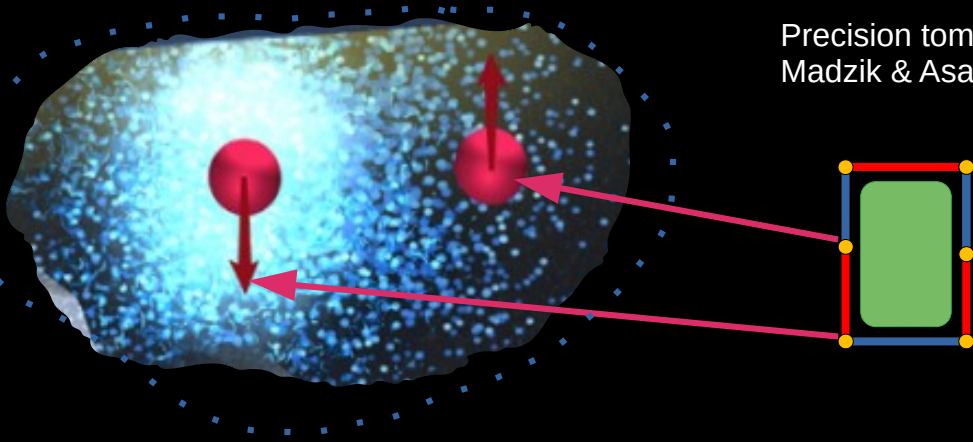


$$\vec{H}_d = -\gamma_e B_0 \hat{S} - \gamma_n B_0 (\hat{I}_{z1} + \hat{I}_{z2}) + A_1 \vec{S} \cdot \vec{I}_1 + A_2 \vec{S} \cdot \vec{I}_2$$



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$$H_{rf}(t) = -\gamma_e \vec{B}_1 \vec{S} \sin(\omega t) - \gamma_n \vec{B}_1 (\vec{I}_1 + \vec{I}_2) \sin(\omega t)$$



## Method 2: Analytical Analysis of Hamiltonian + Numerical simulation

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**Clock Dynamics of  
Hamiltonian + GRAPE  
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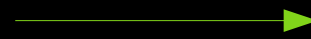
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Clock Dynamics of  
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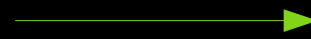
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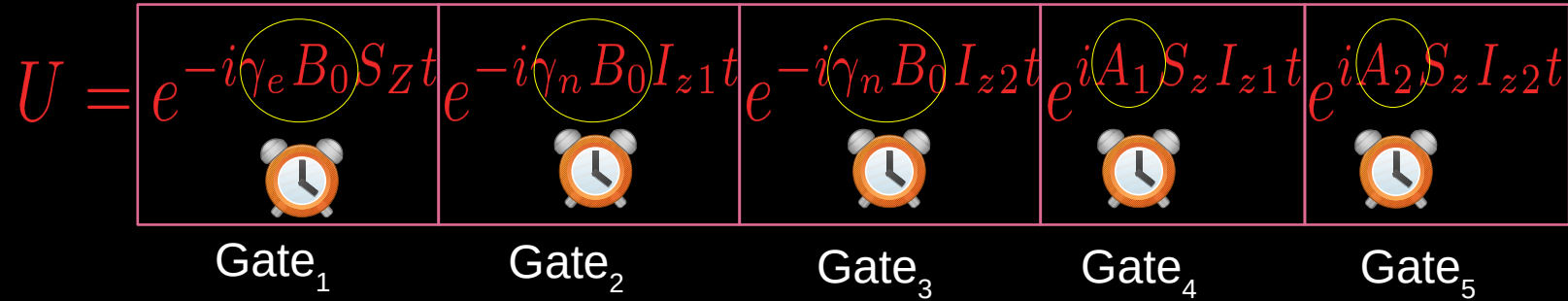
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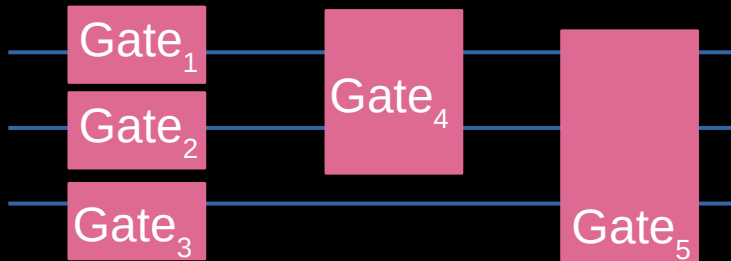
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Gate<sub>1</sub>      Gate<sub>2</sub>      Gate<sub>3</sub>      Gate<sub>4</sub>      Gate<sub>5</sub>

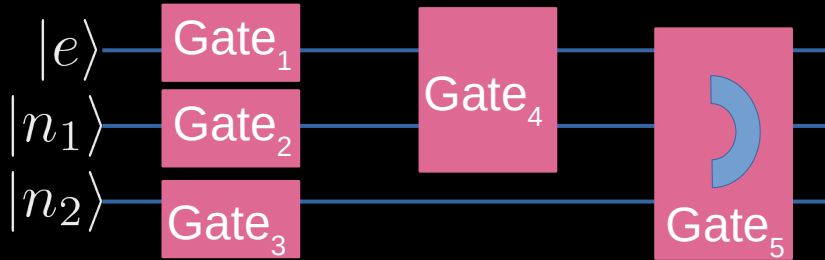
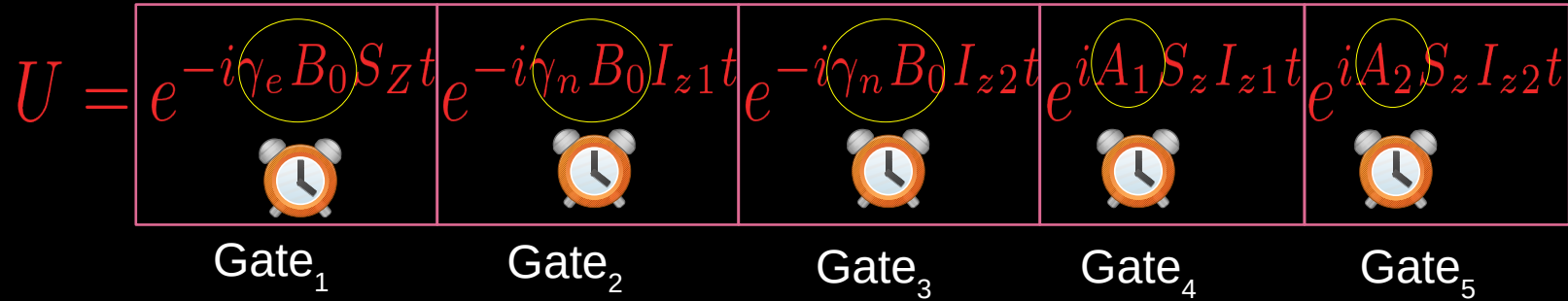


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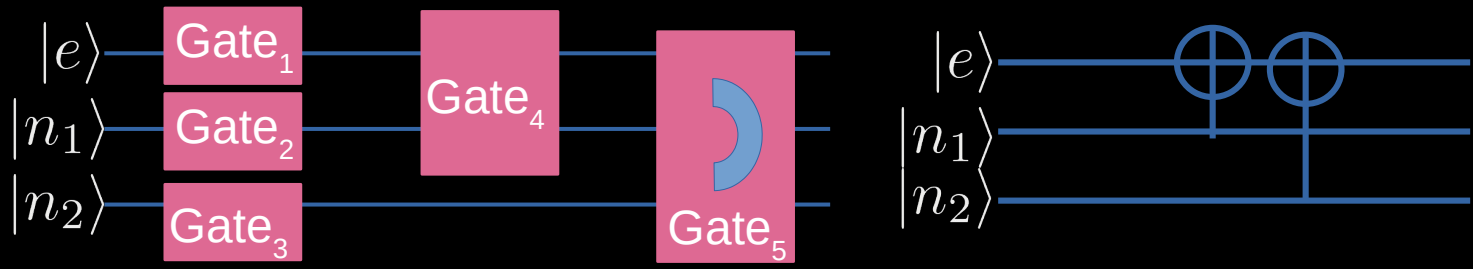
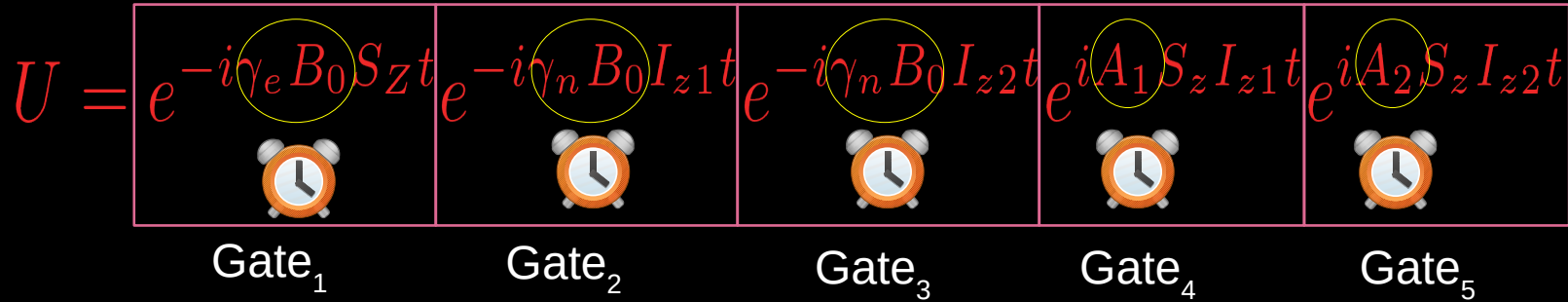


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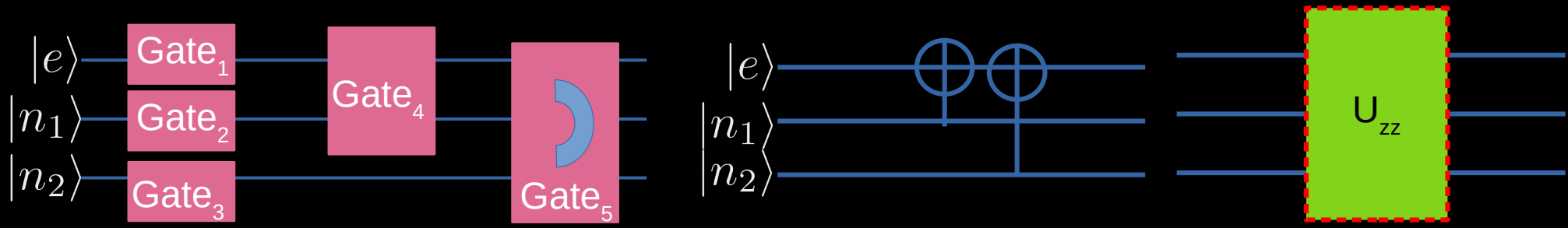
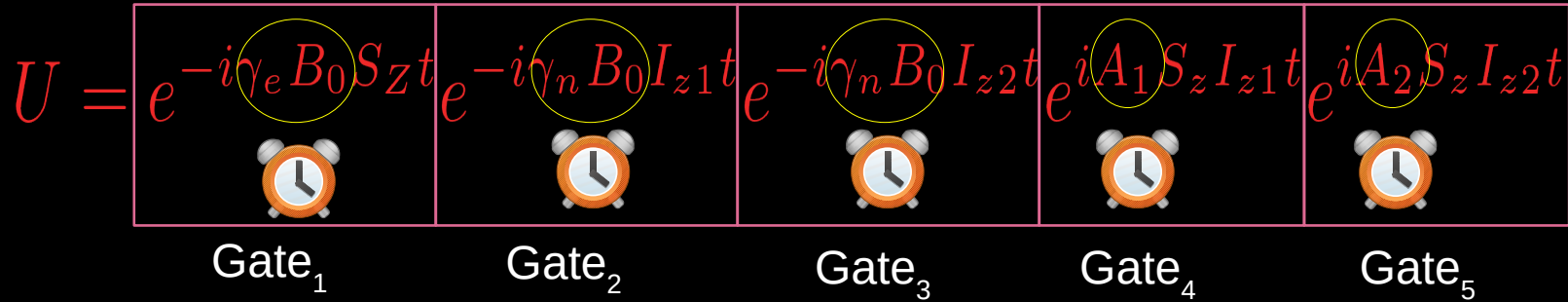


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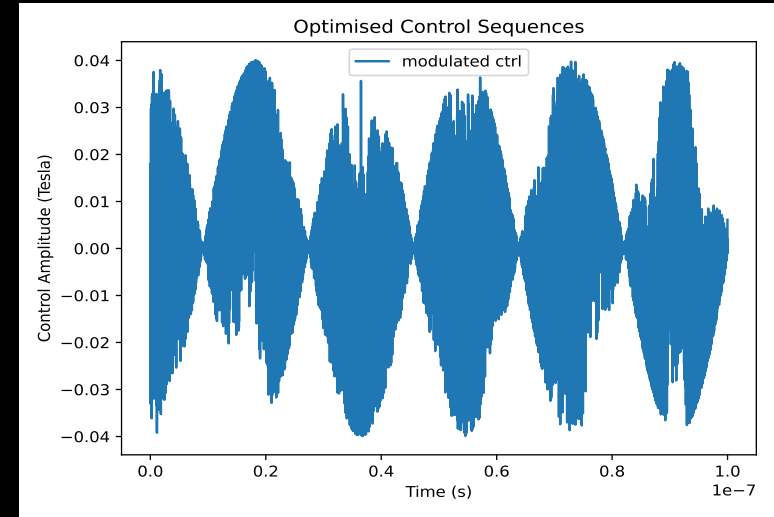
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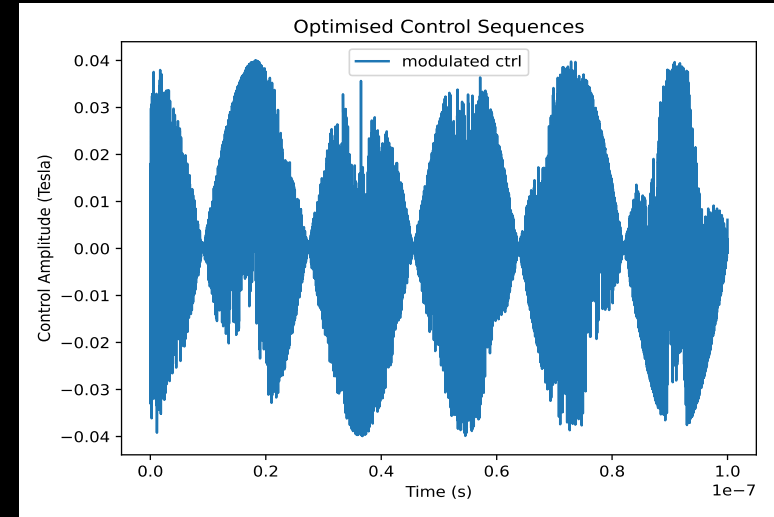
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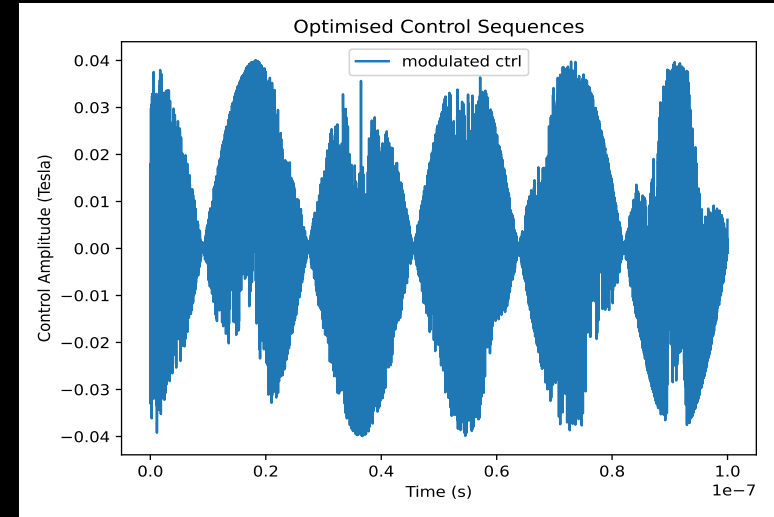


Simulated fidelity error is  $< 10^{-5}$

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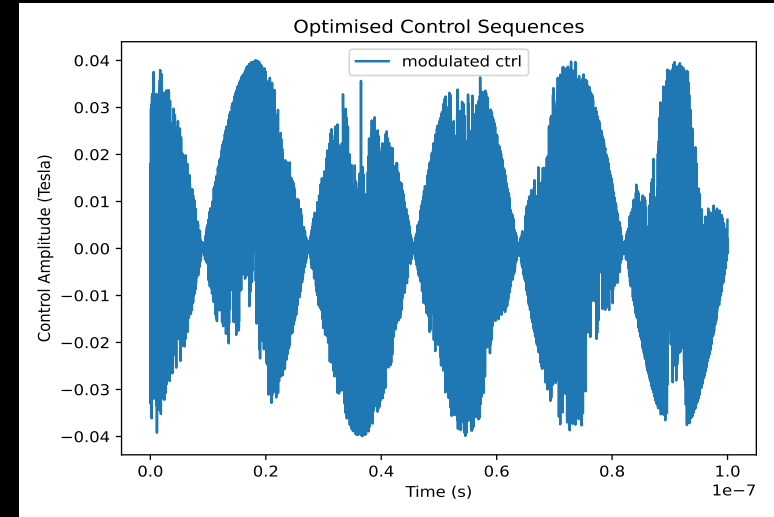
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Hence:  $M_{pp2}$  is one of single step parity check gate + a measurement



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3. Implementing the new gate set for different systems like antimony or 2P2e systems



# 7. CONCLUSION

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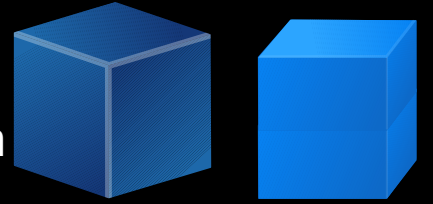


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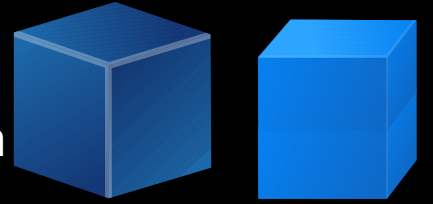
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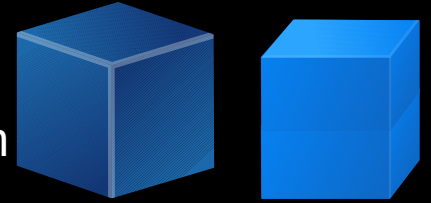
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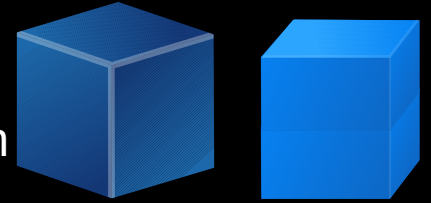
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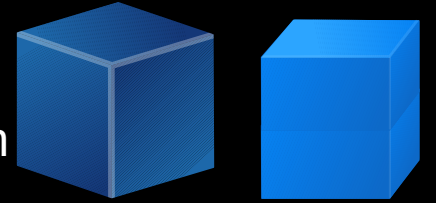


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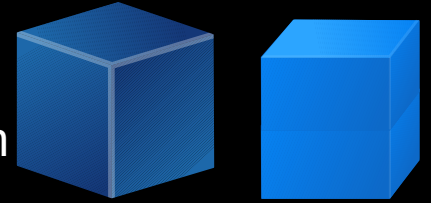


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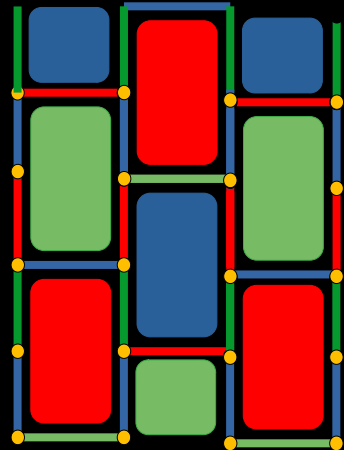


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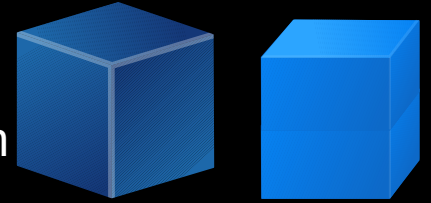
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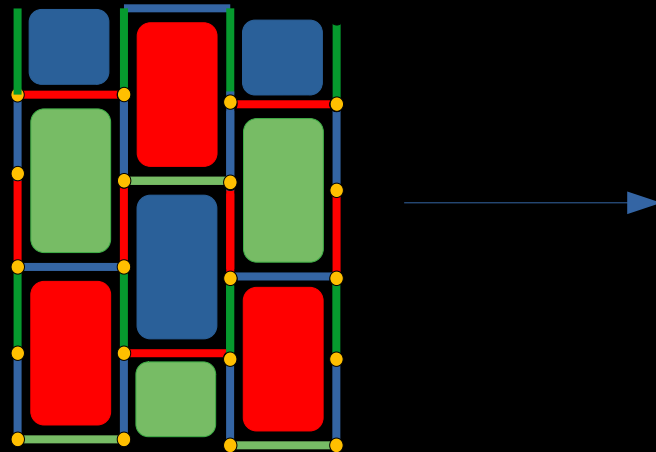
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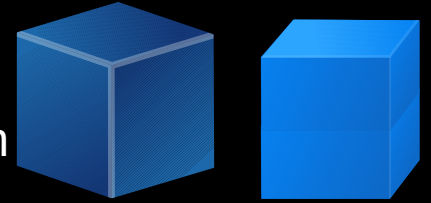
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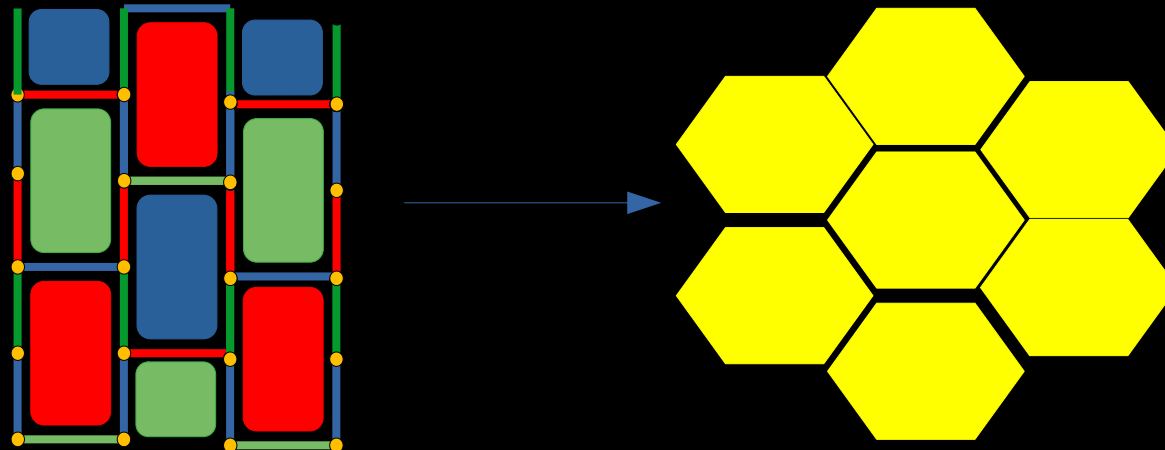
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My lovely team :)



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**THANK YOU FOR LISTENING**