Stabiliser subsystem decompositions for single- and multi-mode Gottesman-Kitaev-Preskill codes

Mackenzie H. Shaw, Andrew C. Doherty and Arne L. Grimsmo

[a] *ARC Centre of Excellence for Engineered Quantum Systems, School of Physics, The University of Sydney, Sydney, NSW 2006, Australia.*

Bosonic codes encode digital quantum information in continuous variable (CV) quantum systems, and are an alternative approach to quantum error correction that has received both theoretical and experimental attention. The Gottesman-Kitaev-Preskill (GKP) code [1] is one of the most intensively studied encodings of this type, and has recently been realized in both trapped ions [3, 4] and microwave resonators [5, 6].
In this work we introduce a new subsystem decomposition for GKP codes, an alternative to the approach of Pantaleoni *et al* [2]. The decomposition has the defining property that a partial trace over the non-logical stabilizer subsystem is equivalent to an ideal decoding of the logical state. The stabilizer subsystem decomposition can be applied to all multi-mode qubit or qudit codes (including concatenated codes), and the description can be updated to incorporate the action of logical Clifford gates on the state. Besides providing a convenient theoretical view on GKP codes, such a decomposition is also of practical use: we use the stabilizer subsystem decomposition to numerically simulate GKP codes subject to realistic noise such as loss and dephasing. In contrast to more conventional Fock basis simulations, the numerical methods we develop are often more efficient the closer the state is to an ideal (infinitely squeezed) GKP state, allowing us to study codes with arbitrarily large photon numbers.
In the case of a square GKP code our treatment is exact. We find that GKP codes are far more resilient against pure loss than against dephasing: a square single-mode codestate with ten decibels of squeezing achieves an average gate infidelity below 10^{-3} for a loss rate up to $\sim$5%, compared to a dephasing rate of up to only $\sim$0.2%. We also find that for both pure loss and dephasing, there is an optimal finite photon number that minimizes the logical error rate. This optimal photon number is much larger for loss than for dephasing at the same rate.

[1] D. Gottesman, A. Kitaev, and J. Preskill, *Phys. Rev. A* **64**, 012310 (2001).

[2] G. Pantaleoni, B. Q. Baragiola, and N. C. Menicucci, *Physical Review Letters* **125**, 040501 (2020).

[3] C. Flümann *et al*, *Nature* **566**, 513 (2019).

[4] B. de Neeve,*et al*, arXiv preprint arXiv:2010.09681 (2020).

[5] P. Campagne-Ibarcq *et al*, *Nature* **584**, 368 (2020).

[6] A. Eickbusch *et al*, arXiv preprint arXiv:2111.06414 (2021).