Optimal quantum linear systems solver via discrete adiabatic theorem

Pedro C. S. Costa[a], Dong An[b,c], Yuval R. Sanders[a,d], Yuan Su[c], Ryan Babbush[c], and Dominic W. Berry[a]

[a] *School of Mathematical and Physical Sciences, Macquarie University, NSW 2109, Australia*

[b] *Joint Center for Quantum Information and Computer Science, University of Maryland*

[c] *Google Research, Venice, CA 90291, United States of America*

[d] *School of Computer Science, University of Technology Sydney, Sydney, NSW 2007, Australia*

Recently, several approaches to solving linear systems on a quantum computer have been formulated in terms of the quantum adiabatic theorem for a continuously varying Hamiltonian [1, 2, 3]. Such approaches enabled near-linear scaling in the condition number κ of the linear system. However, the most efficient of those procedures is still asymptotically sub-optimal by a factor of $\log(\kappa)$. In this work, we develop a quantum algorithm for solving systems of linear equations with complexity $\mathcal{O}(\kappa \log(1/\varepsilon))$. As well as being optimal in κ, our complexity is also optimal in the combined scaling with κ and the precision ε. It is known that there is a lower bound on the complexity in terms of both κ and ε given by $\Omega(\kappa \log(1/\varepsilon))$ [4].

In order to achieve this result, we translate the Hamiltonian into a quantum walk operator $W_T(s)$ via qubitisation. We then apply this discrete walk and prove a new form of the discrete adiabatic theorem. Lastly we use eigenstate filtering on the walk operator. This procedure can be used to generally speed up any adiabatic algorithm, because it avoids the need to simulate time-dependent evolution of a Hamiltonian. In the case of solving linear equations, prior work [2, 3] used a Dyson series that introduced extra overheads to the complexity. Our result is both more efficient and provides an algorithm that is simpler to implement.

The most technically difficult part of the result is the proof of the discrete adiabatic theorem. A result was given in prior work [5], which just gave a result for the scaling of the error in terms of the number of steps. In our work we derive the complete result, most crucially showing the dependence on the gap. Because the gap depends on the condition number, this enables us to give the linear scaling of the complexity in terms of κ.

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