

Equivalent gate noise on macronode cluster state architectures

Blayney Walshe

Ben Q. Baragiola, Rafael N. Alexander, Nicolas C. Menicucci

Experimental efforts

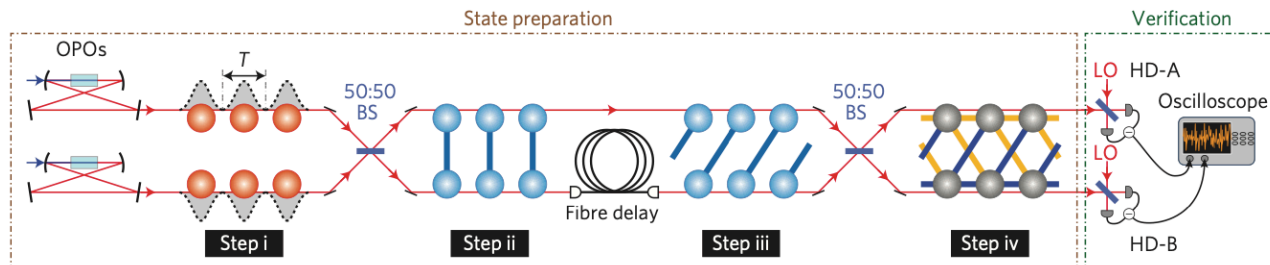
LETTERS

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nature
photonics

Ultra-large-scale continuous-variable cluster states multiplexed in the time domain

Shota Yokoyama¹, Ryuji Ukai¹, Seiji C. Armstrong^{1,2}, Chanond Sornphiphatphong¹, Toshiyuki Kaji¹, Shigenari Suzuki¹, Jun-ichi Yoshikawa¹, Hidehiro Yonezawa¹, Nicolas C. Menicucci³ and Akira Furusawa^{1*}



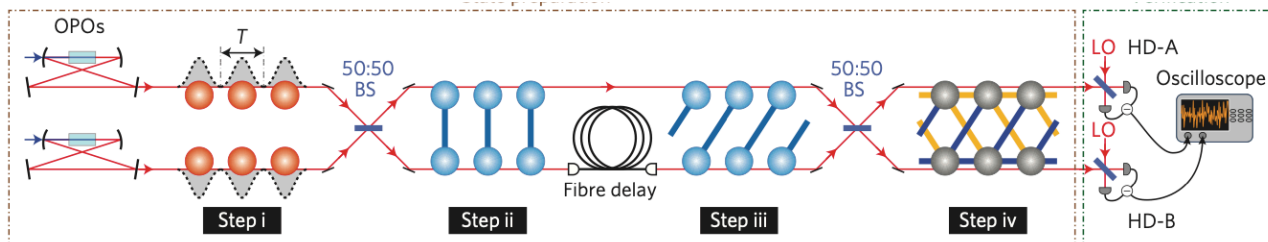
Experimental efforts

Invited Article: Generation of one-million-mode continuous-variable cluster state by unlimited time-domain multiplexing

U Jun-ichi Yoshikawa,¹ Shota Yokoyama,^{1,2} Toshiyuki Kaji,¹
st Chanond Sornphiphatphong,¹ Yu Shiozawa,¹ Kenzo Makino,¹
and Akira Furusawa^{1,a}

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Shi 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan
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and Information Technology, University of New South Wales, Canberra,
Australian Capital Territory 2600, Australia

(Received 23 June 2016; accepted 1 September 2016; published online 27 September 2016)

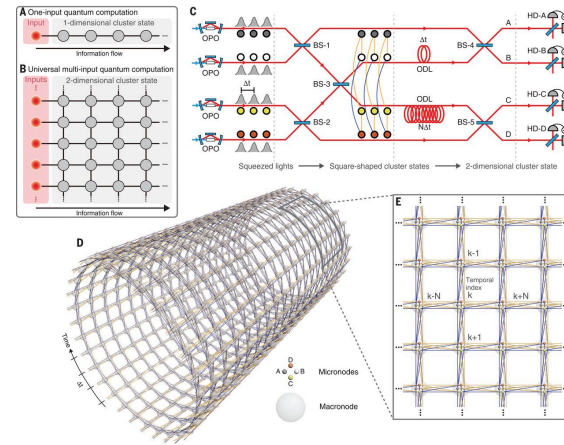


Experimental efforts

QUANTUM COMPUTING

Generation of time-domain-multiplexed two-dimensional cluster state

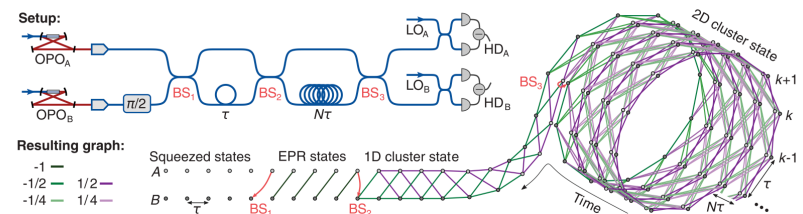
Warit Asavanant¹, Yu Shiozawa¹, Shota Yokoyama², Baramée Charoensombutamon¹, Hiroki Emura¹, Rafael N. Alexander³, Shuntaro Takeda^{1,4}, Jun-ichi Yoshikawa¹, Nicolas C. Menicucci⁵, Hidehiro Yonezawa², Akira Furusawa^{1*}



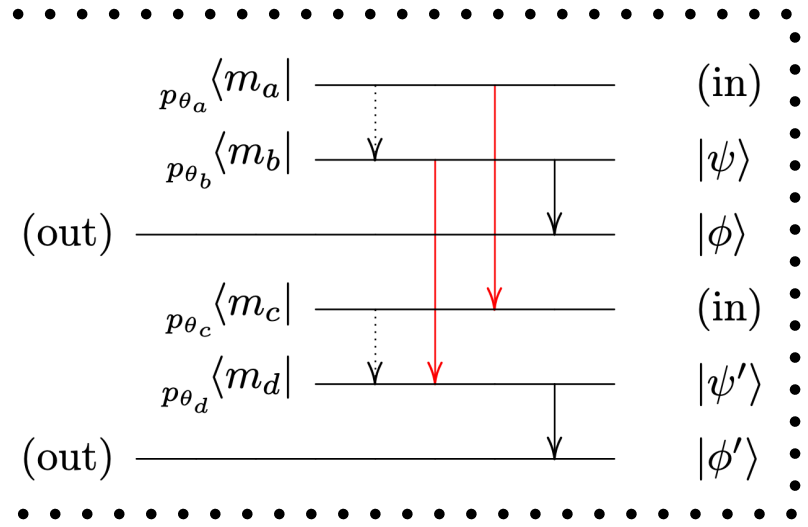
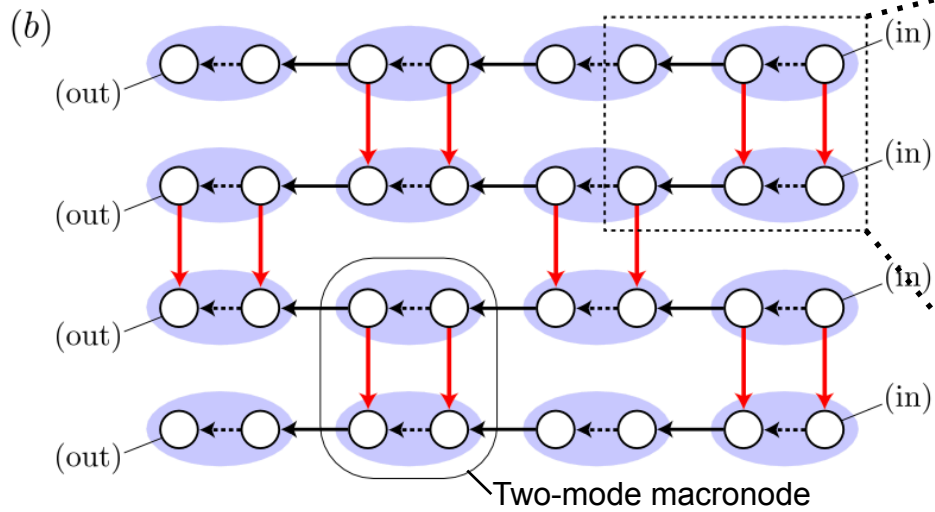
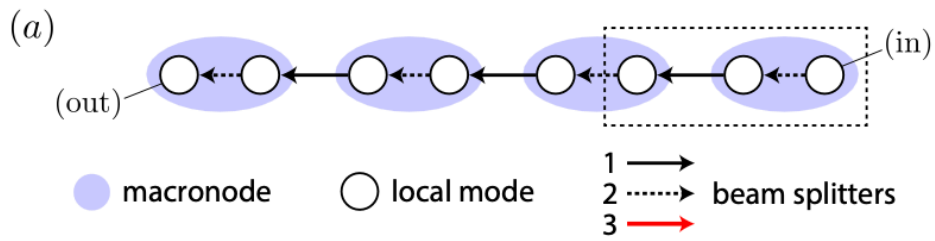
QUANTUM COMPUTING

Deterministic generation of a two-dimensional cluster state

Mikkel V. Larsen*, Xueshi Guo, Casper R. Breum, Jonas S. Neergaard-Nielsen, Ulrik L. Andersen*



Streamline QC



PHYSICAL REVIEW A **104**, 062427 (2021)

Streamlined quantum computing with macronode cluster states

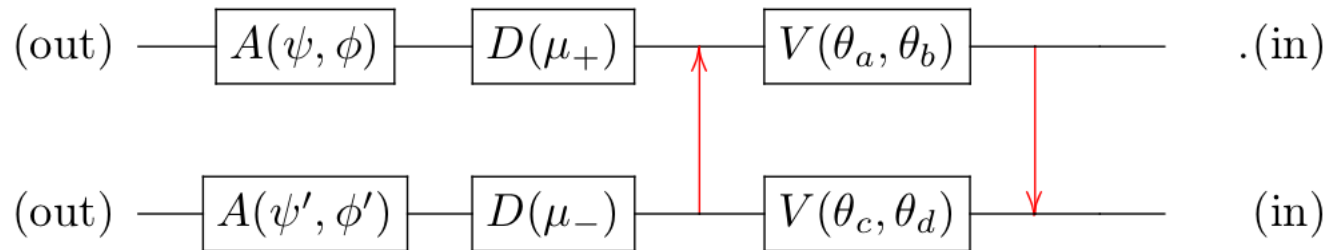
Blayne W. Walshe^{1,*}, Rafael N. Alexander¹, Nicolas C. Menicucci¹, and Ben Q. Baragiola^{1,2}

¹Centre for Quantum Computation and Communication Technology, School of Science, RMIT University, Melbourne, VIC 3000, Australia

²Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa Oiwakecho, Sakyo-ku, Kyoto 606-8502, Japan

(Received 10 September 2021; accepted 19 November 2021; published 16 December 2021)

Two-mode gate



$$\mu_{a,b} := \frac{-m_a e^{i\theta_b} - m_b e^{i\theta_a}}{\sin(2\theta_-)} \quad \mu_{\pm} = \frac{\mu_{c,d} \pm \mu_{a,b}}{\sqrt{2}}$$


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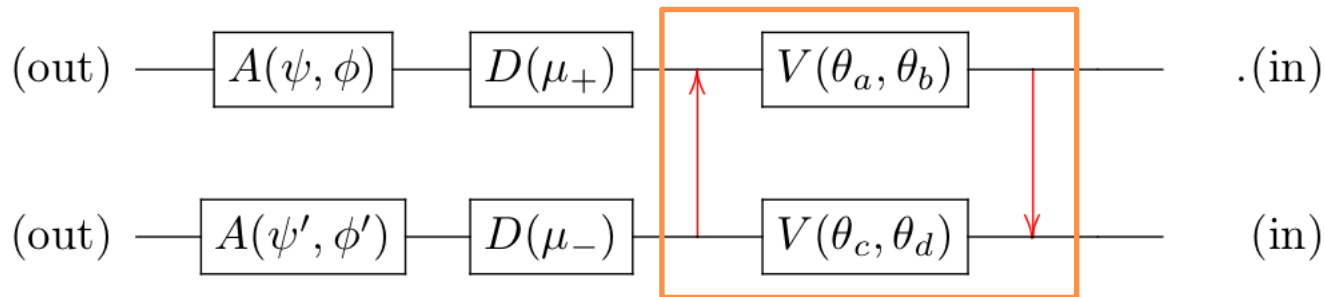
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Two-mode gate



General two-mode gate

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$$\mu_{\pm} = \frac{\mu_{c,d} \pm \mu_{a,b}}{\sqrt{2}}$$

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Streamlined quantum computing with macronode cluster states

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Streamlined QC

Gaussian unitaries that implement the Clifford group on GKP encoded states:

$$\underbrace{\{\hat{I}, \hat{F}, \hat{P}(\pm 1), \hat{C}_Z(\pm 1)\}}_{\text{CV unitaries}} \longmapsto \underbrace{\{\hat{I}, \hat{H}, \sqrt{\hat{Z}}, \hat{C}_Z\}}_{\text{GKP Cliffords}}$$

$\{\theta_a, \theta_b\}$	$\hat{V}(\boldsymbol{\theta})$	Logical Gate
$\{\frac{\pi}{2}, 0\}$	\hat{I}	\bar{I}
$\{\frac{3\pi}{4}, \frac{\pi}{4}\}$	\hat{F}	\bar{H}
$\{\frac{\pi}{2}, \frac{\pi}{2} \mp \chi\}$	$\hat{P}(\pm 1)$	\bar{P}
$\{\theta_a, \theta_b, \theta_c, \theta_d\}$	$\hat{V}^{(2)}(\boldsymbol{\theta})$	Logical Gate
$\{\frac{\pi}{2}, \frac{\pi}{2} \pm \chi, \frac{\pi}{2}, \frac{\pi}{2} \mp \chi\}$	$\hat{C}_Z(\pm 1)$	\bar{C}_Z
$\{0, \frac{\pi}{2}, \frac{\pi}{2}, 0\}$	SWAP	$\overline{\text{SWAP}}$
$\{\frac{\pi}{2}, 0, \frac{\pi}{2}, 0\}$	$\hat{I} \otimes \hat{I}$	$\bar{I} \otimes \bar{I}$
$\{\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\}$	$\hat{F} \otimes \hat{F}$	$\bar{H} \otimes \bar{H}$
$\{\frac{\pi}{2}, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}, \frac{\pi}{2} \mp \chi\}$	$\hat{P}(\pm 1) \otimes \hat{P}(\pm 1)$	$\bar{P} \otimes \bar{P}$

Streamlined QC

Gaussian unitaries that implement the Clifford group on GKP encoded states:

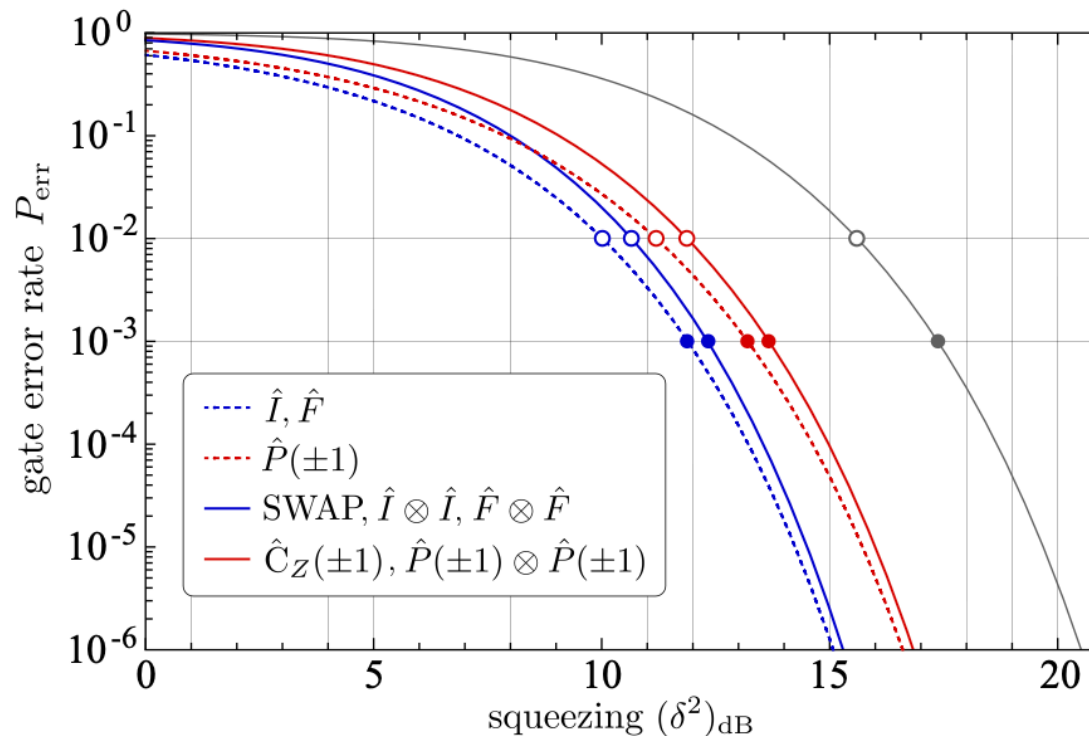
$$\underbrace{\{\hat{I}, \hat{F}, \hat{P}(\pm 1), \hat{C}_Z(\pm 1)\}}_{\text{CV unitaries}} \longmapsto \underbrace{\{\hat{I}, \hat{H}, \sqrt{\hat{Z}}, \hat{C}_Z\}}_{\text{GKP Cliffords}}$$

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$\{\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\}$	$\hat{F} \otimes \hat{F}$	$\bar{H} \otimes \bar{H}$
$\{\frac{\pi}{2}, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}, \frac{\pi}{2} \mp \chi\}$	$\hat{P}(\pm 1) \otimes \hat{P}(\pm 1)$	$\bar{P} \otimes \bar{P}$

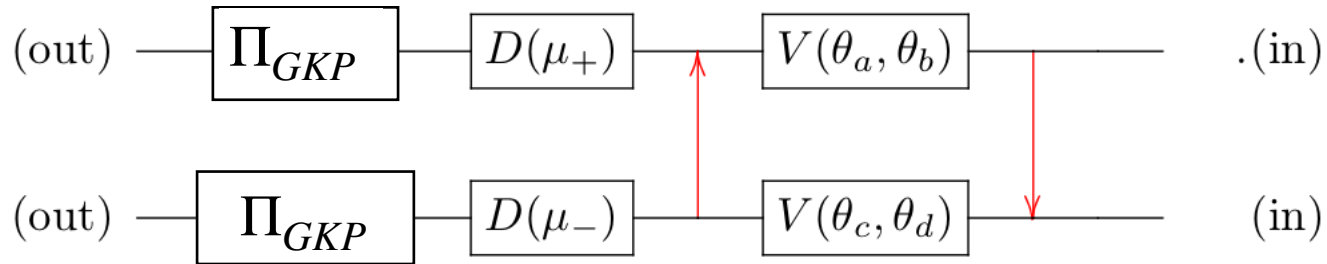
$$\chi = \arctan 2$$

Squeezing level by gate error rate

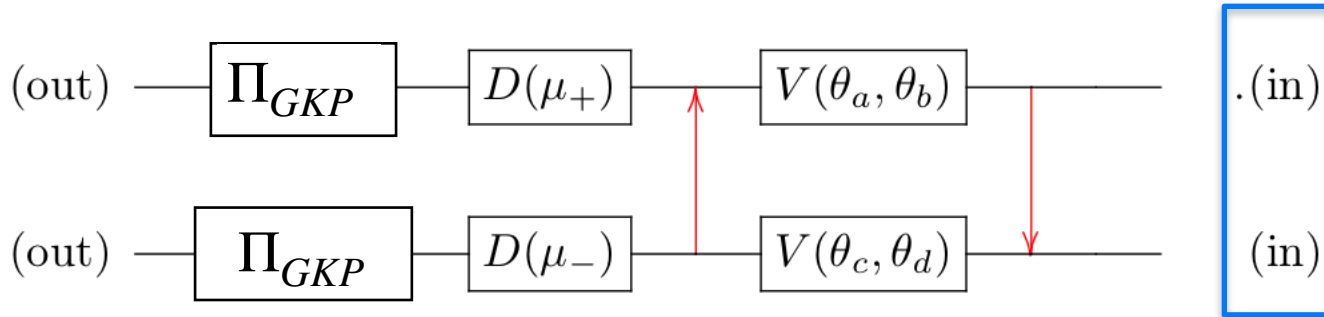
Gate	Error rate: 10^{-2}				Error rate: 10^{-3}			
	Ref. [26]	Ref. [29]	Ref. [20]	ours	Ref. [26]	Ref. [29]	Ref. [20]	ours
\hat{I}	14.0	13.2	11.8	10.0	15.9	15.0	13.6	11.9
\hat{F}	14.8	14.9	11.8	10.0	16.8	16.7	13.6	11.9
$\hat{P}(\pm 1)$	14.4	15.2	12.5	11.2	16.4	17.1	14.5	13.7
$\hat{C}_Z(\pm 1)$	15.6	-	-	11.9	17.4	-	-	13.7
$\hat{F}\hat{F}\hat{C}_Z$	-	16.0	13.2	-	-	17.6	15.0	-



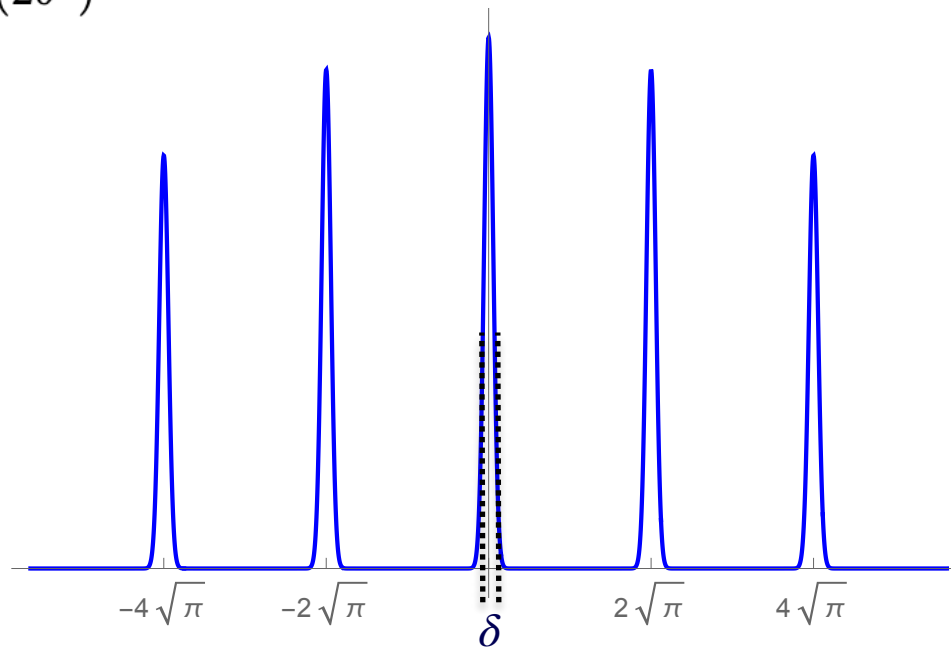
Noise Accumulation



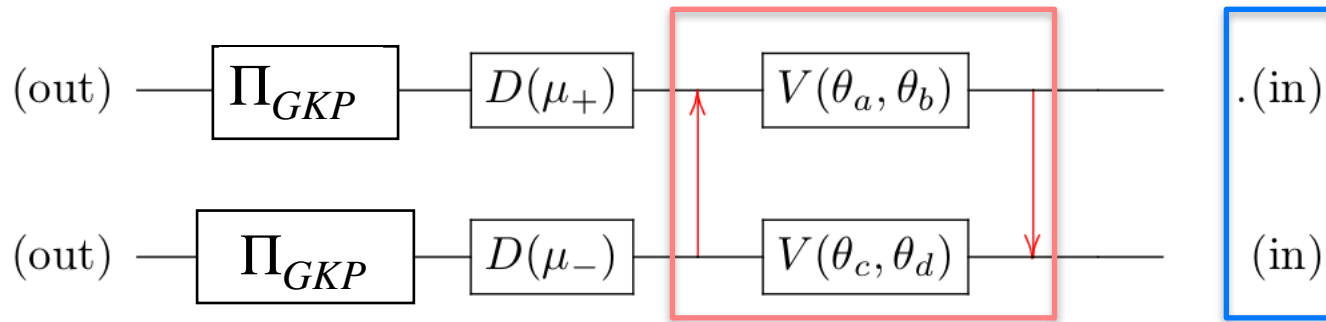
Noise Accumulation



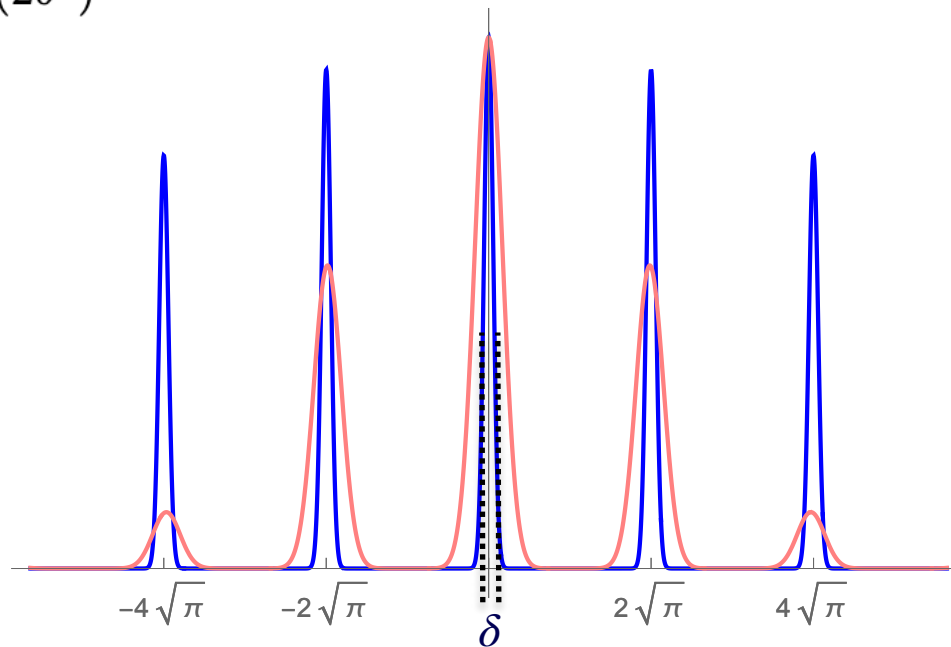
$$(\delta^2)_{\text{dB}} = -10 \log_{10}(2\delta^2)$$



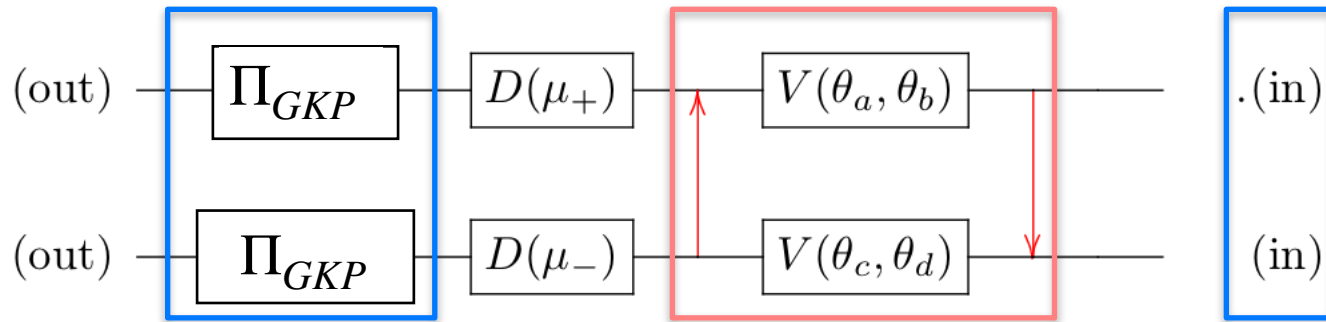
Noise Accumulation



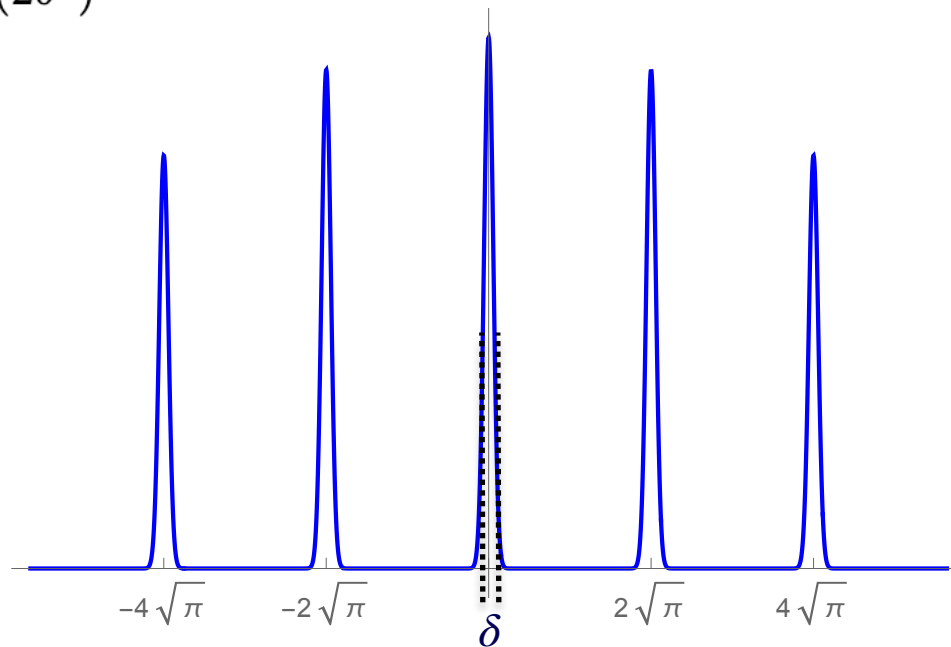
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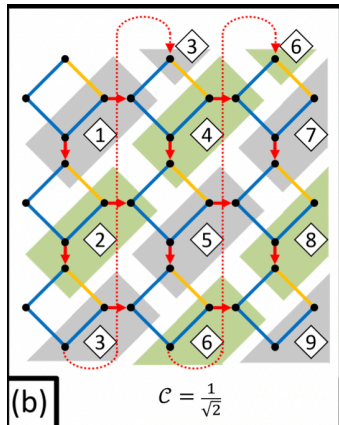
Noise Accumulation



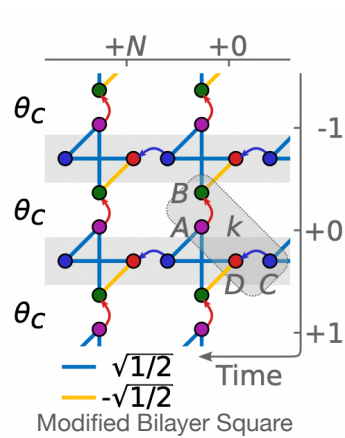
$$(\delta^2)_{\text{dB}} = -10 \log_{10}(2\delta^2)$$



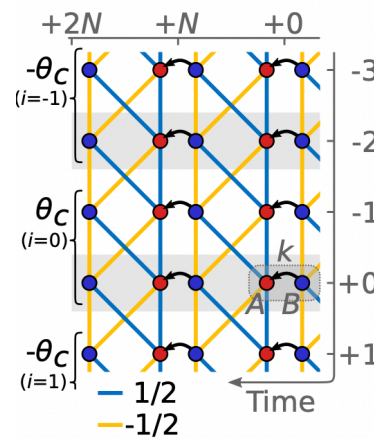
Equivalent noise of two-mode macronode-based cluster states



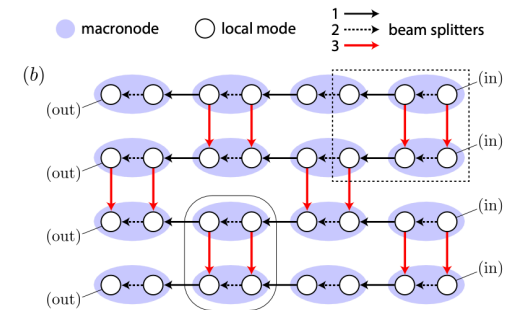
Bilayer Square Lattice.
Alexander, 2018



Modified Bilayer Square Lattice. Alexander, 2018



Double Bilayer Square Lattice. Larsen, 2020

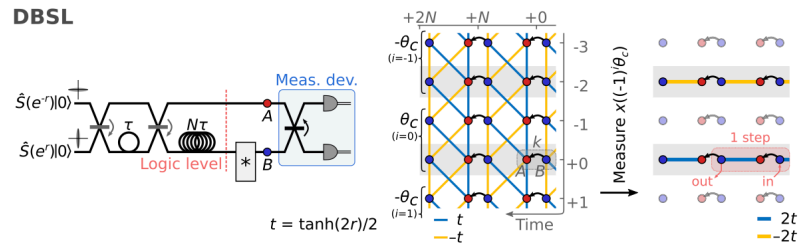


Quad-rail Lattice. Walshe, 2021

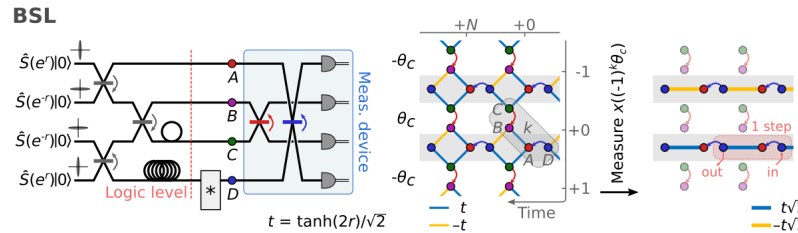
Various groups work with different types of macronode cluster state. Reported squeezing requirements for error rates of 10^{-2} range from 16-17.5 dB

Cluster state projection

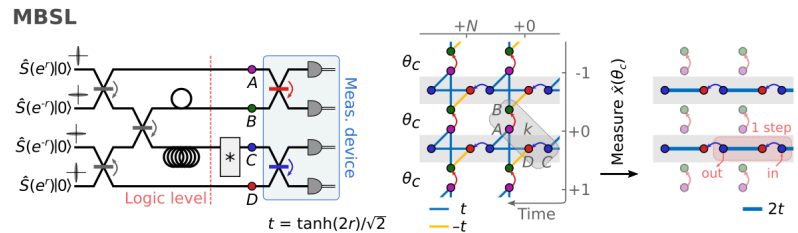
Double bilayer square lattice



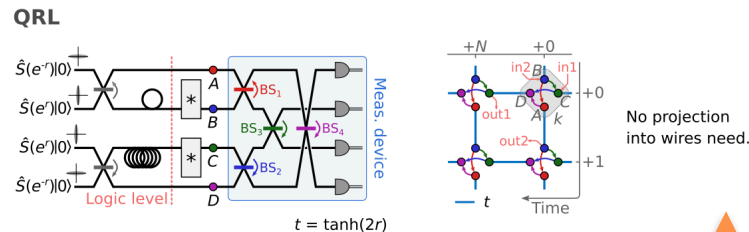
Bilayer square lattice



Modified bilayer square lattice



Quad rail lattice

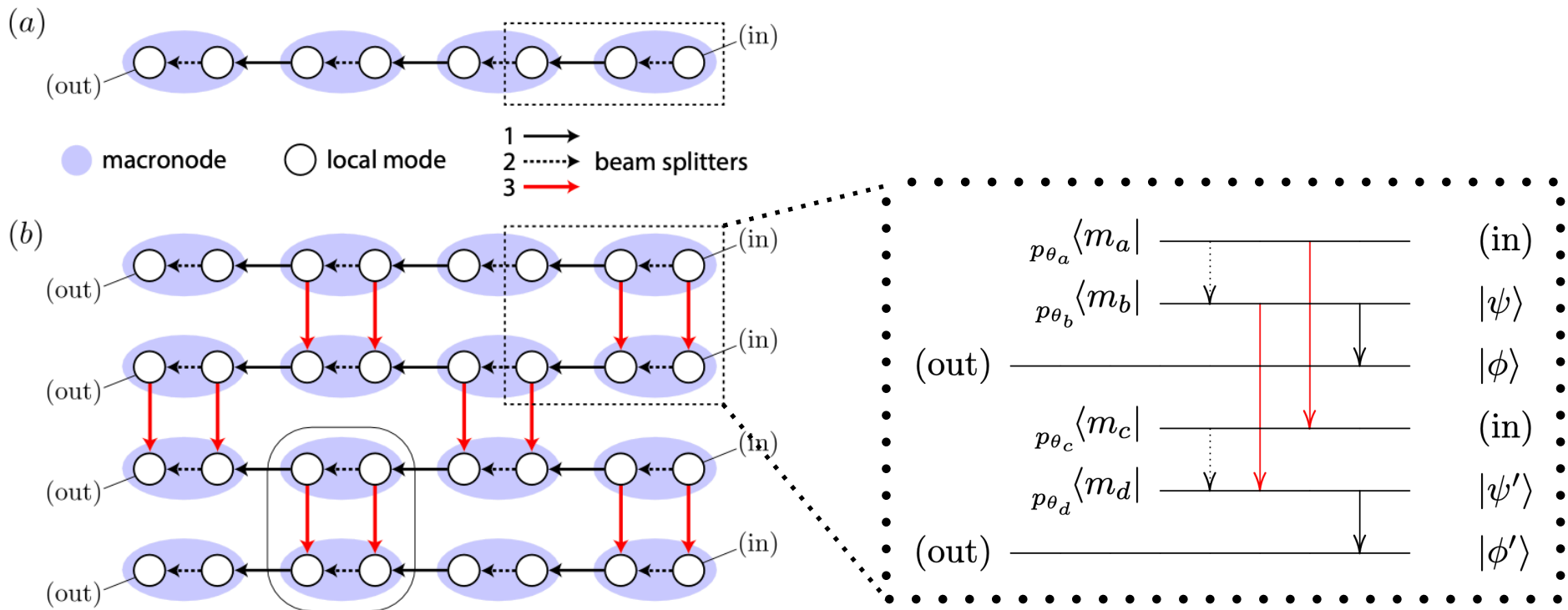


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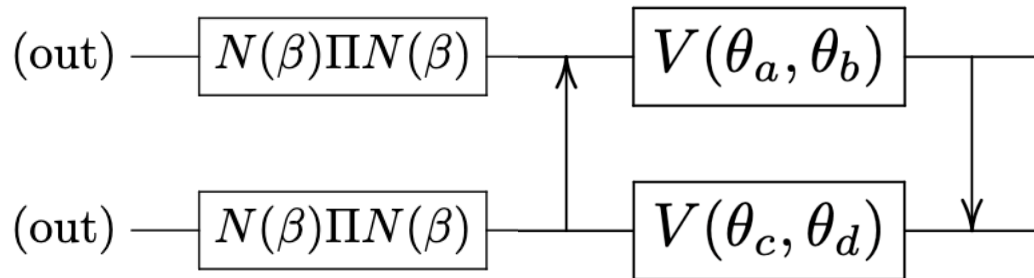
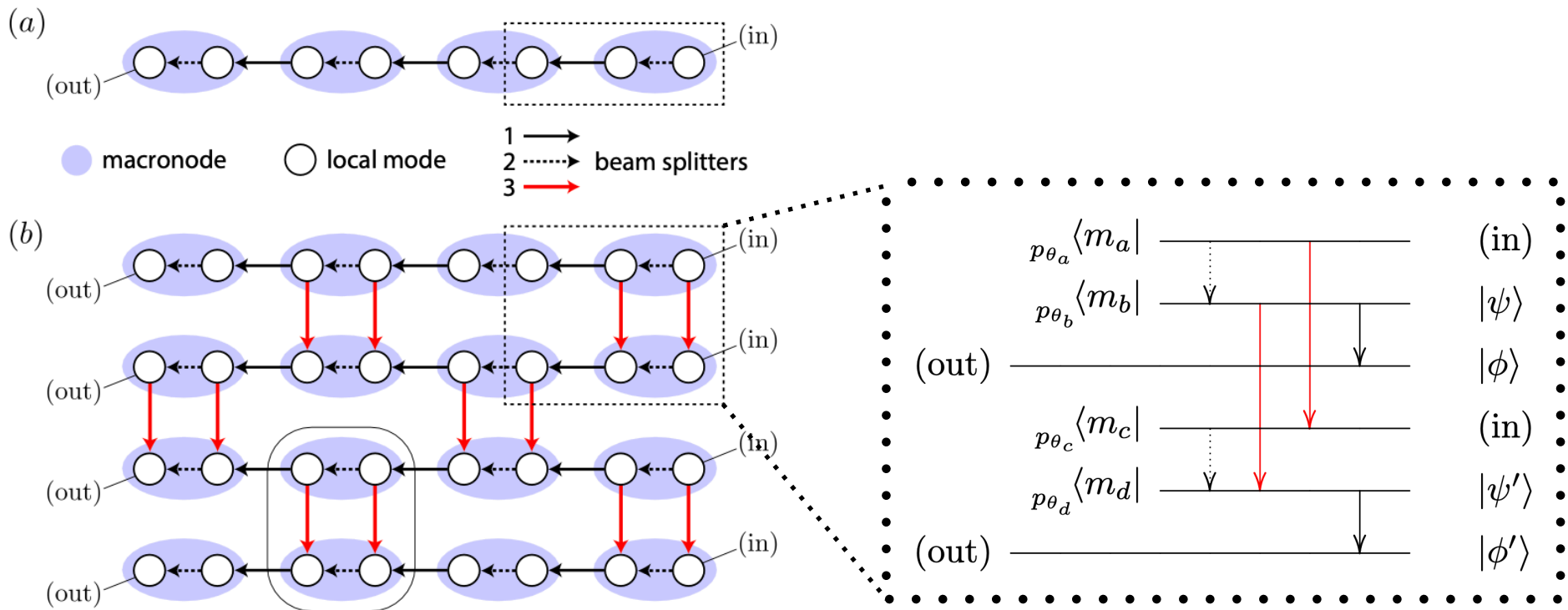
Architecture and noise analysis of continuous-variable quantum gates using two-dimensional cluster states

Mikkel V. Larsen¹, Jonas S. Neergaard-Nielsen, and Ulrik L. Andersen¹
 Center for Macroscopic Quantum States (bigQ), Department of Physics, Technical University of Denmark, Fysikvej, 2800 Kongens Lyngby, Denmark

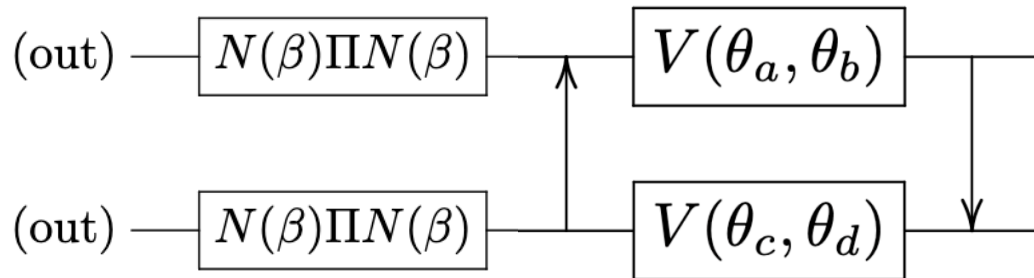
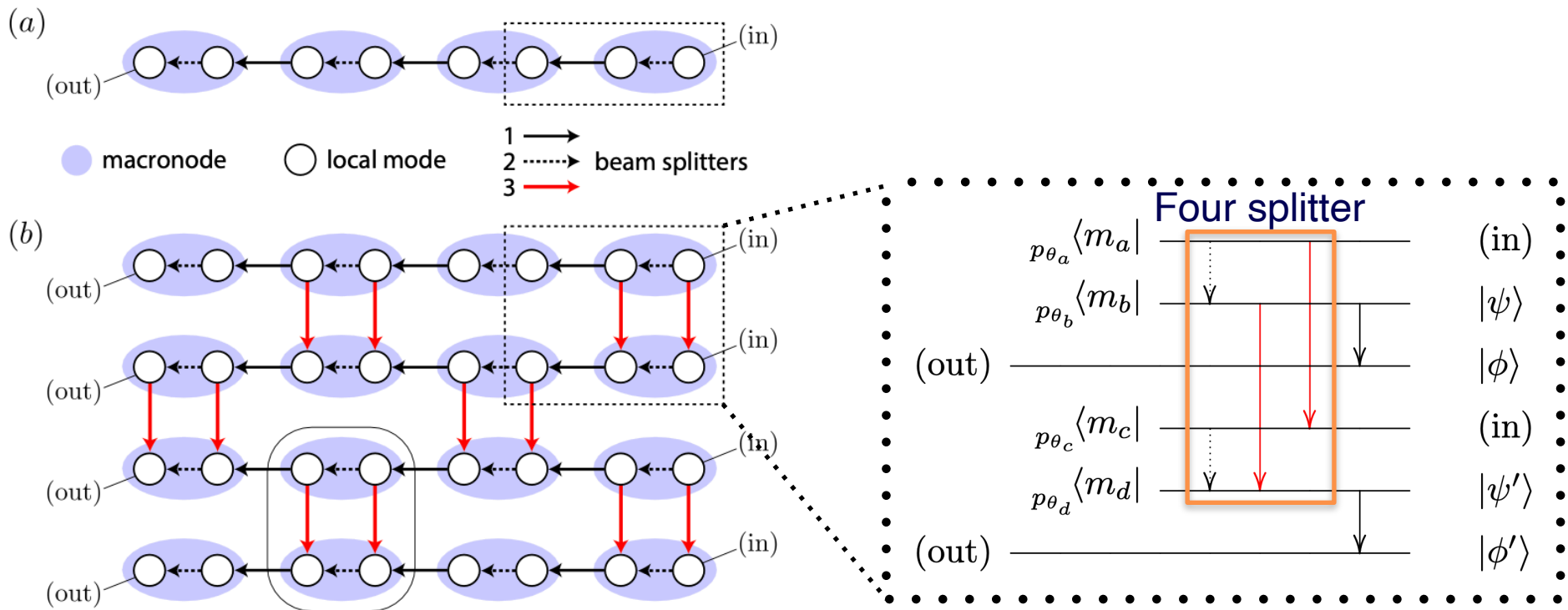
Arbitrary Streamlined QC



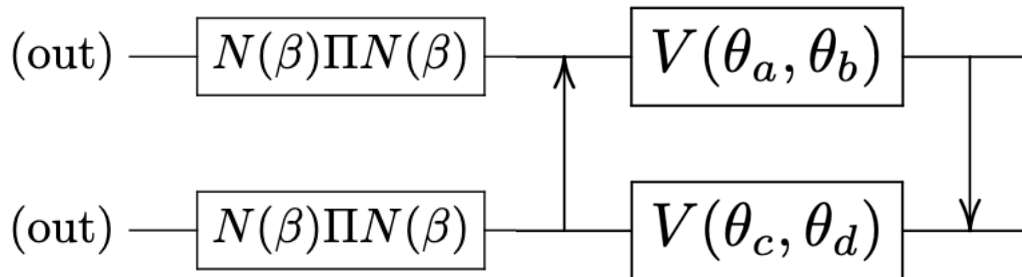
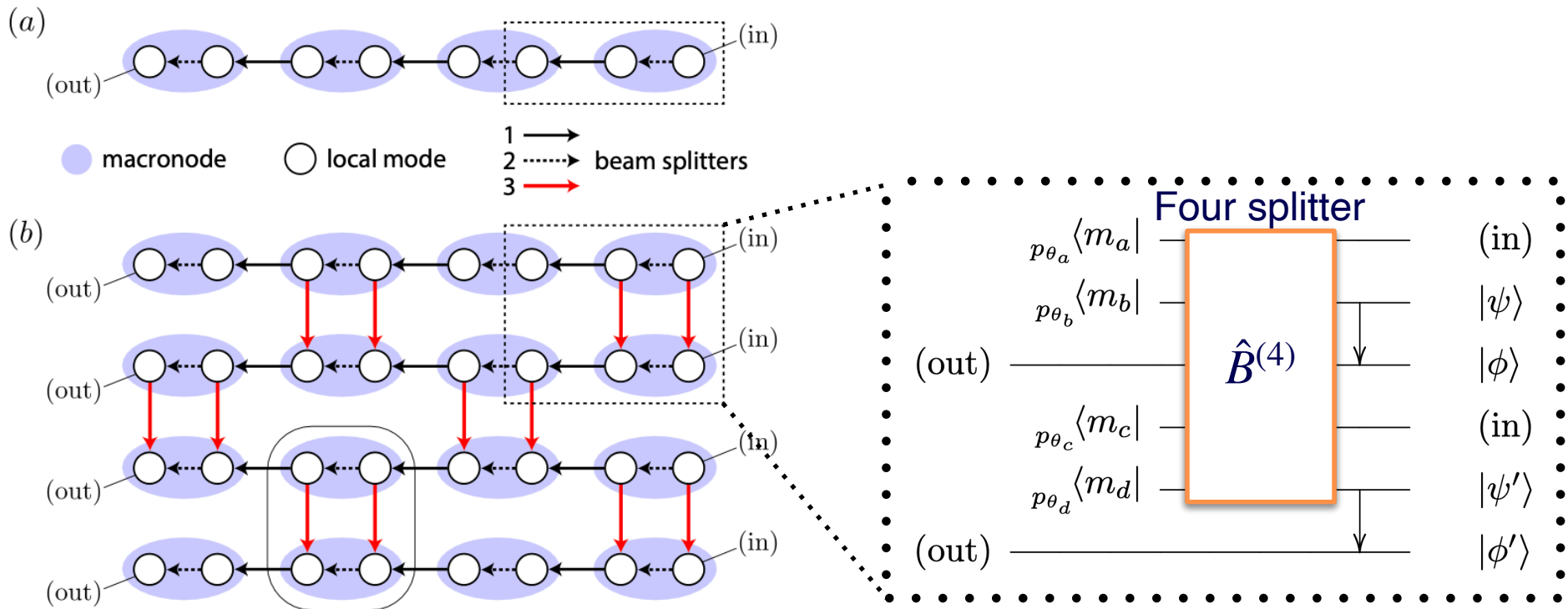
Arbitrary Streamlined QC



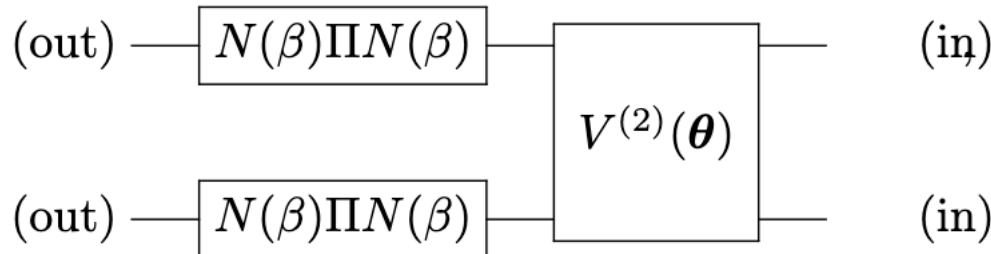
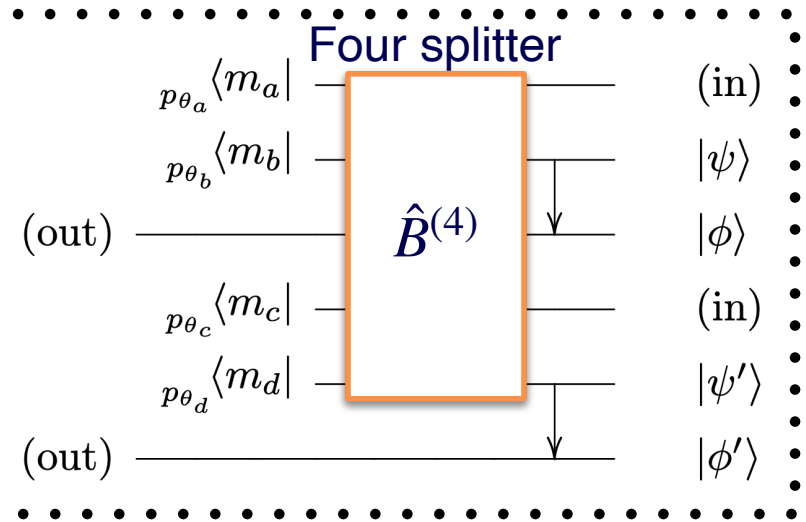
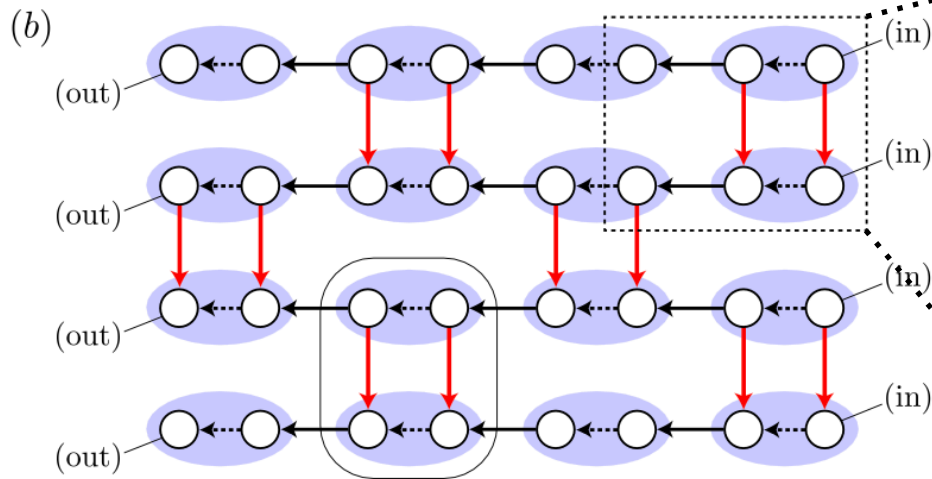
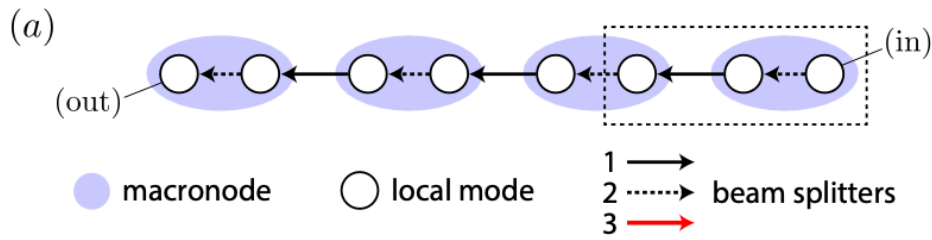
Arbitrary Streamlined QC



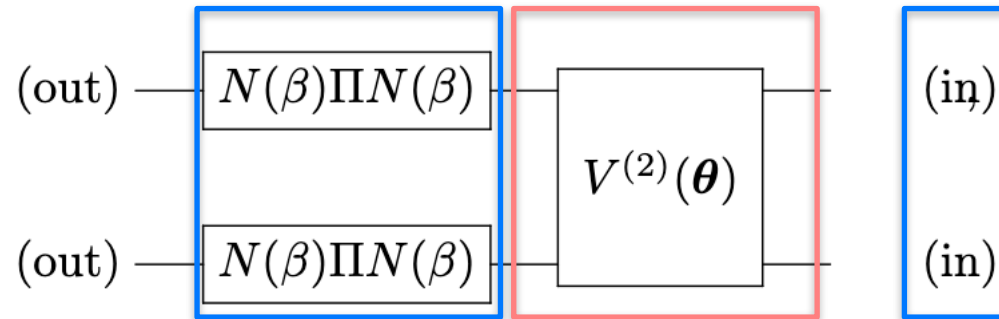
Arbitrary Streamlined QC



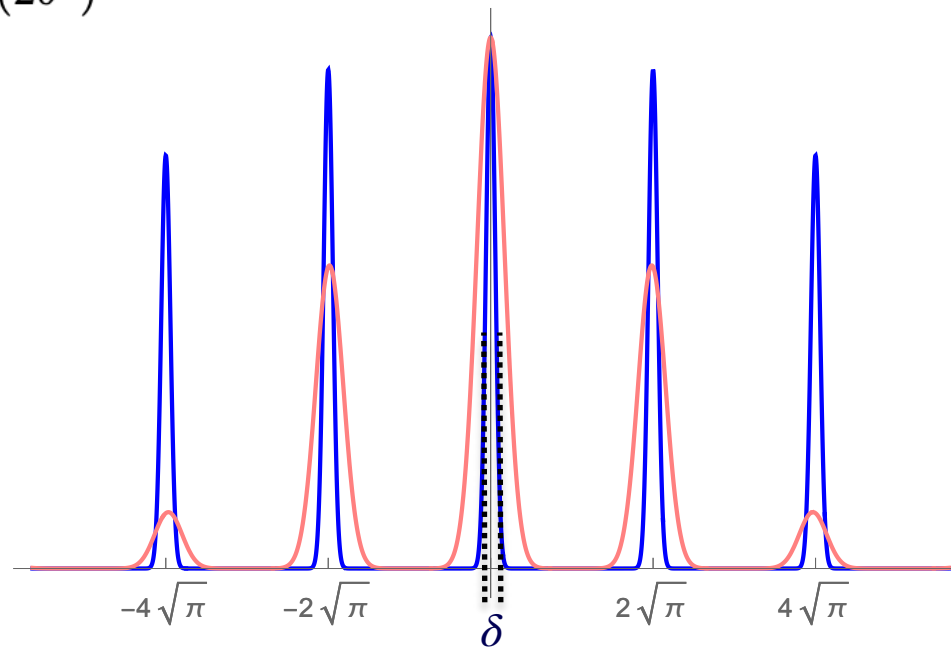
Arbitrary Streamlined QC



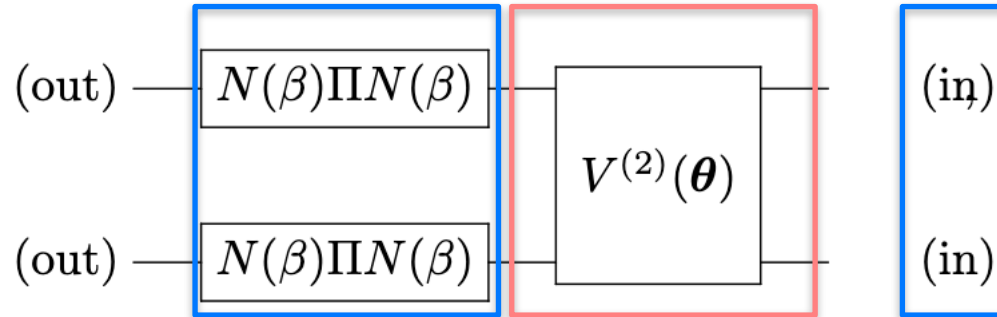
Arbitrary Streamlined QC



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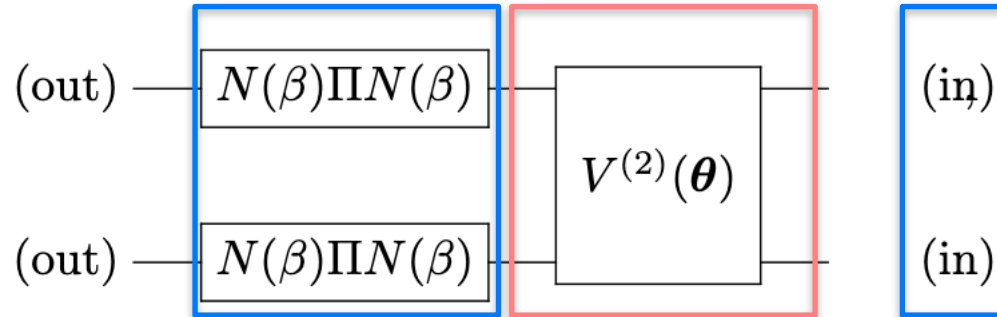


Arbitrary Streamlined QC



The only piece that depends on the beam splitter network is $\hat{V}^{(2)}$

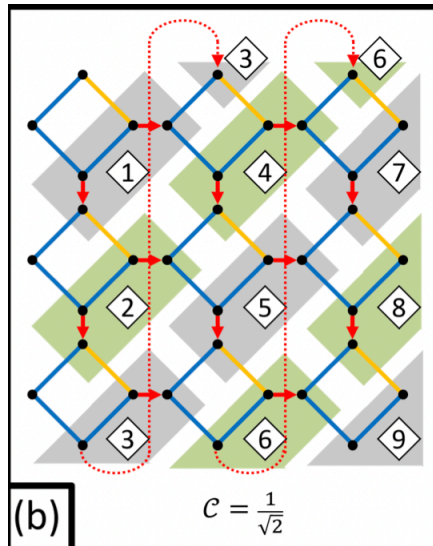
Arbitrary Streamlined QC



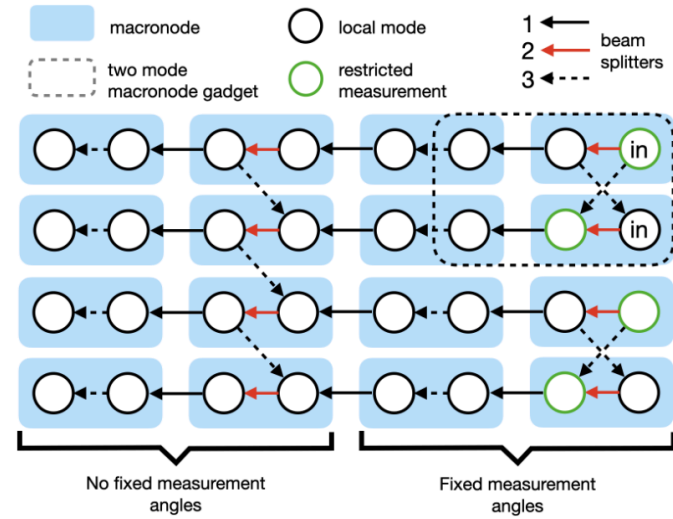
The only piece that depends on the beam splitter network is $\hat{V}^{(2)}$

If we can do the same gate in a single step in each architecture, we will have the same noise properties in each architecture.

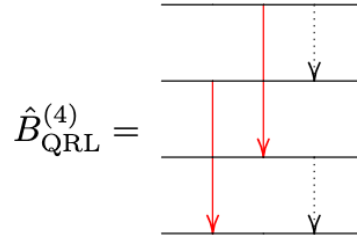
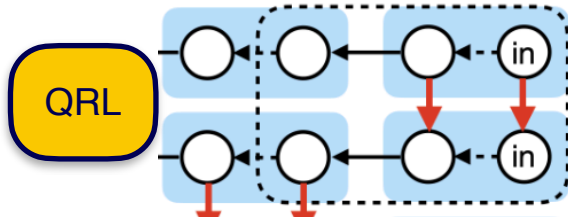
Bilayer square lattice four splitter



=



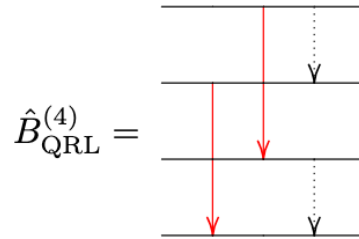
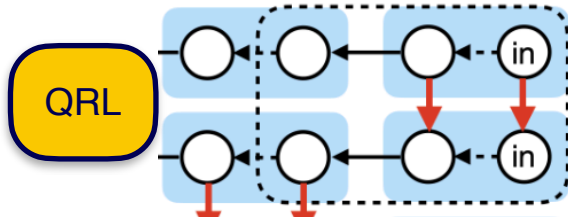
Four splitter



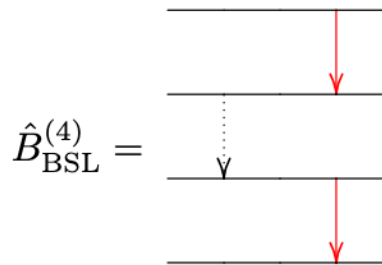
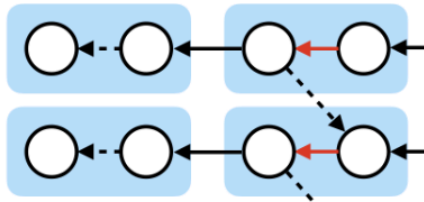
$$\hat{B}_{\text{QRL}}^{(4)} =$$

$$\mathbf{B}_{\text{QRL}}^{(4)} = \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Four splitter

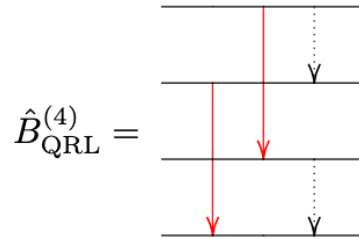
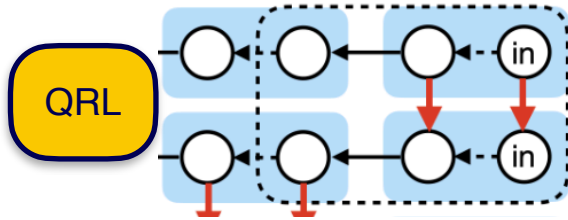


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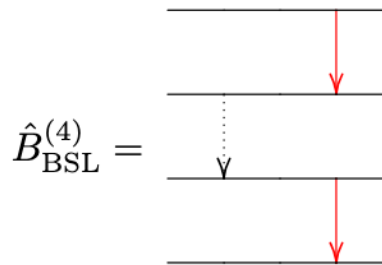
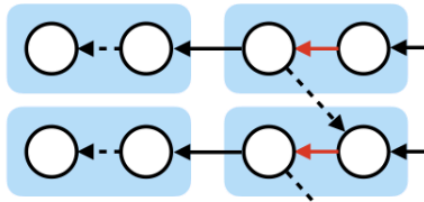


$$\mathbf{B}_{\text{BSL}}^{(4)} = \frac{1}{2} \begin{bmatrix} \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 0 & 0 & \sqrt{2} & \sqrt{2} \end{bmatrix}$$

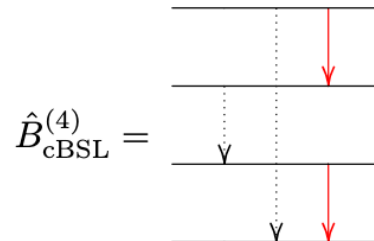
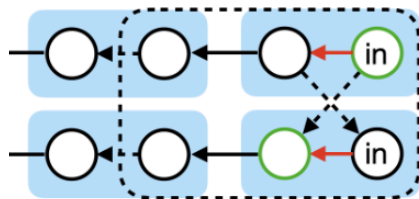
Four splitter



$$\mathbf{B}_{\text{QRL}}^{(4)} = \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

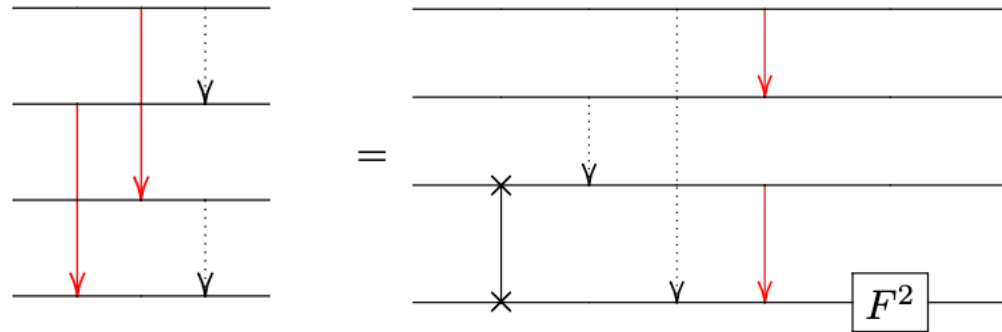


$$\mathbf{B}_{\text{BSL}}^{(4)} = \frac{1}{2} \begin{bmatrix} \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 0 & 0 & \sqrt{2} & \sqrt{2} \end{bmatrix}$$



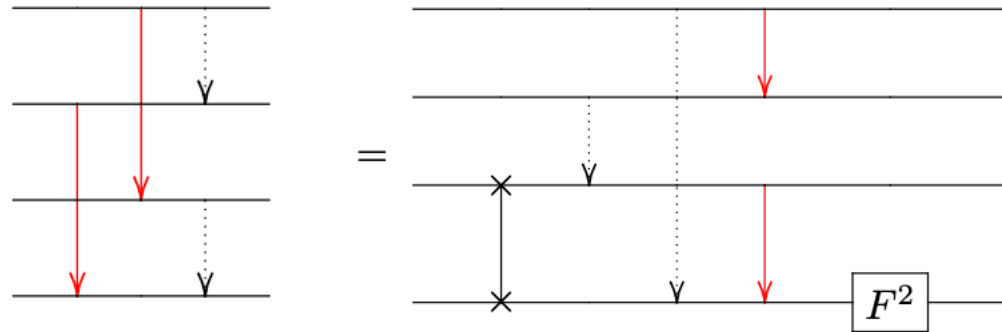
$$\mathbf{B}_{\text{cBSL}}^{(4)} = \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

Four splitter equivalence



All four splitters belong to a single equivalence class that permits 3 operations:

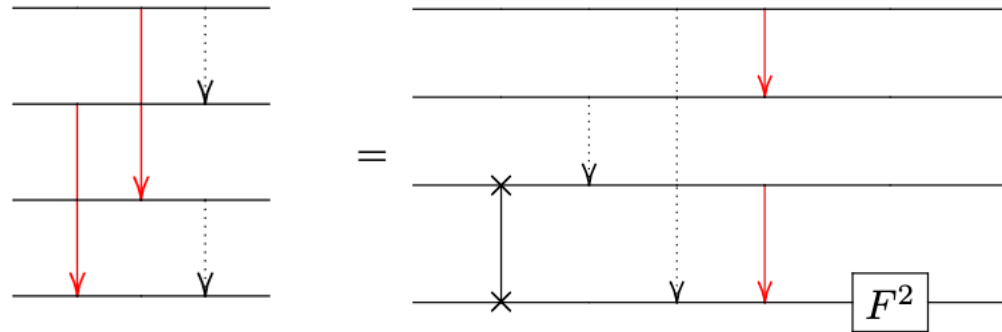
Four splitter equivalence



All four splitters belong to a single equivalence class that permits 3 operations:

1. SWAP gates *after* the four splitter (to the left)

Four splitter equivalence

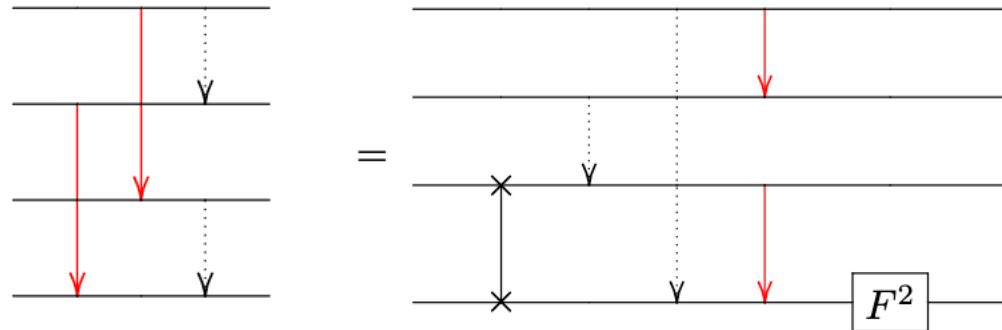


All four splitters belong to a single equivalence class that permits 3 operations:

1. SWAP gates *after* the four splitter (to the left)

$$\frac{1}{2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Four splitter equivalence



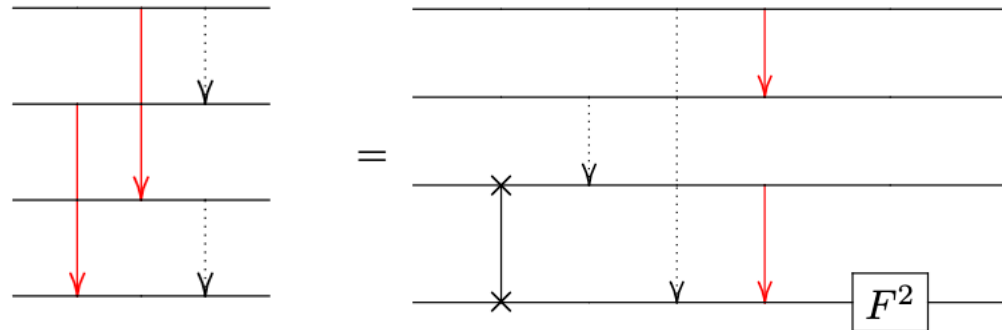
All four splitters belong to a single equivalence class that permits 3 operations:

1. SWAP gates *after* the four splitter (to the left)

1. SWAP $\frac{1}{2}$

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Four splitter equivalence



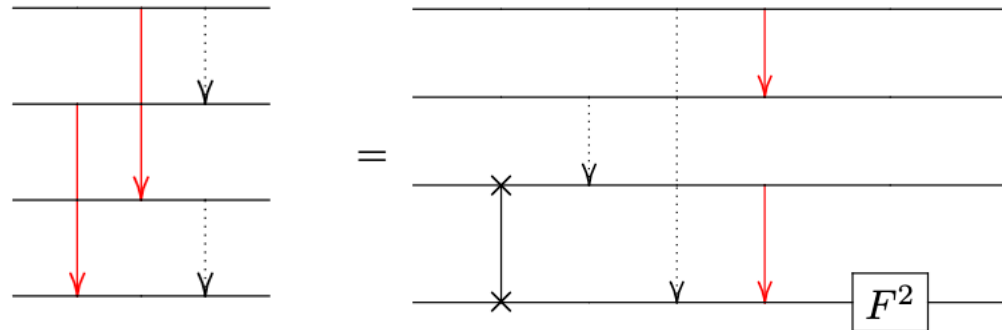
All four splitters belong to a single equivalence class that permits 3 operations:

1. SWAP gates *after* the four splitter (to the left)
2. \hat{F}^2 gates on any wires *after* the four splitter (to the left)

1. SWAP $\frac{1}{2}$

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Four splitter equivalence

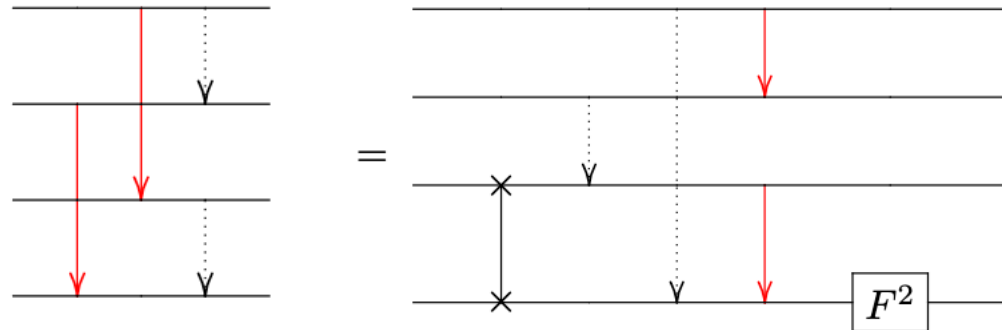


All four splitters belong to a single equivalence class that permits 3 operations:

1. SWAP gates *after* the four splitter (to the left)
2. \hat{F}^2 gates on any wires *after* the four splitter (to the left)

$$\begin{array}{l}
 \text{1. SWAP} \\
 \frac{1}{2} \\
 \text{2. } \hat{F}^2
 \end{array}
 \begin{bmatrix}
 1 & -1 & -1 & 1 \\
 1 & 1 & -1 & -1 \\
 1 & -1 & 1 & -1 \\
 -1 & -1 & -1 & -1
 \end{bmatrix}$$

Four splitter equivalence

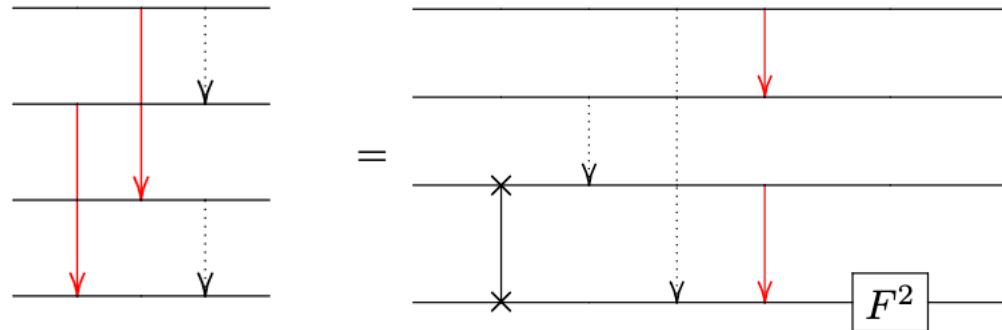


All four splitters belong to a single equivalence class that permits 3 operations:

1. SWAP gates *after* the four splitter (to the left)
2. \hat{F}^2 gates on any wires *after* the four splitter (to the left)
3. An \hat{F}^2 on any single wire *before* the four splitter (to the right)

$$\begin{array}{l}
 \text{1. SWAP} \\
 \frac{1}{2} \\
 \text{2. } \hat{F}^2
 \end{array}
 \begin{bmatrix}
 1 & -1 & -1 & 1 \\
 1 & 1 & -1 & -1 \\
 1 & -1 & 1 & -1 \\
 -1 & -1 & -1 & -1
 \end{bmatrix}$$

Four splitter equivalence

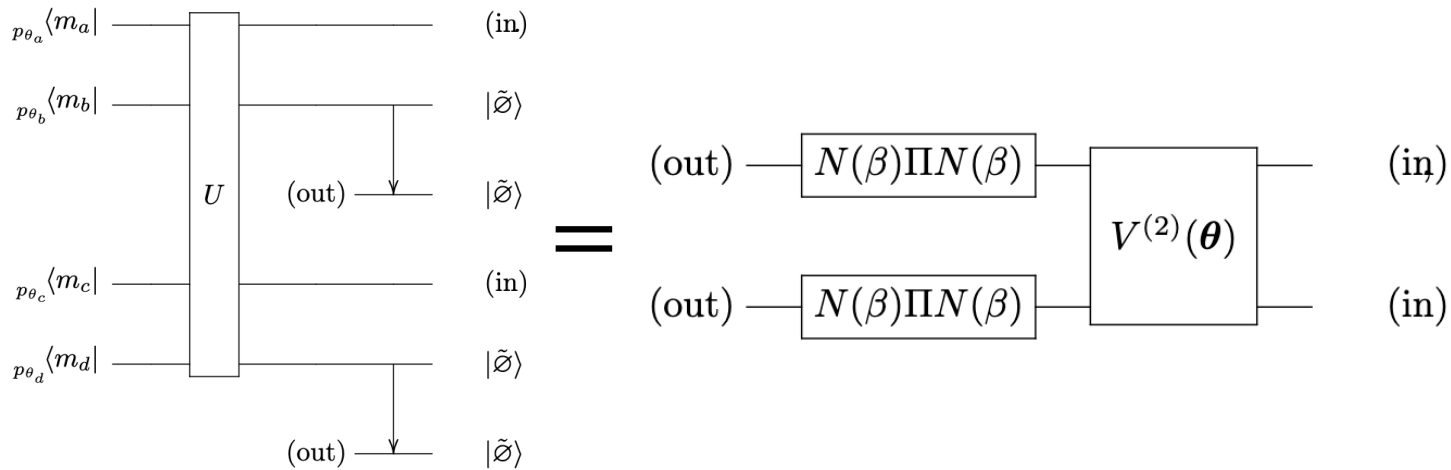


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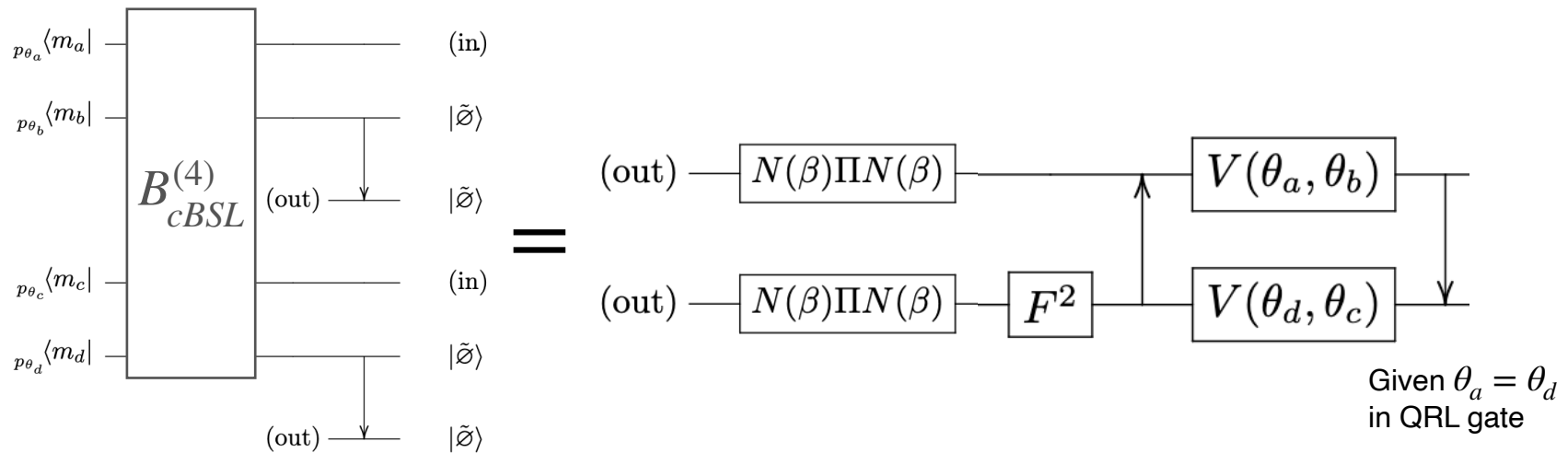
1. SWAP gates *after* the four splitter (to the left)
2. \hat{F}^2 gates on any wires *after* the four splitter (to the left)
3. An \hat{F}^2 on any single wire *before* the four splitter (to the right)

$$\begin{array}{l}
 \text{1. SWAP} \\
 \frac{1}{2} \\
 \text{2. } \hat{F}^2 \\
 \text{3. } \hat{F}^2
 \end{array}
 \begin{bmatrix}
 1 & -1 & -1 & -1 \\
 1 & 1 & -1 & +1 \\
 1 & -1 & 1 & +1 \\
 -1 & -1 & -1 & +1
 \end{bmatrix}$$

BSL reduction



BSL reduction



Dictionary of single step two-mode gates

These two-mode gates can each be done in a single step, minimising the noise added by the operation. Noise which is independent of the architecture chosen.

Architecture	QRL	BSL	DBSL	Mikkel-splitter
Mapping	$\{\theta_a, \theta_b, \theta_c, \theta_d\}$	$\{\theta_a, \theta_b, \theta_d, \theta_c\} \theta_a = \theta_c$	$\{\theta_a, \theta_c, \theta_d, \theta_b\} \theta_a = \theta_d$	$\{\theta_a, \theta_b, \theta_d, \theta_c\} \theta_b = \theta_d$
$\hat{C}_Z[1]$	$\{\frac{\pi}{2}, \frac{\pi}{2} \pm \chi, \frac{\pi}{2}, \frac{\pi}{2} \mp \chi\}$	$\{\frac{\pi}{2}, \frac{\pi}{2} \pm \chi, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}\}$	$\{\frac{\pi}{2}, \frac{\pi}{2} \pm \chi, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}\}$	**
SWAP	$\{0, \frac{\pi}{2}, \frac{\pi}{2}, 0\}$	**	**	**
$\hat{I} \otimes \hat{I}$	$\{\frac{\pi}{2}, 0, \frac{\pi}{2}, 0\}$	$\{\frac{\pi}{2}, 0, 0, \frac{\pi}{2}\}$	$\{\frac{\pi}{2}, 0, 0, \frac{\pi}{2}\}$	$\{\frac{\pi}{2}, 0, 0, \frac{\pi}{2}\}$
$\hat{F} \otimes \hat{F}$	$\{\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\}$	$\{\frac{3\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\}$	$\{\frac{3\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\}$	$\{\frac{3\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\}$
$\hat{P}(1) \otimes \hat{P}(1)$	$\{\frac{\pi}{2}, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}, \frac{\pi}{2} \mp \chi\}$	$\{\frac{\pi}{2}, \frac{\pi}{2} \mp \chi, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}\}$	$\{\frac{\pi}{2}, \frac{\pi}{2} \mp \chi, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}\}$	$\{\frac{\pi}{2}, \frac{\pi}{2} \mp \chi, \frac{\pi}{2} \mp \chi, \frac{\pi}{2}\}$

Additional Slides

Typical thresholds

PRL 112, 120504 (2014)

PHYSICAL REVIEW LETTERS

week ending
28 MARCH 2014

Fault-Tolerant Measurement-Based Quantum Computing with Continuous-Variable Cluster States

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School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia

(Received 29 October 2013; published 26 March 2014)

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~15.6 – 20.5 dB

for qubit error rates 10^{-2} – 10^{-6}

(depends on qubit code employed)

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PHYSICAL REVIEW X 8, 021054 (2018)

High-Threshold Fault-Tolerant Quantum Computation with Analog Quantum Error Correction

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*Graduate School of Information Science and Technology, Hokkaido University,
Kita14-Nishi9, Kita-ku, Sapporo 060-0814, Japan*

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PHYSICAL REVIEW A 101, 012316 (2020)

Editors' Suggestion

Fault-tolerant bosonic quantum error correction with the surface-Gottesman-Kitaev-Preskill code

Kyungjoo Noh^{1,*} and Christopher Chamberland^{2,†}

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Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer

J. Eli Bourassa^{1,2,*}, Rafael N. Alexander^{1,3,4,*}, Michael Vasmer^{5,6}, Ashlesha Patil^{1,7}, Ilan Tzitrin^{1,2}, Takaya Matsuura^{1,8}, Daiqin Su¹, Ben Q. Baragjola^{1,4}, Saikat Guha^{1,7}, Guillaume Dauphinais¹, Krishna K. Sabapathy¹, Nicolas C. Menicucci^{1,4}, and Ish Dhand¹

Quantum 5, 392 (2021)

PRX QUANTUM 2, 030325 (2021)

Fault-Tolerant Continuous-Variable Measurement-based Quantum Computation Architecture

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PRX Quantum 2, 030325 (2021)

PRX QUANTUM 2, 040353 (2021)

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PRX Quantum 2, 040353 (2021)

~10 – 18 dB
based on topological codes

The teleported gate

$$|\text{EPR}\rangle := \begin{array}{c} \bullet \\ | \text{---} | \\ | \text{---} | \\ \oplus \\ | \text{---} | \end{array} \begin{array}{l} |0\rangle_p \\ |0\rangle_q \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{\frac{1}{\sqrt{2\pi}} I} \\ \text{---} \end{array}$$

$$\begin{array}{c} | \text{---} | \\ | \text{---} | \\ \downarrow \\ | \text{---} | \end{array} \begin{array}{l} |t\rangle_p \\ |s\rangle_q \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{\frac{1}{\sqrt{\pi}} D(s + it)} \\ \text{---} \end{array}$$

The teleported gate

$$|\text{EPR}\rangle := \begin{array}{c} \bullet \\ | \text{---} | \\ | \text{---} | \\ \oplus \\ | \text{---} | \end{array} \begin{array}{l} |0\rangle_p \\ |0\rangle_q \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{\frac{1}{\sqrt{2\pi}} I} \end{array}$$

$$\begin{array}{c} | \text{---} | \\ | \text{---} | \\ \downarrow \\ | \text{---} | \end{array} \begin{array}{l} |t\rangle_p \\ |s\rangle_q \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{\frac{1}{\sqrt{\pi}} D(s + it)} \end{array}$$

$$\begin{array}{c} | \text{---} | \\ | \text{---} | \\ \downarrow \\ | \text{---} | \end{array} \begin{array}{l} |\psi\rangle \\ |\phi\rangle \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{???} \end{array}$$

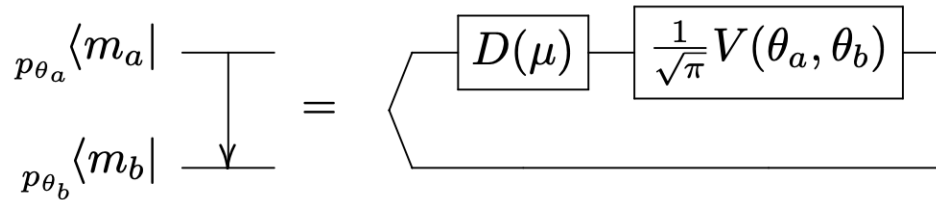
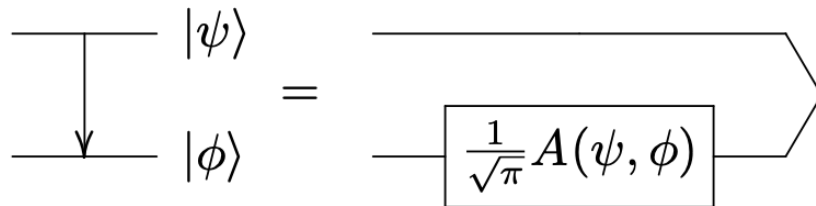
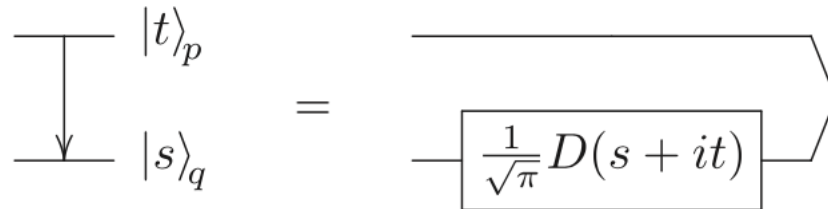
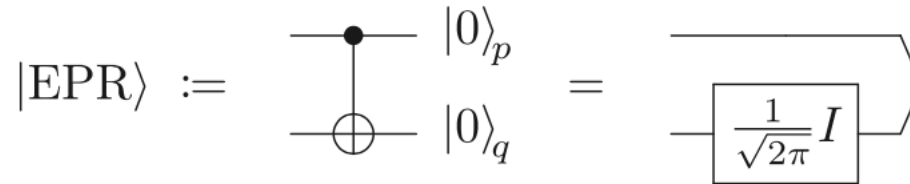
The teleported gate

$$|\text{EPR}\rangle := \begin{array}{c} \bullet \\ |0\rangle_p \\ \oplus \\ |0\rangle_q \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{\frac{1}{\sqrt{2\pi}} I} \end{array}$$

$$\begin{array}{c} |t\rangle_p \\ \downarrow \\ |s\rangle_q \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{\frac{1}{\sqrt{\pi}} D(s + it)} \end{array}$$

$$\begin{array}{c} |\psi\rangle \\ \downarrow \\ |\phi\rangle \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \boxed{\frac{1}{\sqrt{\pi}} A(\psi, \phi)} \end{array}$$

The teleported gate



The teleported gate

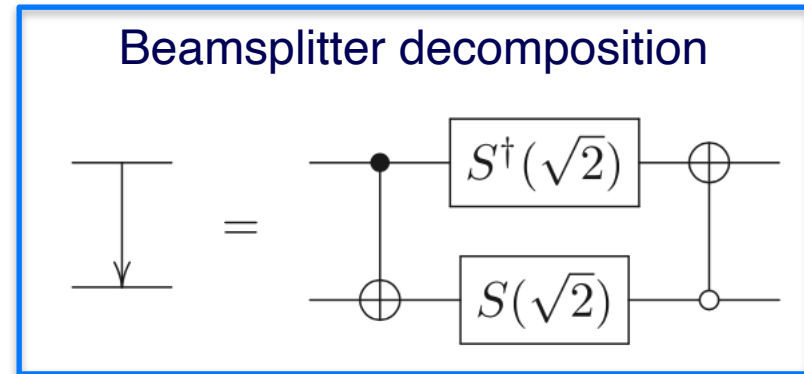
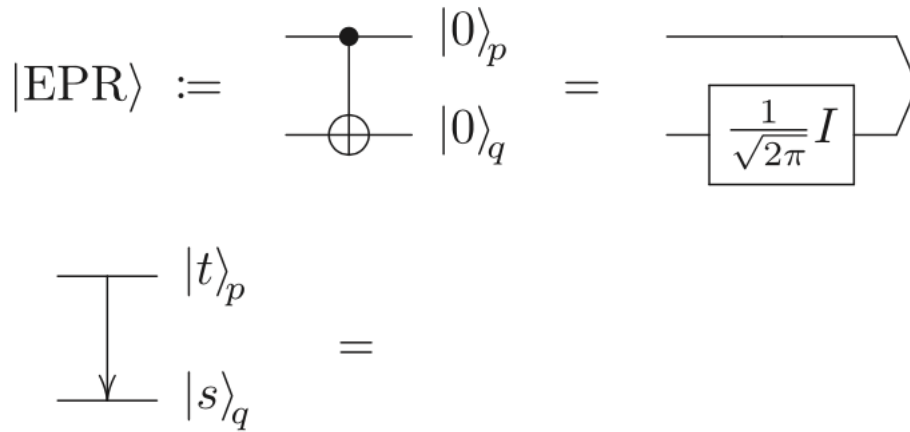
$$|\text{EPR}\rangle := \begin{array}{c} \text{---} \bullet \text{---} \\ | \text{---} \oplus \text{---} \\ |0\rangle_p \\ |0\rangle_q \end{array} = \begin{array}{c} \text{---} \text{---} \\ \boxed{\frac{1}{\sqrt{2\pi}} I} \\ \text{---} \text{---} \end{array}$$

The teleported gate

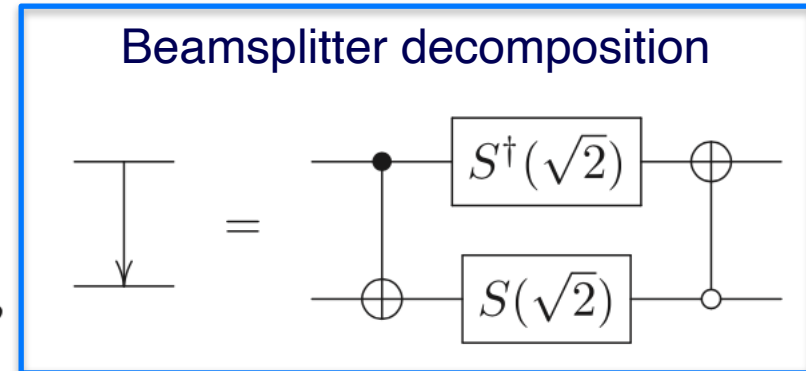
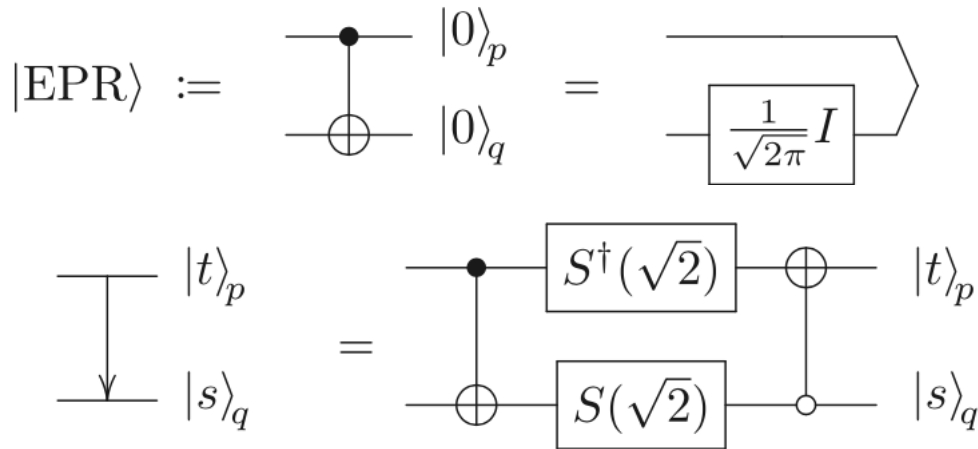
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$$\begin{array}{c} \text{---} \\ | \text{---} | \\ \downarrow \\ | \text{---} | \end{array} \begin{array}{l} |t\rangle_p \\ |s\rangle_q \end{array} =$$

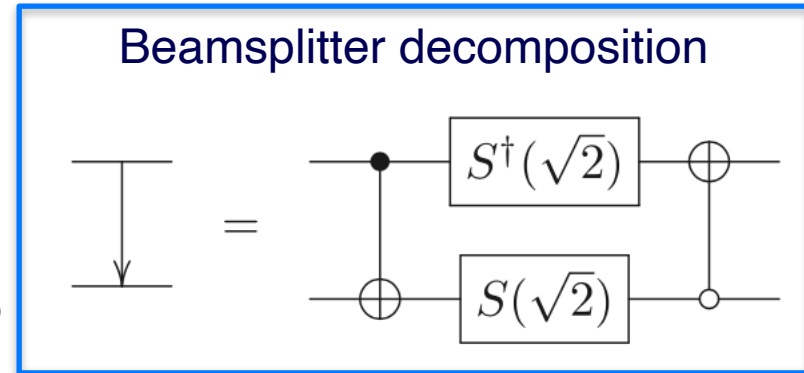
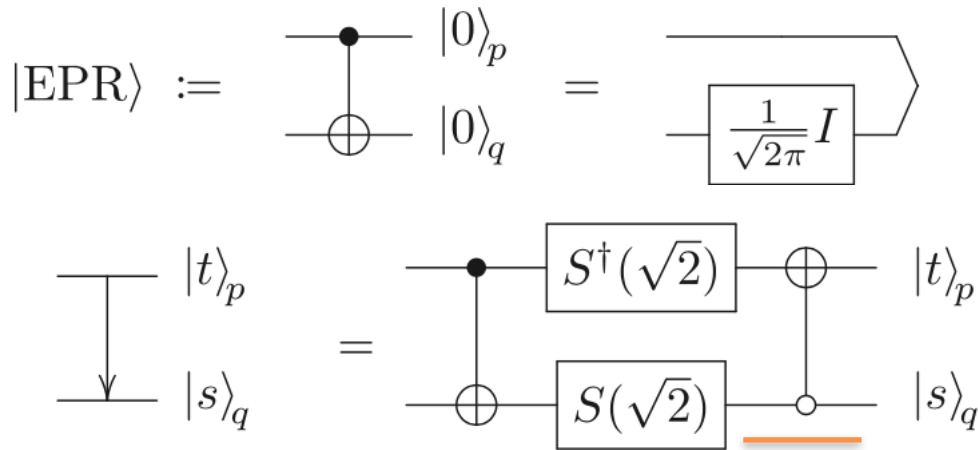
The teleported gate



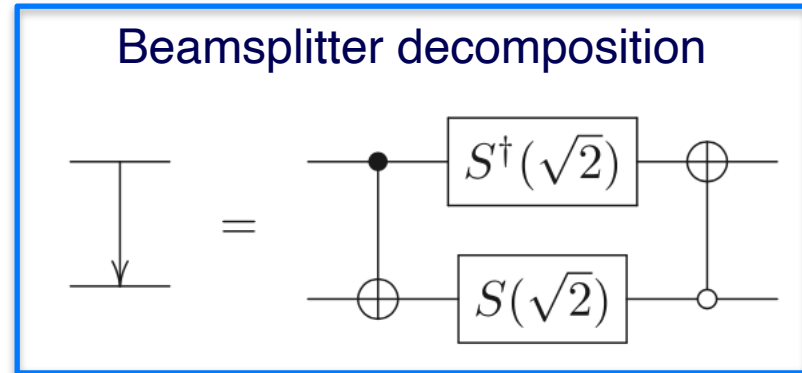
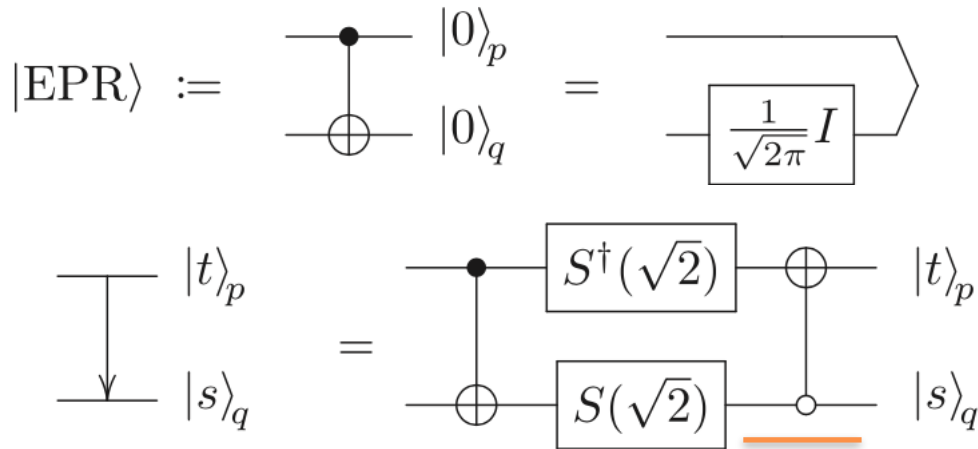
The teleported gate



The teleported gate

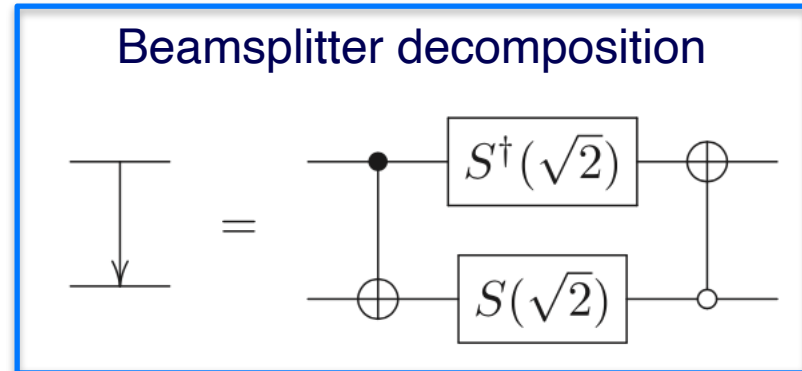
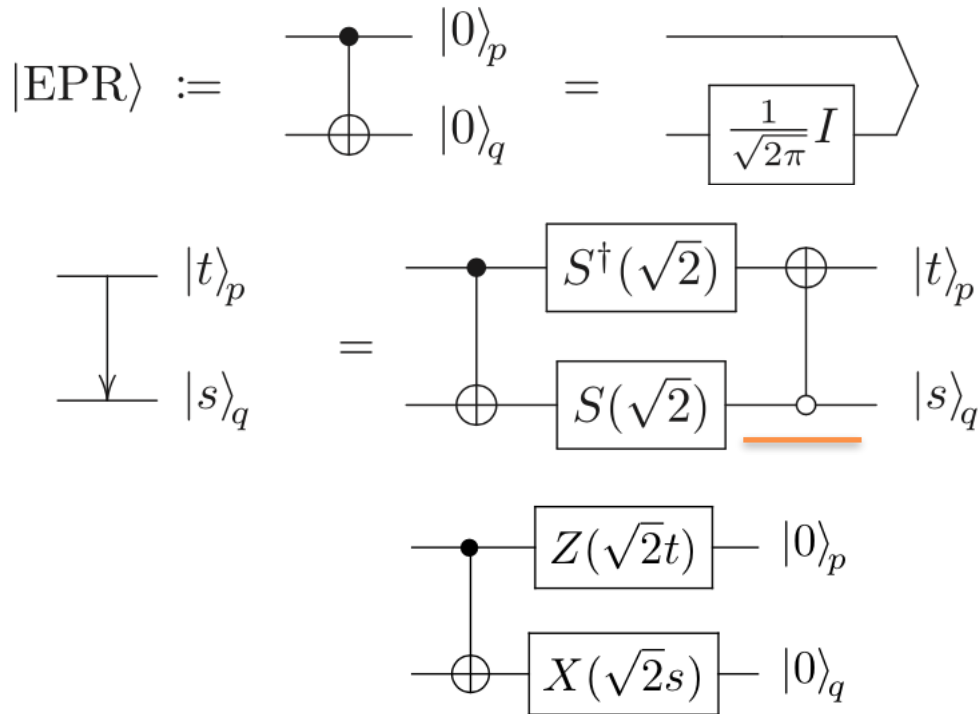


The teleported gate



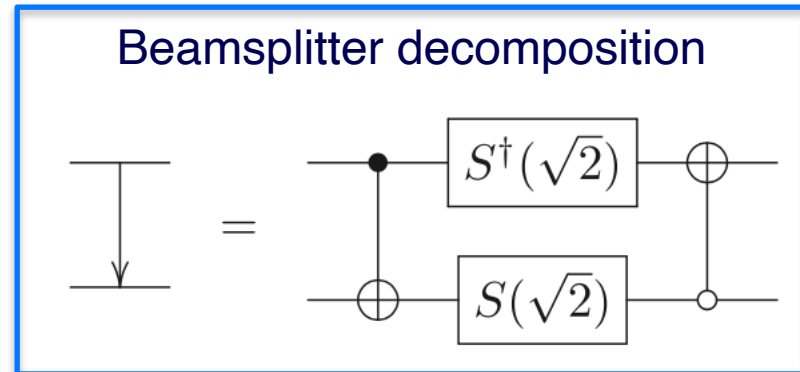
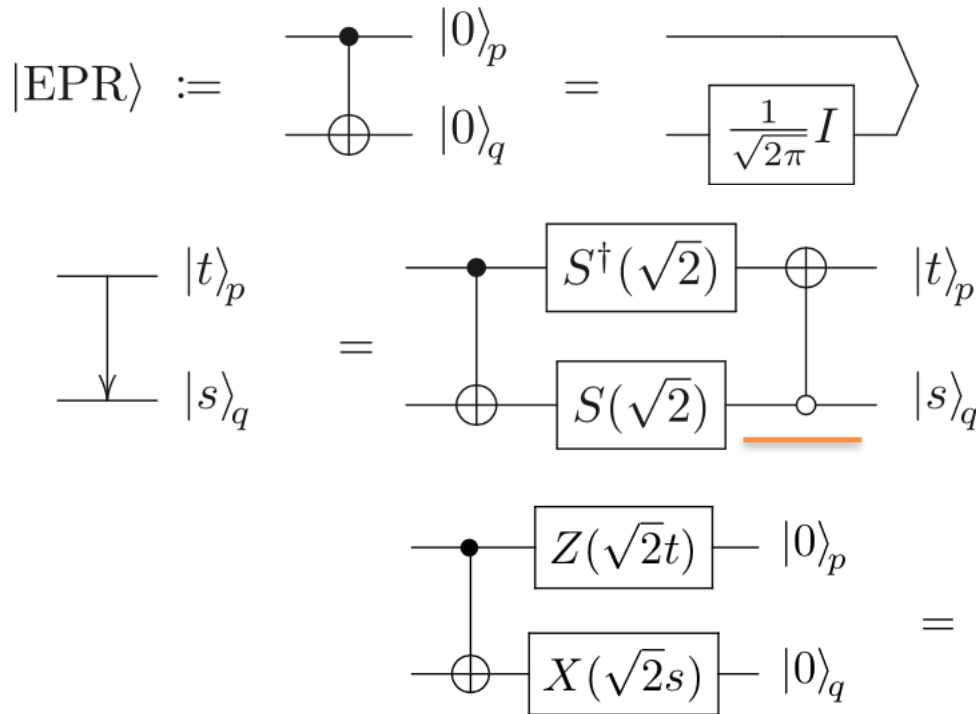
$$e^{i\hat{p}_1 \otimes \hat{q}_2} \rightarrow e^{ist}$$

The teleported gate



$$e^{i\hat{p}_1 \otimes \hat{q}_2} \rightarrow e^{ist}$$

The teleported gate



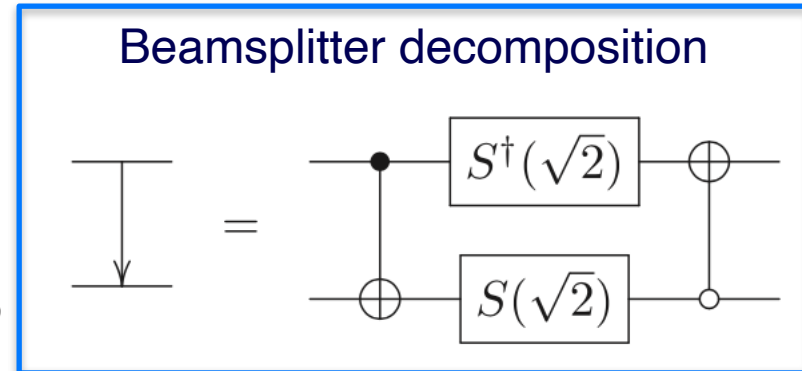
$$e^{i\hat{p}_1 \otimes \hat{q}_2} \rightarrow e^{ist}$$

The teleported gate

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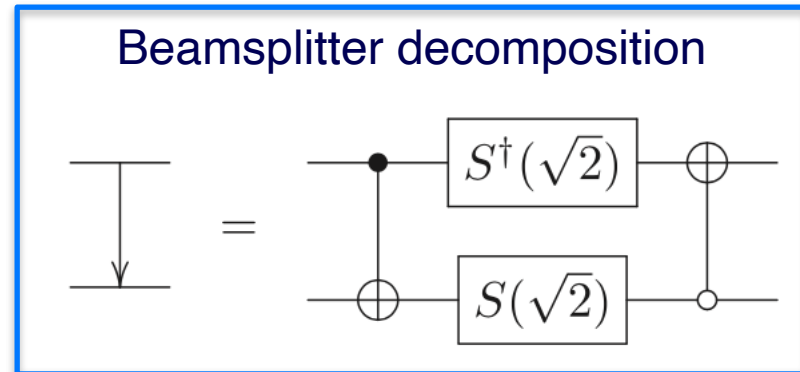
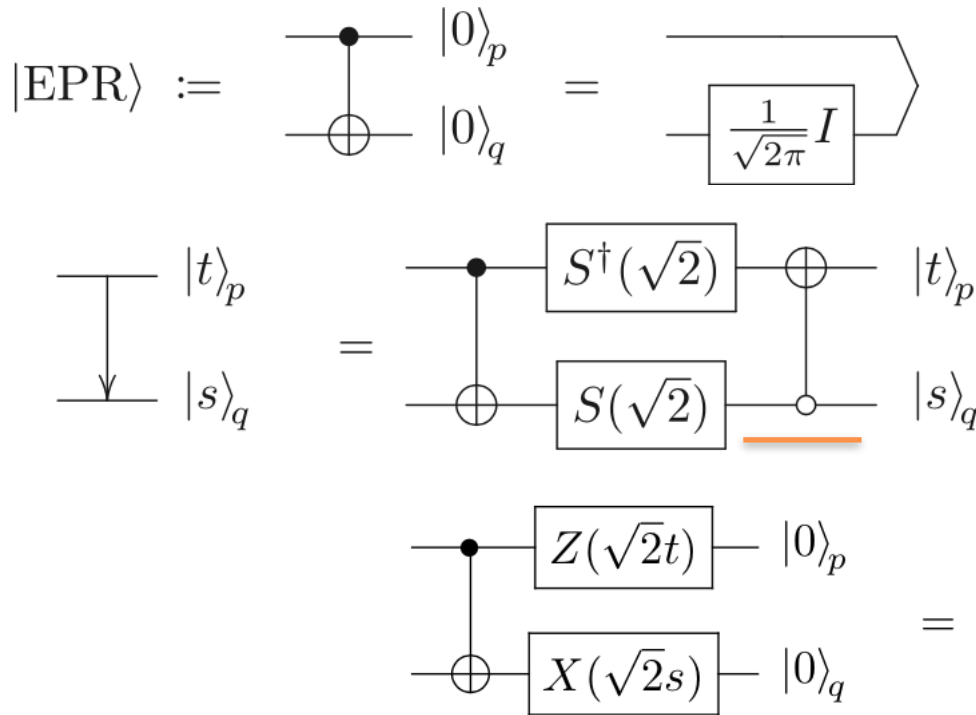
$$\begin{array}{c} \text{---} \\ | \text{---} | \\ | \text{---} | \\ \text{---} \end{array} \begin{array}{l} |t\rangle_p \\ |s\rangle_q \end{array} = \begin{array}{c} \bullet \\ | \text{---} | \\ | \oplus | \\ | \text{---} | \\ \bullet \end{array} \begin{array}{l} S^\dagger(\sqrt{2}) \\ S(\sqrt{2}) \end{array} \begin{array}{c} | \oplus | \\ | \circ | \\ | \text{---} | \\ | \text{---} | \end{array} \begin{array}{l} |t\rangle_p \\ |s\rangle_q \end{array}$$

$$\begin{array}{c} \bullet \\ | \text{---} | \\ | \oplus | \\ | \text{---} | \\ \bullet \end{array} \begin{array}{l} Z(\sqrt{2}t) \\ X(\sqrt{2}s) \end{array} \begin{array}{l} |0\rangle_p \\ |0\rangle_q \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{l} Z(\sqrt{2}t) \\ X(\sqrt{2}s) \end{array}$$

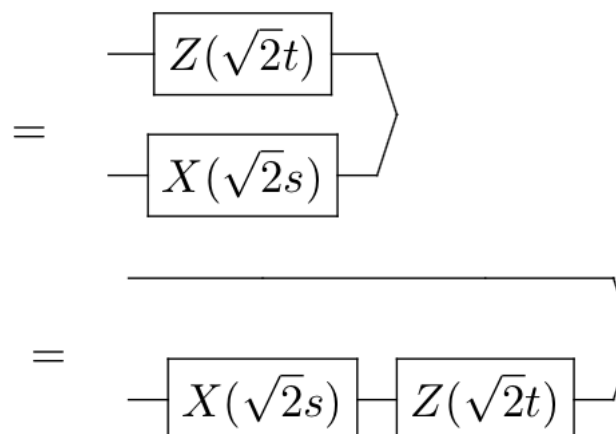


$$e^{i\hat{p}_1 \otimes \hat{q}_2} \rightarrow e^{ist}$$

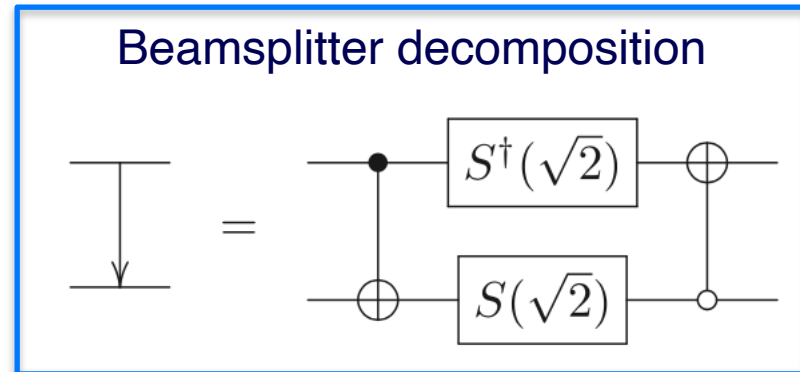
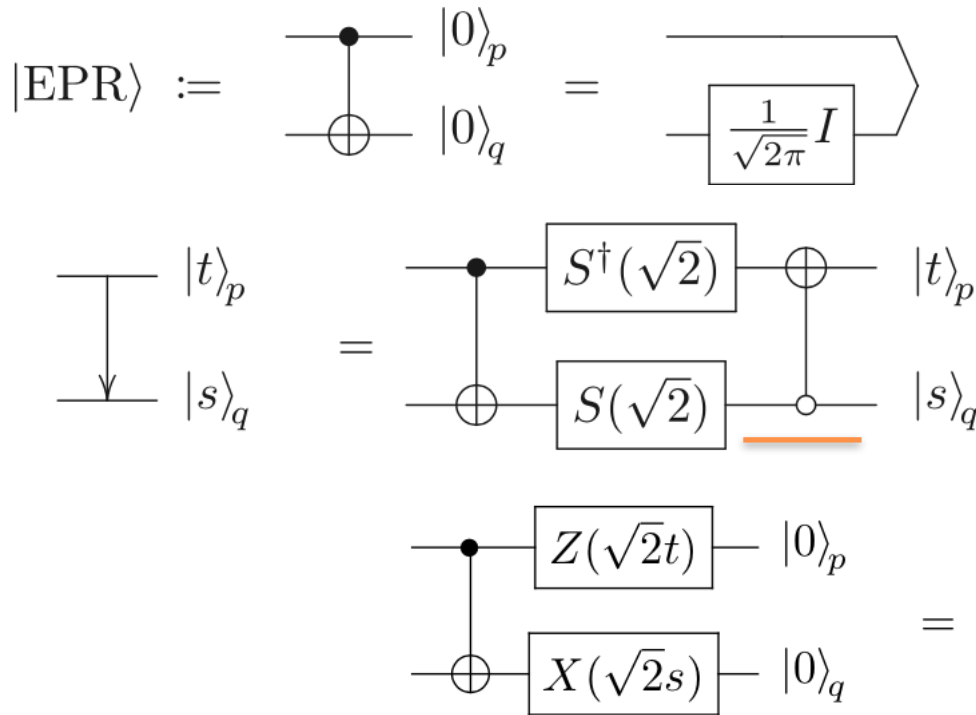
The teleported gate



$$e^{i\hat{p}_1 \otimes \hat{q}_2} \rightarrow e^{ist}$$

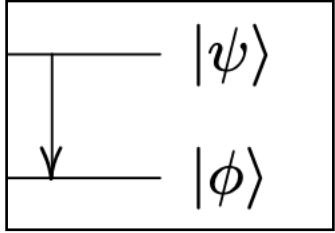


The teleported gate



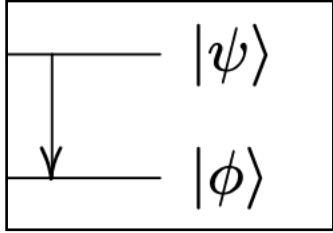
$$\hat{D}(\alpha) = e^{i\alpha_R\alpha_I} \hat{X}(\sqrt{2}\alpha_R)\hat{Z}(\sqrt{2}\alpha_I)$$

The teleported gate



$$\hat{B}_{1,2} |\psi\rangle_1 |\phi\rangle_2$$

The teleported gate

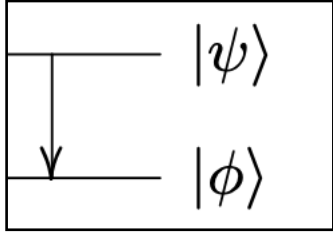


$$|\psi\rangle = \int dt \widetilde{\psi}(t) |t\rangle_p$$

$$|\phi\rangle = \int ds \phi(s) |s\rangle_q$$

$$\hat{B}_{1,2} |\psi\rangle_1 |\phi\rangle_2$$

The teleported gate

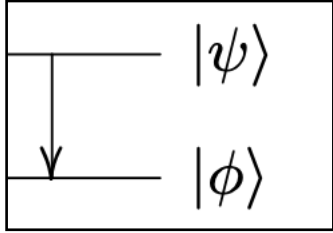


$$|\psi\rangle = \int dt \widetilde{\psi}(t) |t\rangle_p \quad |\phi\rangle = \int ds \phi(s) |s\rangle_q$$

Substitute into the equation:

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The teleported gate

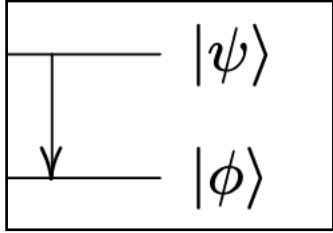


$$|\psi\rangle = \int dt \widetilde{\psi}(t) |t\rangle_p \quad |\phi\rangle = \int ds \phi(s) |s\rangle_q$$

Substitute into the equation:

$$\hat{B}_{1,2} |\psi\rangle_1 |\phi\rangle_2 = \hat{B}_{1,2} \int dt ds \widetilde{\psi}(t) \phi(s) |t\rangle_{p_1} |s\rangle_{q_2}$$

The teleported gate

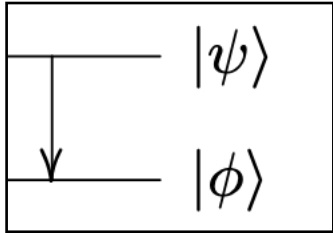


$$|\psi\rangle = \int dt \widetilde{\psi}(t) |t\rangle_p \quad |\phi\rangle = \int ds \phi(s) |s\rangle_q$$

Substitute into the equation:

$$\begin{aligned} \hat{B}_{1,2} |\psi\rangle_1 |\phi\rangle_2 &= \hat{B}_{1,2} \int dt ds \widetilde{\psi}(t) \phi(s) |t\rangle_{p_1} |s\rangle_{q_2} \\ &= \int dt ds \widetilde{\psi}(t) \phi(s) \hat{B}_{1,2} |t\rangle_{p_1} |s\rangle_{q_2} \end{aligned}$$

The teleported gate

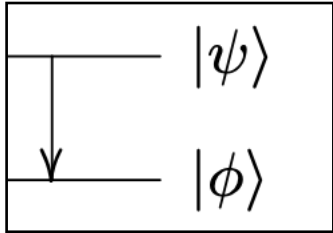


$$|\psi\rangle = \int dt \widetilde{\psi}(t) |t\rangle_p \quad |\phi\rangle = \int ds \phi(s) |s\rangle_q$$

Substitute into the equation:

$$\begin{aligned} \hat{B}_{1,2} |\psi\rangle_1 |\phi\rangle_2 &= \hat{B}_{1,2} \int dt ds \widetilde{\psi}(t) \phi(s) |t\rangle_{p_1} |s\rangle_{q_2} \\ &= \int dt ds \widetilde{\psi}(t) \phi(s) \hat{B}_{1,2} |t\rangle_{p_1} |s\rangle_{q_2} \\ &= \int dt ds \widetilde{\psi}(t) \phi(s) \hat{D}_2(s + it) |EPR\rangle \end{aligned}$$

The teleported gate

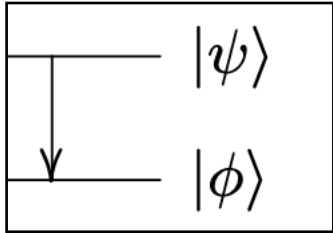


$$|\psi\rangle = \int dt \widetilde{\psi}(t) |t\rangle_p \quad |\phi\rangle = \int ds \phi(s) |s\rangle_q$$

Substitute into the equation:

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The teleported gate

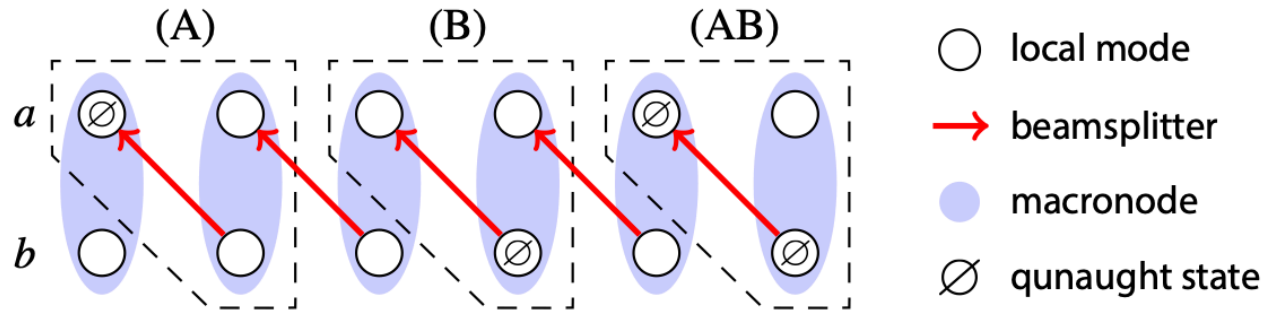


$$|\psi\rangle = \int dt \widetilde{\psi}(t) |t\rangle_p \quad |\phi\rangle = \int ds \phi(s) |s\rangle_q$$

Substitute into the equation:

$$\begin{aligned} \hat{B}_{1,2} |\psi\rangle_1 |\phi\rangle_2 &= \hat{B}_{1,2} \int dt ds \widetilde{\psi}(t) \phi(s) |t\rangle_{p_1} |s\rangle_{q_2} \\ &= \int dt ds \widetilde{\psi}(t) \phi(s) \hat{B}_{1,2} |t\rangle_{p_1} |s\rangle_{q_2} \\ &= \int dt ds \widetilde{\psi}(t) \phi(s) \hat{D}_2(s + it) |EPR\rangle \\ &= \int d^2\alpha \widetilde{\psi}(\alpha_I) \phi(\alpha_R) \hat{D}_2(\alpha) |EPR\rangle \\ &= \hat{A}_2(\psi, \phi) |EPR\rangle \end{aligned}$$

Error correction with the teleported gate



$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I) \phi(\alpha_R) \hat{D}(\alpha)$$

(A)

$$\begin{array}{c} |0\rangle_p \\ \downarrow \\ |\emptyset\rangle \end{array} = \text{---} \boxed{2^{\frac{1}{4}} \text{III}_{\sqrt{\pi}}(\hat{p})} \text{---}$$

(B)

$$\begin{array}{c} |\emptyset\rangle \\ \downarrow \\ |0\rangle_q \end{array} = \text{---} \boxed{2^{\frac{1}{4}} \text{III}_{\sqrt{\pi}}(\hat{q})} \text{---}$$

(AB)

$$\begin{array}{c} |\emptyset\rangle \\ \downarrow \\ |\emptyset\rangle \end{array} = \text{---} \boxed{\frac{1}{\sqrt{2}} \Pi_{\text{GKP}}} \text{---}$$

Two qunaught states (AB)

$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I)\phi(\alpha_R)\hat{D}(\alpha)$$

Two qunaught states (AB)

$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I)\phi(\alpha_R)\hat{D}(\alpha)$$

$$\hat{A}(\emptyset, \emptyset) = \iint d^2\alpha \text{III}_{\sqrt{2\pi}}(\alpha_R)\text{III}_{\sqrt{2\pi}}(\alpha_I)\hat{D}(\alpha)$$

Two qunaught states (AB)

$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I)\phi(\alpha_R)\hat{D}(\alpha)$$

$$\hat{A}(\emptyset, \emptyset) = \iint d^2\alpha \text{III}_{\sqrt{2\pi}}(\alpha_R)\text{III}_{\sqrt{2\pi}}(\alpha_I)\hat{D}(\alpha)$$

$$\hat{D}(\alpha) = e^{i\alpha_R\alpha_I}\hat{X}(\sqrt{2}\alpha_R)\hat{Z}(\sqrt{2}\alpha_I) \quad e^{i\alpha_R\alpha_I} \rightarrow e^{i2\pi} = 1$$

Two qunaught states (AB)

$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I)\phi(\alpha_R)\hat{D}(\alpha)$$

$$\hat{A}(\emptyset, \emptyset) = \iint d^2\alpha \text{III}_{\sqrt{2\pi}}(\alpha_R)\text{III}_{\sqrt{2\pi}}(\alpha_I)\hat{D}(\alpha)$$

$$\hat{D}(\alpha) = e^{i\alpha_R\alpha_I}\hat{X}(\sqrt{2}\alpha_R)\hat{Z}(\sqrt{2}\alpha_I) \quad e^{i\alpha_R\alpha_I} \rightarrow e^{i2\pi} = 1$$

$$\hat{A}(\emptyset, \emptyset) = \pi\sqrt{2}\text{III}_{\sqrt{\pi}}(\hat{p})\text{III}_{\sqrt{\pi}}(\hat{q})$$

Two qunaught states (AB)

$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I) \phi(\alpha_R) \hat{D}(\alpha)$$

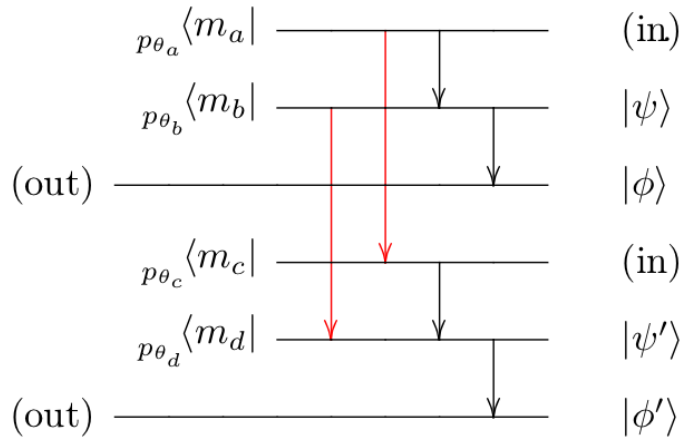
$$\hat{A}(\emptyset, \emptyset) = \iint d^2\alpha \text{III}_{\sqrt{2\pi}}(\alpha_R) \text{III}_{\sqrt{2\pi}}(\alpha_I) \hat{D}(\alpha)$$

$$\hat{D}(\alpha) = e^{i\alpha_R\alpha_I} \hat{X}(\sqrt{2}\alpha_R) \hat{Z}(\sqrt{2}\alpha_I) \quad e^{i\alpha_R\alpha_I} \rightarrow e^{i2\pi} = 1$$

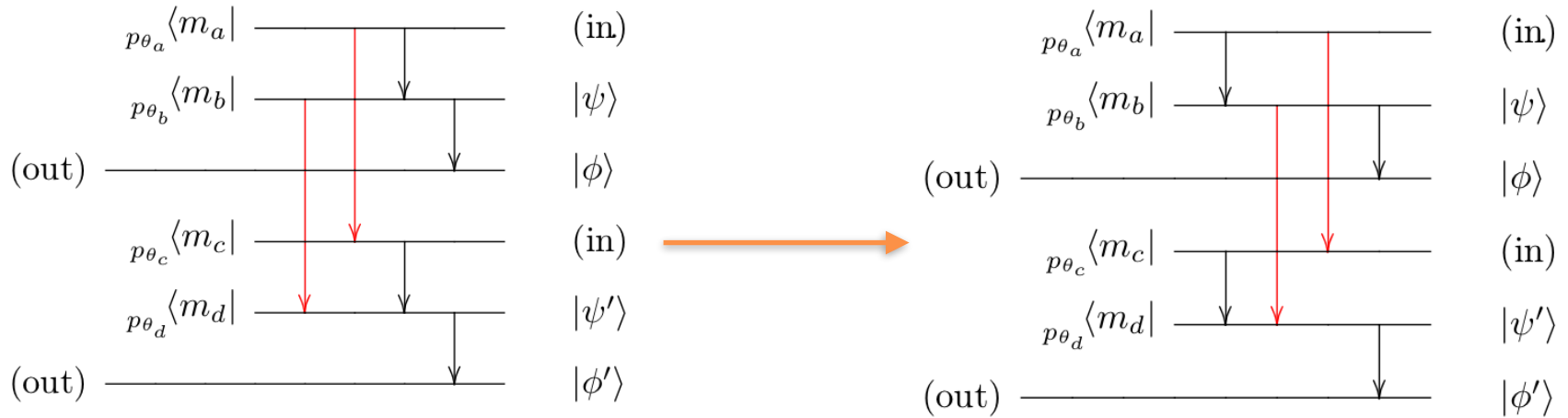
$$\hat{A}(\emptyset, \emptyset) = \pi \sqrt{2} \text{III}_{\sqrt{\pi}}(\hat{p}) \text{III}_{\sqrt{\pi}}(\hat{q})$$

$$= \sqrt{\frac{\pi}{2}} \hat{\Pi}_{\text{GKP}}$$

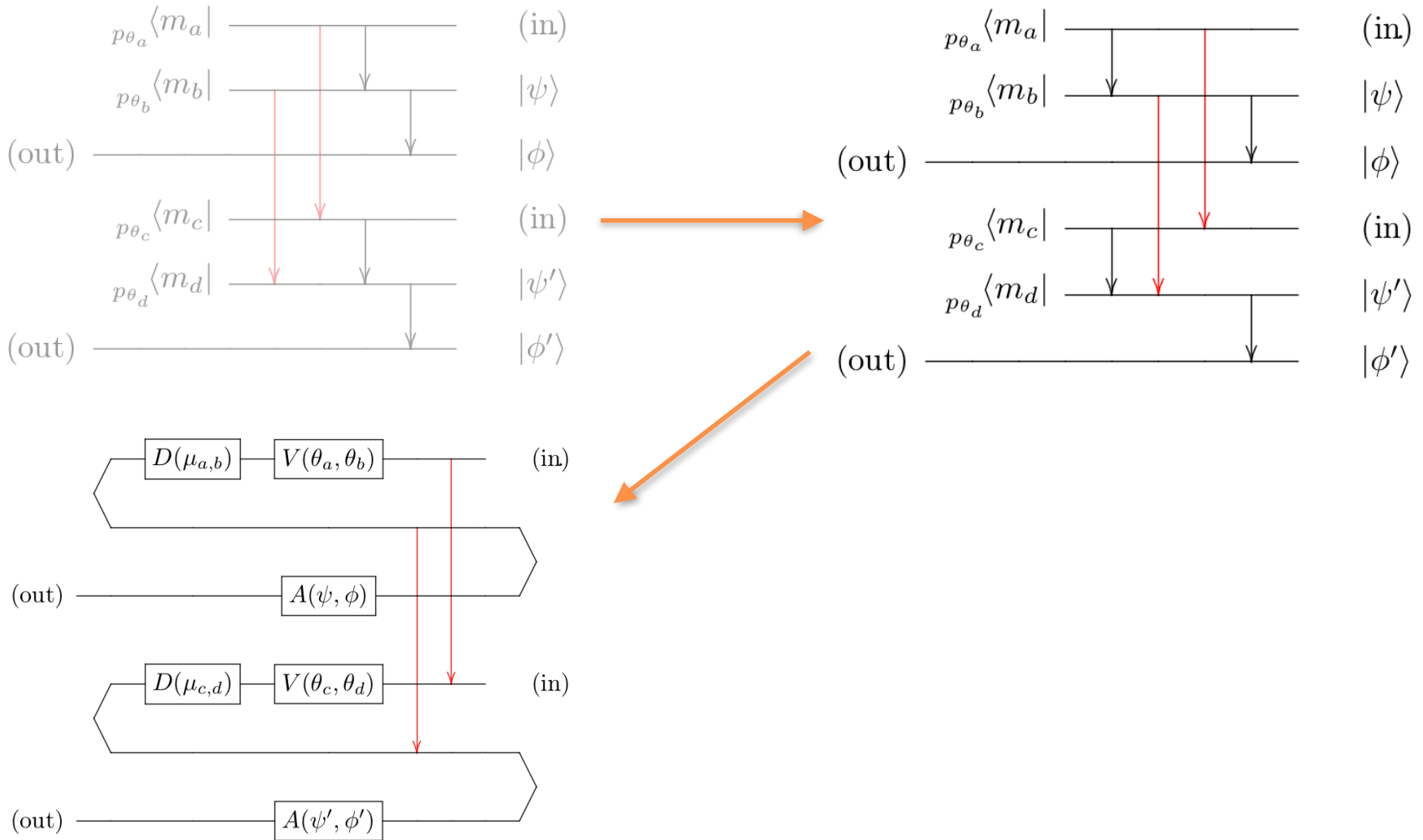
Two-mode gate



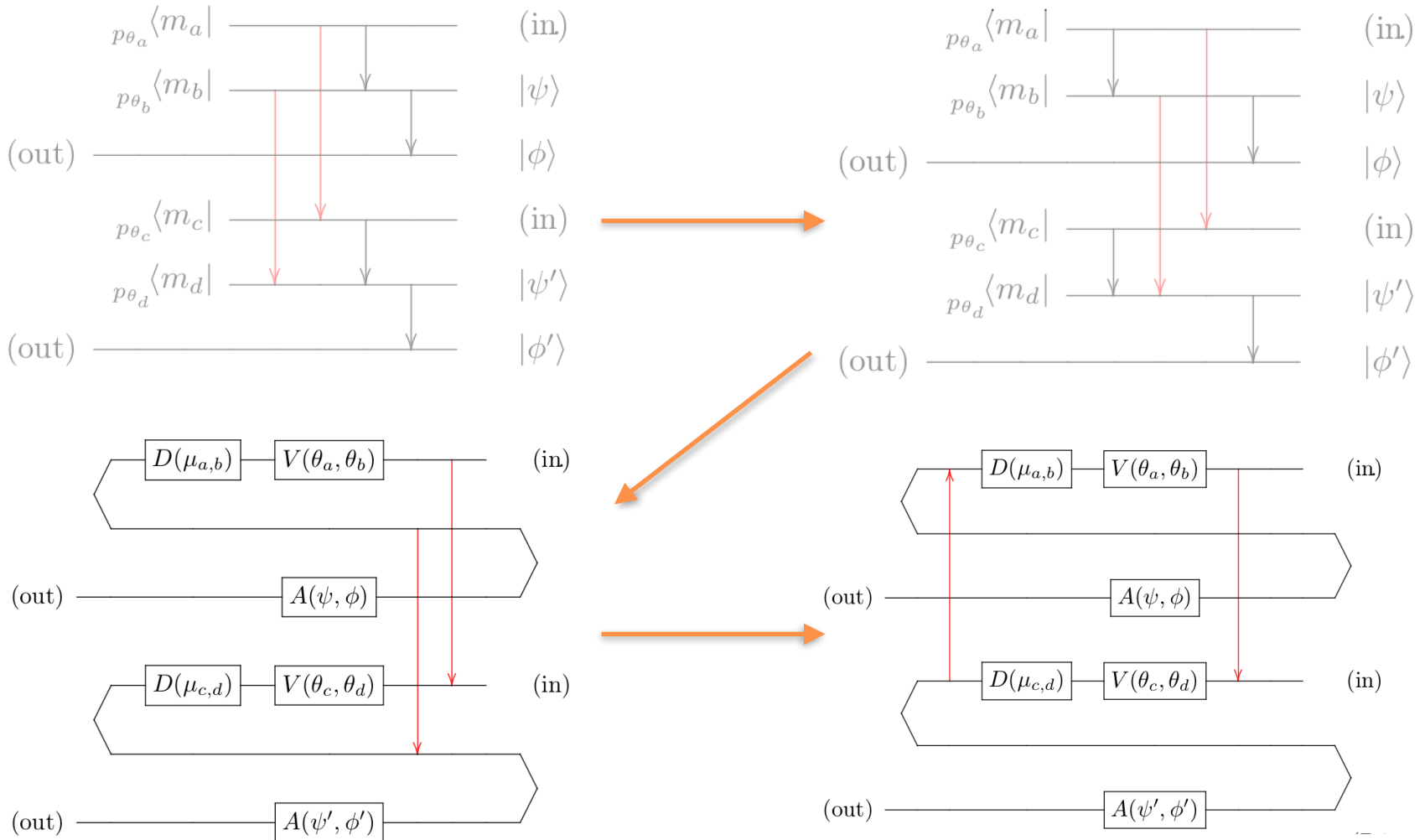
Two-mode gate



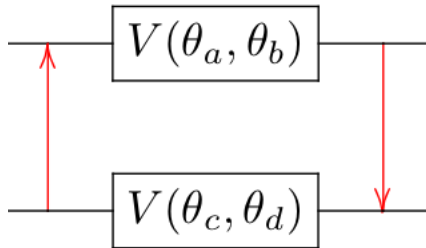
Two-mode gate



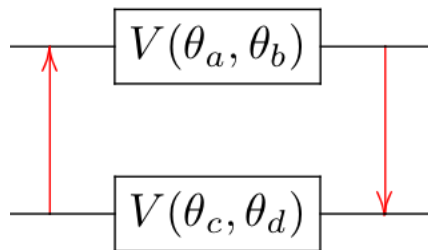
Two-mode gate



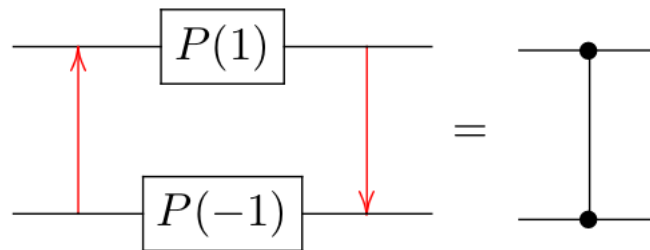
Two-mode gate



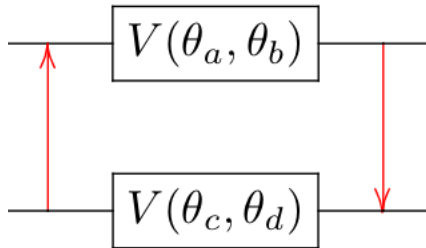
Two-mode gate



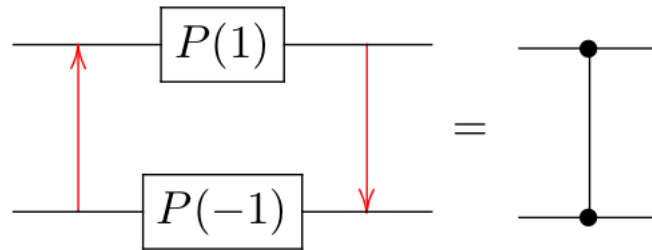
Cz gate:



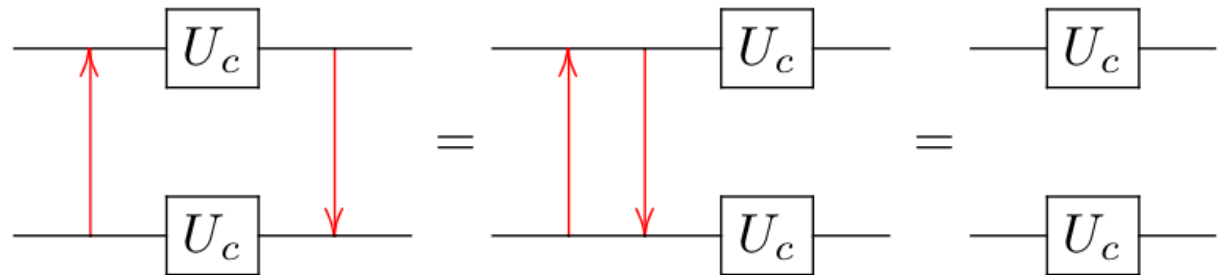
Two-mode gate



Cz gate:



Disentangled gate:



GKP with GBS

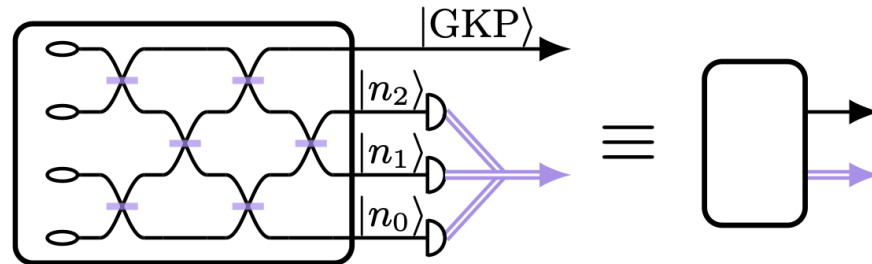


Figure 1: GBS devices for state preparation. (left) A single integrated photonic device implementing GBS-based preparation of non-Gaussian states based on the schemes presented in Refs. [27–30]. The emitted light from one output port is in a chosen non-Gaussian state subject to obtaining the correct click pattern $\{n_i\}$ at the PNR detectors connected to the remaining output ports. The double purple lines represent classical logic, which is used to trigger a switch on the emitted port. (right) A simplified representation of a single GBS device.

Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer

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Virtual beam splitter

$$\begin{array}{l} \theta \langle m_1 | \text{---} \\ \theta \langle m_2 | \text{---} \end{array} \begin{array}{c} \text{---} \\ \downarrow \\ \text{---} \end{array} = \begin{array}{l} \theta \langle \frac{1}{\sqrt{2}}(m_1 + m_2) | \text{---} \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 - m_1) | \text{---} \end{array}$$

$$\begin{array}{l} \theta \langle m_1 | \text{---} \\ \theta \langle m_2 | \text{---} \end{array} = \begin{array}{l} \theta \langle \frac{1}{\sqrt{2}}(m_1 - m_2) | \text{---} \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 + m_1) | \text{---} \end{array} \begin{array}{c} \text{---} \\ \vdots \\ \text{---} \\ \downarrow \\ \text{---} \end{array}$$

Virtual beam splitter

$$\begin{array}{c} \theta \langle m_1 | \text{---} \\ \quad \quad \quad \downarrow \\ \theta \langle m_2 | \text{---} \end{array} = \begin{array}{c} \theta \langle \frac{1}{\sqrt{2}}(m_1 + m_2) | \text{---} \\ \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 - m_1) | \text{---} \end{array}$$

$$\begin{array}{c} \theta \langle m_1 | \text{---} \\ \\ \theta \langle m_2 | \text{---} \end{array} = \begin{array}{c} \theta \langle \frac{1}{\sqrt{2}}(m_1 - m_2) | \text{---} \\ \quad \quad \quad \vdots \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 + m_1) | \text{---} \end{array}$$

$$\hat{B}_{1,2} |s\rangle_{q_1} |t\rangle_{q_2} = e^{-iq_1 p_2} \hat{S}_1^\dagger(\sqrt{2}) \hat{S}_2(\sqrt{2}) e^{ip_1 q_2} |s\rangle_{q_1} |t\rangle_{q_2} \quad (10)$$

$$= e^{-iq_1 p_2} \hat{S}_1^\dagger(\sqrt{2}) \hat{S}_2(\sqrt{2}) |s - t\rangle_{q_1} |t\rangle_{q_2} \quad (11)$$

$$= e^{-iq_1 p_2} \left| \frac{1}{\sqrt{2}}(s - t) \right\rangle_{q_1} \left| \sqrt{2}t \right\rangle_{q_2} \quad (12)$$

$$= \left| \frac{1}{\sqrt{2}}(s - t) \right\rangle_{q_1} \left| \frac{1}{\sqrt{2}}(s + t) \right\rangle_{q_2}$$

Virtual beam splitter

$$\begin{array}{l} \theta \langle m_1 | \text{---} \\ \theta \langle m_2 | \text{---} \end{array} \begin{array}{c} \text{---} \\ \downarrow \\ \text{---} \end{array} = \begin{array}{l} \theta \langle \frac{1}{\sqrt{2}}(m_1 + m_2) | \text{---} \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 - m_1) | \text{---} \end{array}$$

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$$= \left| \frac{1}{\sqrt{2}}(s - t) \right\rangle_{q_1} \left| \frac{1}{\sqrt{2}}(s + t) \right\rangle_{q_2}$$

$$\hat{B}_{1,2}^\dagger \mathbf{x} \hat{B}_{1,2} = \mathbf{S}_{\hat{B}_{1,2}} \mathbf{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} \hat{q}_1 - \hat{q}_2 \\ \hat{q}_1 + \hat{q}_2 \\ \hat{p}_1 - \hat{p}_2 \\ \hat{p}_1 + \hat{p}_2 \end{bmatrix}$$

Virtual beam splitter

$$\begin{array}{l} \theta \langle m_1 | \text{---} \\ \theta \langle m_2 | \text{---} \end{array} \begin{array}{c} \text{---} \\ \downarrow \\ \text{---} \end{array} = \begin{array}{l} \theta \langle \frac{1}{\sqrt{2}}(m_1 + m_2) | \text{---} \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 - m_1) | \text{---} \end{array}$$

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$$\hat{B}_{1,2} |s\rangle_{q_1} |t\rangle_{q_2} = e^{-iq_1 p_2} \hat{S}_1^\dagger(\sqrt{2}) \hat{S}_2(\sqrt{2}) e^{ip_1 q_2} |s\rangle_{q_1} |t\rangle_{q_2} \quad (10)$$

$$= e^{-iq_1 p_2} \hat{S}_1^\dagger(\sqrt{2}) \hat{S}_2(\sqrt{2}) |s-t\rangle_{q_1} |t\rangle_{q_2} \quad (11)$$

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$$\hat{B}_{1,2}^\dagger \mathbf{x} \hat{B}_{1,2} = \mathbf{S}_{\hat{B}_{1,2}} \mathbf{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} \hat{q}_1 - \hat{q}_2 \\ \hat{q}_1 + \hat{q}_2 \\ \hat{p}_1 - \hat{p}_2 \\ \hat{p}_1 + \hat{p}_2 \end{bmatrix} \quad \text{--- You can know } \hat{q}_1 \text{ and } \hat{q}_2 \text{ perfectly}$$

Virtual beam splitter

$$\begin{array}{l} \theta \langle m_1 | \text{---} \\ \theta \langle m_2 | \text{---} \end{array} \begin{array}{c} \text{---} \\ \downarrow \\ \text{---} \end{array} = \begin{array}{l} \theta \langle \frac{1}{\sqrt{2}}(m_1 + m_2) | \text{---} \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 - m_1) | \text{---} \end{array}$$

$$\begin{array}{l} \theta \langle m_1 | \text{---} \\ \theta \langle m_2 | \text{---} \end{array} = \begin{array}{l} \theta \langle \frac{1}{\sqrt{2}}(m_1 - m_2) | \text{---} \\ \theta \langle \frac{1}{\sqrt{2}}(m_2 + m_1) | \text{---} \end{array}$$

$$\hat{B}_{1,2} |s\rangle_{q_1} |t\rangle_{q_2} = e^{-iq_1 p_2} \hat{S}_1^\dagger(\sqrt{2}) \hat{S}_2(\sqrt{2}) e^{ip_1 q_2} |s\rangle_{q_1} |t\rangle_{q_2} \quad (10)$$

$$= e^{-iq_1 p_2} \hat{S}_1^\dagger(\sqrt{2}) \hat{S}_2(\sqrt{2}) |s-t\rangle_{q_1} |t\rangle_{q_2} \quad (11)$$

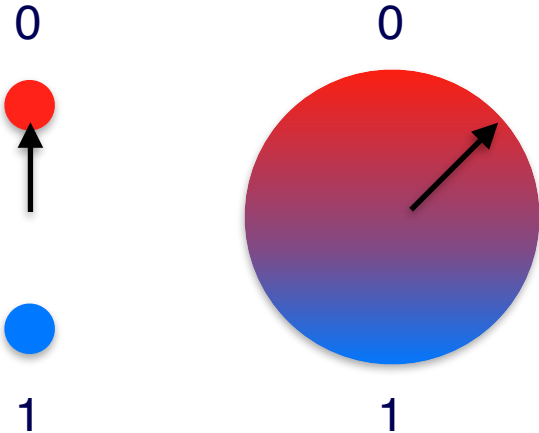
$$= e^{-iq_1 p_2} \left| \frac{1}{\sqrt{2}}(s-t) \right\rangle_{q_1} \left| \sqrt{2}t \right\rangle_{q_2} \quad (12)$$

$$= \left| \frac{1}{\sqrt{2}}(s-t) \right\rangle_{q_1} \left| \frac{1}{\sqrt{2}}(s+t) \right\rangle_{q_2}$$

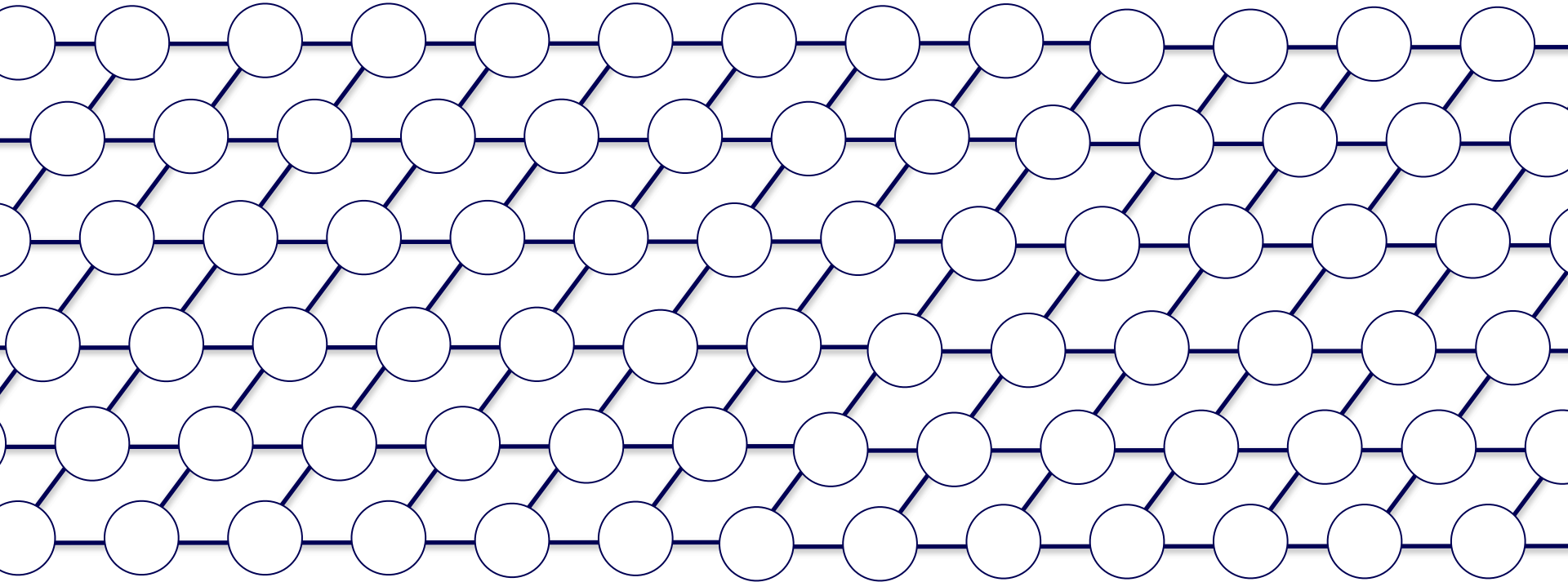
$$\hat{B}_{1,2}^\dagger \mathbf{x} \hat{B}_{1,2} = \mathbf{S}_{\hat{B}_{1,2}} \mathbf{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} \hat{q}_1 - \hat{q}_2 \\ \hat{q}_1 + \hat{q}_2 \\ \hat{p}_1 - \hat{p}_2 \\ \hat{p}_1 + \hat{p}_2 \end{bmatrix}$$

You can know \hat{q}_1 and \hat{q}_2 perfectly
 You cannot know \hat{q}_1 and \hat{p}_1 , or \hat{q}_2 and \hat{p}_2 at the same time

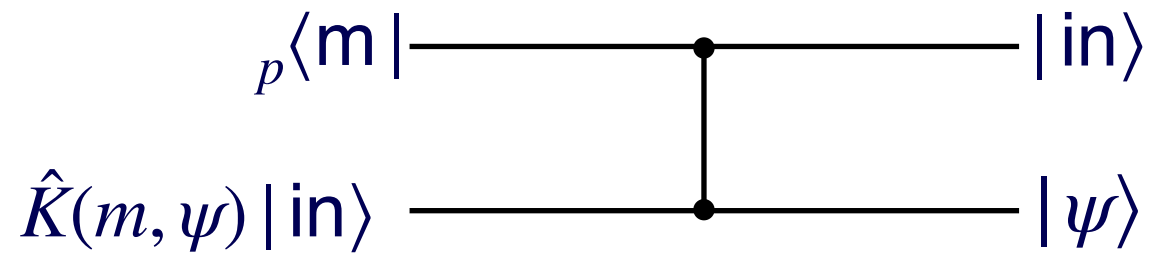
Classic vs. Quantum



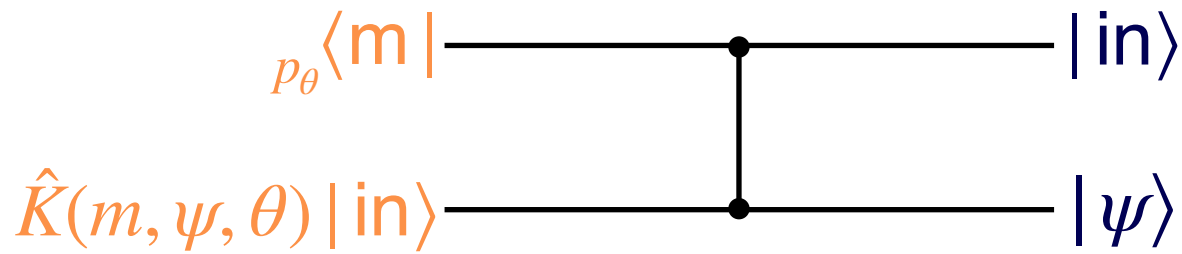
Continuous variable measurement-based quantum computing



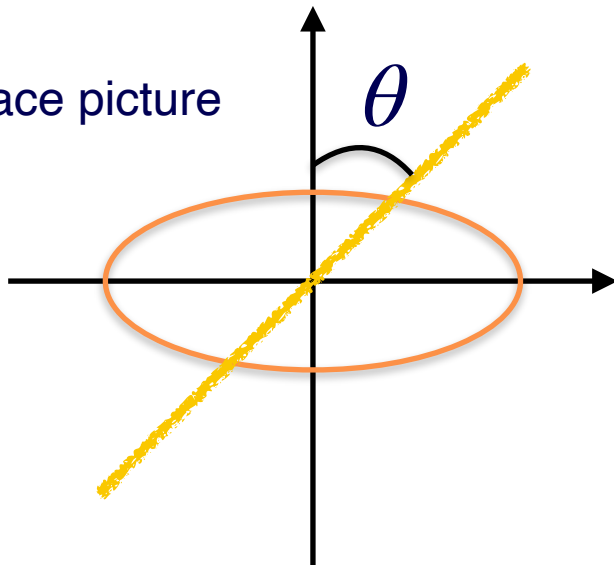
Teleportation



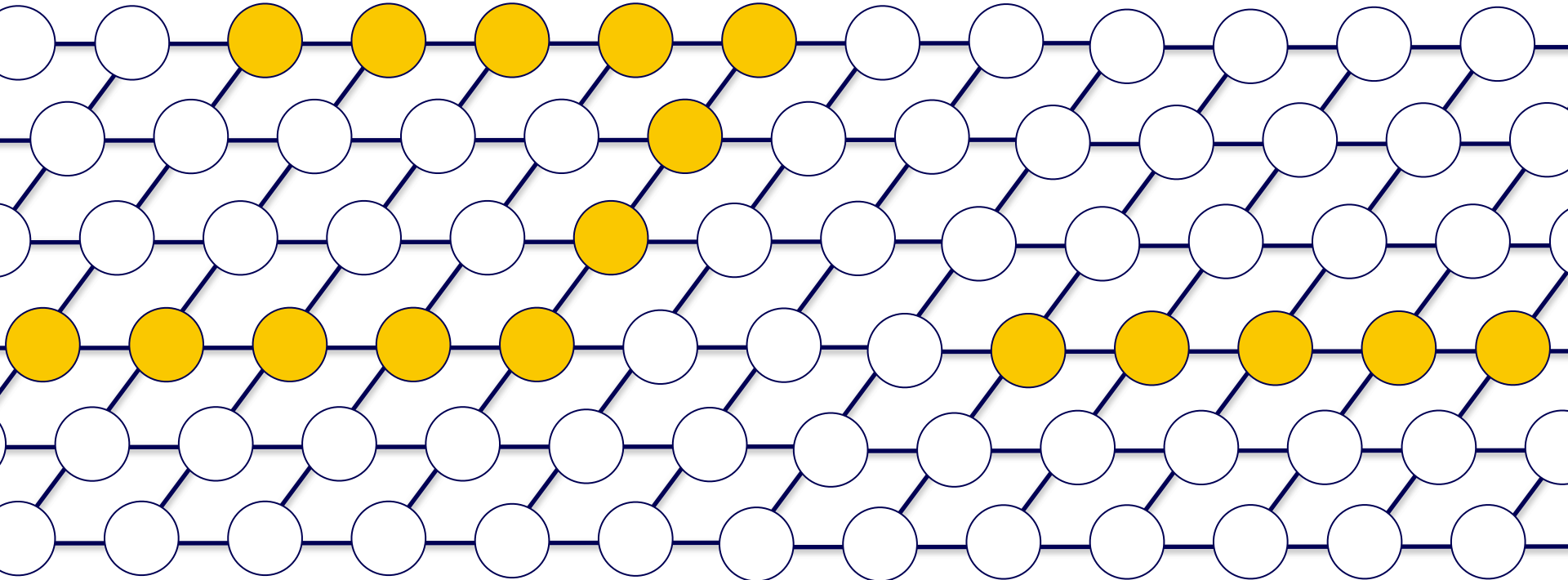
Teleportation



Phase-space picture



Two- and one-mode gates



Defining a qubit

PHYSICAL REVIEW A, VOLUME 64, 012310

Encoding a qubit in an oscillator

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Error analysis for encoding a qubit in an oscillator

S. Glancy* and E. Knill†

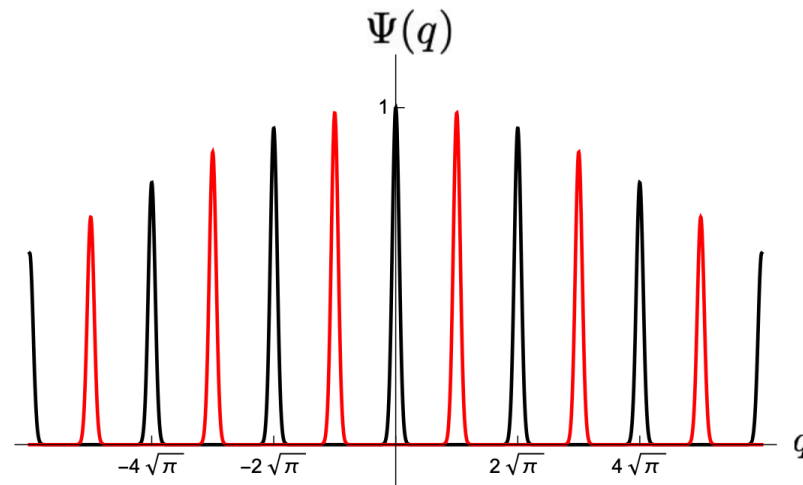
Mathematical and Computing Science Division, Information Technology Laboratory,

National Institute of Standards and Technology, Boulder, Colorado 80301, USA

(Received 14 October 2005; published 19 January 2006)

■ $\langle \Psi(q) | \tilde{0}_L \rangle$

■ $\langle \Psi(q) | \tilde{1}_L \rangle$



Defining a qubit

PHYSICAL REVIEW A, VOLUME 64, 012310

Encoding a qubit in an oscillator

Daniel Gottesman,^{1,2,*} Alexei Kitaev,^{1,†} and John Preskill^{3,‡}

¹Microsoft Corporation, One Microsoft Way, Redmond, Washington 98052

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³Institute for Quantum Information, California Institute of Technology, Pasadena, California 91125

(Received 9 August 2000; published 11 June 2001)

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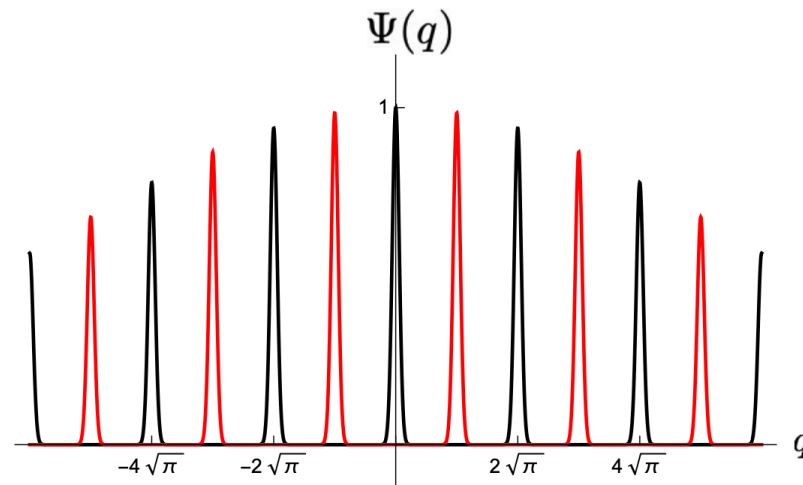
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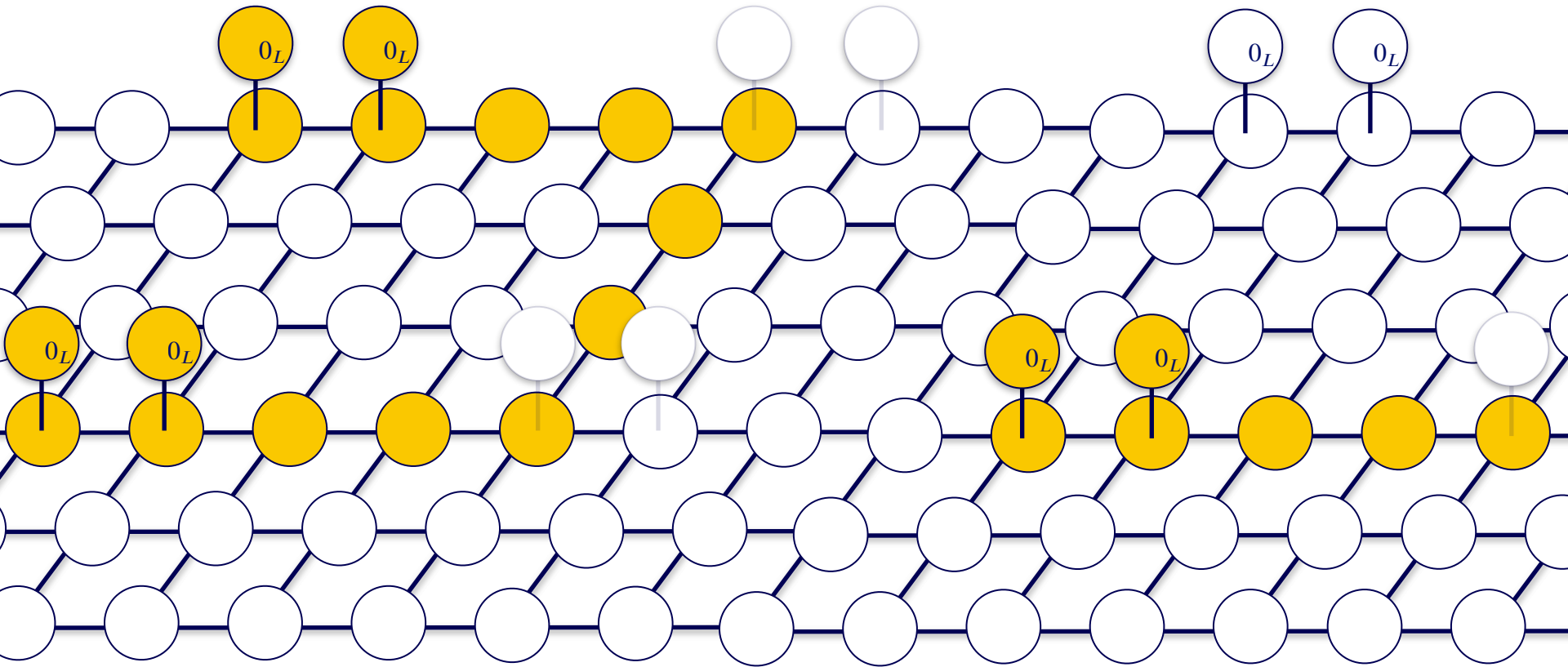
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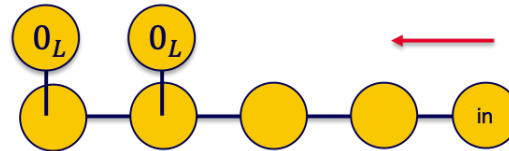


$$|\emptyset\rangle := \int ds \text{III}_{\sqrt{2\pi}}(s) |s\rangle_q$$

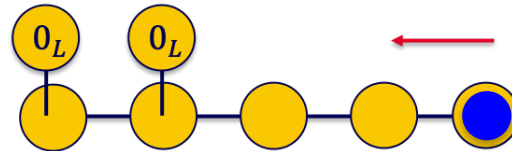
GKP error correction



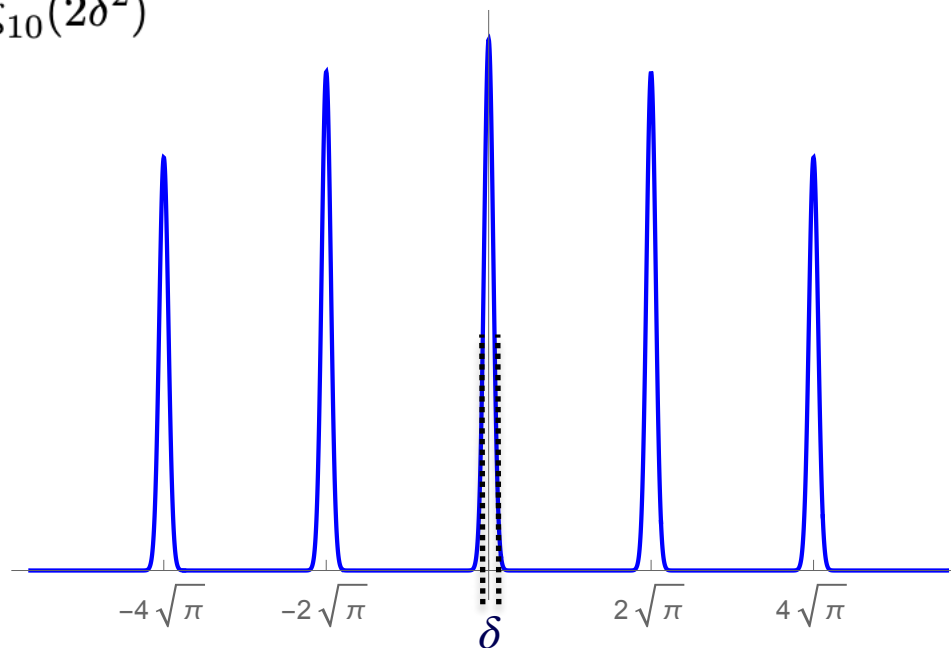
Noise Accumulation



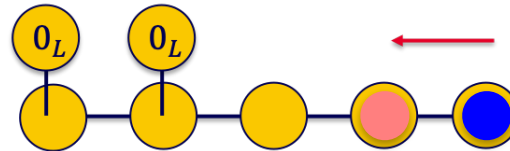
Noise Accumulation



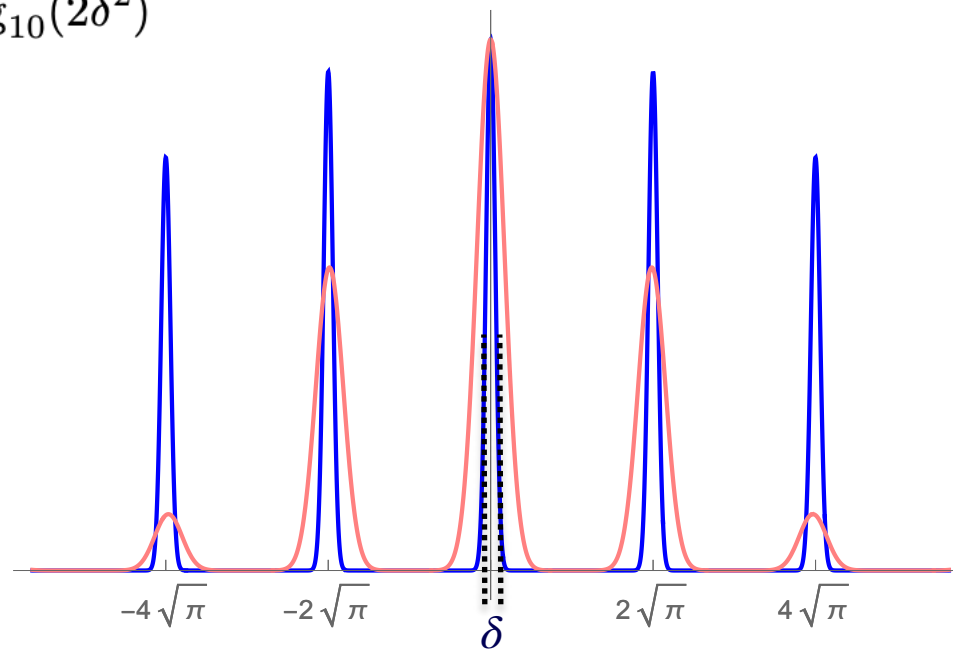
$$(\delta^2)_{\text{dB}} = -10 \log_{10}(2\delta^2)$$



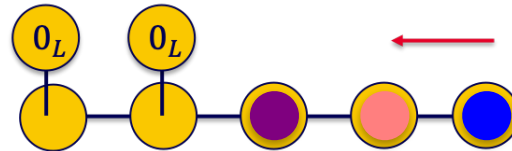
Noise Accumulation



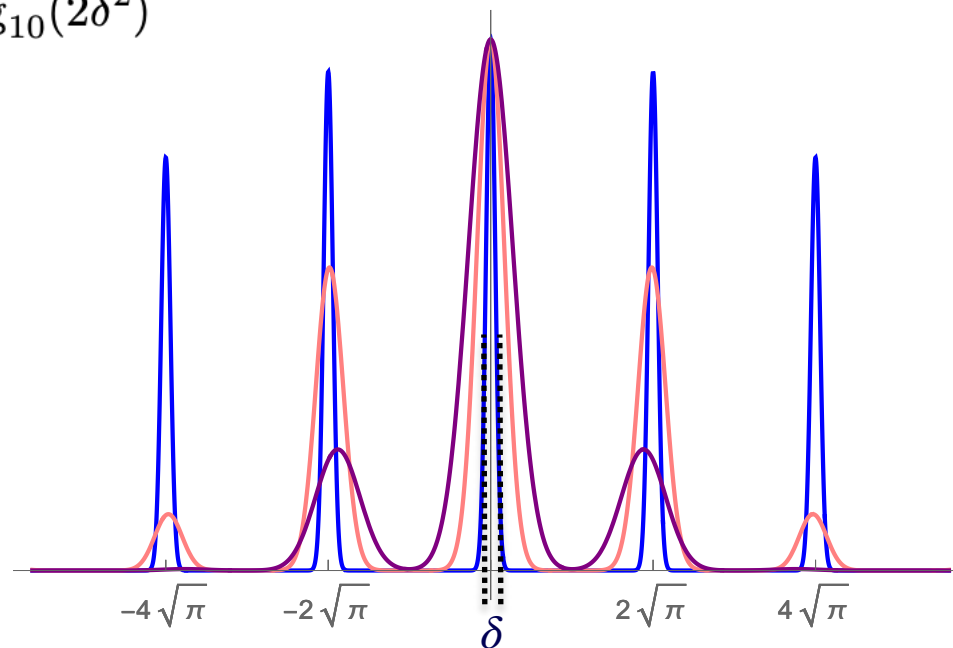
$$(\delta^2)_{\text{dB}} = -10 \log_{10}(2\delta^2)$$



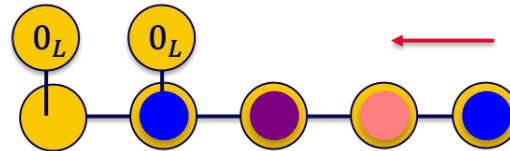
Noise Accumulation



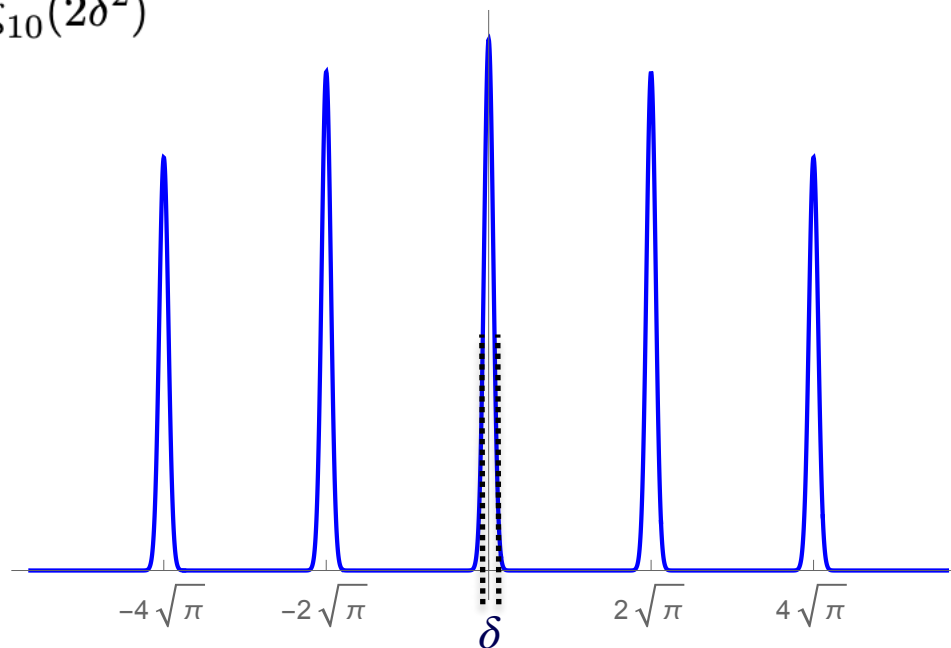
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Noise Accumulation



$$(\delta^2)_{\text{dB}} = -10 \log_{10}(2\delta^2)$$



Fault-tolerant continuous variable measurement-based quantum computing

PRL 112, 120504 (2014)

PHYSICAL REVIEW LETTERS

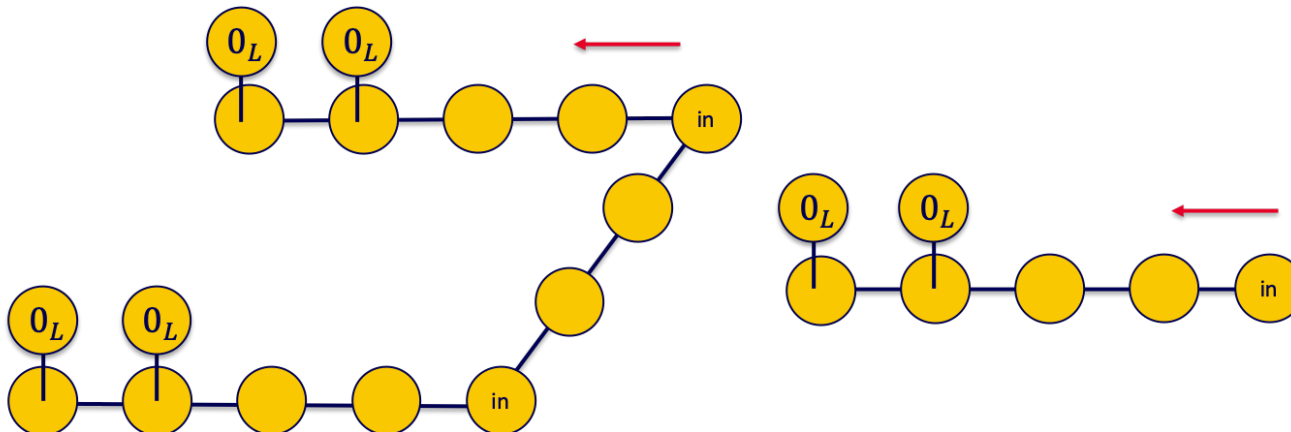
week ending
28 MARCH 2014

Fault-Tolerant Measurement-Based Quantum Computing with Continuous-Variable Cluster States

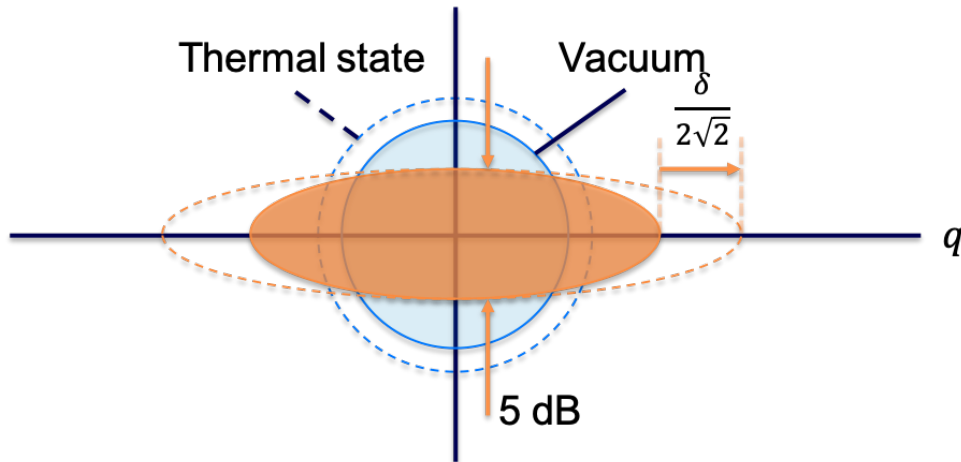
Nicolas C. Menicucci*

School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia

(Received 29 October 2013; published 26 March 2014)



Robust fault tolerance



Error Rate	Squeezing (dB)
10^{-6}	20.5 dB
10^{-3}	17.4 dB
1 %	15.6 dB

PHYSICAL REVIEW A **100**, 010301(R) (2019)

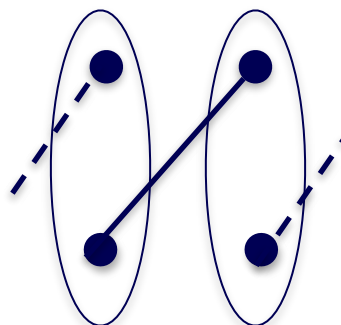
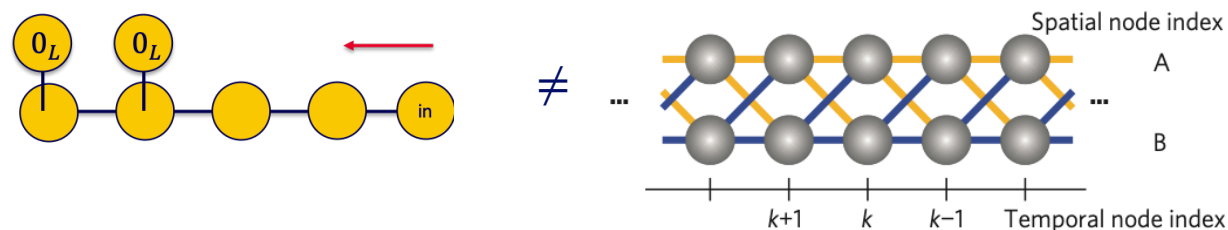
Rapid Communications

Robust fault tolerance for continuous-variable cluster states with excess antisqueezing

Blayne W. Walsh,^{*} Lucas J. Mensen, Ben Q. Baragiola, and Nicolas C. Menicucci
 Centre for Quantum Computation and Communication Technology, School of Science, RMIT University, Melbourne, VIC 3000, Australia

(Received 15 March 2019; published 22 July 2019)

Using the experimentally accessible resource



Simplified view

PHYSICAL REVIEW A **90**, 062324 (2014)

Noise analysis of single-mode Gaussian operations using continuous-variable cluster states

Rafael N. Alexander,^{1,*} Seiji C. Armstrong,^{2,3} Ryuji Ukai,² and Nicolas C. Menicucci¹

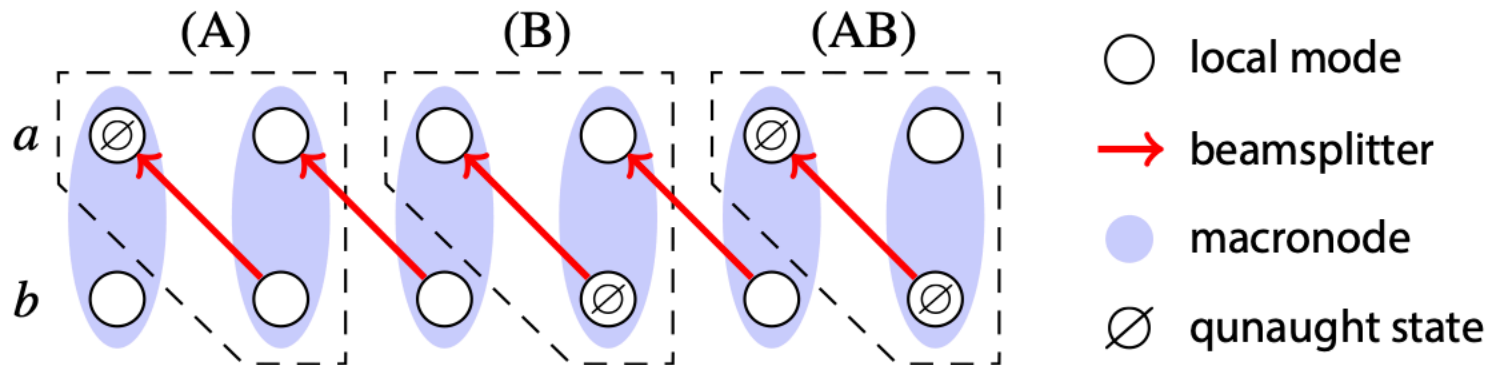
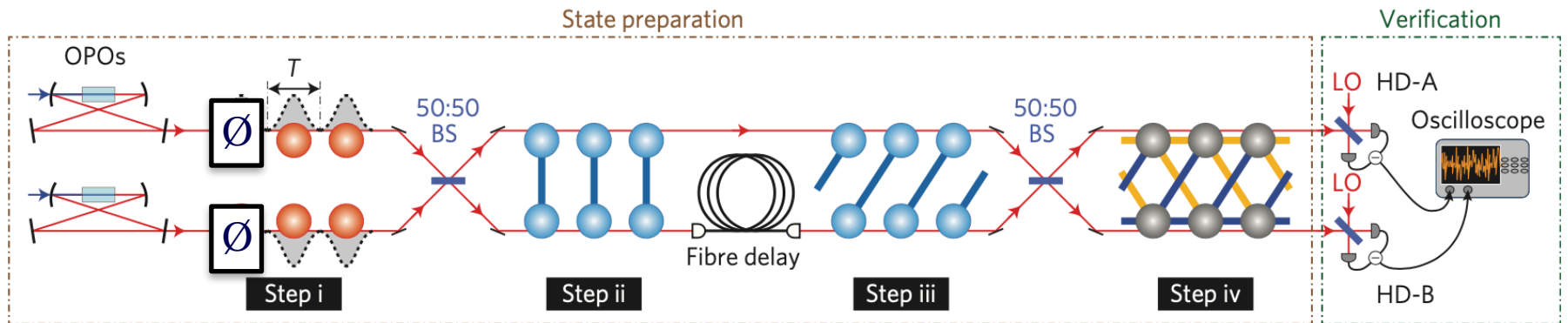
¹*School of Physics, The University of Sydney, NSW, 2006, Australia*

²*Department of Applied Physics, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

³*Centre for Quantum Computation and Communication Technology, Department of Quantum Science, The Australian National University, Canberra, ACT 0200, Australia*

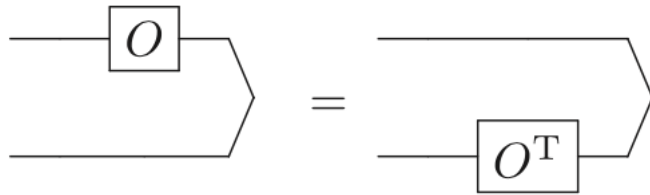
(Received 15 November 2013; revised manuscript received 12 October 2014; published 15 December 2014)

Error Correction on the Macronode Wire

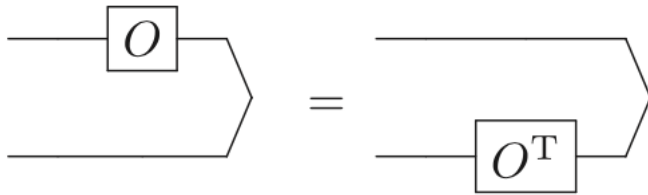


$$|\emptyset\rangle := \int ds \text{III}_{\sqrt{2\pi}}(s) |s\rangle_q$$

Bounce rules



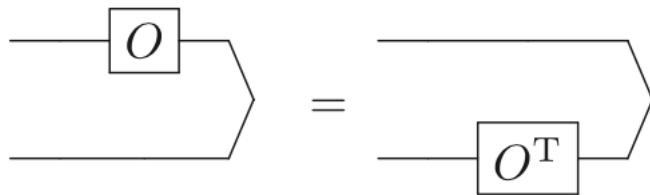
Bounce rules



$$\hat{q}^T = \hat{q}$$

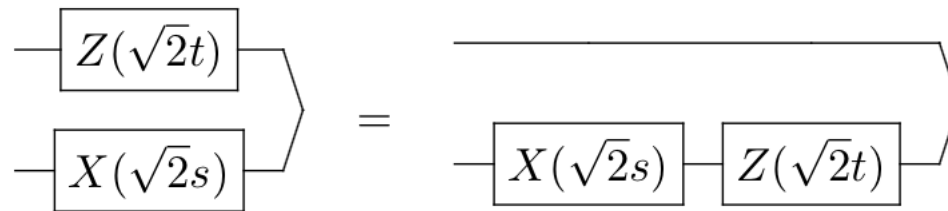
$$\hat{p}^T = -\hat{p}$$

Bounce rules

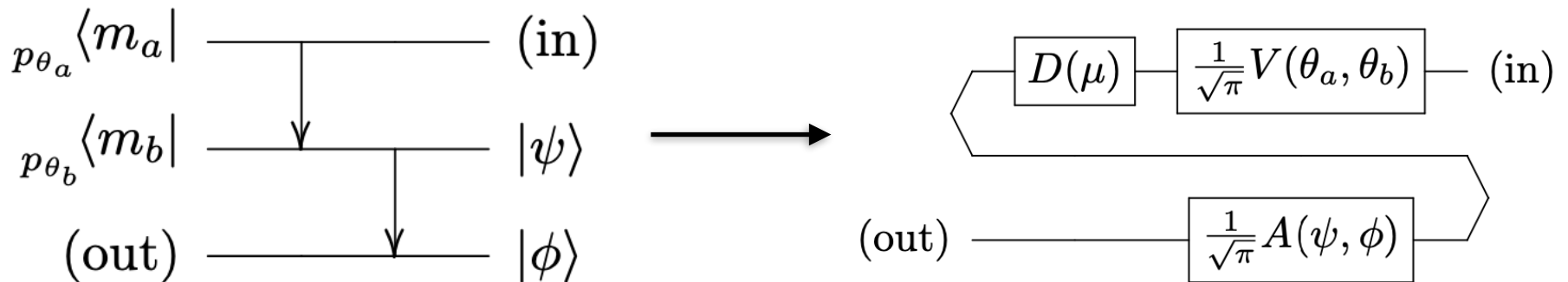
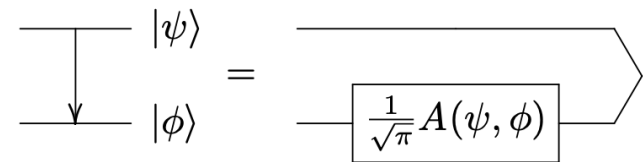
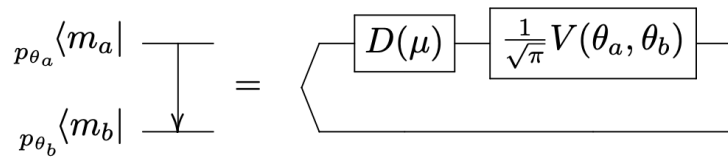


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Circuit identities



PHYSICAL REVIEW A **102**, 062411 (2020)

Continuous-variable gate teleportation and bosonic-code error correction

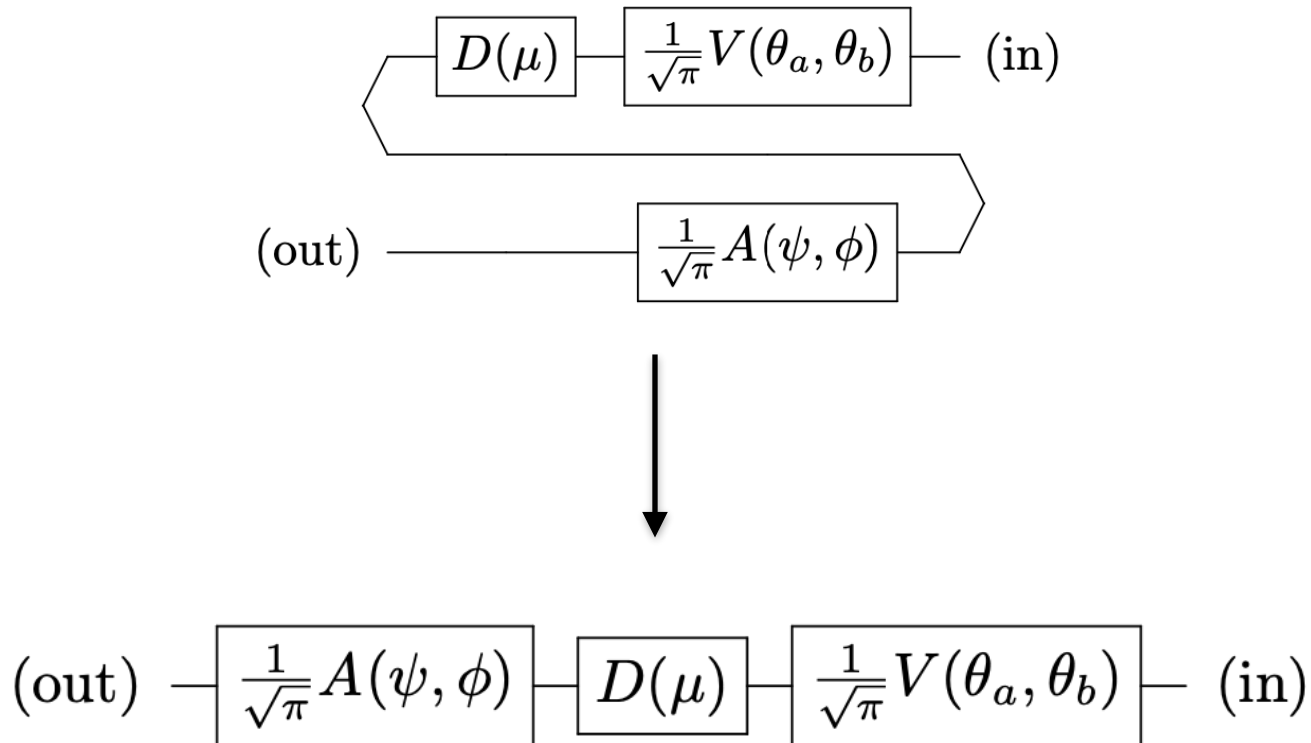
Blayne W. Walsh^{1,*}, Ben Q. Baragiola¹, Rafael N. Alexander^{1,2} and Nicolas C. Menicucci¹

¹Centre for Quantum Computation and Communication Technology, School of Science, RMIT University, Melbourne, Victoria 3000, Australia

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Kraus operator for a macronode measurement



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Kraus operator for a macronode measurement

$$\hat{K}(m_a, m_b) = \frac{1}{\pi} \hat{A}(\psi, \phi) \hat{D}(\mu) \hat{V}(\theta_a, \theta_b)$$


PHYSICAL REVIEW A **102**, 062411 (2020)

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$$\hat{V}(\theta_a, \theta_b) := \hat{R}(\theta_+ - \frac{\pi}{2}) \hat{S}(\tan \theta_-) \hat{R}(\theta_+)$$

$$\theta_{\pm} = \frac{\theta_a \pm \theta_b}{2}$$

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$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I) \phi(\alpha_R) \hat{D}(\alpha)$$

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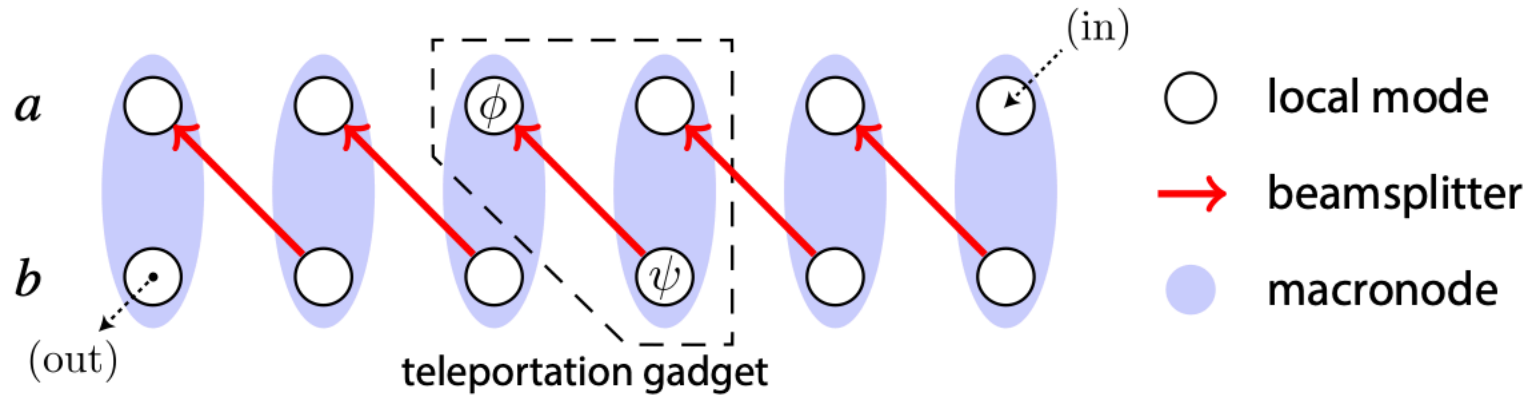
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The teleported gate



$$\hat{A}(\psi, \phi) := \iint d^2\alpha \tilde{\psi}(\alpha_I) \phi(\alpha_R) \hat{D}(\alpha)$$

$$\begin{array}{c} \text{---} |\emptyset\rangle \\ \text{---} |\emptyset\rangle \\ \text{---} \end{array} = \text{---} \left[\frac{1}{\sqrt{2}} \Pi_{\text{GKP}} \right] \text{---}$$

PHYSICAL REVIEW A **102**, 062411 (2020)

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