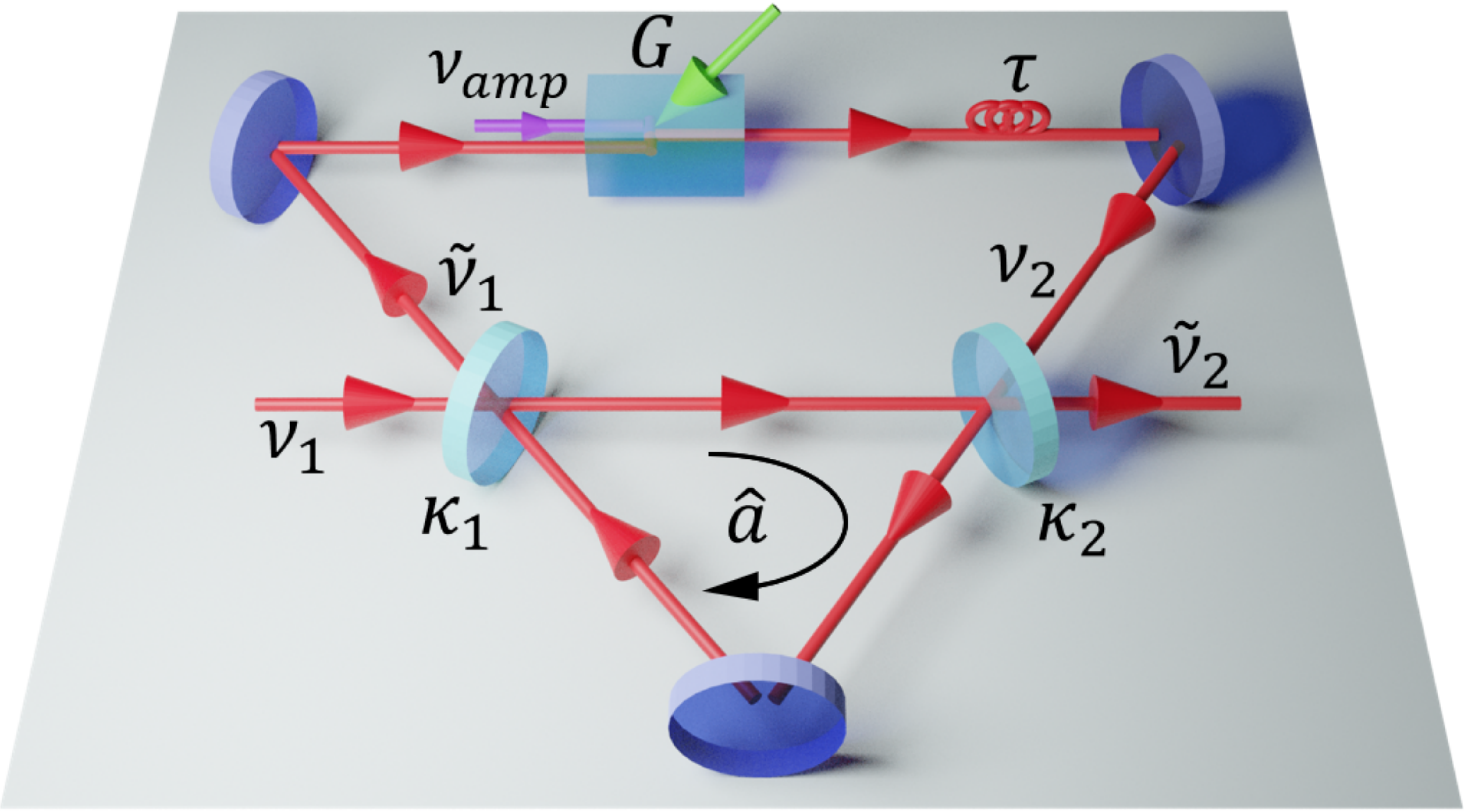
Quantum self-oscillation with time-delay feedback



OIST

Dr Yanan Liu

Griffith University

Prof. Bill Munro

NTT Japan

Prof. Jason Twamley

OIST Japan

Philosophical Magazine
Series 7
(1926-1955)

Taylor & Francis

The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science
Series 7

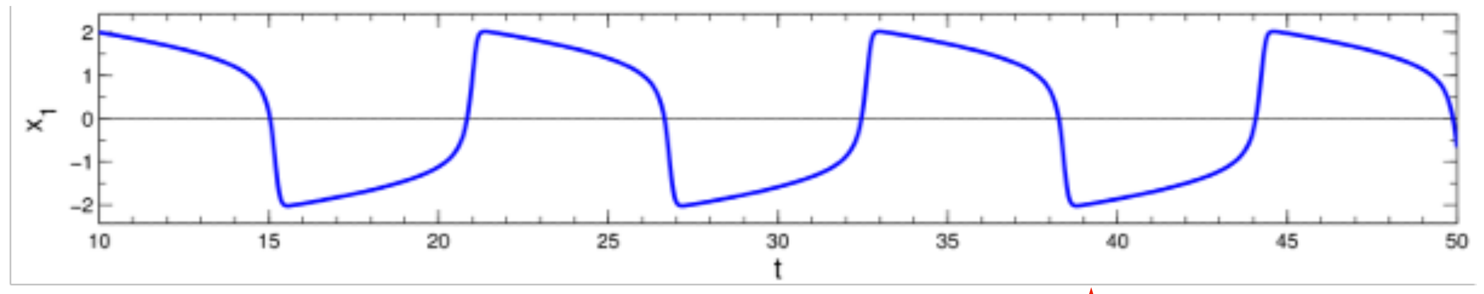
ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: https://www.tandfonline.com/loi/tphm18

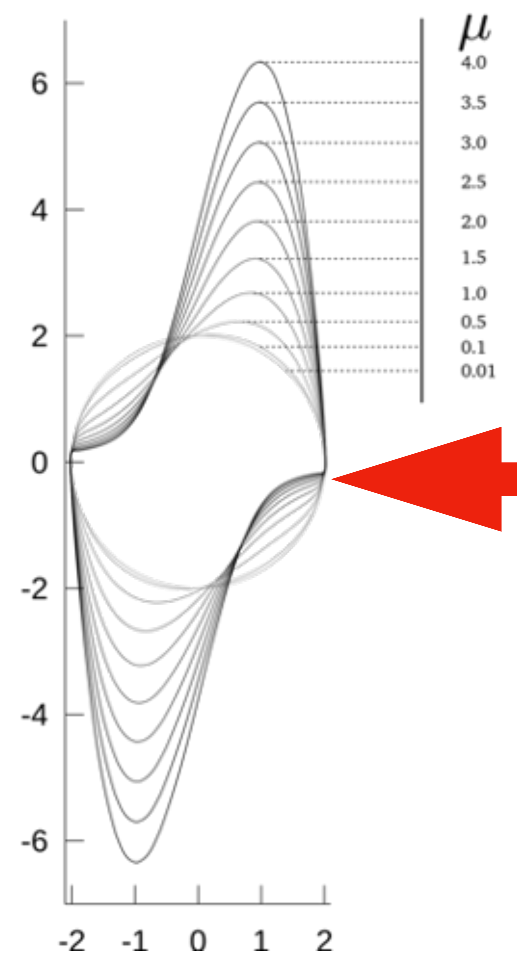
LXXXVIII. *On "relaxation-oscillations"*

Balth. van der Pol Jun. D.Sc

To cite this article: Balth. van der Pol Jun. D.Sc (1926) LXXXVIII. *On "relaxation-oscillations"* , The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 2:11, 978-992, DOI: 10.1080/14786442608564127

To link to this article: https://doi.org/10.1080/14786442608564127





$$\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + x = 0$$

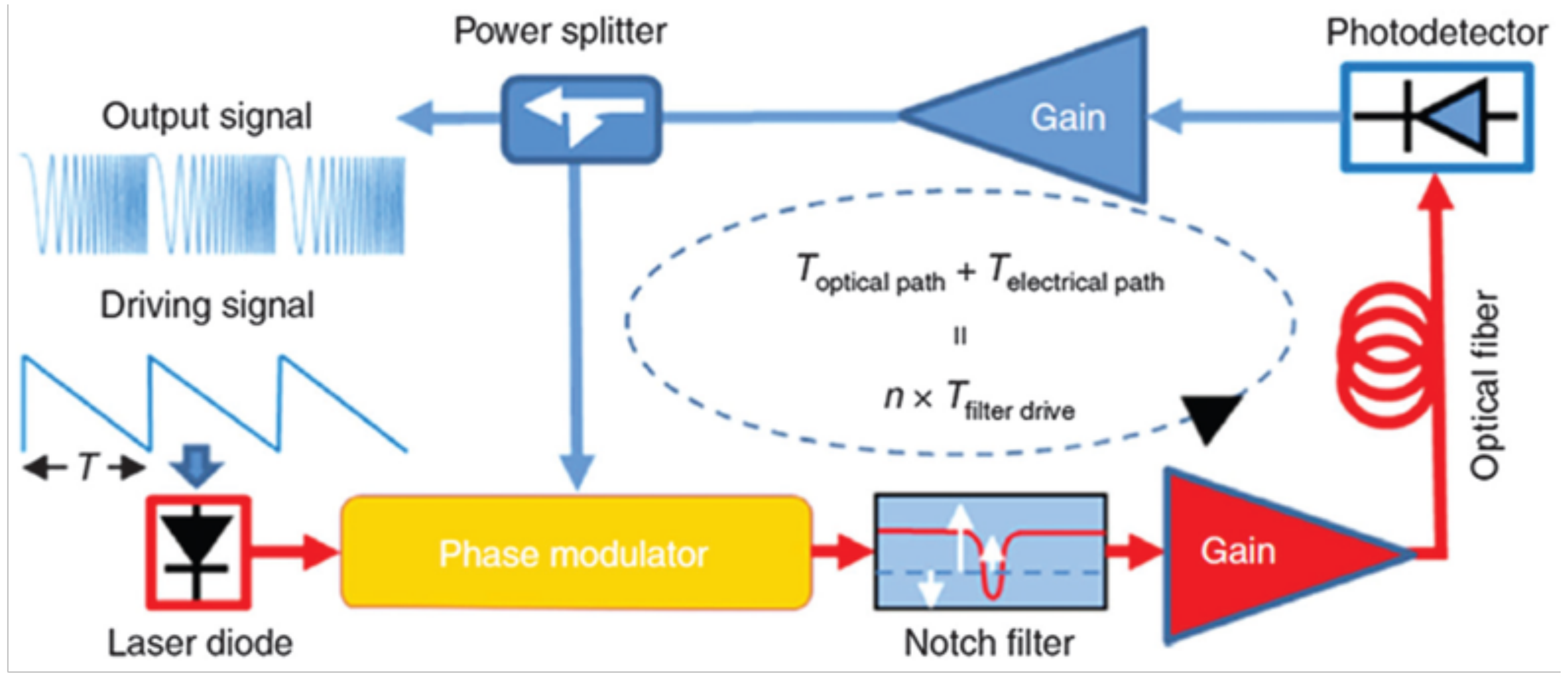
- ✓Clock
- ✓Communication link
- ✓Radar
- ✓Signal processing
- ✓Remote sensing

CLASSICAL SELF-OSCILLATORS--Generate a periodic oscillating signal

One popular way: Time-delay feedback

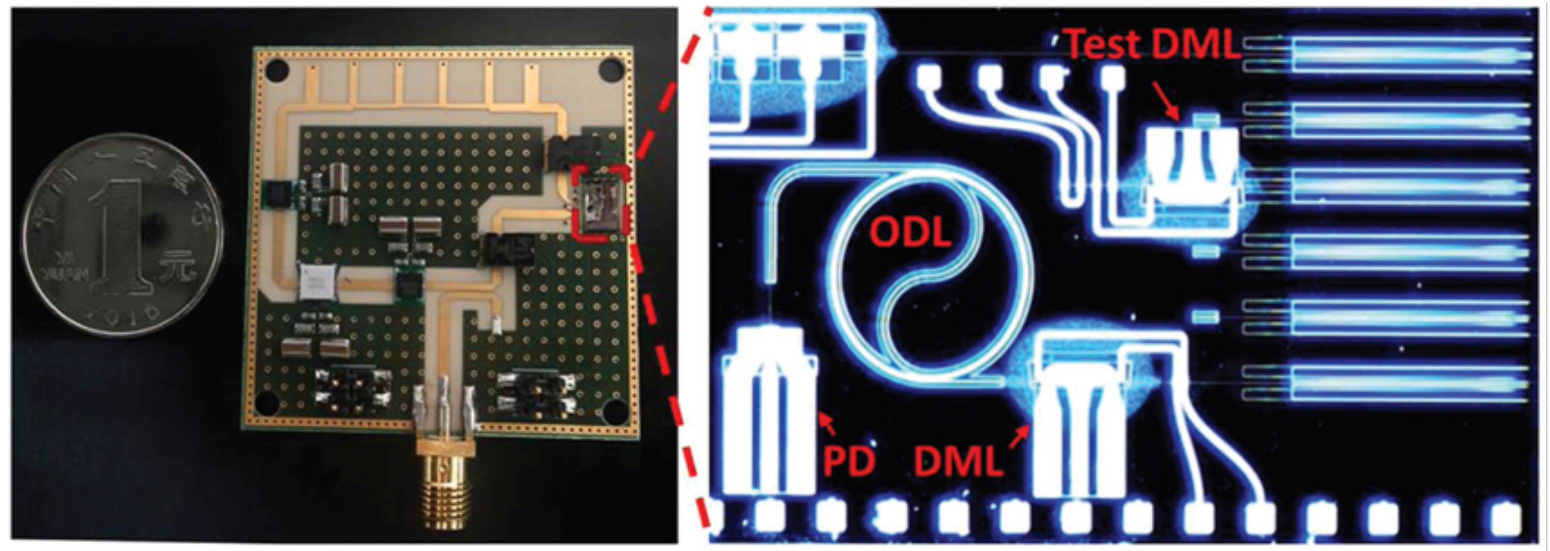
Optoelectronic oscillators (OEO) [2]

High-quality oscillator with less phase noise



Integrated OEO (Photonics integrated circuits) [3]

Low phase noise!!!



[1] van der Pol, Balth. "LXXXVIII. On "relaxation-oscillations"." The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 2.11 (1926): 978-992.
[2] Hao, Tengfei, et al. "Recent advances in optoelectronic oscillators." *Advanced Photonics* 2.4 (2020): 044001.
[3] Tang, Jian, et al. "Integrated optoelectronic oscillator." *Optics express* 26.9 (2018): 12257-12265.

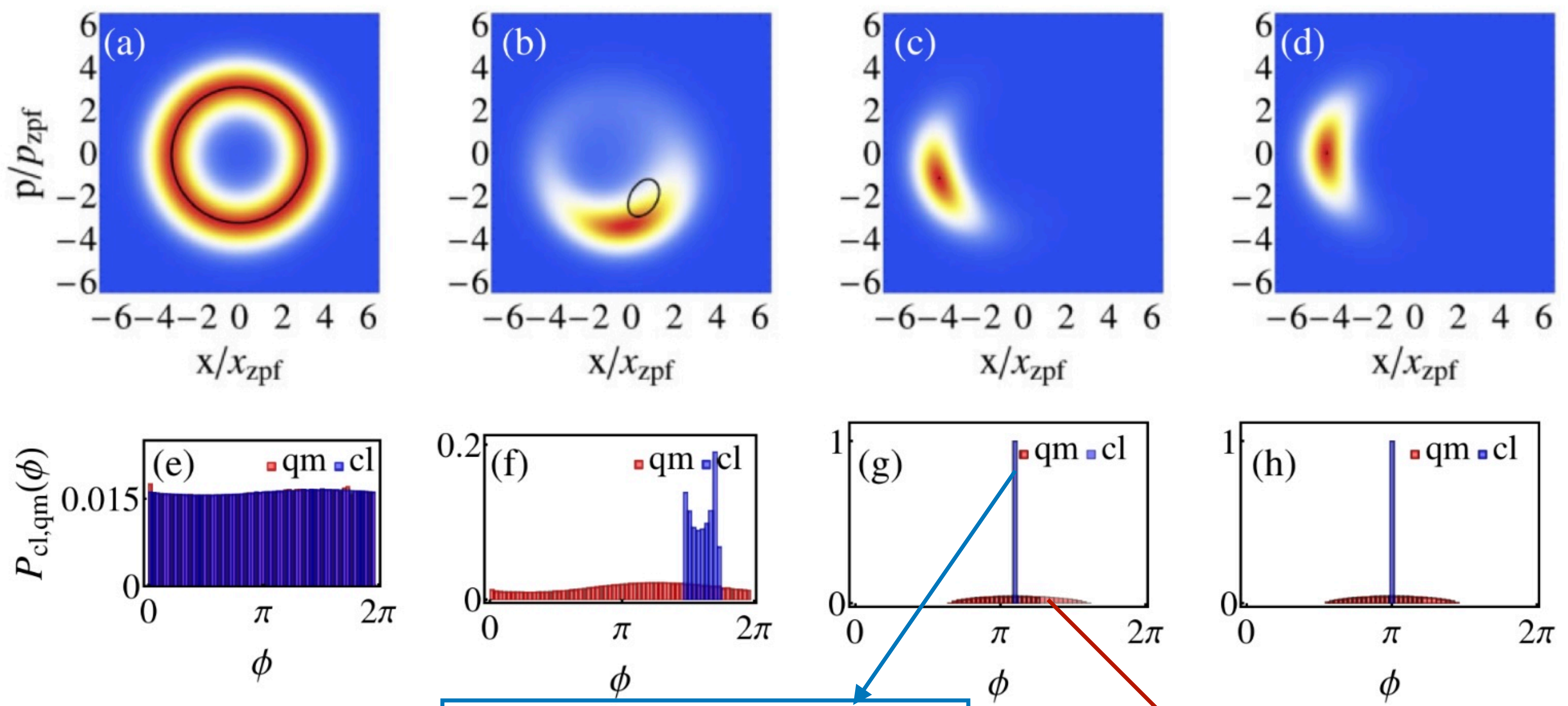
QUANTUM SELF-OSCILLATORS—With time delay

Quantum van der Pol Oscillator [1,2,3]

$$\dot{\rho} = -i[H,\rho] + \gamma_1 \mathscr{D}[\hat{b}^\dagger]\rho + \gamma_2 \mathscr{D}[\hat{b}^2]\rho$$

Negative damping (Gain)

Nonlinear damping



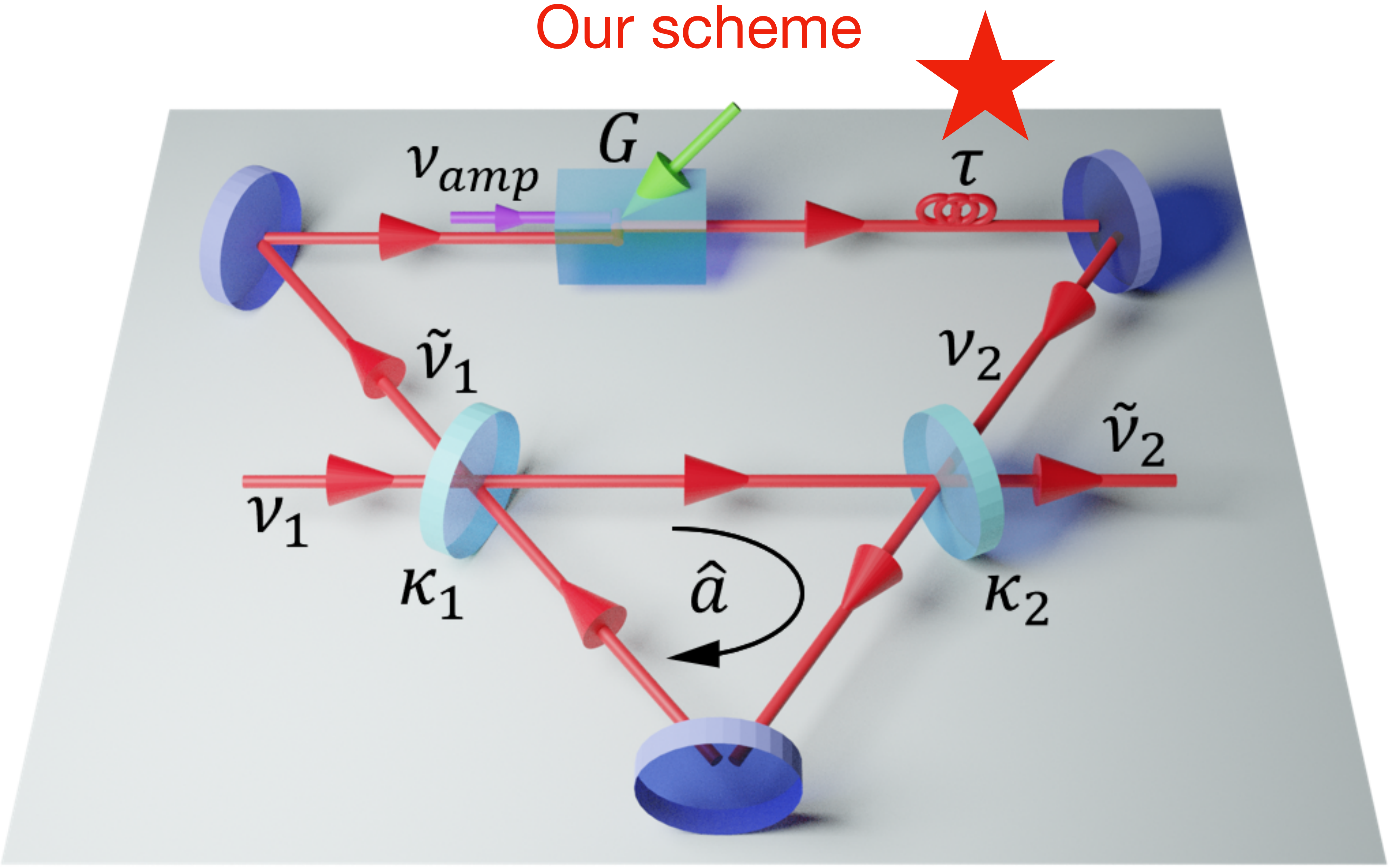
Classical phase locking

Quantum phase diffusion

PRL 112(9) 2014

$$H = -i\Delta\hat{b}^\dagger\hat{b} + i\Omega(\hat{b} - \hat{b}^\dagger)$$

Our scheme

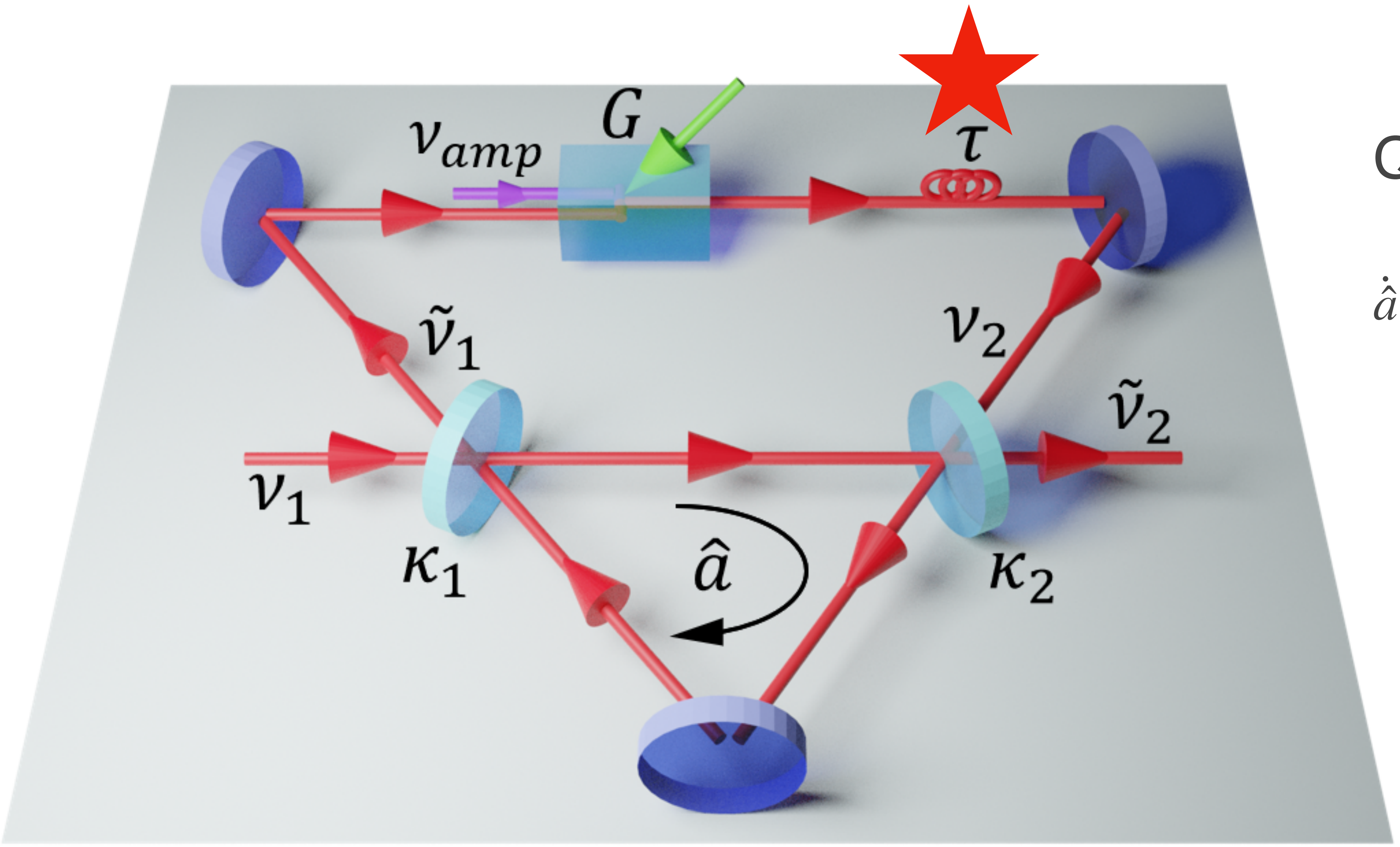


1: a ring cavity mode $\hat{a}$, two partially reflective mirrors with decay rates κ_1, κ_2

2: One output field $\tilde{\nu}_1$ is being amplified and delayed

3: amplified and delayed signal is fed back to cavity

[1] Dutta, S. and Cooper, N.R., 2019. Critical response of a quantum van der Pol oscillator. *Physical Review Letters*, *123*(25), p.250401.
[2] Lee, T.E. and Sadeghpour, H.R., 2013. Quantum synchronization of quantum van der Pol oscillators with trapped ions. *Physical review letters*, *111*(23), p.234101.
[3] Walter, S., Nunnenkamp, A. and Bruder, C., 2014. Quantum synchronization of a driven self-sustained oscillator. *Physical review letters*, *112*(9), p.094102.



Quantum Delay Differential Equation (DDE):

$$\dot{\hat{a}}(t) = -\kappa\hat{a}(t) - e^{i\phi}\sqrt{G\kappa_1\kappa_2}\hat{a}(t-\tau) - \sqrt{\kappa_1}\nu_1(t) - \sqrt{G\kappa_2}\nu_1(t-\tau) - e^{i\phi}\sqrt{(G-1)\kappa_2}\nu_{amp}(t-\tau)$$

Total decay rate: $\kappa = \dfrac{\kappa_1 + \kappa_2}{2}$

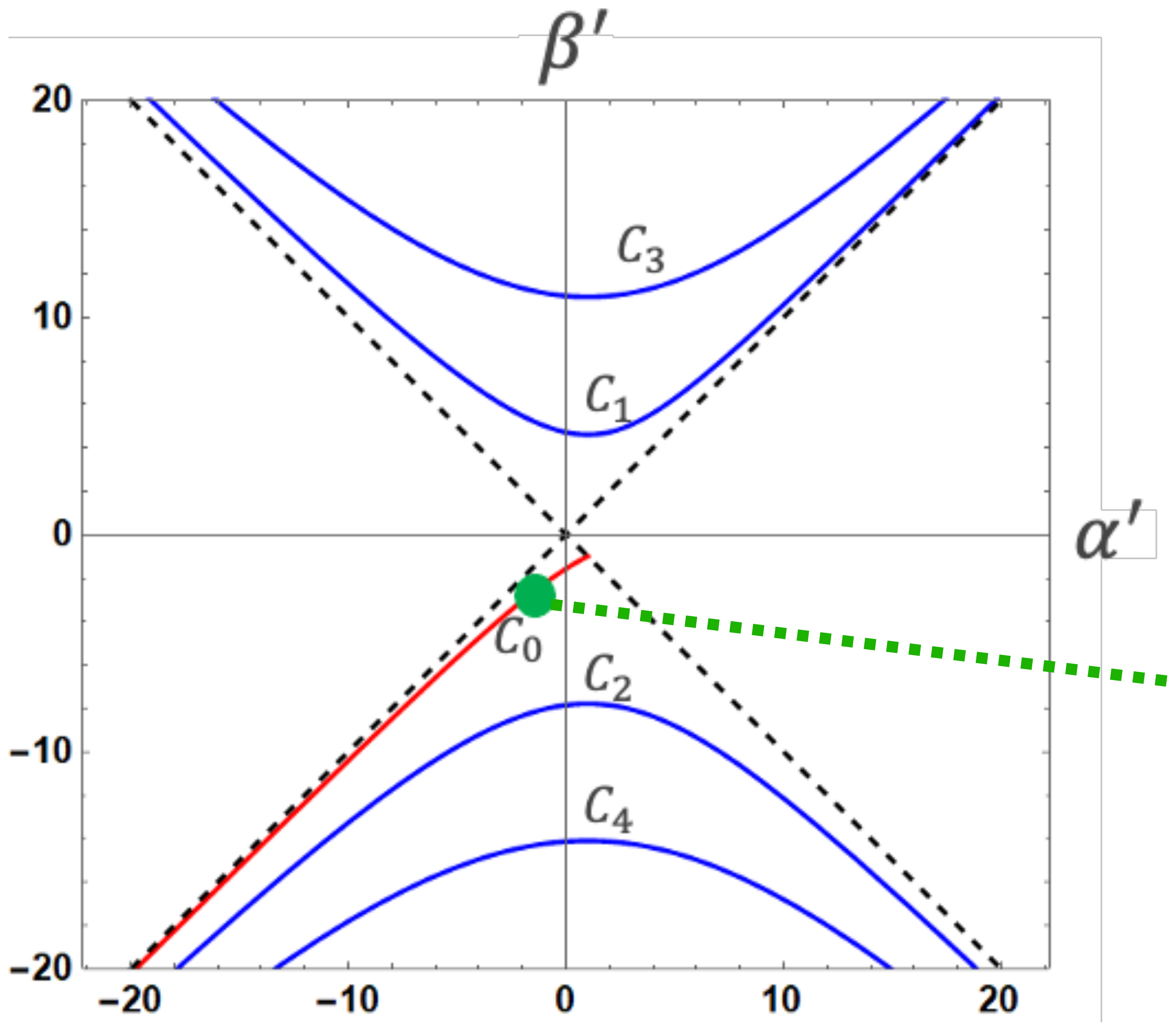
Amplification gain: G

Amplifier noise: ν_{amp}

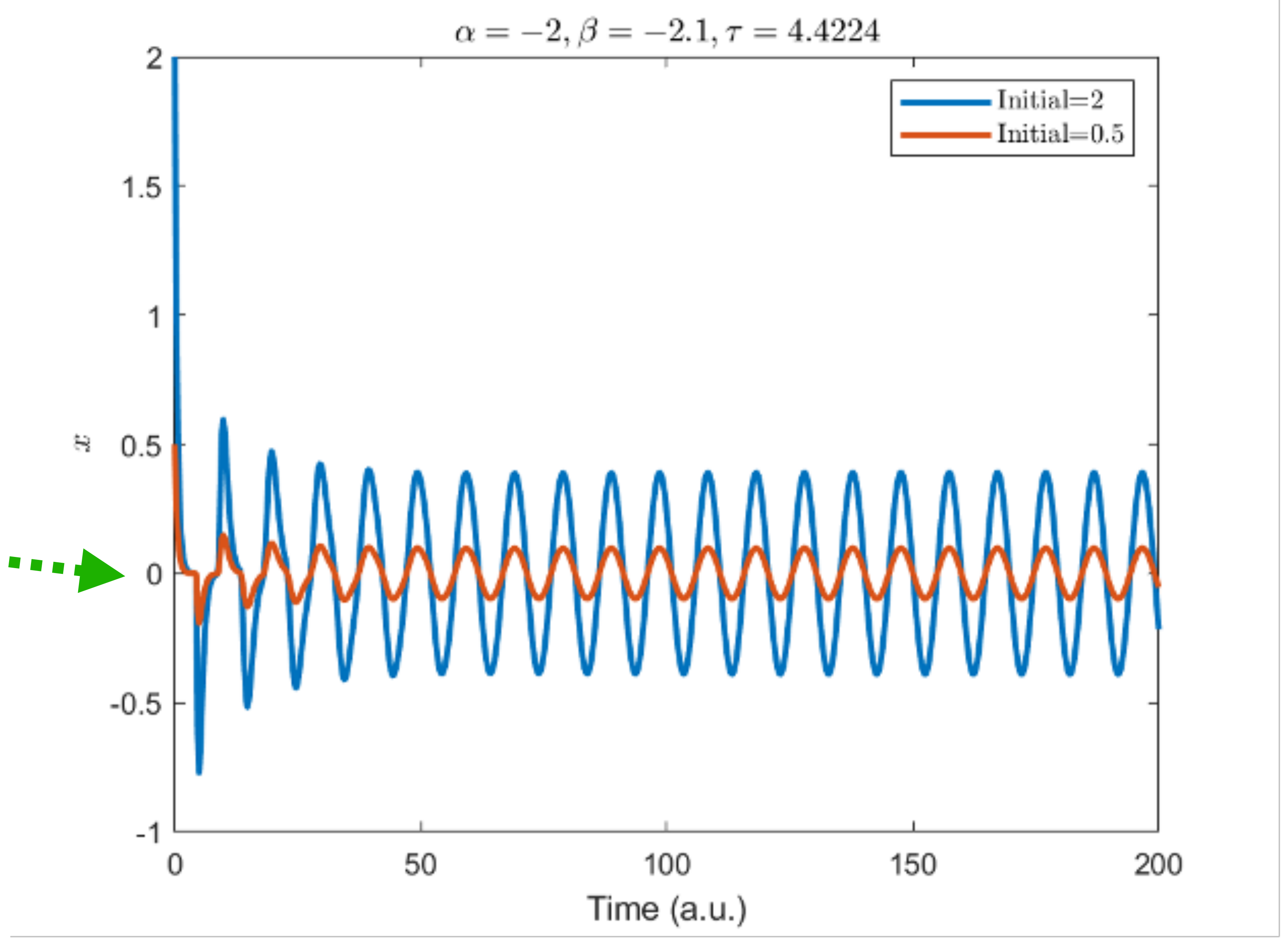
Compare with the classical DDE:

$$\dot{x} = \alpha x(t) + \beta x(t-\tau)$$

$$\langle \dot{\hat{a}}(t) \rangle = -\kappa \langle \hat{a}(t) \rangle - e^{i\phi}\sqrt{G\kappa_1\kappa_2}\langle \hat{a}(t-\tau) \rangle$$

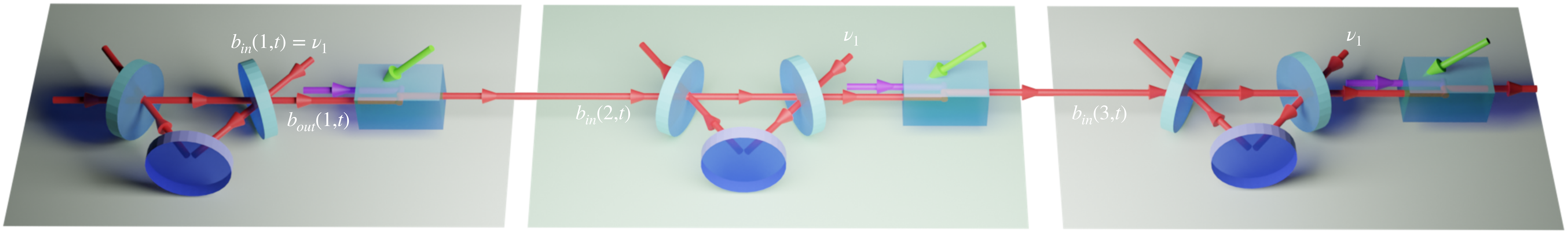


$\alpha' = \alpha\tau, \beta' = \beta\tau$



- Parameters on the red line
- Continuous oscillation

CASCADED THEORY—with amplifier in between



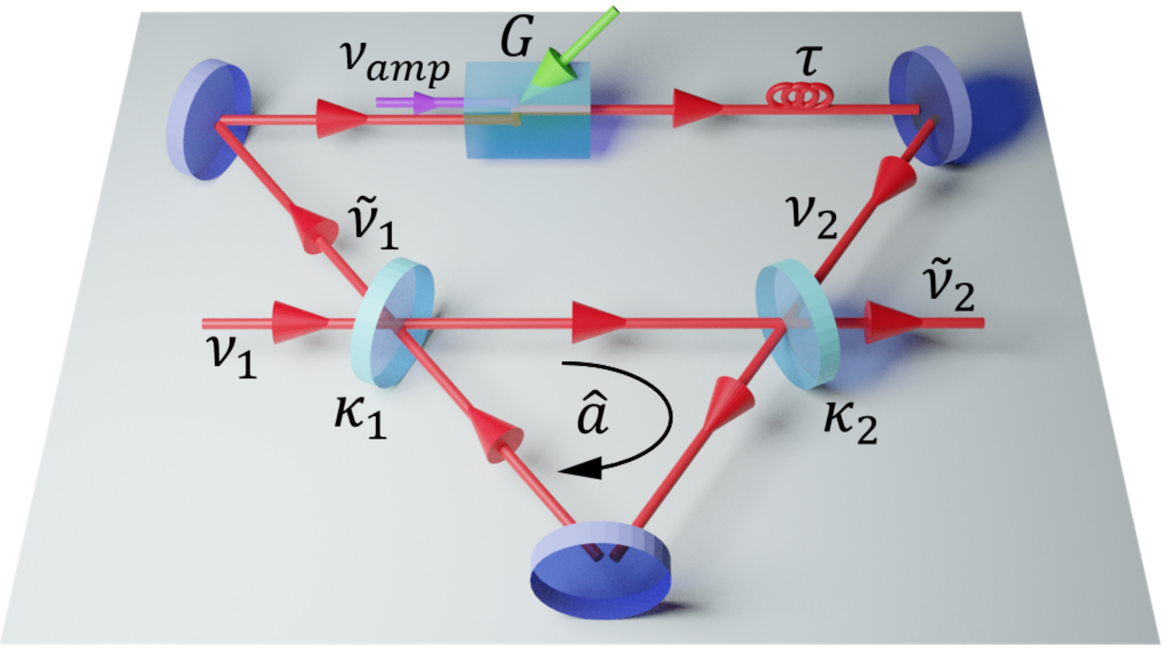
$t \in [0,\tau]$ $t \in [\tau, 2\tau]$ $t \in [2\tau, 3\tau]$

Vacuum fields
$\bar{N}_{n-m} = G - 1, \text{ for } n = 1,2,\cdots,m$

$$\frac{\rho(t)}{dt} = \mathscr{L}_m \rho(t), t \in [m\tau, (m+1)\tau]$$

Different from [1], unless $G = 1$

$$\mathscr{L}_m \rho = (\kappa_1 + \kappa_2) \sum_{n=0}^{m} \mathscr{D}[\hat{a}_n]\rho + \sum_{n=0}^{m-1} \kappa_1 \bar{N}_{n-m} \left(\mathscr{D}[\hat{a}_n]\rho + \mathscr{D}[\hat{a}_n^\dagger]\rho \right) - \sqrt{G\kappa_1\kappa_2} \sum_{n=1}^{m} \left\{ [\hat{a}_{n-1}^\dagger, \hat{a}_n \rho] + [\rho \hat{a}_n^\dagger, \hat{a}_{n-1}] \right\}$$



[1] Whalen, S., 2015. Open quantum systems with time-delayed interactions (Doctoral dissertation, ResearchSpace@ Auckland).

$$\frac{\rho(t)}{dt} = \mathscr{L}_m \rho(t), t \in [m\tau, (m+1)\tau]$$

$$\mathscr{L}_m \rho = (\kappa_1 + \kappa_2) \sum_{n=0}^{m} \mathscr{D}[\hat{a}_n]\rho + \sum_{n=0}^{m-1} \kappa_1 \bar{N}_{n-m} \left(\mathscr{D}[\hat{a}_n]\rho + \mathscr{D}[\hat{a}_n^\dagger]\rho \right) - \sqrt{G\kappa_1\kappa_2} \sum_{n=1}^{m} \left\{ [\hat{a}_{n-1}^\dagger, \hat{a}_n \rho] + [\rho \hat{a}_n^\dagger, \hat{a}_{n-1}] \right\}$$

Calculate the ODEs of mean field $\langle \hat{a}_0 \rangle$

In $[0,\tau]$,

$$\langle \dot{\hat{a}}_0 \rangle = -\kappa \langle \hat{a}_0 \rangle$$

In $[\tau, 2\tau]$,

$$\langle \dot{\hat{a}}_0 \rangle = -\kappa \langle \hat{a}_0 \rangle - \eta \langle \hat{a}_1 \rangle$$
$$\langle \dot{\hat{a}}_1 \rangle = -\kappa \langle \hat{a}_1 \rangle$$

$$\vdots$$

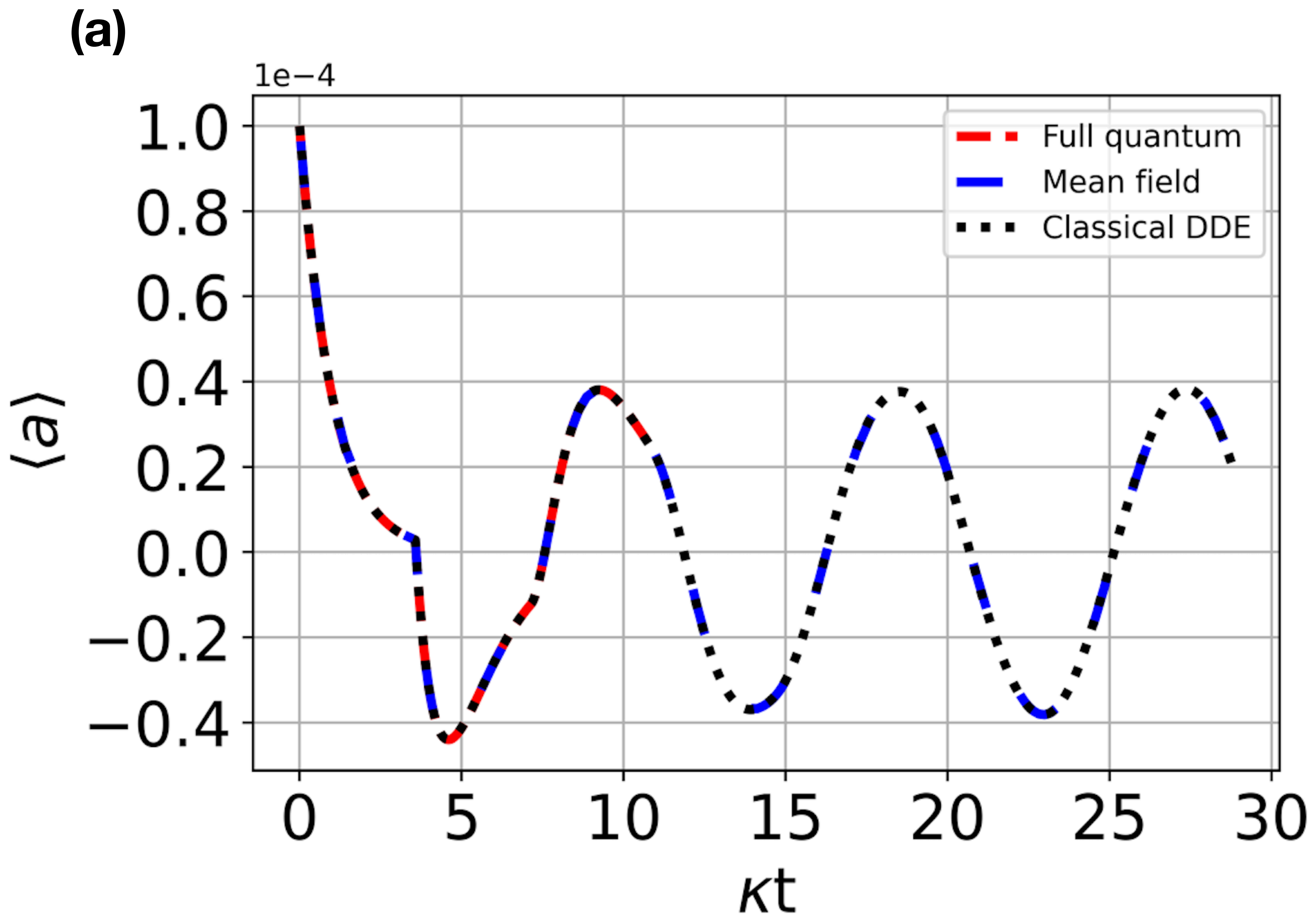
In $[i\tau, (i+1)\tau]$,

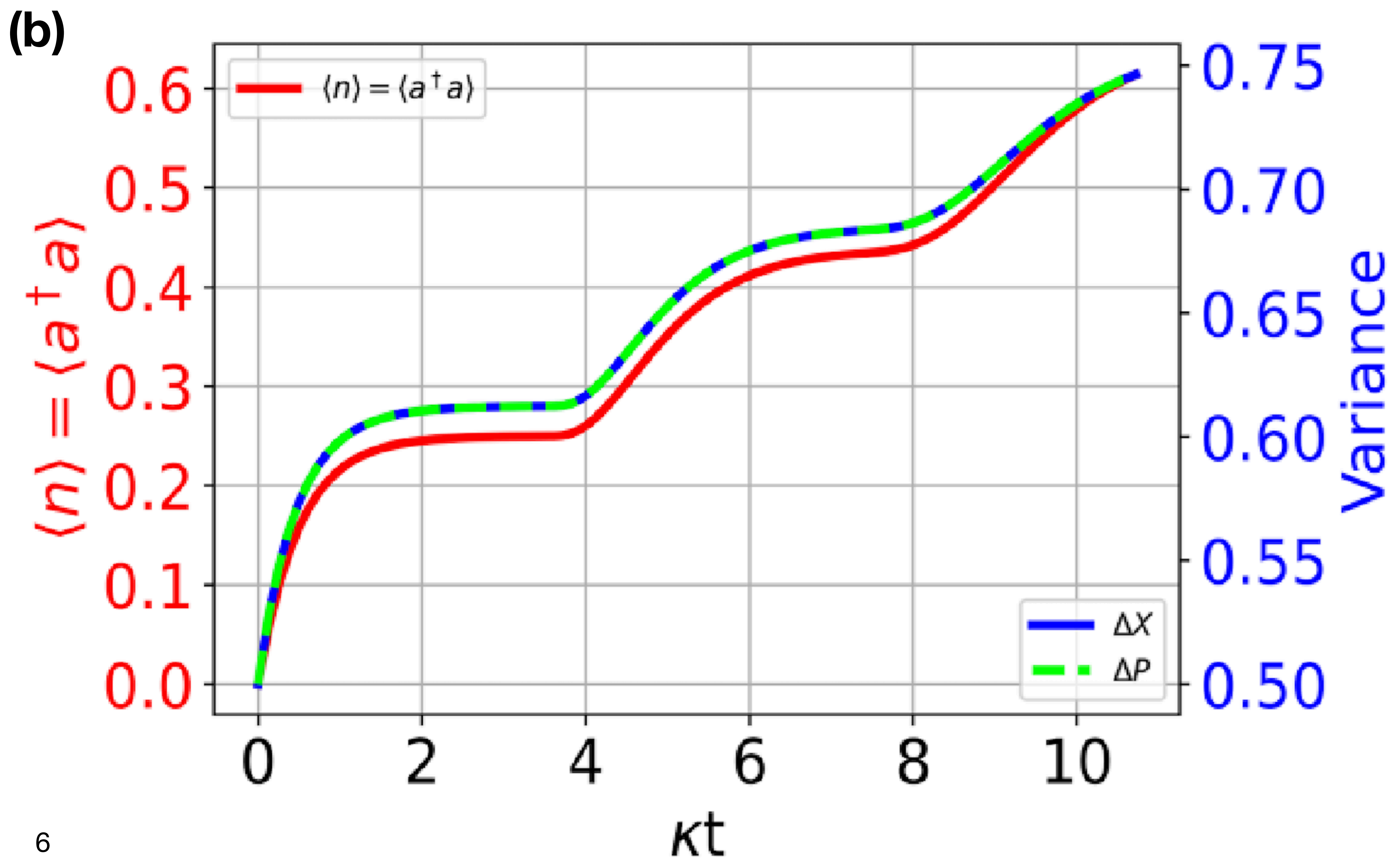
$$\langle \dot{\hat{a}}_0 \rangle = -\kappa \langle \hat{a}_0 \rangle - \eta \langle \hat{a}_1 \rangle$$
$$\langle \dot{\hat{a}}_1 \rangle = -\kappa \langle \hat{a}_1 \rangle - \eta \langle \hat{a}_2 \rangle$$
$$\vdots$$
$$\langle \dot{\hat{a}}_{m-1} \rangle = -\kappa \langle \hat{a}_{m-1} \rangle - \eta \langle \hat{a}_m \rangle$$
$$\langle \dot{\hat{a}}_m \rangle = -\kappa \langle \hat{a}_m \rangle$$

1. $\langle \hat{a} \rangle$ shows perfect oscillation
2. Energy (photon number) increases infinitely
3. Get self-oscillation without phase diffusion

Initial state $\rho = \underbrace{\rho_s \otimes \rho_s \otimes \cdots \rho_s}_{m \text{ times}}$

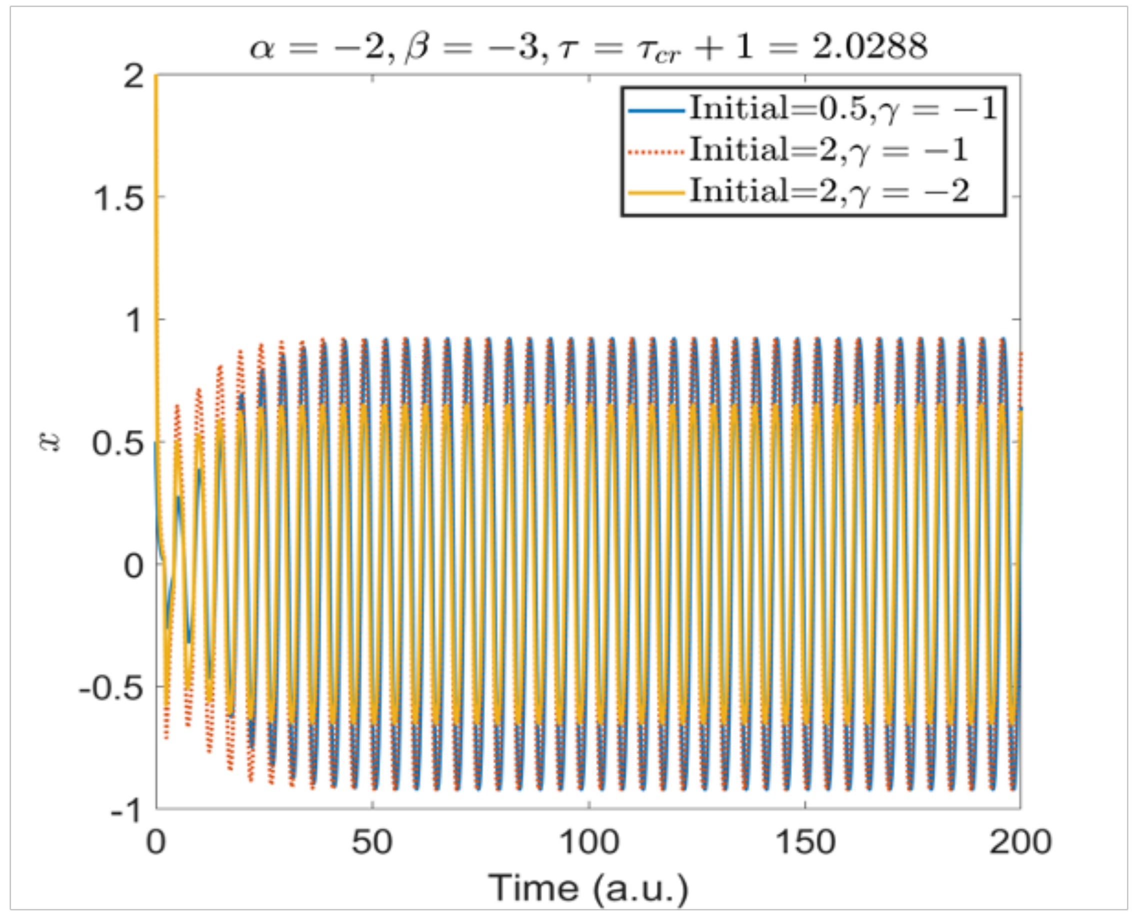
Annihilation operator $\hat{a}_n = \underbrace{I \otimes I \otimes \cdots \otimes}_{n \text{ times}} \hat{a} \otimes \underbrace{I \otimes \cdots \otimes I}_{m-n-1 \text{ times}}$

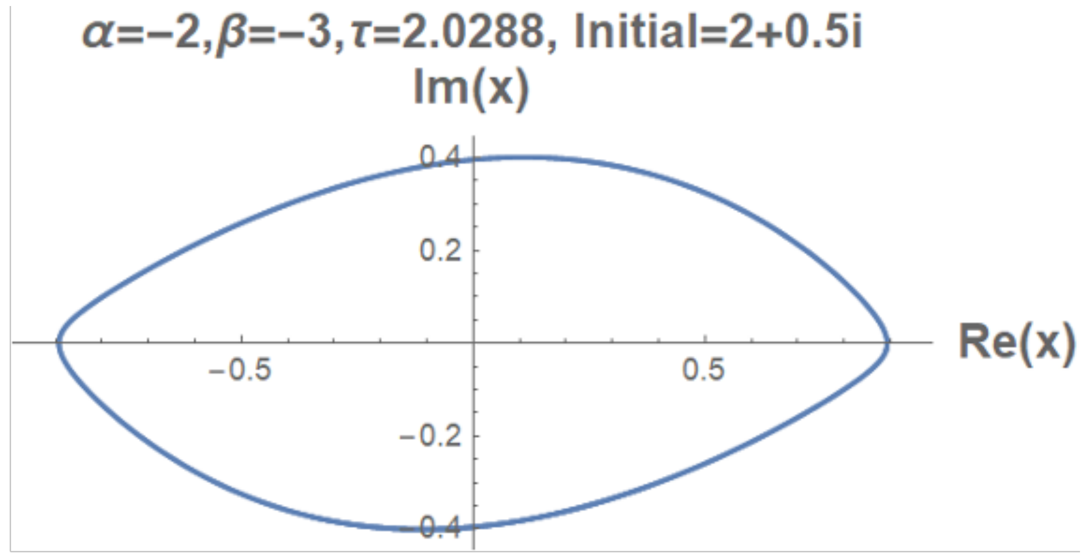




Nonlinear quantum oscillators

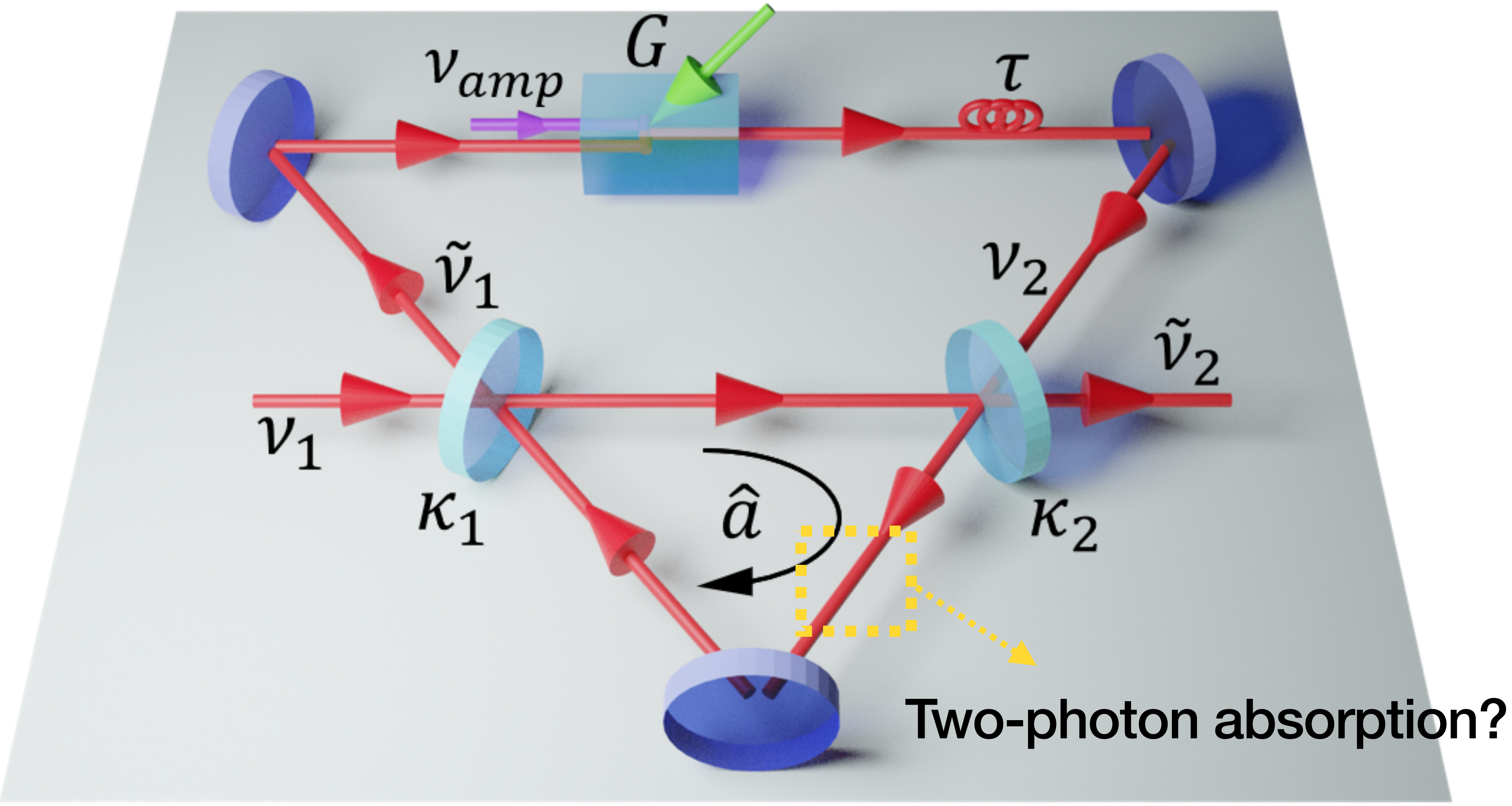
Reason from nonlinear DDE: $\dot{x} = \alpha x(t) + \beta x(t-\tau) + \gamma x^3(t)$





Advantages:

- Nonlinear damping rate γ can tune the oscillating amplitude
- Limit cycle in phase space
- Can have relaxation oscillation



This two-photon absorption has the ability to absorb two photons from the cavity at the same time, thereby bringing a nonlinear dissipation to the cavity dynamics

$$\frac{\rho(t)}{dt} = \mathcal{L}_m \rho(t), t \in [m\tau, (m+1)\tau]$$

$$\mathcal{L}_m \rho = \gamma \sum_{n=0}^{m} \mathcal{D}[\hat{a}^2]\rho + (\kappa_1 + \kappa_2) \sum_{n=0}^{m} \mathcal{D}[\hat{a}_n]\rho + \sum_{n=0}^{m-1} \kappa_1 \bar{N}_{n-m} \left(\mathcal{D}[\hat{a}_n]\rho + \mathcal{D}[\hat{a}_n^\dagger]\rho \right) - \sqrt{G\kappa_1\kappa_2} \sum_{n=1}^{m} \left\{ [\hat{a}_{n-1}^\dagger, \hat{a}_n\rho] + [\rho\hat{a}_n^\dagger, \hat{a}_{n-1}] \right\}$$

Mean-field simulation for nonlinear quantum oscillators

$$\frac{\rho(t)}{dt} = \mathscr{L}_m \rho(t), t \in [m\tau, (m+1)\tau]$$

$$\mathscr{L}_m \rho = \boxed{\gamma \sum_{n=0}^{m} \mathscr{D}[\hat{a}^2]\rho} + (\kappa_1 + \kappa_2) \sum_{n=0}^{m} \mathscr{D}[\hat{a}_n]\rho + \sum_{n=0}^{m-1} \kappa_1 \bar{N}_{n-m} \left(\mathscr{D}[\hat{a}_n]\rho + \mathscr{D}[\hat{a}_n^\dagger]\rho \right) - \sqrt{G\kappa_1\kappa_2} \sum_{n=1}^{m} \left\{ [\hat{a}_{n-1}^\dagger, \hat{a}_n \rho] + [\rho \hat{a}_n^\dagger, \hat{a}_{n-1}] \right\}$$

Equations of mean-field

In $[0, \tau]$,

5 Eqns

$$\langle \dot{a_0} \rangle = -\gamma \langle a_0 \rangle - 2\chi \langle a_0^\dagger a_0 a_0 \rangle$$

$$\left\langle a_0^\dagger \dot{a_0} a_0 \right\rangle = (-3\gamma - 2\chi) \langle a_0^\dagger a_0 a_0 \rangle - 6\chi \langle a_0^\dagger a_0^\dagger a_0 a_0 a_0 \rangle$$

$$\langle a_0^\dagger a_0^\dagger a_0 a_0 a_0 \rangle \to \langle a_0 \rangle, \langle a_0^\dagger \rangle, \langle a_0 a_0 \rangle, \langle a_0^\dagger a_0 \rangle, \langle a_0^\dagger a_0^\dagger \rangle, \langle a_0 a_0 a_0 \rangle, \langle a_0^\dagger a_0 a_0 \rangle, \langle a_0^\dagger a_0^\dagger a_0 \rangle$$

$\vdots$

In $[\tau, 2\tau]$,

18 Eqns

$$\langle \dot{a_0} \rangle = -\gamma \langle a_0 \rangle - 2\chi \langle a_0^\dagger a_0 a_0 \rangle - \eta \langle a_1 \rangle$$

$$\langle \dot{a_1} \rangle = -\gamma \langle a_1 \rangle - 2\chi \langle a_1^\dagger a_1 a_1 \rangle$$

$$\left\langle a_0^\dagger \dot{a_0} a_0 \right\rangle = (-3\gamma - 2\chi) \langle a_0^\dagger a_0 a_0 \rangle - 6\chi \langle a_0^\dagger a_0^\dagger a_0 a_0 a_0 \rangle$$

$$\langle a_1 \dot{a_0^\dagger} a_0 \rangle = -3\gamma a_1 a_0^\dagger a_0 - 4\chi a_1 a_0^\dagger a_0^\dagger a_0 a_0 - 2\chi a_1^\dagger a_1 a_1 a_0^\dagger a_0 - \eta a_1 a_1 a_0^\dagger - \eta a_1^\dagger a_1 a_0$$

$\vdots$

$$\langle a_0^\dagger a_0^\dagger a_0 a_0 a_0 \rangle \to \langle a_0 \rangle, \langle a_0^\dagger \rangle, \langle a_0 a_0 \rangle, \langle a_0^\dagger a_0 \rangle, \langle a_0^\dagger a_0^\dagger \rangle, \langle a_0 a_0 a_0 \rangle, \langle a_0^\dagger a_0 a_0 \rangle, \langle a_0^\dagger a_0^\dagger a_0 \rangle$$

$\vdots$

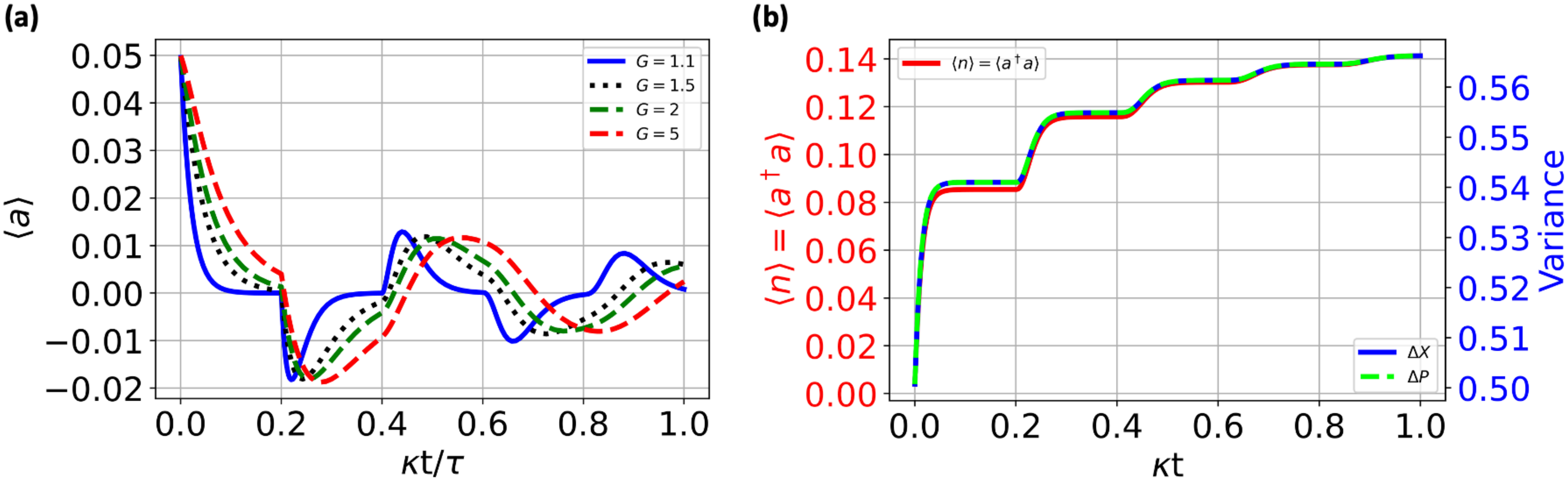
Becomes expensive than Linear case!

But still cheaper than full quantum dimension calculation!

Polynomial of time periods and truncated order

[1] Plankensteiner, David, Christoph Hotter, and Helmut Ritsch. *Quantum* 6 (2022): 617.

Nonlinear quantum oscillators



- $\langle \hat{a} \rangle$ shows a decay oscillation with time, this is more likely due to the phase diffusion of quantum nonlinear damping $\langle \hat{a}^\dagger \hat{a} \hat{a} \rangle$, which comes from the two-photon absorption.

We guess this is due to the difference between two equations:

$$\langle \dot{\hat{a}} \rangle = -\kappa \langle \hat{a} \rangle - \sqrt{G\kappa_1\kappa_2} \langle \hat{a}(t-\tau) \rangle - \gamma \langle \hat{a}^\dagger \hat{a} \hat{a} \rangle, \ \langle \hat{a}^\dagger \hat{a} \hat{a} \rangle \neq \langle \hat{a}^\dagger \rangle \langle \hat{a} \rangle \langle \hat{a} \rangle$$

We could achieve non-decay self-oscillation without phase diffusion in linear system but not in nonlinear system

Conclusions

- A linear quantum self-oscillator with time delay feedback can generate a perfect periodic oscillating signal
- Cascaded theory is generalised to the cases with feedback gain
- A mean-field calculation is used to do simulations for longer time
- Is that possible to have a nonlinear quantum self-oscillator without phase diffusion?