

Transversal Injection: A method for direct encoding of ancilla states for non-Clifford gates using stabiliser codes

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Quantum Error Correction

Differences to Classical

- Bit *and* Phase errors
- Cannot simply read the full state of the system
- No-cloning theorem

Solutions

- Different approach to detecting errors
- Redundancy via encoding



Stabiliser Codes

7-Qubit Steane Code

$$|0_L\rangle = \frac{1}{\sqrt{8}}(|0000000\rangle + |1010101\rangle + |0111011\rangle + |1100110\rangle \\ + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle)$$

$$|1_L\rangle = \frac{1}{\sqrt{8}}(|1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle \\ + |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle)$$

$$K1 = IIIXXXX$$

$$K2 = IXXIIXX$$

$$K3 = XIXIXIX$$

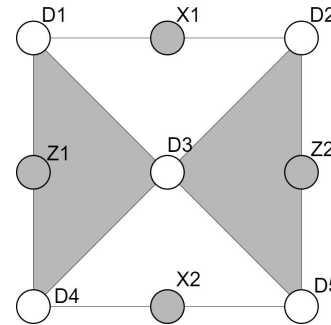
$$K4 = IIIZZZZ$$

$$K5 = IZZIIZZ$$

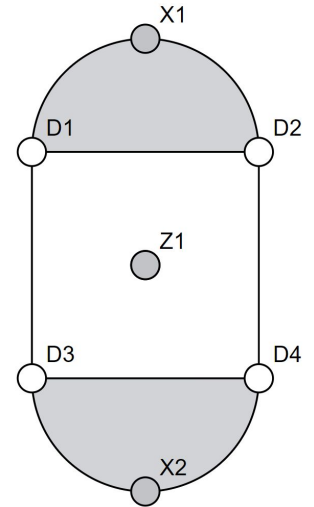
$$K6 = ZIZIZIZ$$

Planar Surface Code Layout

(distance = 2)



Unrotated



Rotated



Relevant Theorems

- Threshold Theorem

With a sufficiently low physical error rate, concatenation of QEC codes can achieve an arbitrary degree of accuracy.

- Eastin-Knill No-Go Theorem

For a given QEC code there are a set of *native* operations that can be applied transversally. This set will always fall short of achieving universality.



Transversal Injection

Standard QEC

- Initialised in zero or plus state
- Requires state injection, distillation and teleportation for “magic” states

Transversal Injection

- A method for encoding non-Pauli ancillary states **directly** on the surface code
- Used as resource states for enacting logical gates
- Improved injection - resulting in dramatic savings elsewhere



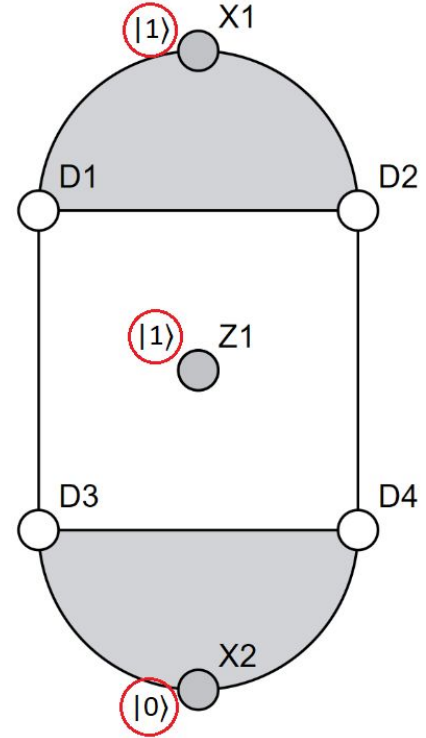
Definitions

Stabiliser Trajectory

- String of bits corresponding to observed eigenvalues
- $\{X_1, X_2, Z_1\}$ e.g. $\{1, 0, 1\}$

Transversal Operation

- A gate applied across all data qubits
- E.g. A Hadamard applied to D_1, D_2, D_3, D_4





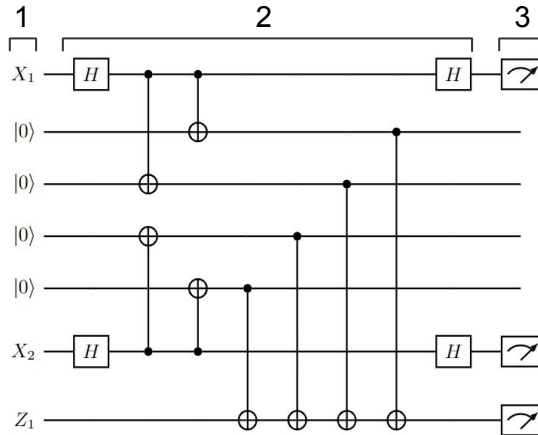
Standard Surface Code Encoding

- 1) Qubit initialisation
- 2) Apply stabilisers
- 3) Syndrome Extraction

Initial state = $|0000\rangle$

In the trivial case X: $\{0, 0\}$, Z: $\{0\}$

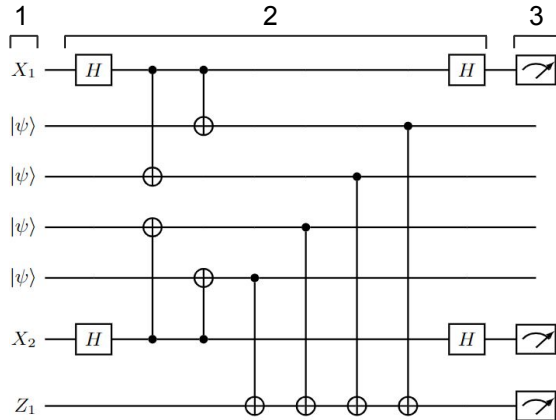
Final state = $\frac{1}{2} (|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle)$





Transversal Injection

- 1) All qubits initialised using a transversal rotation
- 2) Apply stabilisers
- 3) Syndrome Extraction



Initial state of each qubit = $\alpha|0\rangle + \beta|1\rangle$

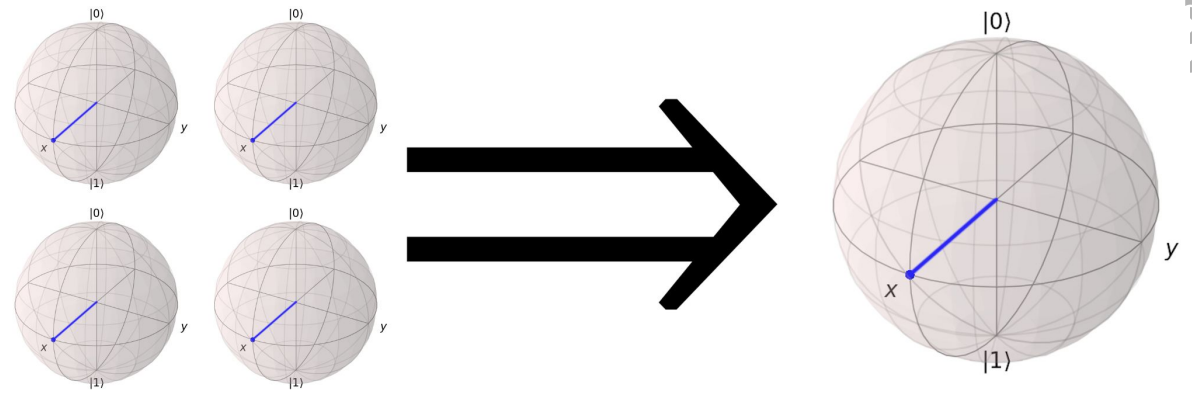
In the trivial case X: {0, 0}, Z: {0}

Final state =

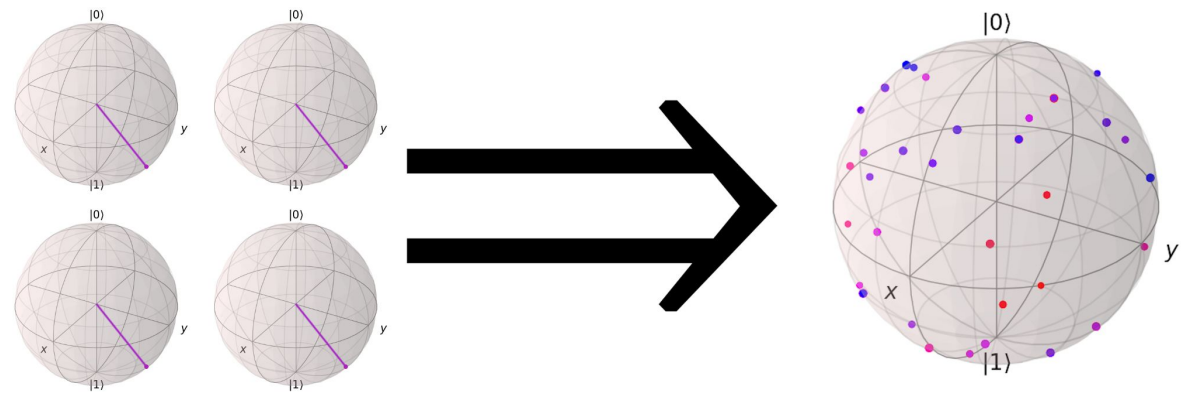
$$\begin{aligned}
 &(\alpha^4 + 2\alpha^2\beta^2 + \beta^4) \times (|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle) \\
 &\quad (2\alpha^2\beta^2) \quad \times (|0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle)
 \end{aligned}$$

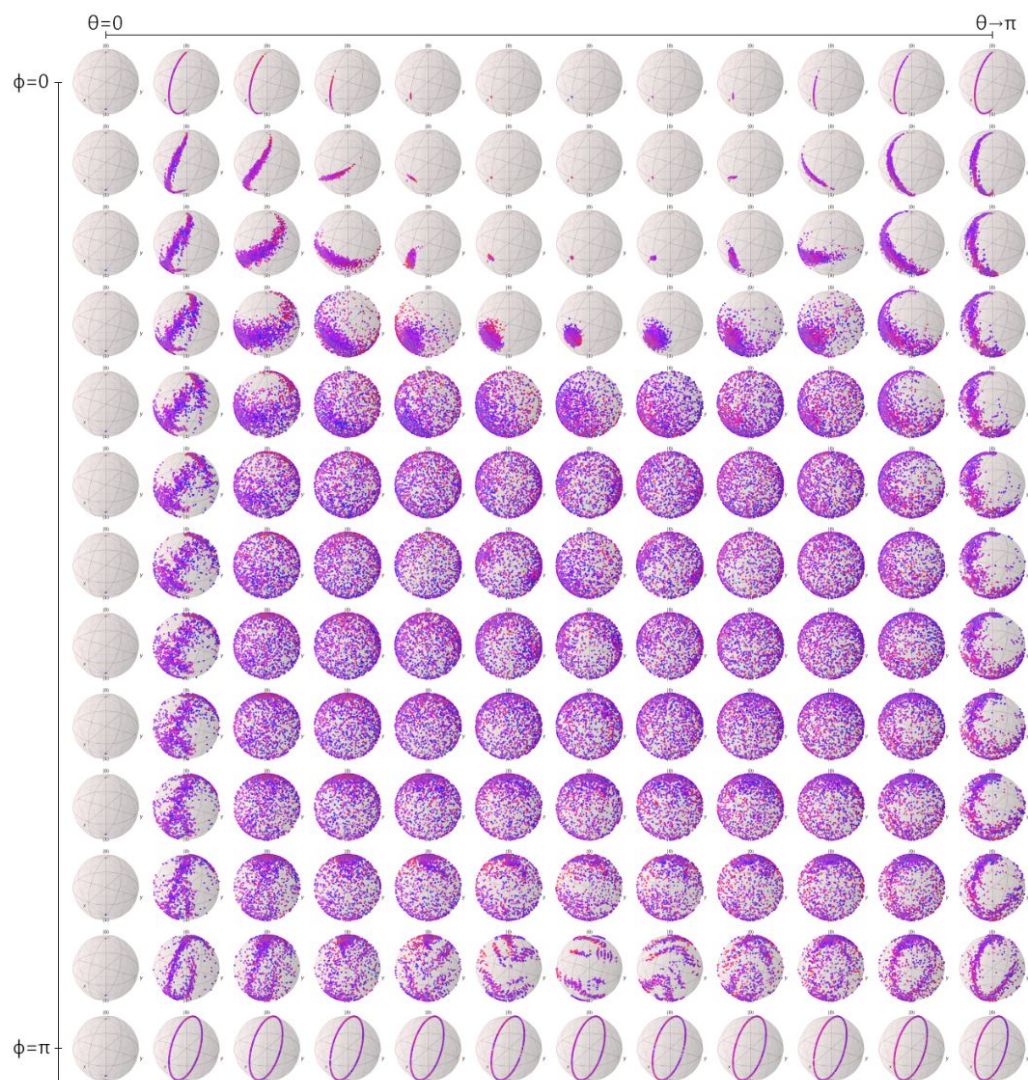


$|+\rangle$ initialisation:



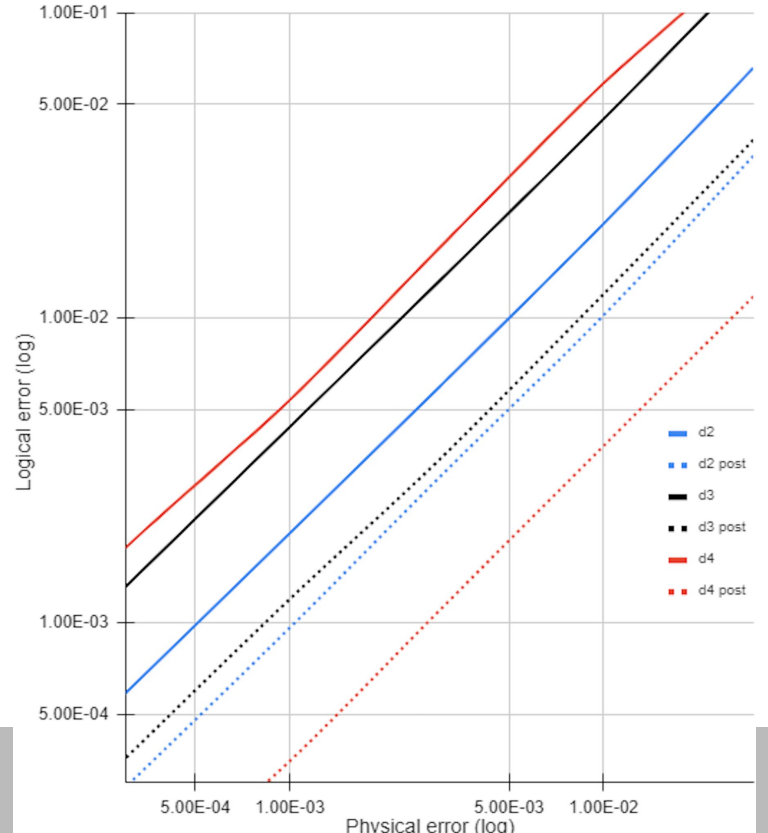
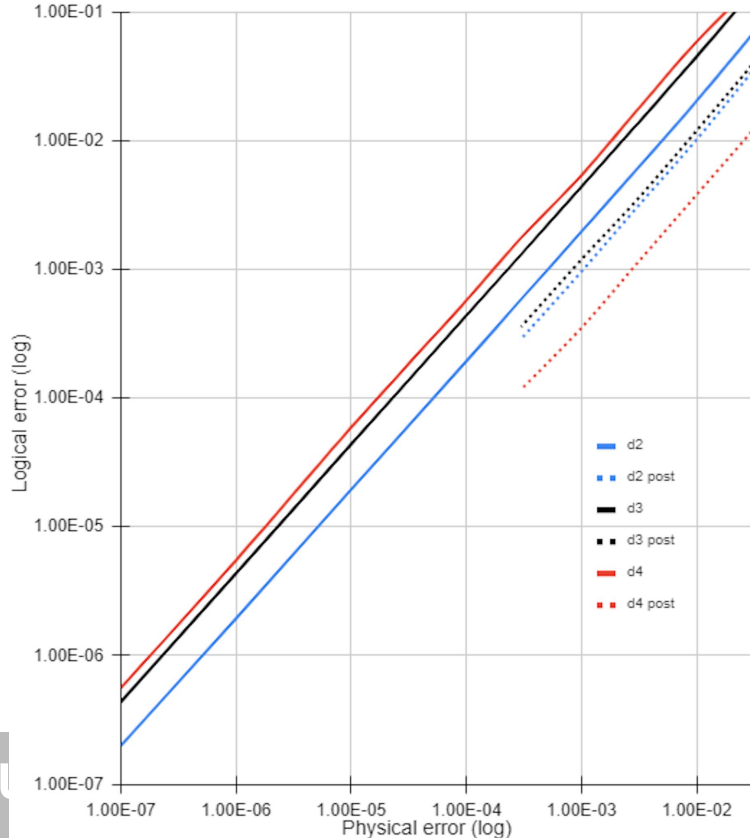
Transversal injection:







Numerical simulations





Eastin-Knill Theorem

(arXiv:0811.4262)

“For any nontrivial local-error-detecting quantum code, the set of transversal, logical unitary operators is not universal.”

Transversal Injection:

- Does not introduce new transversal gates
- Is still bound by EK Theorem
 - Sacrifices error-detection capacity of the code



Future Work

- Experimental Verification
- Tweaking initial parameters
- Investigating patterns/redundancy in trajectories
- Benchmarking at distance 5+
- Non-Pauli state distillation
- Circuit compilation and algorithm benchmarking



Questions?