

Symmetry Invariant Encodings for Quantum Machine Learning

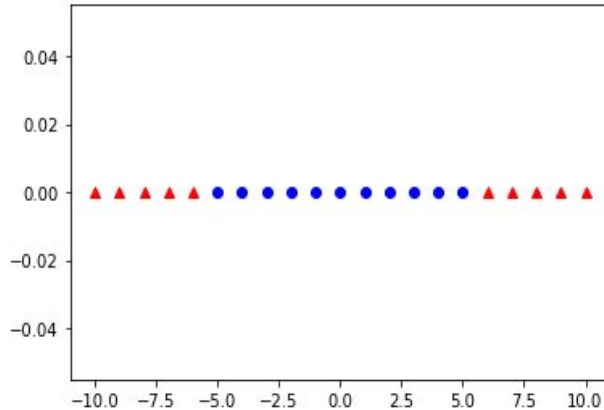
Jamie Heredge, Charles Hill, Lloyd
Hollenberg and Martin Seviar

Classical Support Vector Machines

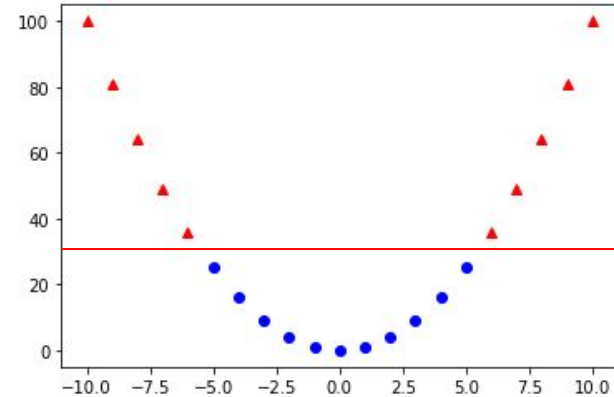
General Idea:

Encode to a higher dimensional space where the classification is easier.

Can't separate in 1D

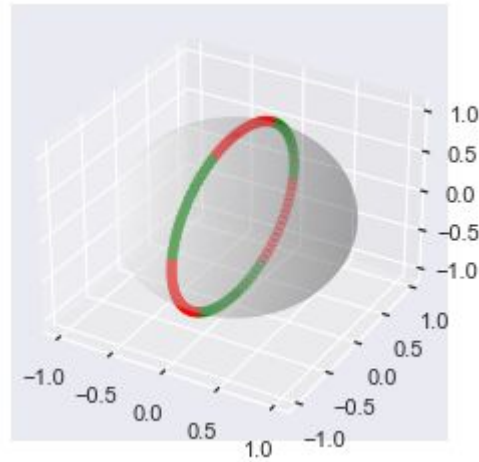


Separable in 2D



Single Qubit Encoding Example

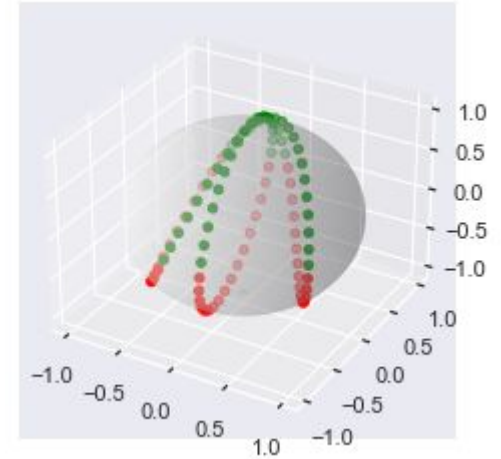
Classical 1 dimensional data



Encoding provided by a single x rotation gate on an initial 0 state

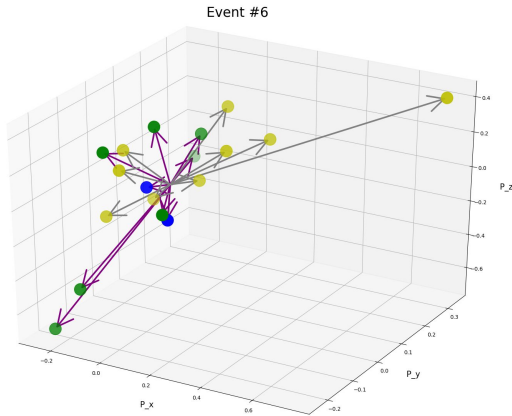


Encoding provided by circuit specific to the data



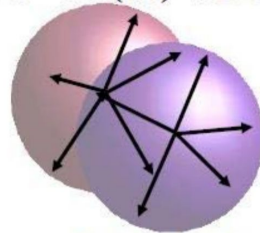
Pointcloud Data

Particle Decay Data



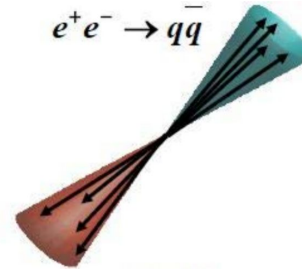
Particle Event Classification

$$e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}$$



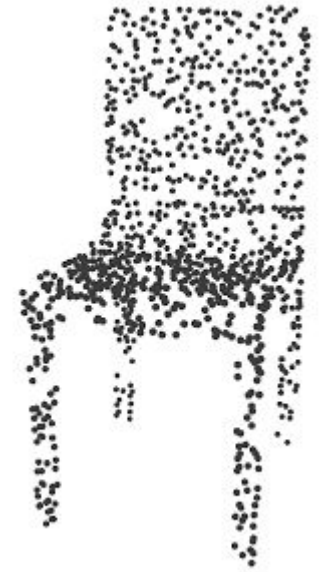
Spherical

$$e^+e^- \rightarrow q\bar{q}$$



Jet-like

3D Object Classification

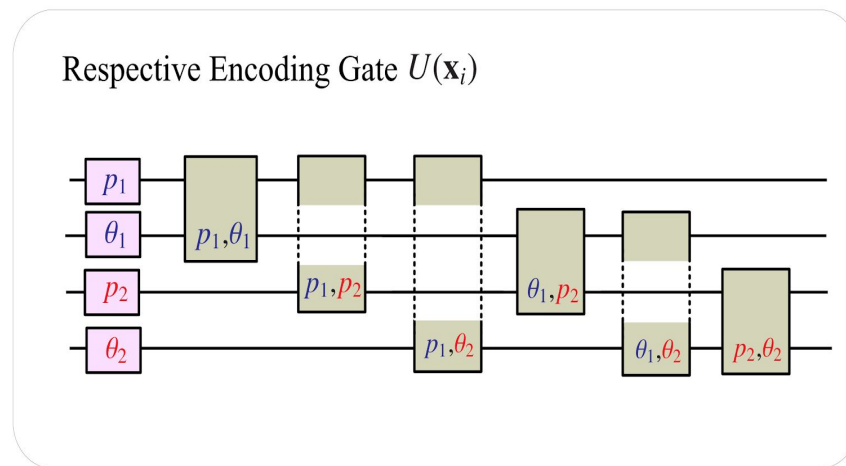
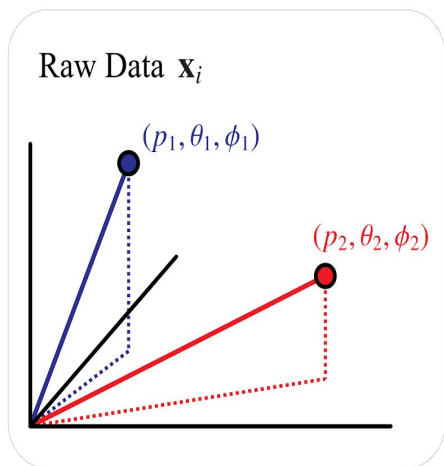


Encoding Step

$$|\Phi(z)\rangle = \mathcal{U}_{\Phi(z)}|0^n\rangle$$

Combinatorial Encoding

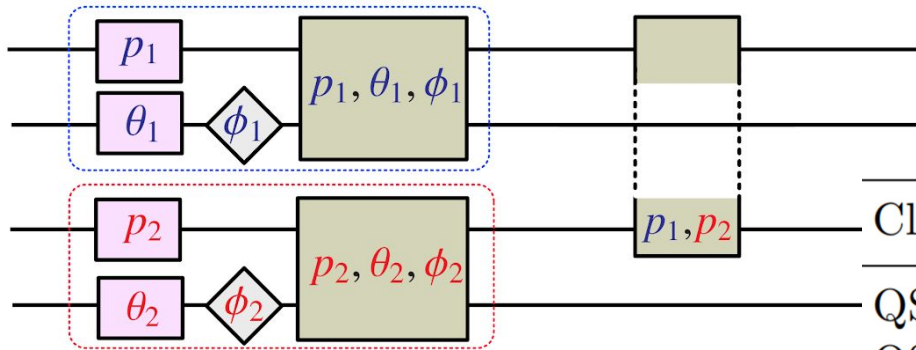
- Large number of gates
- Generic approach



Particle Physics Quantum Support Vector Machines

Particles entangled individually

Particles entangled with each other through their momenta



Classification Algorithm	AUC
QSVM - Combinatorial Encoding	0.827
QSVM - Bloch Sphere Encoding	0.861
QSVM - Separate Particle Encoding	0.877
Classical SVM - RBF Kernel	0.865
Classical FastBDT	0.821

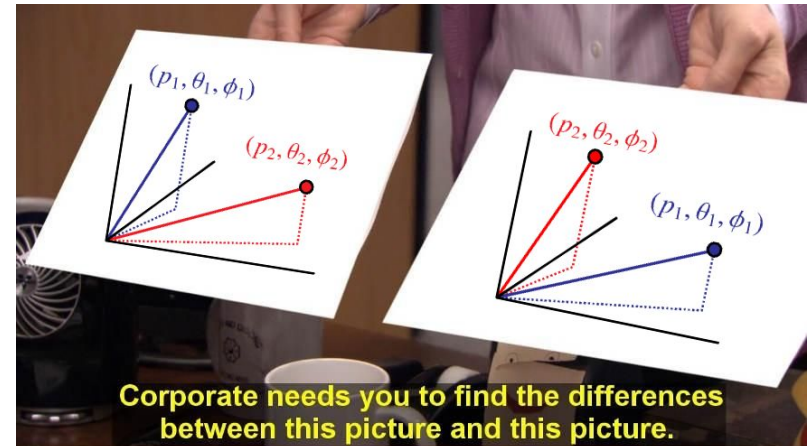
Point Order Permutation

Problem with QSVM is overfitting to training. Especially as dimension of space grows with more points.

We need a way to reduce expressibility

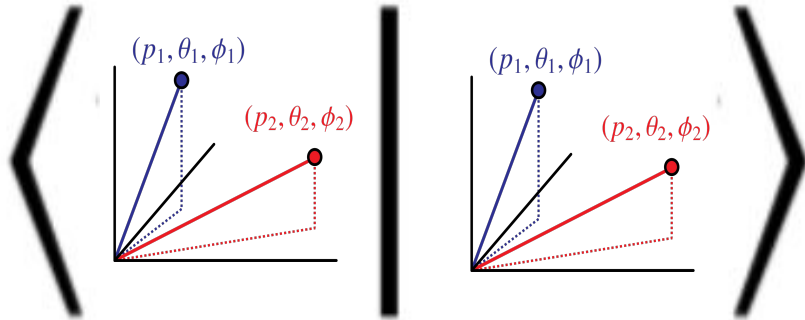
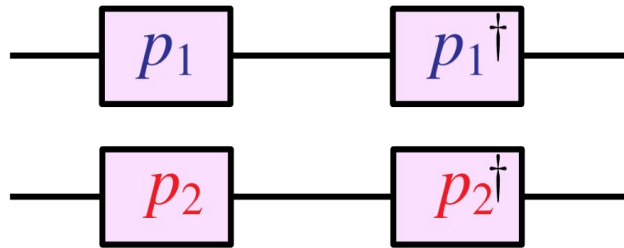
One way we can do this while still respecting properties of the underlying data is exploiting symmetries

We focus on Point Re-ordering
Permutation Symmetry.

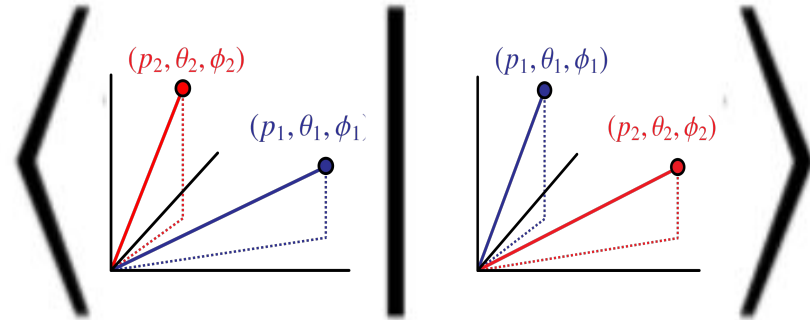
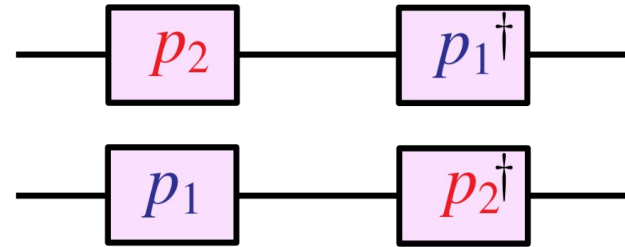


Different Results Under Point Order Permutation

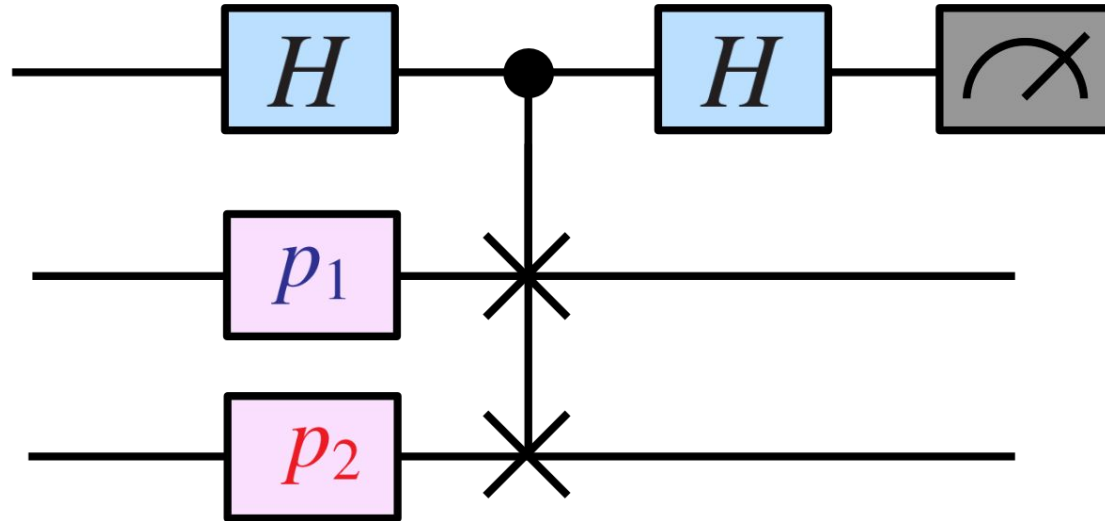
Always measure 0 state = Perfect overlap



Non-perfect overlap = Data looks different!
But they are the same picture!



Symmetric State Preparation

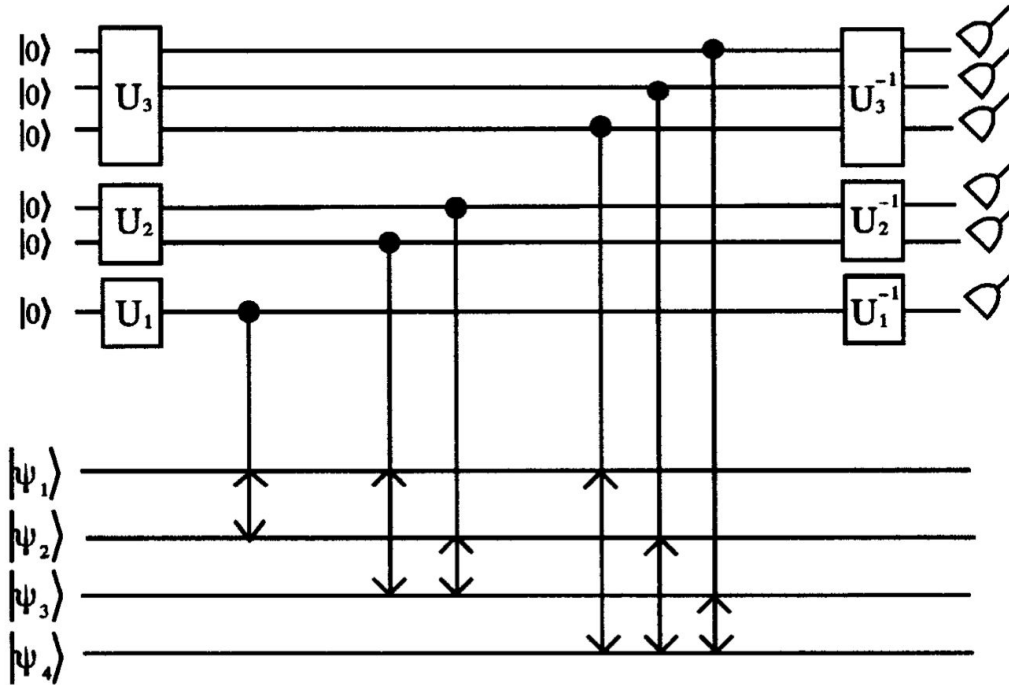


Measure 0 State =
Symmetric

Measure 1 State =
Anti-symmetric

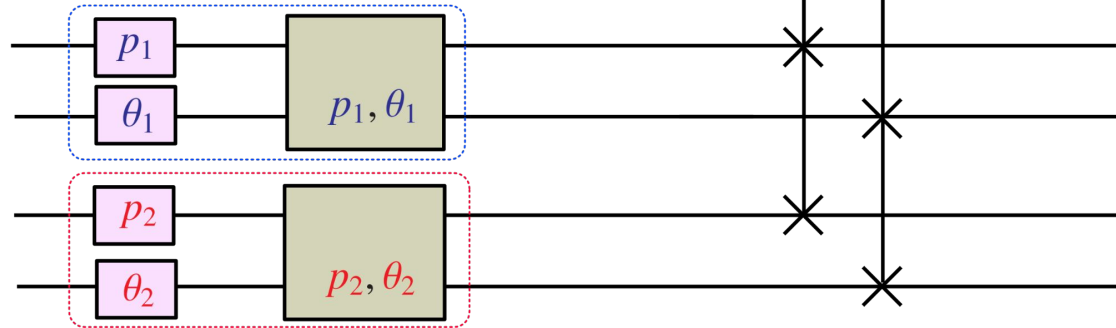
$$|\psi\rangle = \frac{1}{2} \left(|p_1\rangle|p_2\rangle + |p_2\rangle|p_1\rangle \right) |0\rangle + \frac{1}{2} \left(|p_1\rangle|p_2\rangle - |p_2\rangle|p_1\rangle \right) |1\rangle$$

Generalised Symmetrisation



Symmetrised Encoding

Particles Entangled Separately

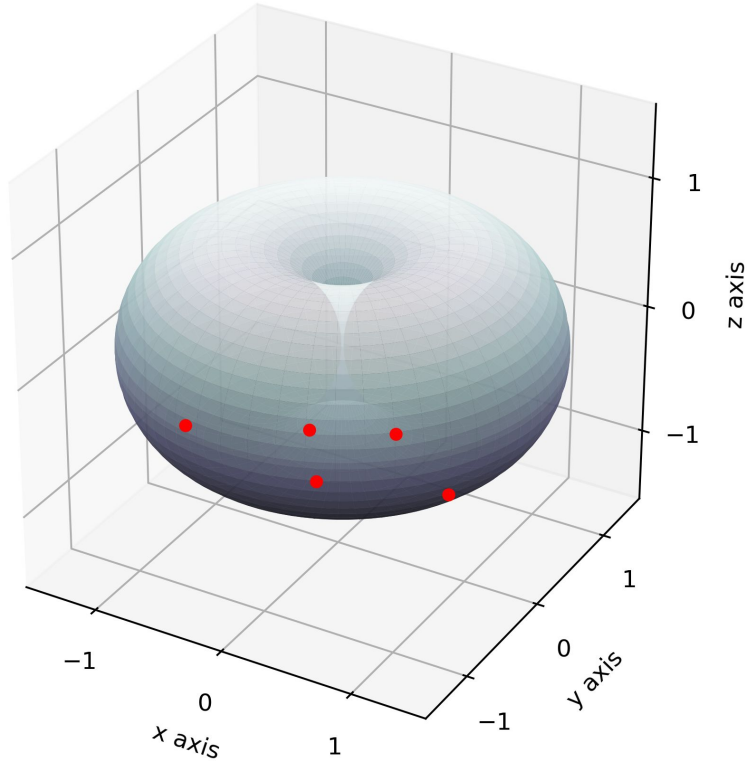


Symmetric Superposition State Created

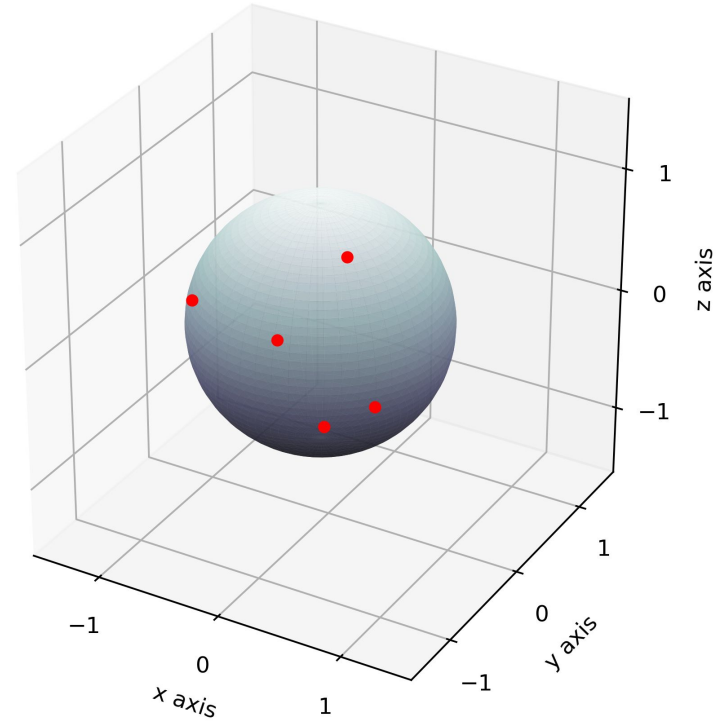
$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|p_1\rangle |p_2\rangle + |p_2\rangle |p_1\rangle \right)$$

Pointcloud Shape Classification Data

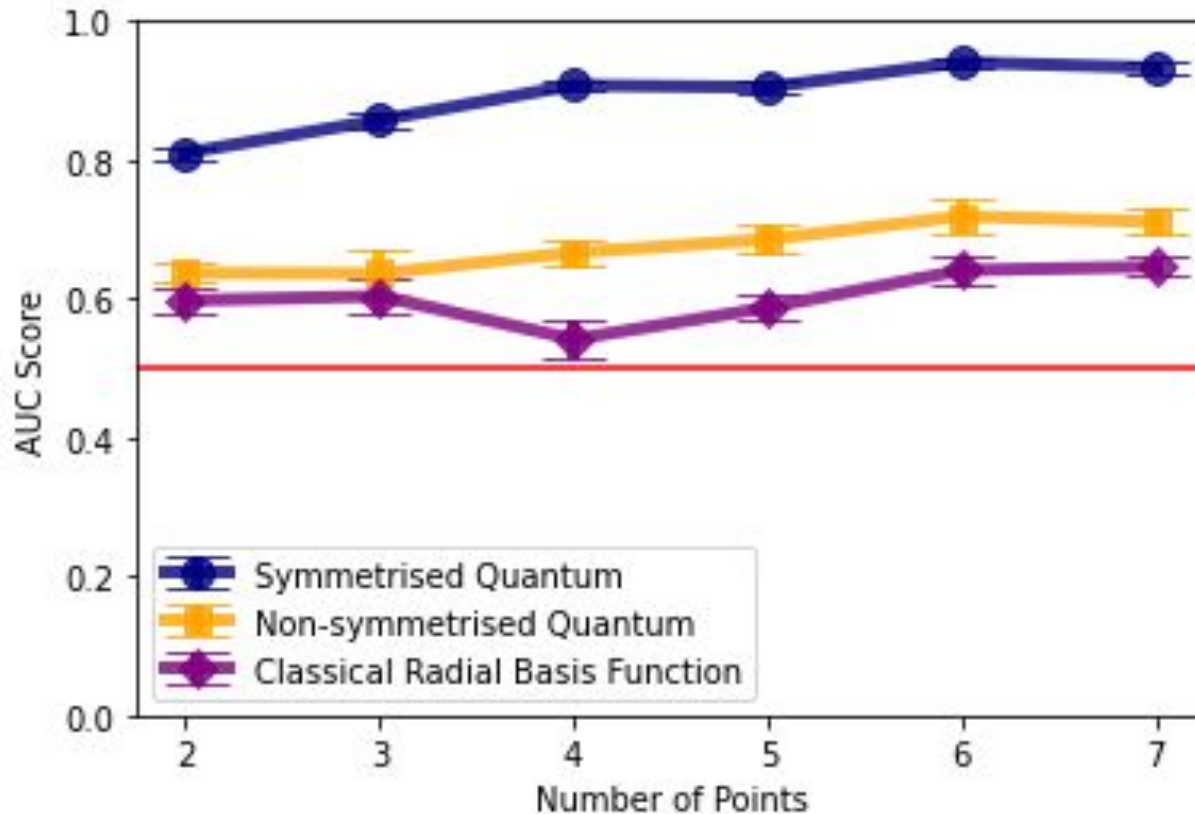
Torus Distribution



Sphere Distribution



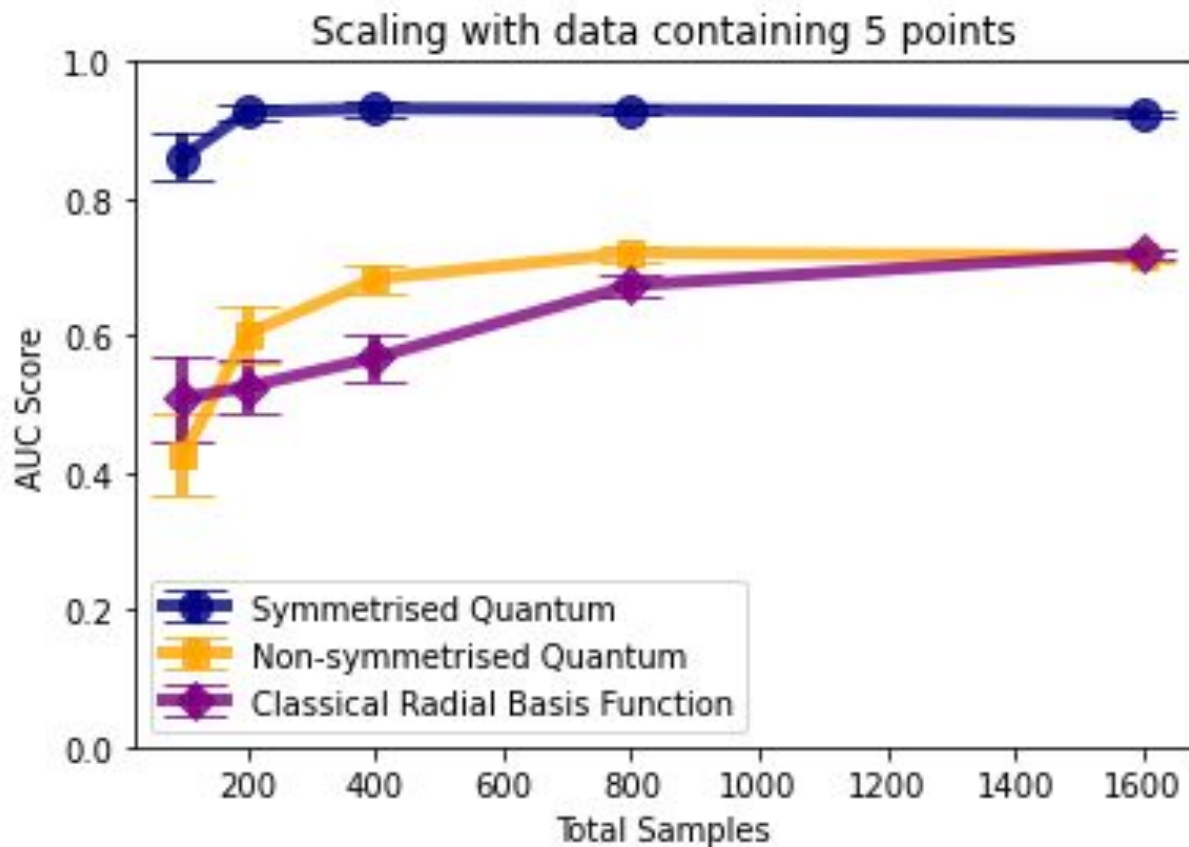
Scaling with Increased Points



Dataset size of 200

Average AUC over
10 randomised
experiments

Scaling with Dataset Size



Summary and Future Work

- Scaling in number on ancillary qubits is n^2
- Try improve the algorithm originally suggested (Have found a way for 3 points)
- Investigate whether anti-symmetrised data can still be useful
- Find approximately symmetrised methods through generic state preparation.

Encoding Circuit (3 point Pointclouds ,200 datasamples)	Average AUC Score (100 Experiments) (Error \pm 0.01)
Symmetrised QSVM	0.856
Separate Particle QSVM	0.579
Combinatorial QSVM	0.451
Classical Pointnet (Symmetrised Classical)	0.750
Classical RBF SVM	0.684

- [1] Vojtech Havlicek, Antonio D. C´orcoles, Kristan Temme, Aram W. Harrow, Abhinav Kandala, Jerry M. Chow, and Jay M. Gambetta. Supervised learning with quantum enhanced feature spaces, *Nature* volume 567, 209–212 (2019)
- [2] Heredge, Jamie & Hill, Charles & Hollenberg, Lloyd & Sevier, Martin. (2021). Quantum Support Vector Machines for Continuum Suppression in B Meson Decays. *Computing and Software for Big Science*. 5. 10.1007/s41781-021-00075-x.
- [3] Stabilisation of Quantum Computations by Symmetrisation, Adriano Barenco, Andre` Berthiaume, David Deutsch, Artur Ekert, Richard Jozsa, Chiara Macchiavello (1996)