THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

CENTRE FOR
QUANTUM COMPUTATION
& COMMUNICATION
TECHNOLOGY

Joshua Guanzon,
Austin Lund, Timothy Ralph,
Matthew Winnel, Deepesh Singh,
Nedasadat Hosseinidehaj

Quantum Amplification using Linear Optical Tools

Noiseless Linear Amplifier

Highest Quality Quantum Amplifier

Noiseless = ???

Linear = ???

Amplifier = increases amplitude.

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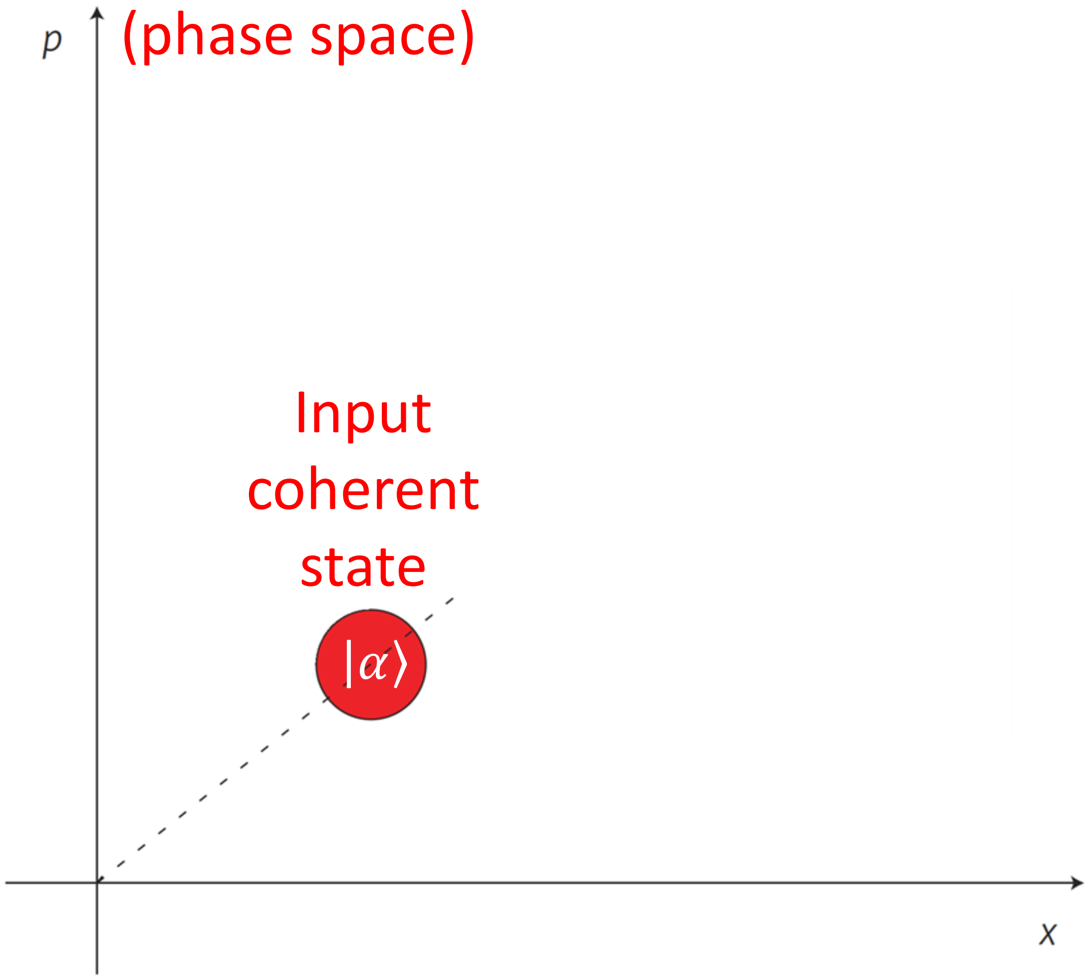

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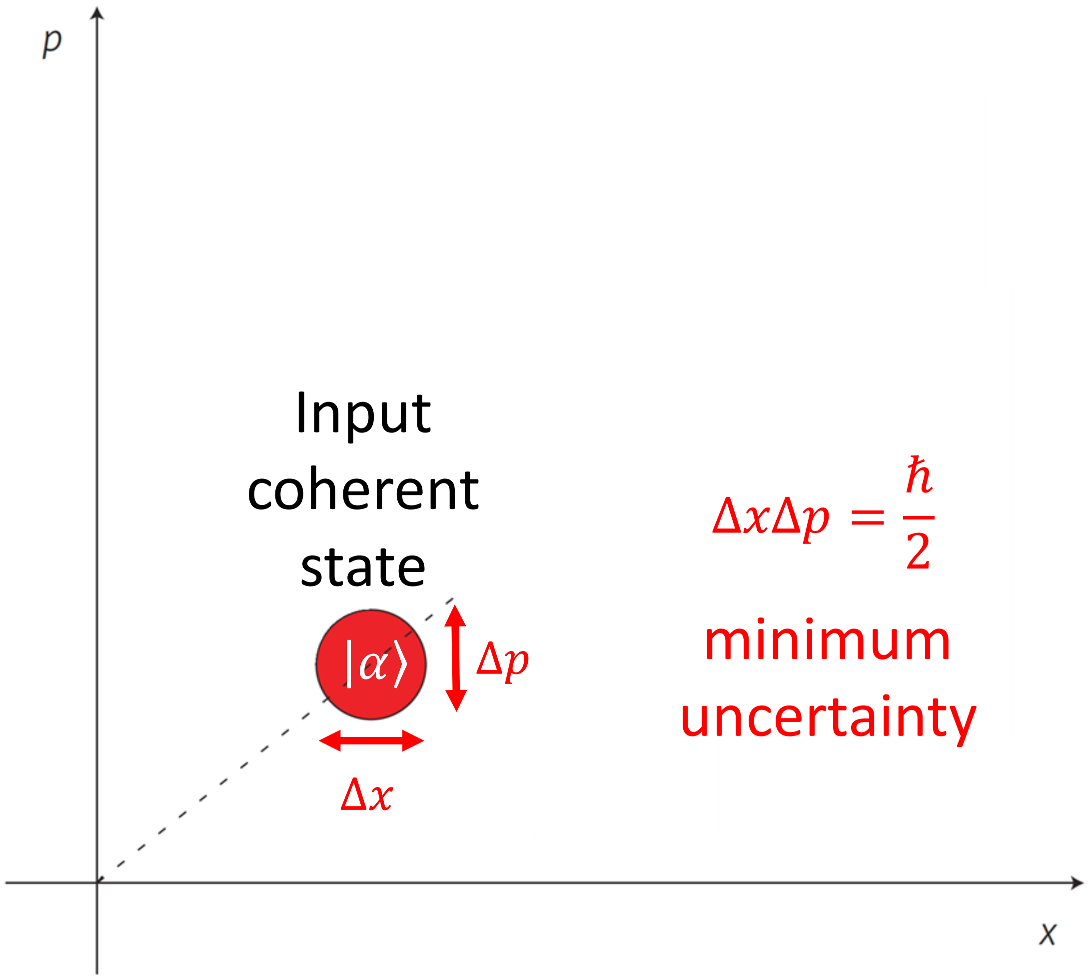

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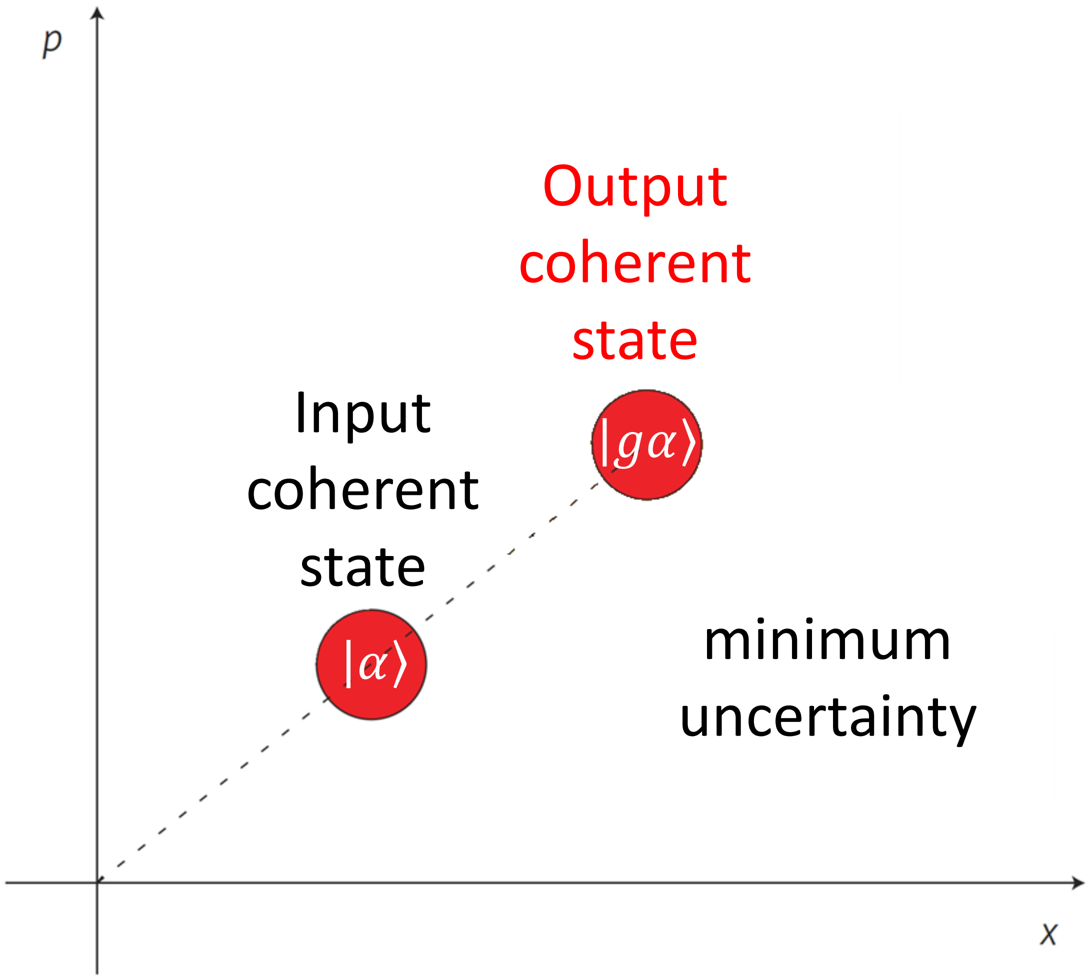

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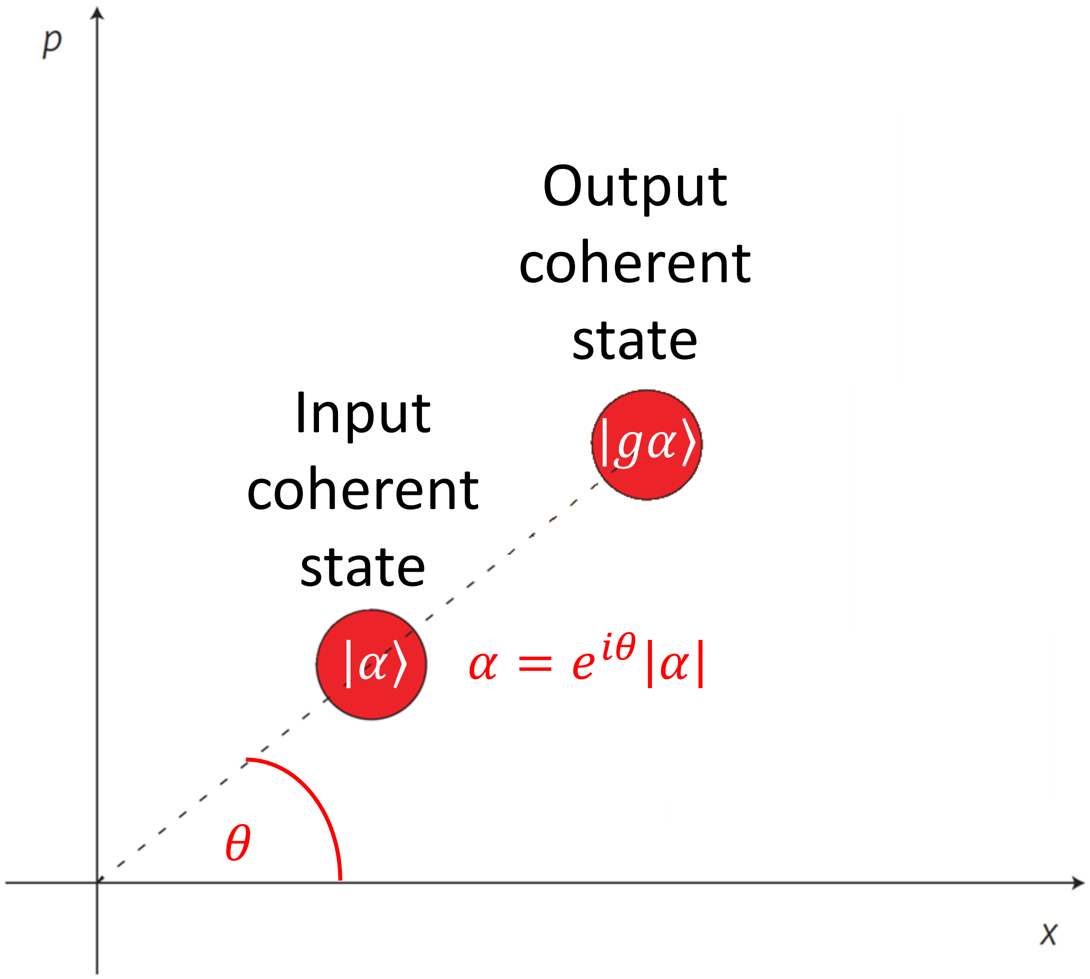

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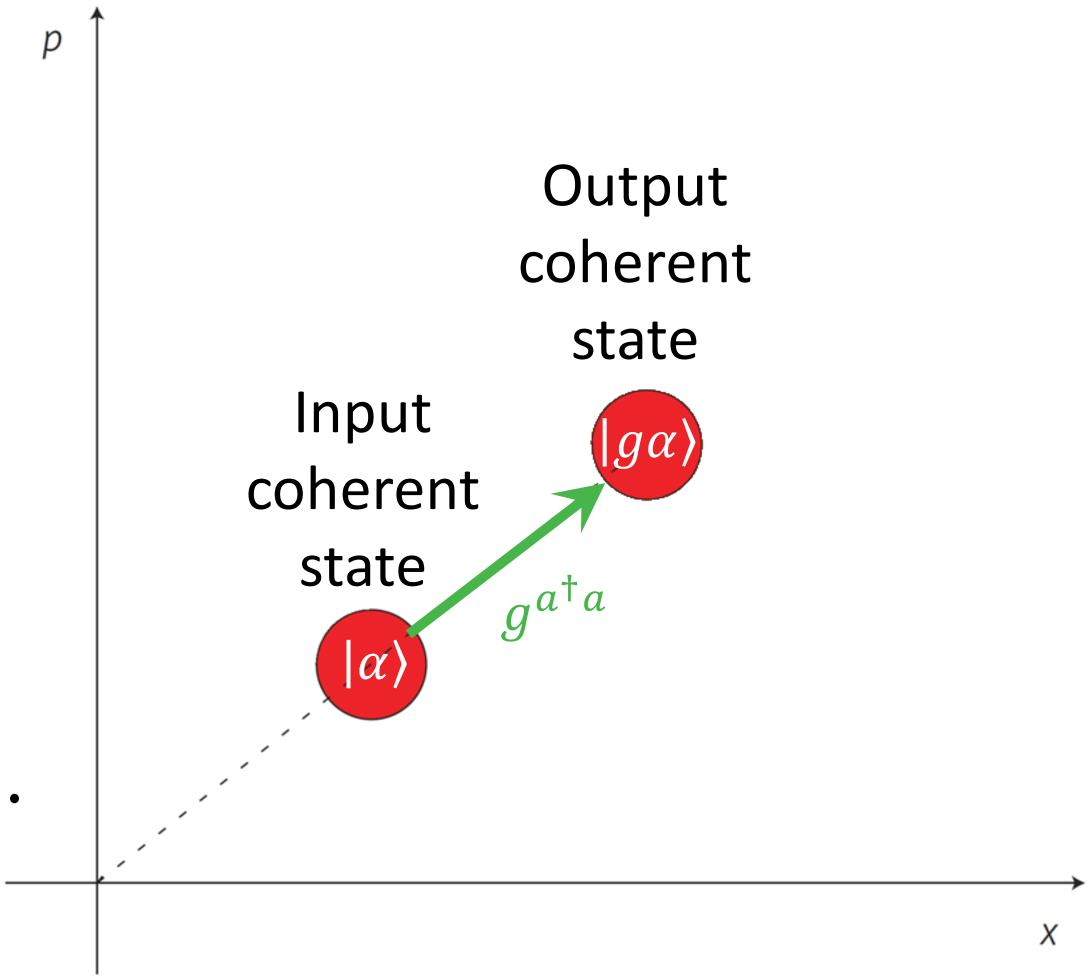
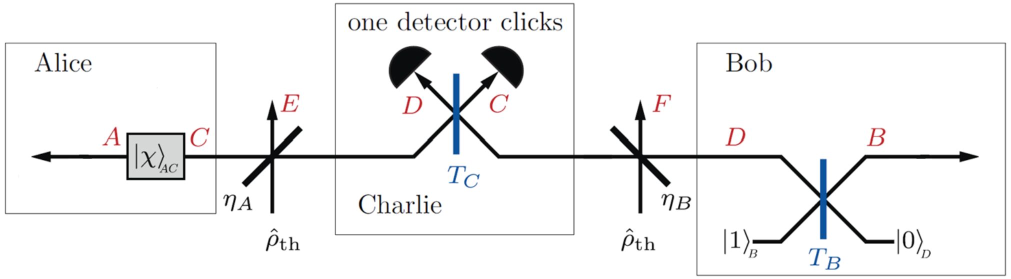
$$|g\alpha\rangle \propto g^{a^\dagger a}|\alpha\rangle$$

$$g^{a^\dagger a} = |0\rangle\langle 0| + g|1\rangle\langle 1| + g^2|2\rangle\langle 2| + \cdots$$



Noiseless Linear Amplifier

Applications



- Quantum repeaters/relays
- Quantum key distribution
- Distillation of entanglement

Noiseless Linear Amplifier

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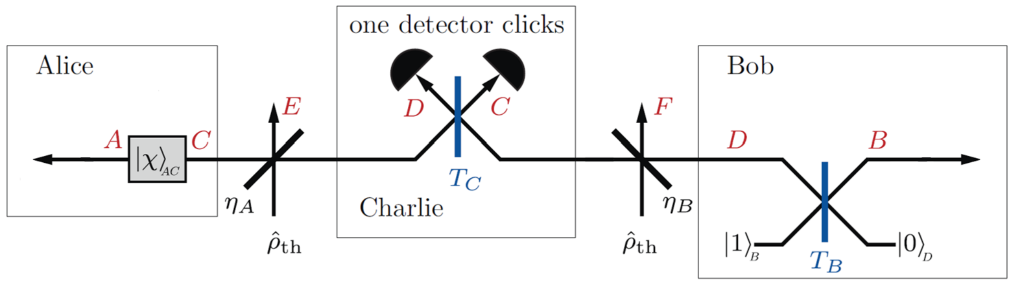
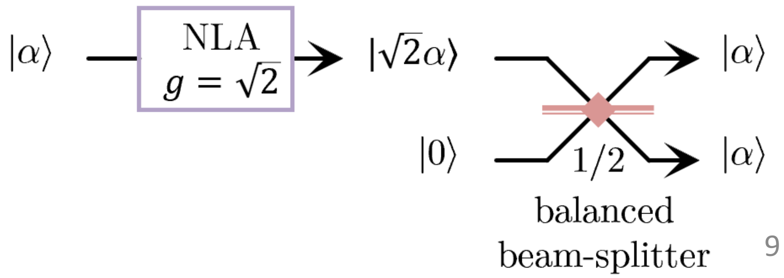


- Quantum repeaters/relays
- Quantum key distribution
- Distillation of entanglement
- Probabilistic cloning



Noiseless Linear Amplifier

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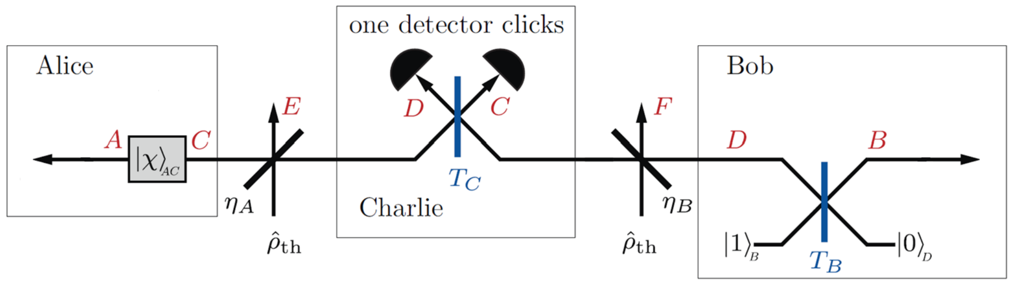
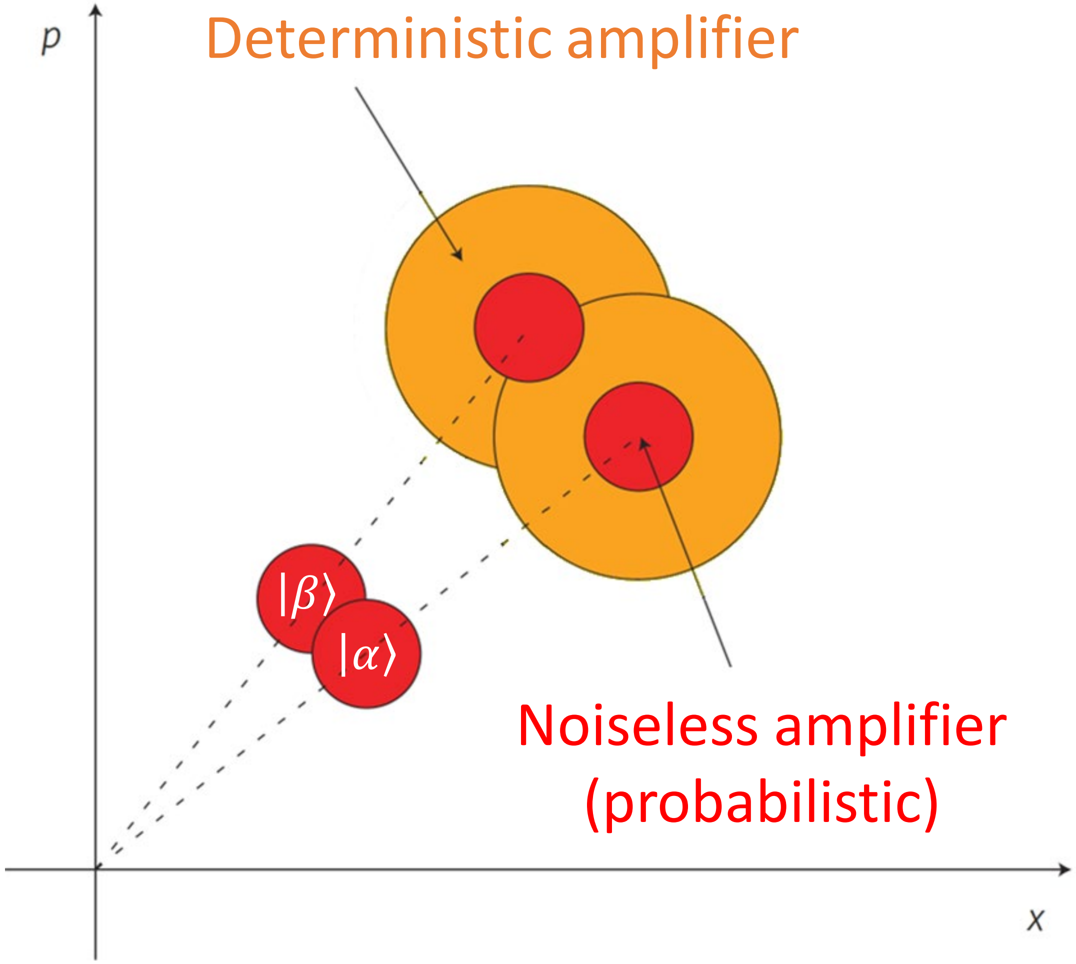
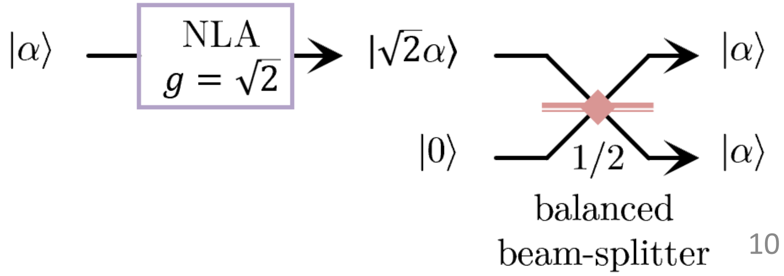




- Quantum repeaters/relays
- Quantum key distribution
- Distillation of entanglement
- Probabilistic cloning
- Quantum metrology
- Quantum state discrimination



doi.org/10.1038/nphoton.2010.260

Noiseless Linear Amplifier

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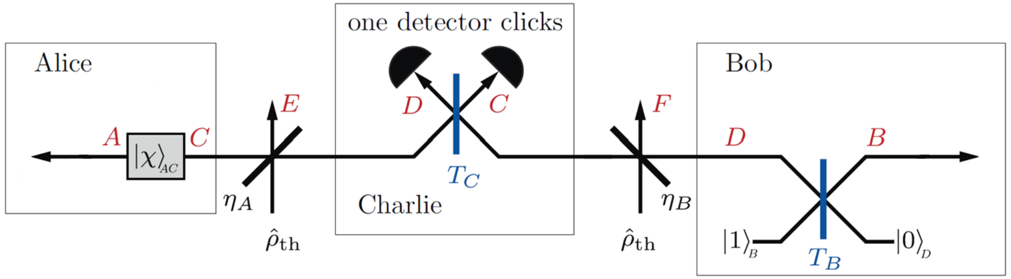
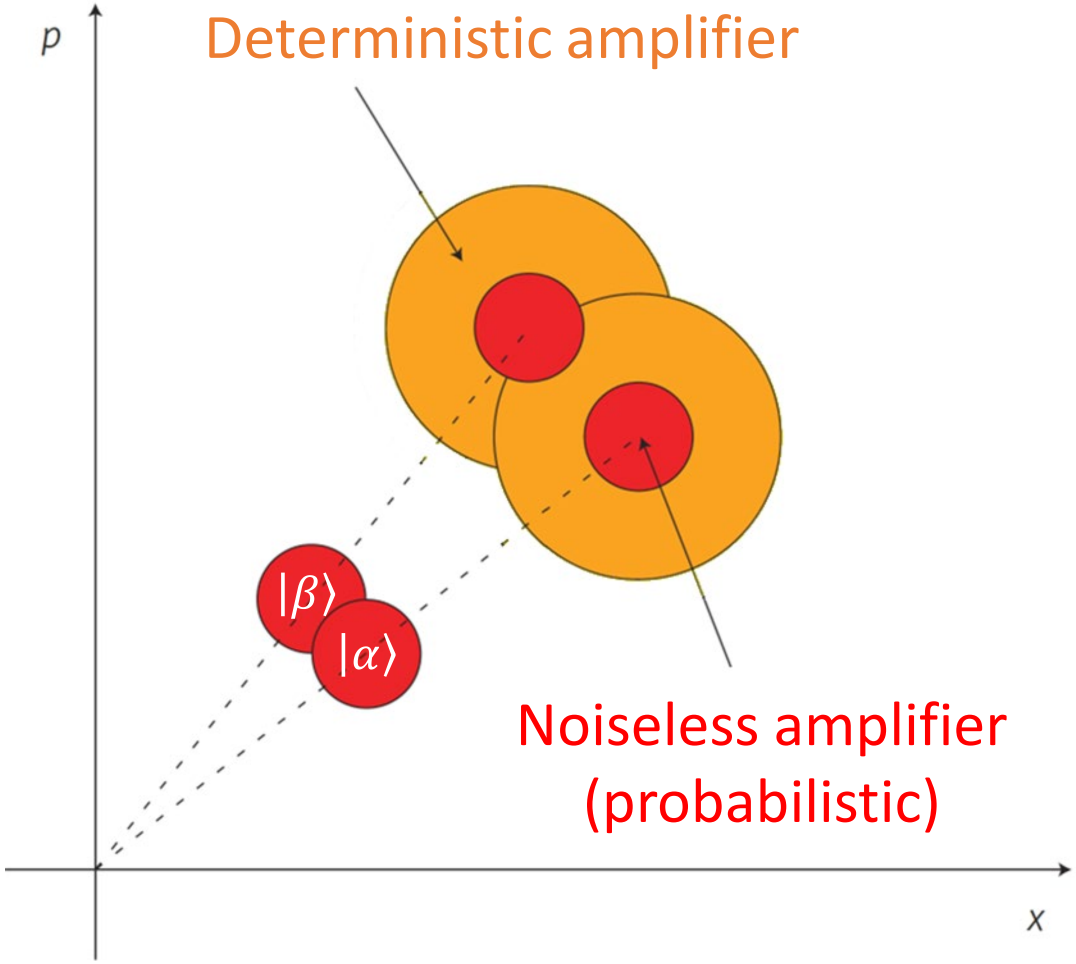
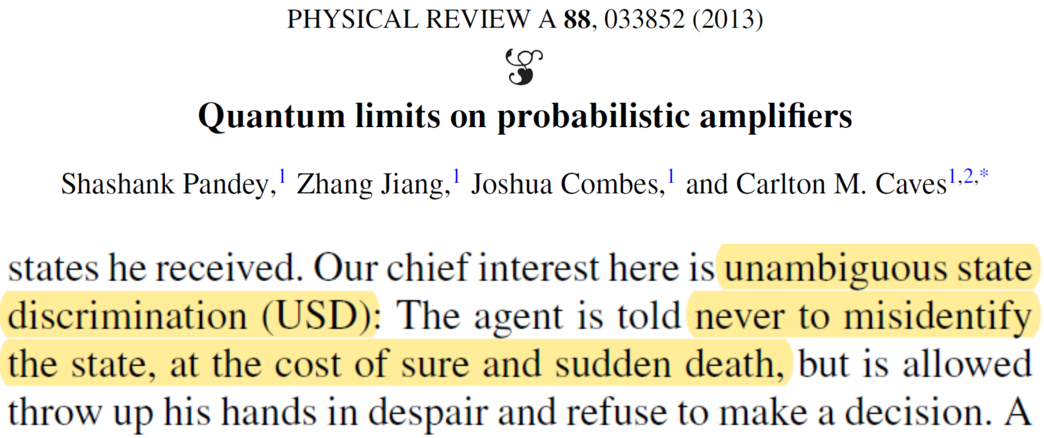




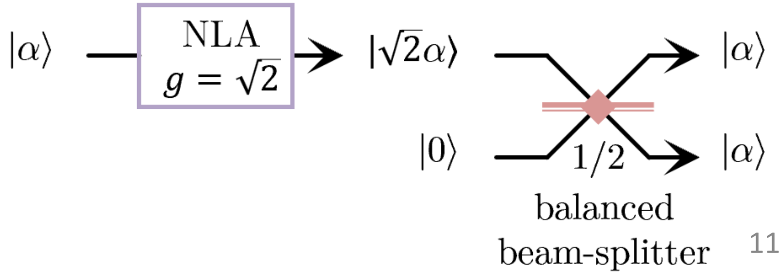

PHYSICAL REVIEW A **88**, 033852 (2013)

Quantum limits on probabilistic amplifiers

Shashank Pandey,[1] Zhang Jiang,[1] Joshua Combes,[1] and Carlton M. Caves[1,2,*]

states he received. Our chief interest here is unambiguous state discrimination (USD): The agent is told never to misidentify the state, at the cost of sure and sudden death, but is allowed throw up his hands in despair and refuse to make a decision. A

- Quantum state discrimination



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How can we implement an NLA?

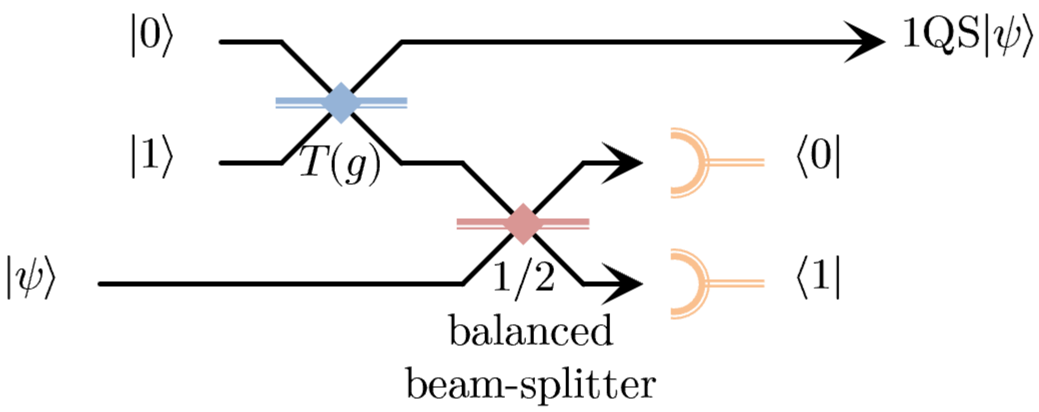
What we want:

$$g^{a^\dagger a} = |0\rangle\langle 0| + g|1\rangle\langle 1| + g^2|2\rangle\langle 2| + \cdots$$

If we restrict ourselves to inputs with $\ll 1$ photon then:

$$g^{a^\dagger a} \approx |0\rangle\langle 0| + g|1\rangle\langle 1|$$

One-Photon Quantum Scissor



$$1\mathrm{QS} \propto |0\rangle\langle 0| + g|1\rangle\langle 1|$$

Nondeterministic Noiseless Linear Amplification of Quantum Systems

AIP Conference Proceedings **1110**, 155 (2009); https://doi.org/10.1063/1.3131295

T. C. Ralph[a] *and* A. P. Lund[b]

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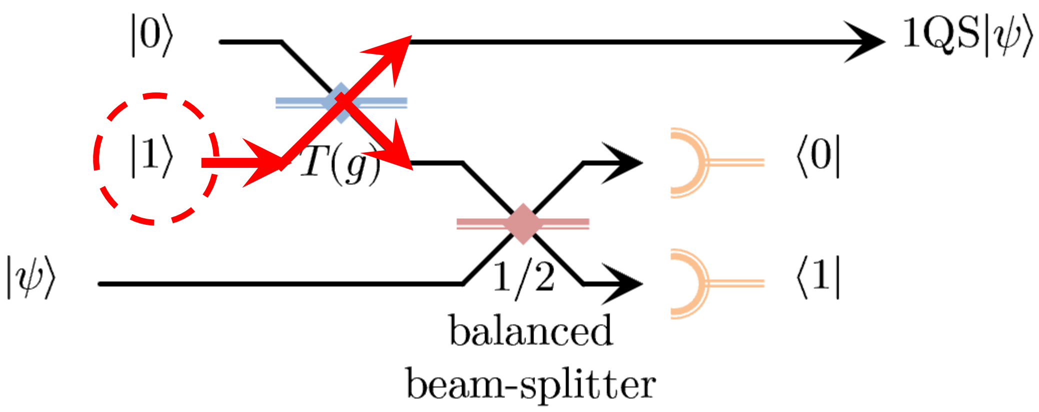

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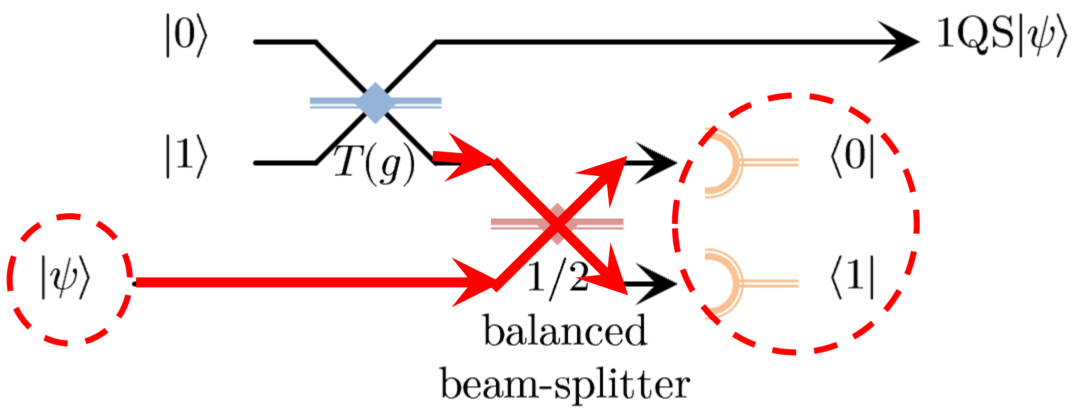

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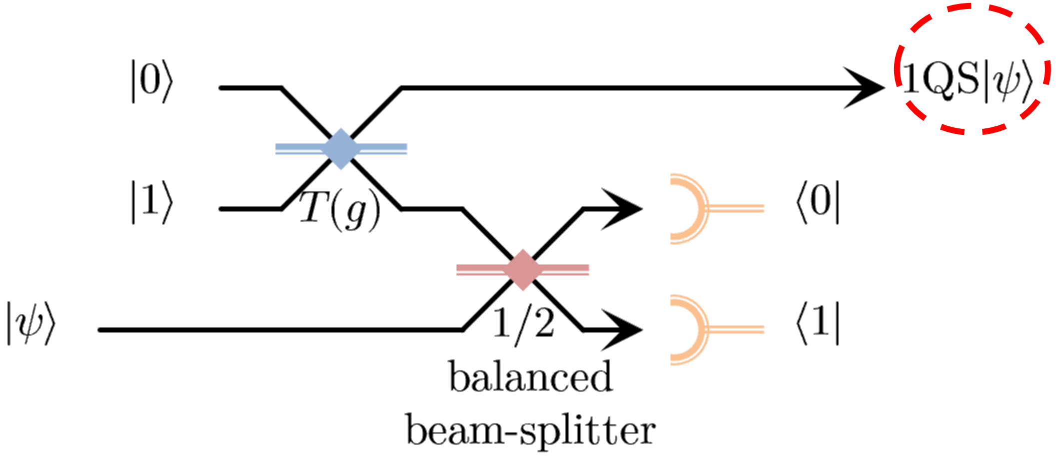

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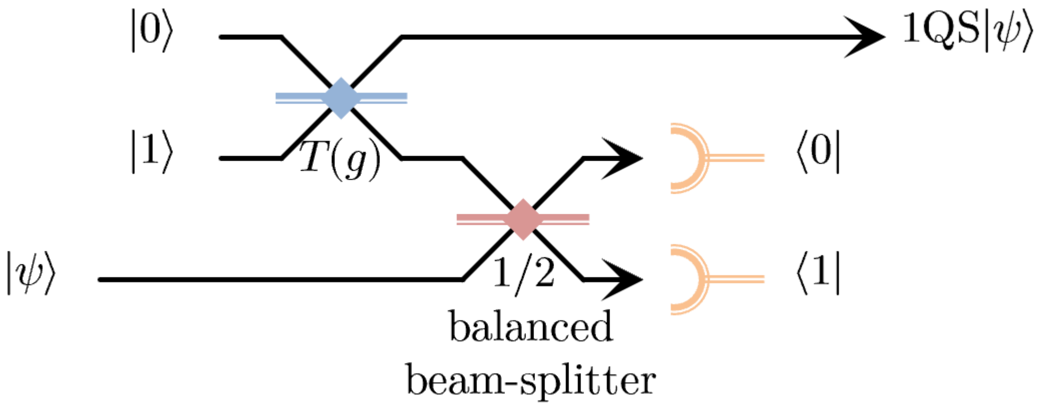

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How can we amplify more than one photon?

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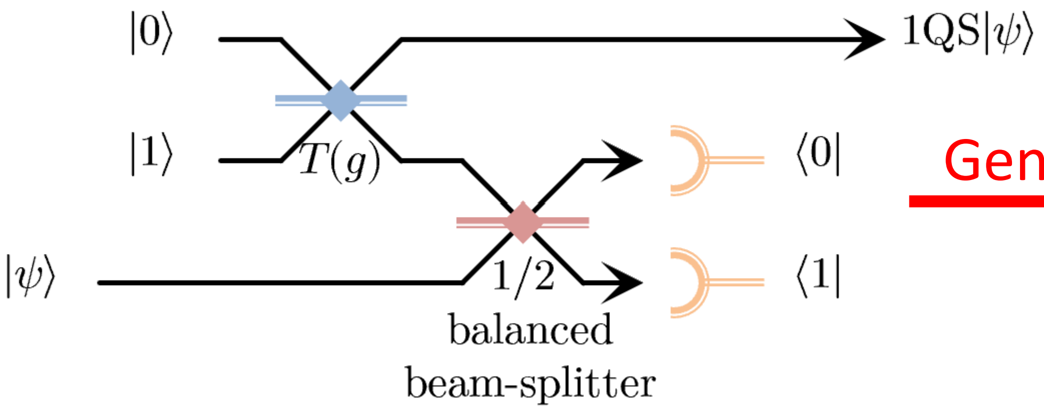
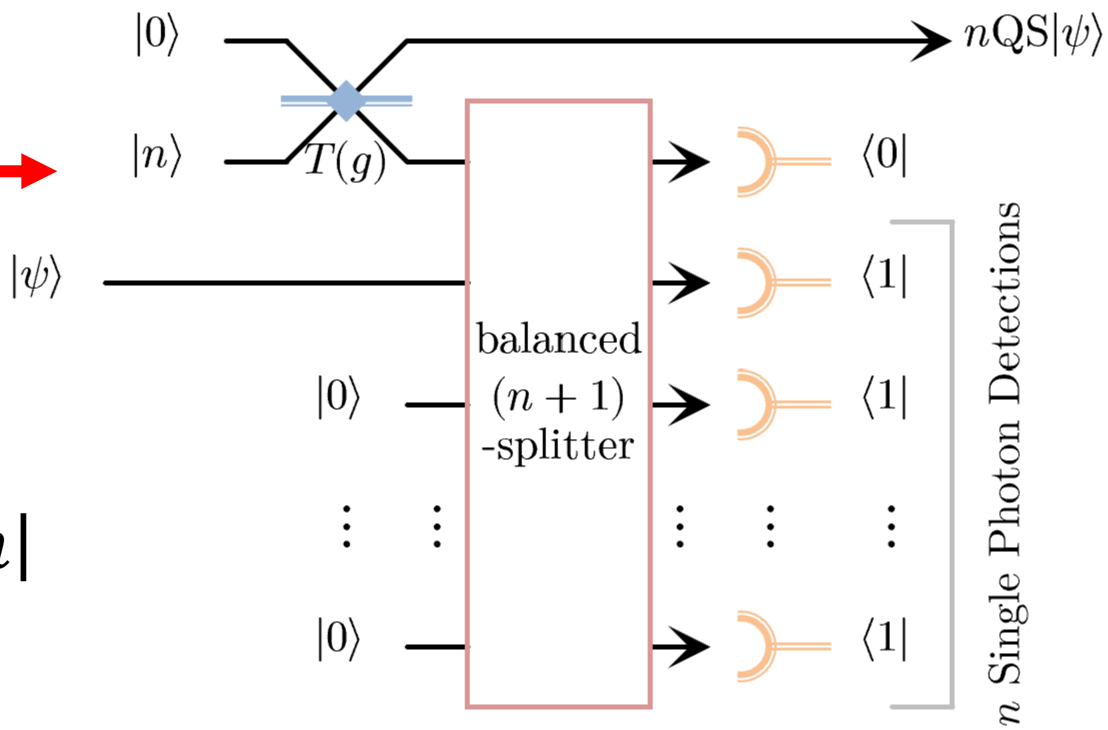


Generalisation



$$nQS \propto |0\rangle\langle 0| + g|1\rangle\langle 1| + \cdots + g^n|n\rangle\langle n|$$

PHYSICAL REVIEW LETTERS **128,** 160501 (2022)

Ideal Quantum Teleamplification up to a Selected Energy Cutoff Using Linear Optics

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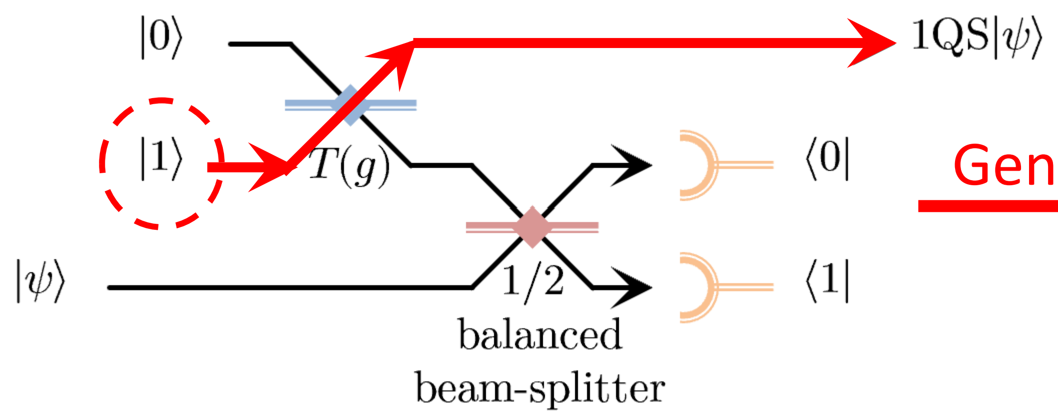

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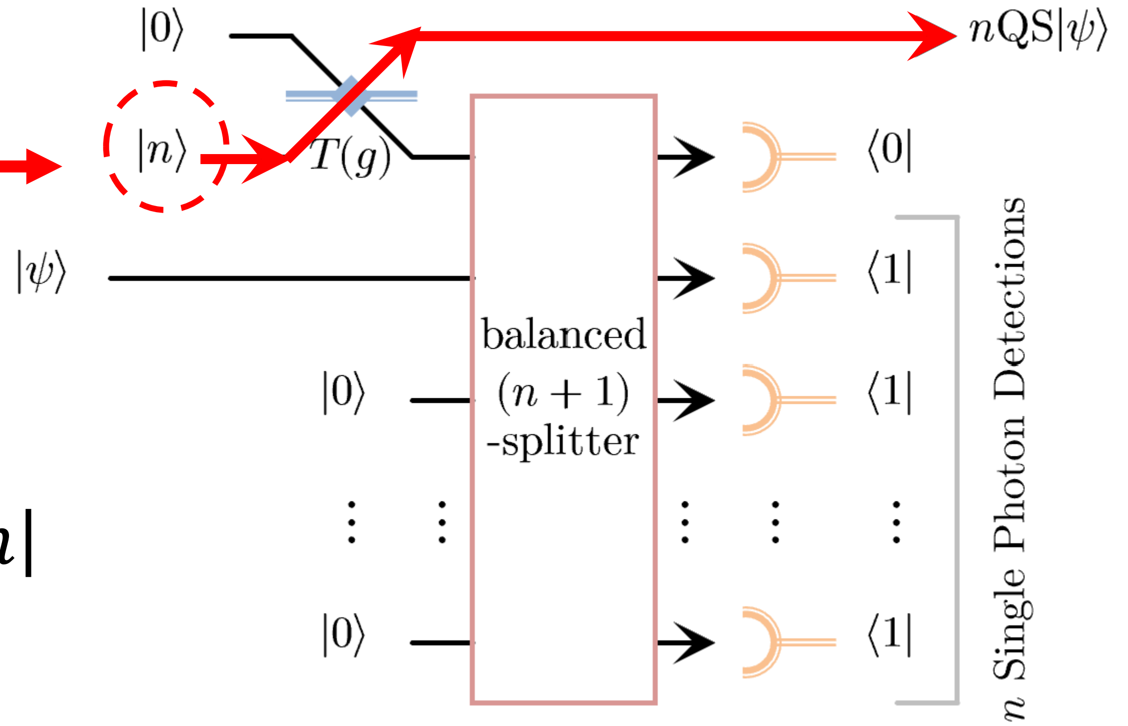

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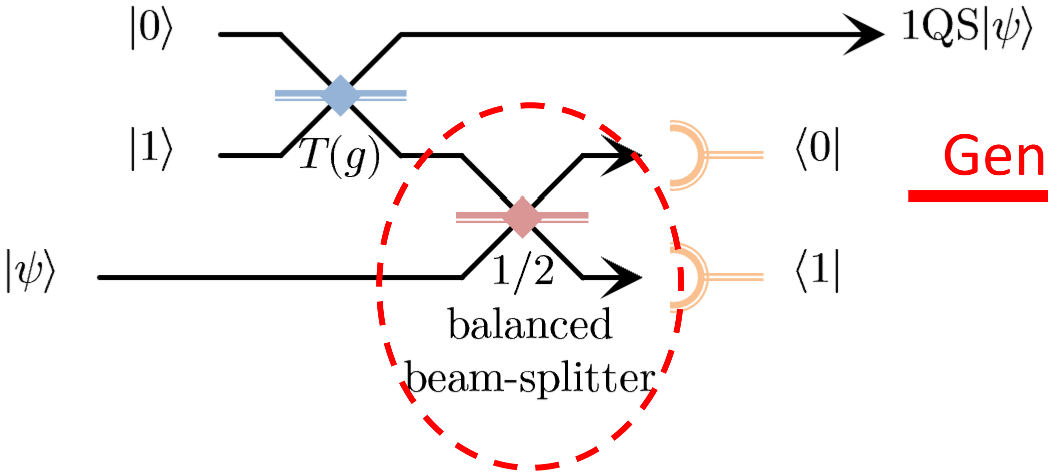

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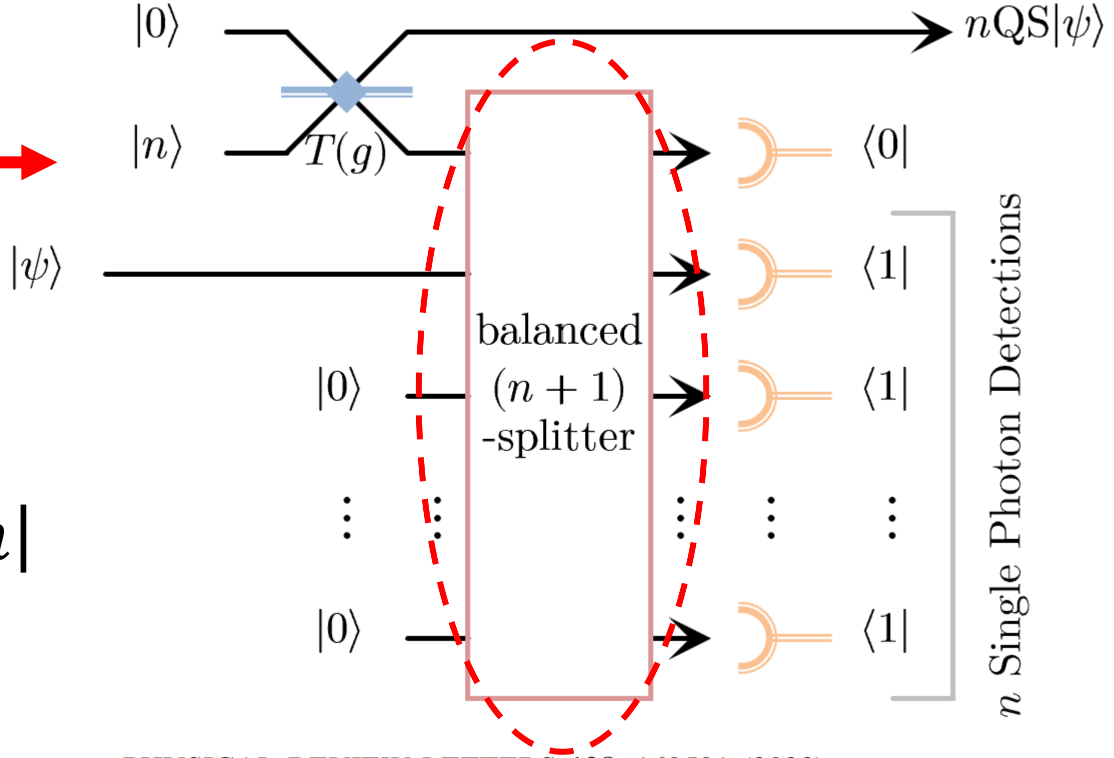

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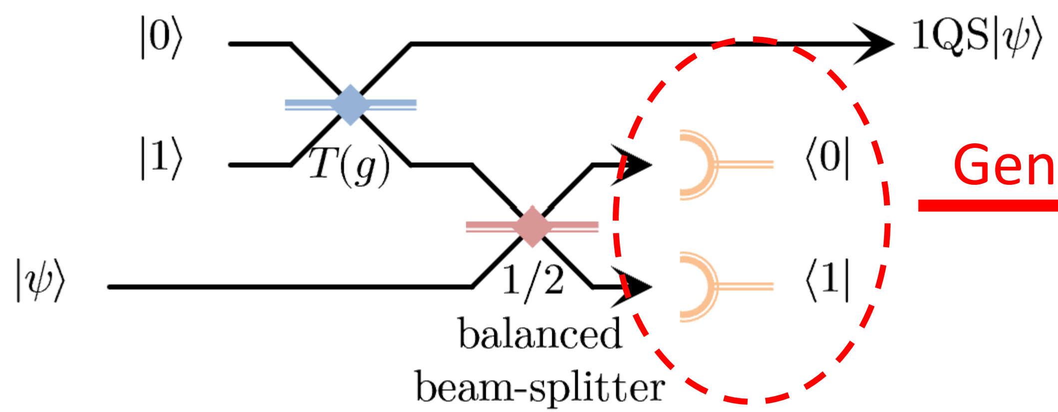

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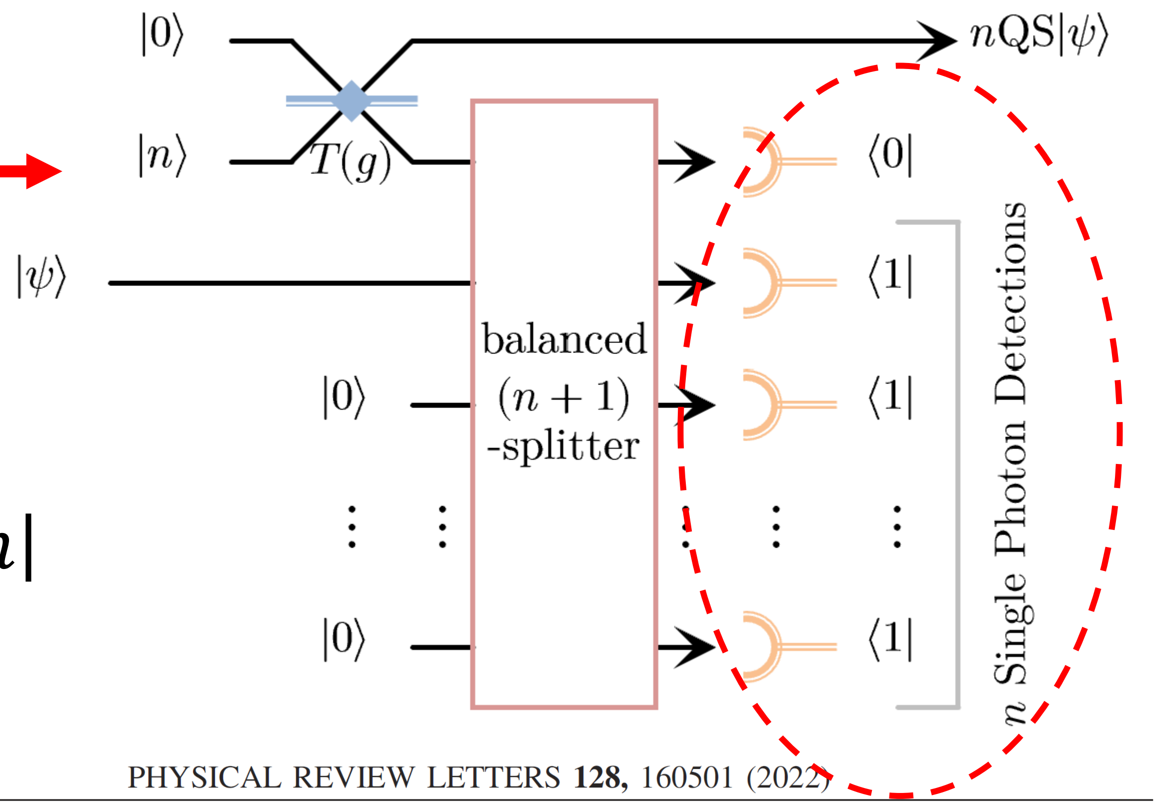

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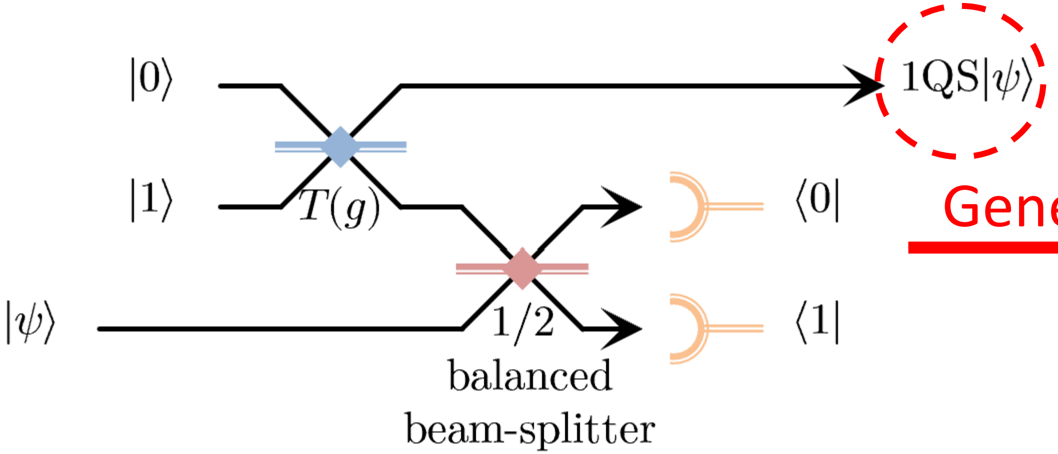

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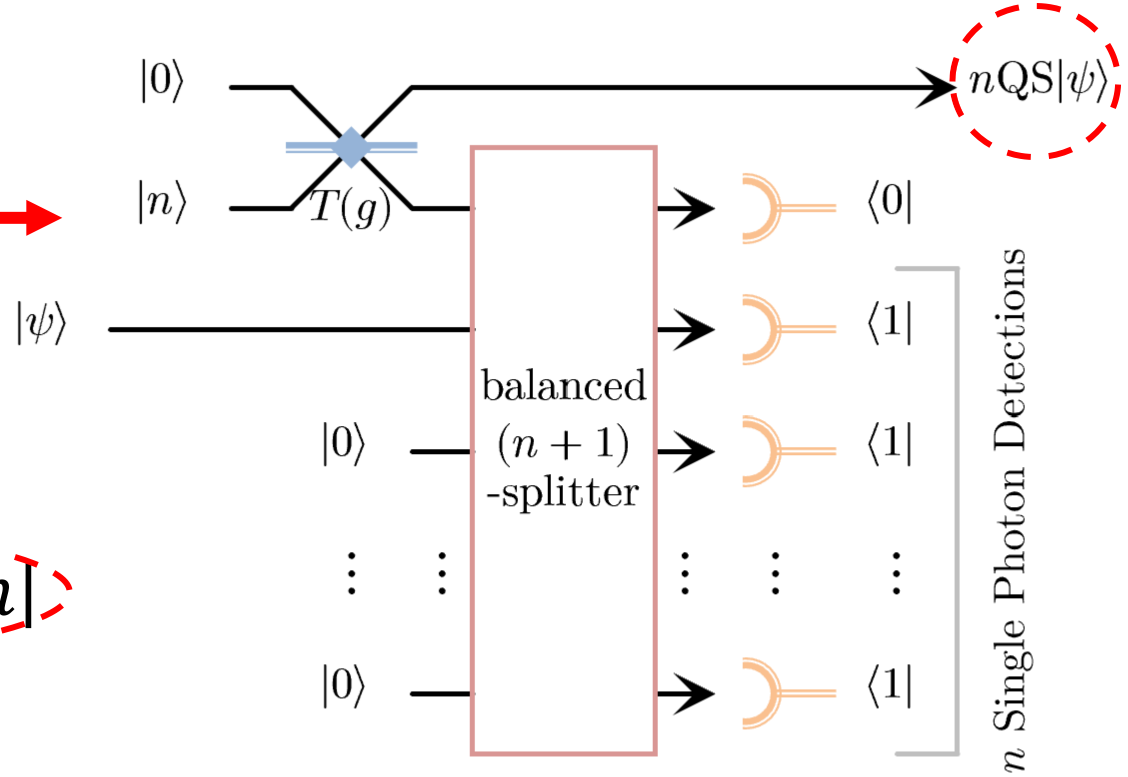

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Ideal Quantum Teleamplification up to a Selected Energy Cutoff Using Linear Optics

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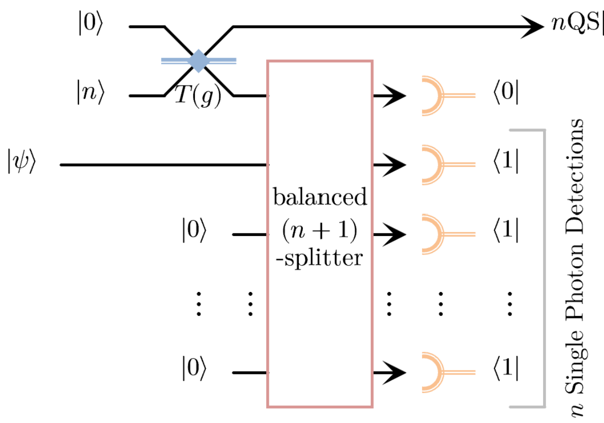
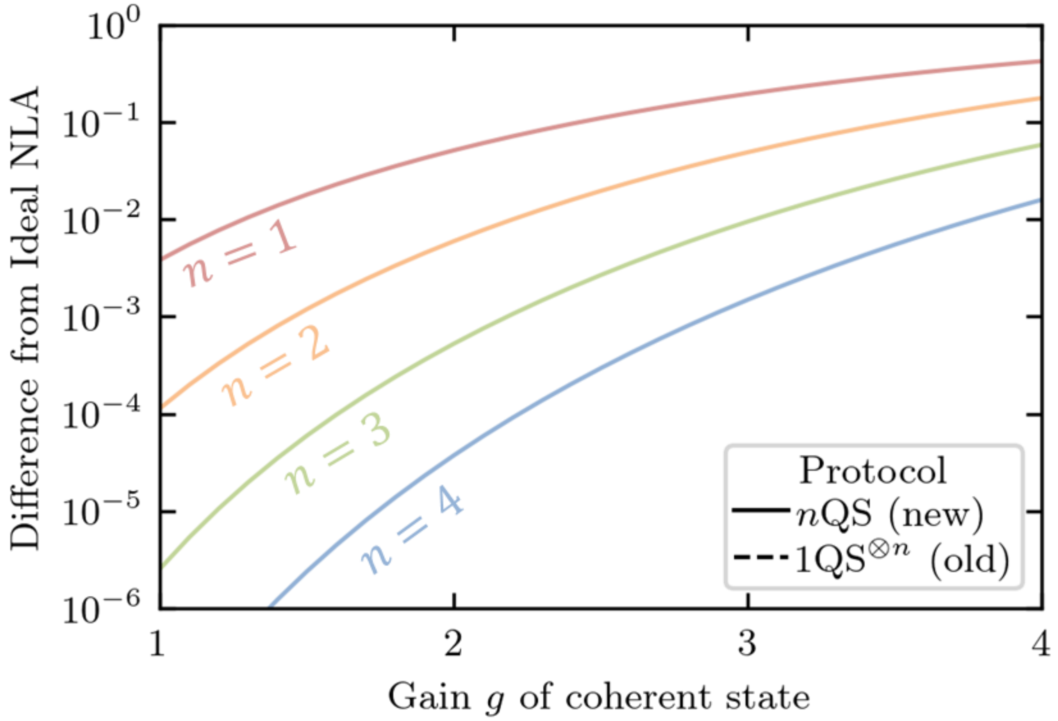
nQS for Amplification

Solid lines is our nQS:



$$nQS \propto \sum_{j=0}^{n} g^j |j\rangle\langle j|$$

How close to ideal amplifier $g^{a^\dagger a}$?



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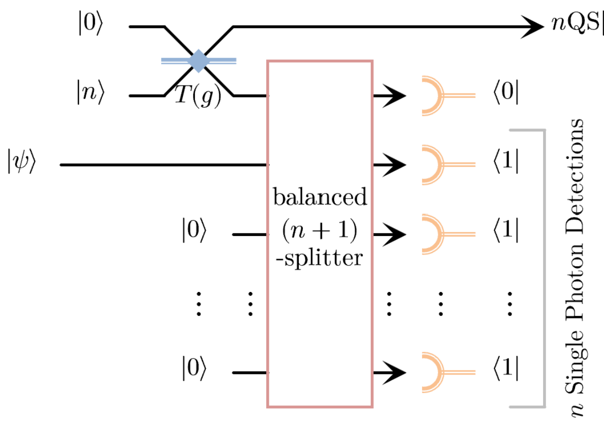

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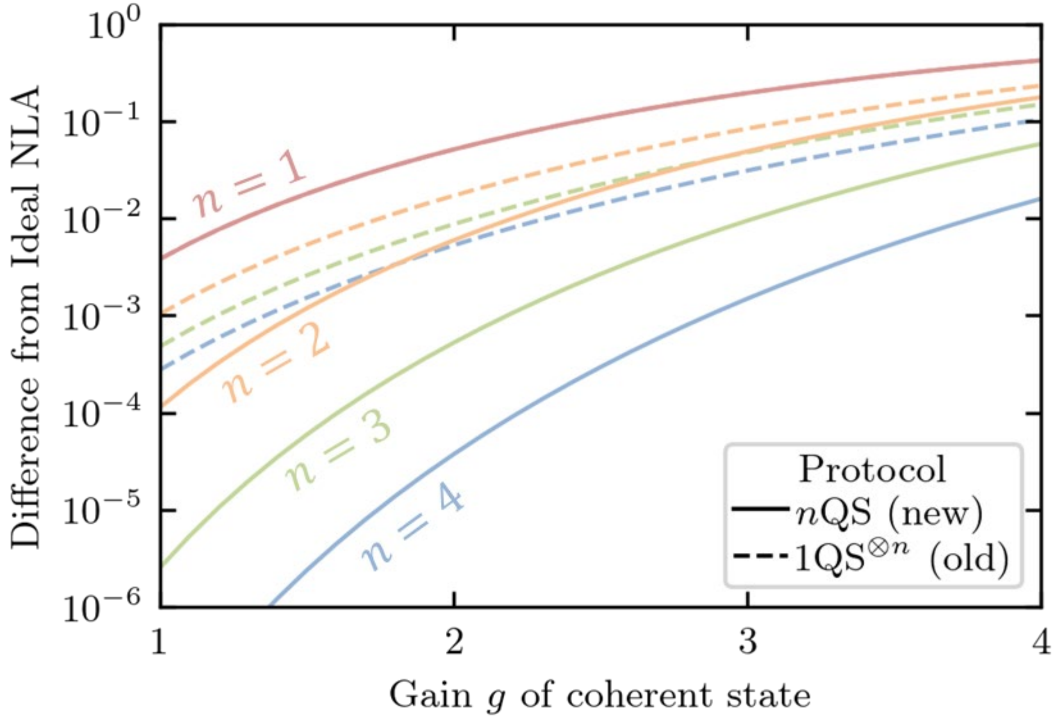

Dashed lines are an old alternative:

LETTERS
PUBLISHED ONLINE: 28 MARCH 2010 | DOI: 10.1038/NPHOTON.2010.35
nature photonics

Heralded noiseless linear amplification and distillation of entanglement

G. Y. Xiang[1], T. C. Ralph[2], A. P. Lund[1,2], N. Walk[2] and G. J. Pryde[1]*

PHYSICAL REVIEW LETTERS **128,** 160501 (2022)

Ideal Quantum Teleamplification up to a Selected Energy Cutoff Using Linear Optics

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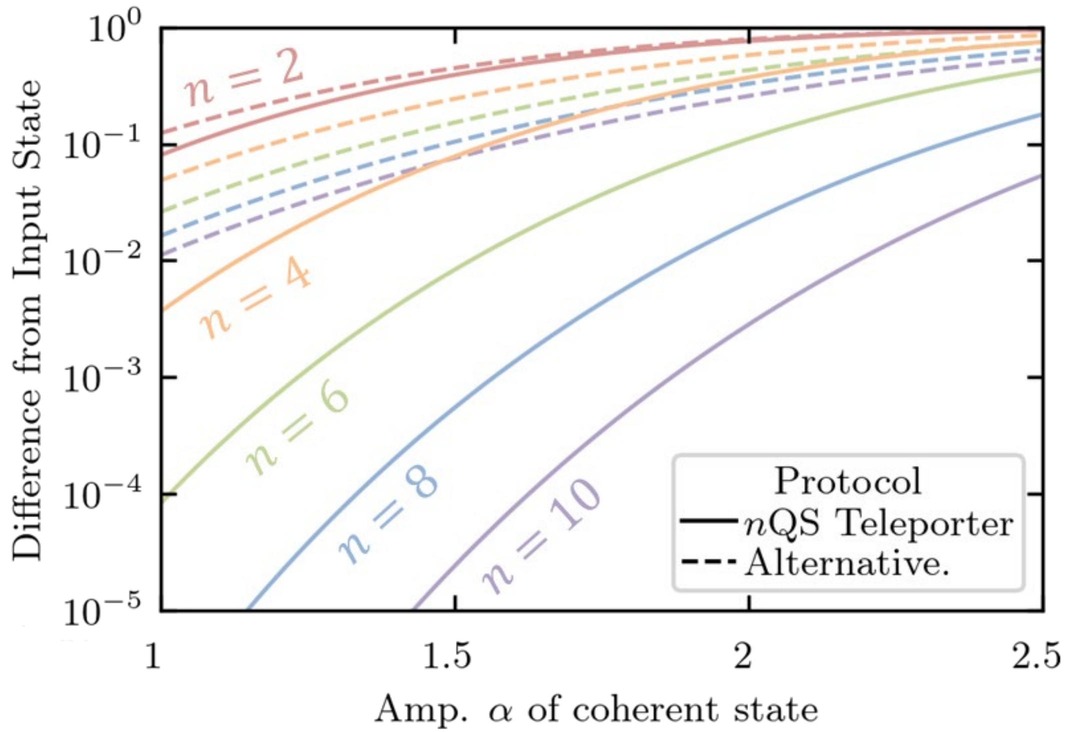
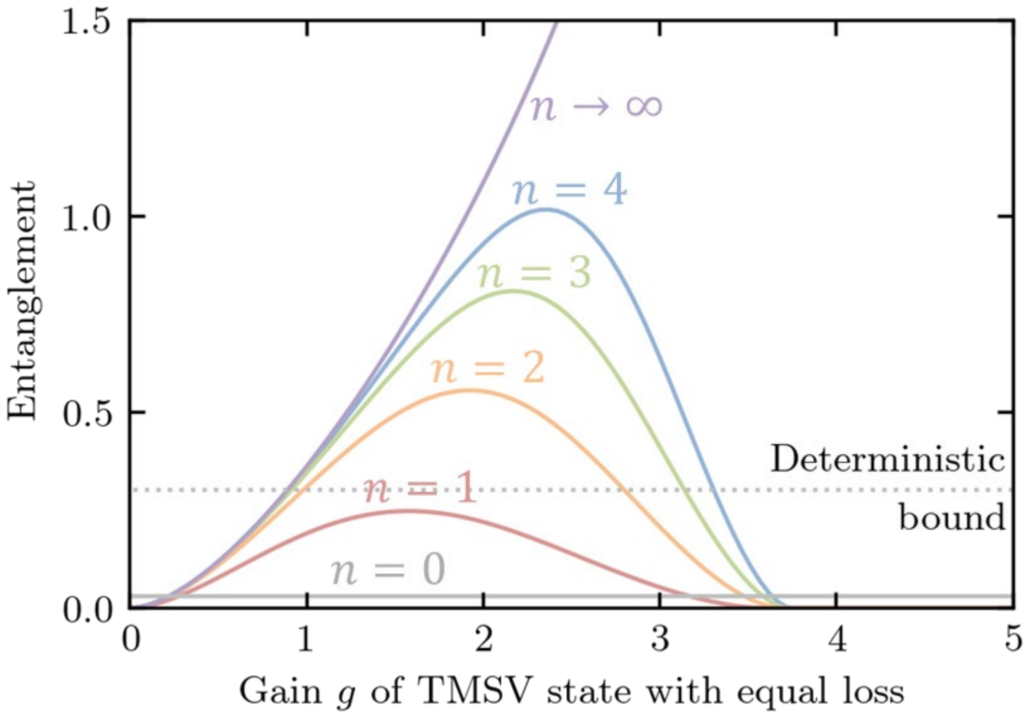
nQS for Teleportation & Entangl. Distillation





$$nQS(g = 1) \propto \sum_{j=0}^{n} |j\rangle\langle j|$$

PHYSICAL REVIEW LETTERS **128,** 160501 (2022)

Ideal Quantum Teleamplification up to a Selected Energy Cutoff Using Linear Optics

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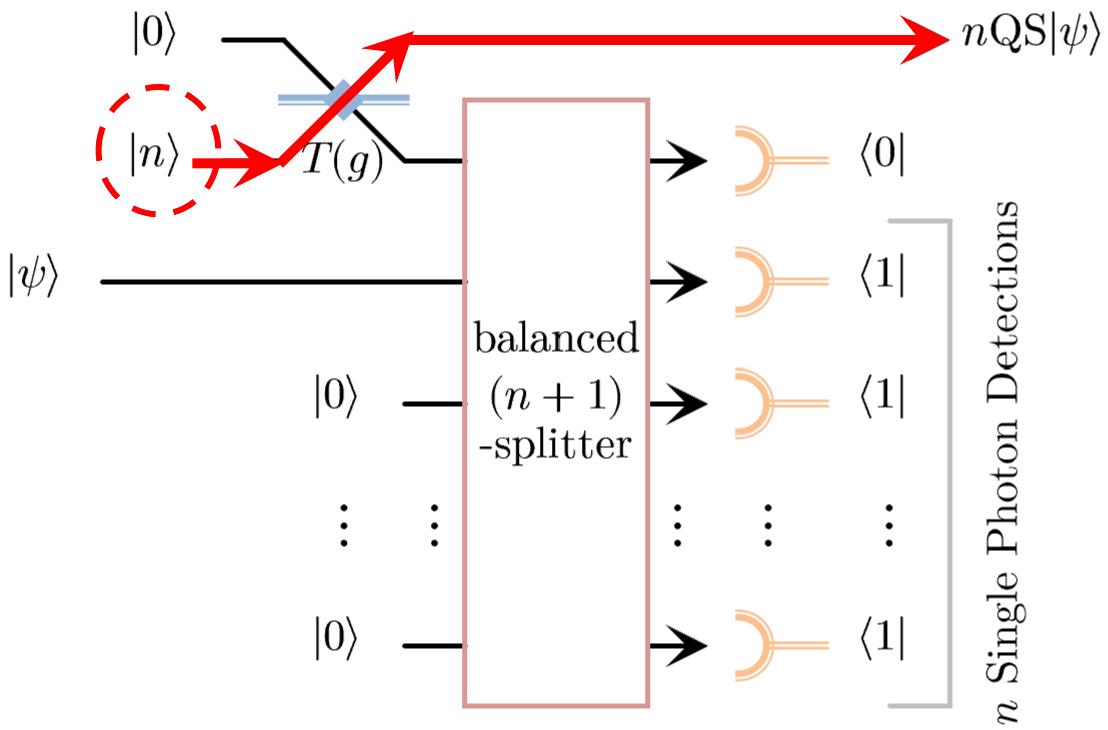
Limitation 1: nQS is only up to n photons



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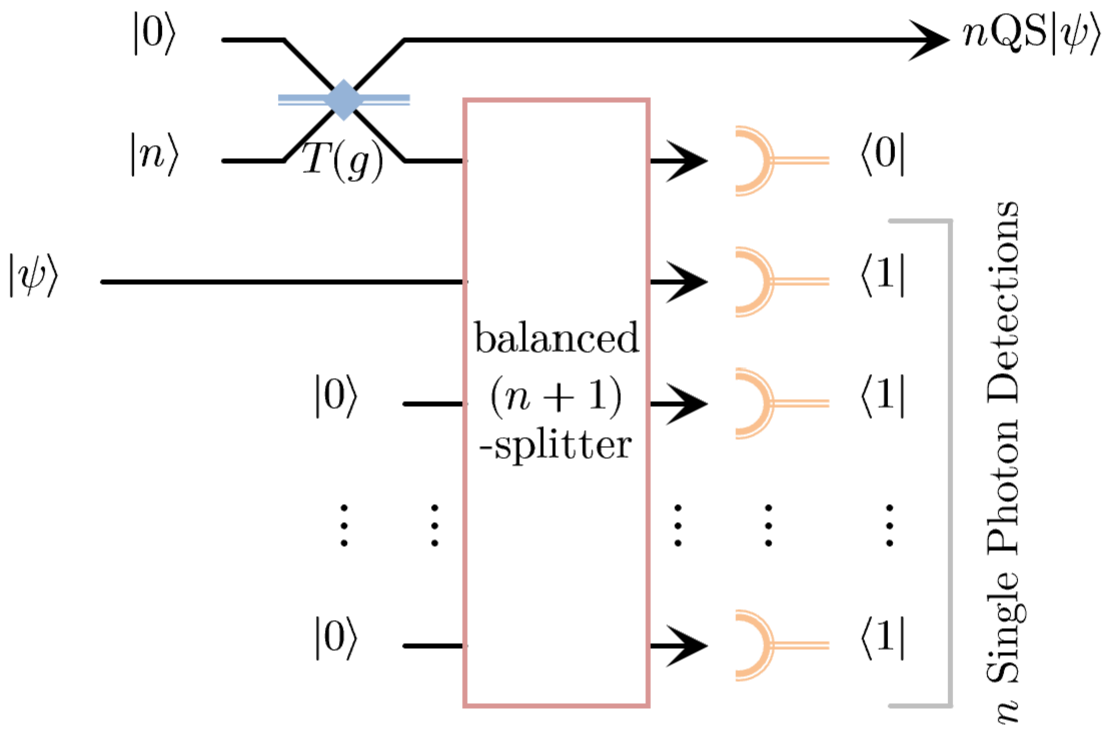

$$|\text{cat}_{\beta,n+1}\rangle \xrightarrow[\beta\to 0]{} |n\rangle$$

small amplitude cat ≈ number state

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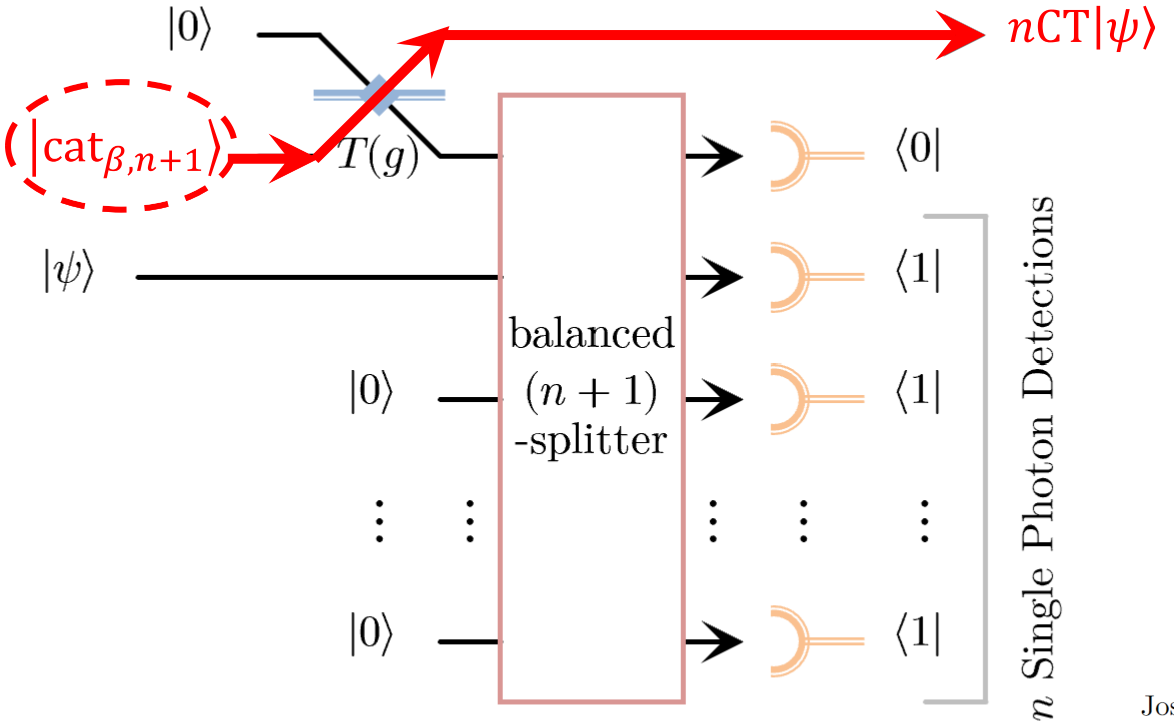


$$\left|\mathrm{cat}_{\beta,n+1}\right\rangle \xrightarrow[\beta\to 0]{} |n\rangle$$

small amplitude cat ≈ number state

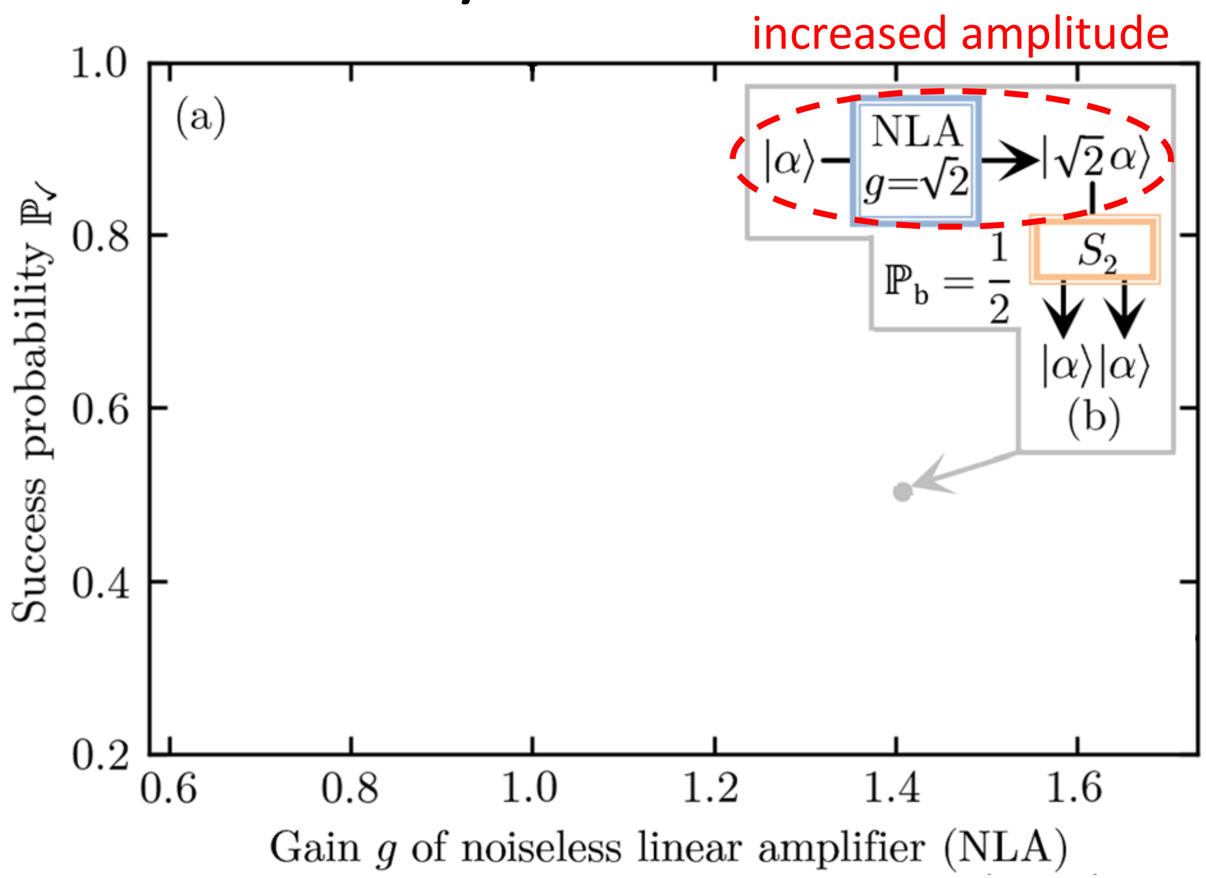
Idea is to generalise resource to cat state.

Noiseless Linear Amplification and Loss-Tolerant Quantum Relay using Coherent State Superpositions

Joshua J. Guanzon,[1,*] Matthew S. Winnel,[1] Austin P. Lund,[2,1] and Timothy C. Ralph[1]

arXiv:2211.08035

Limitation 2: Success Probability Bounded



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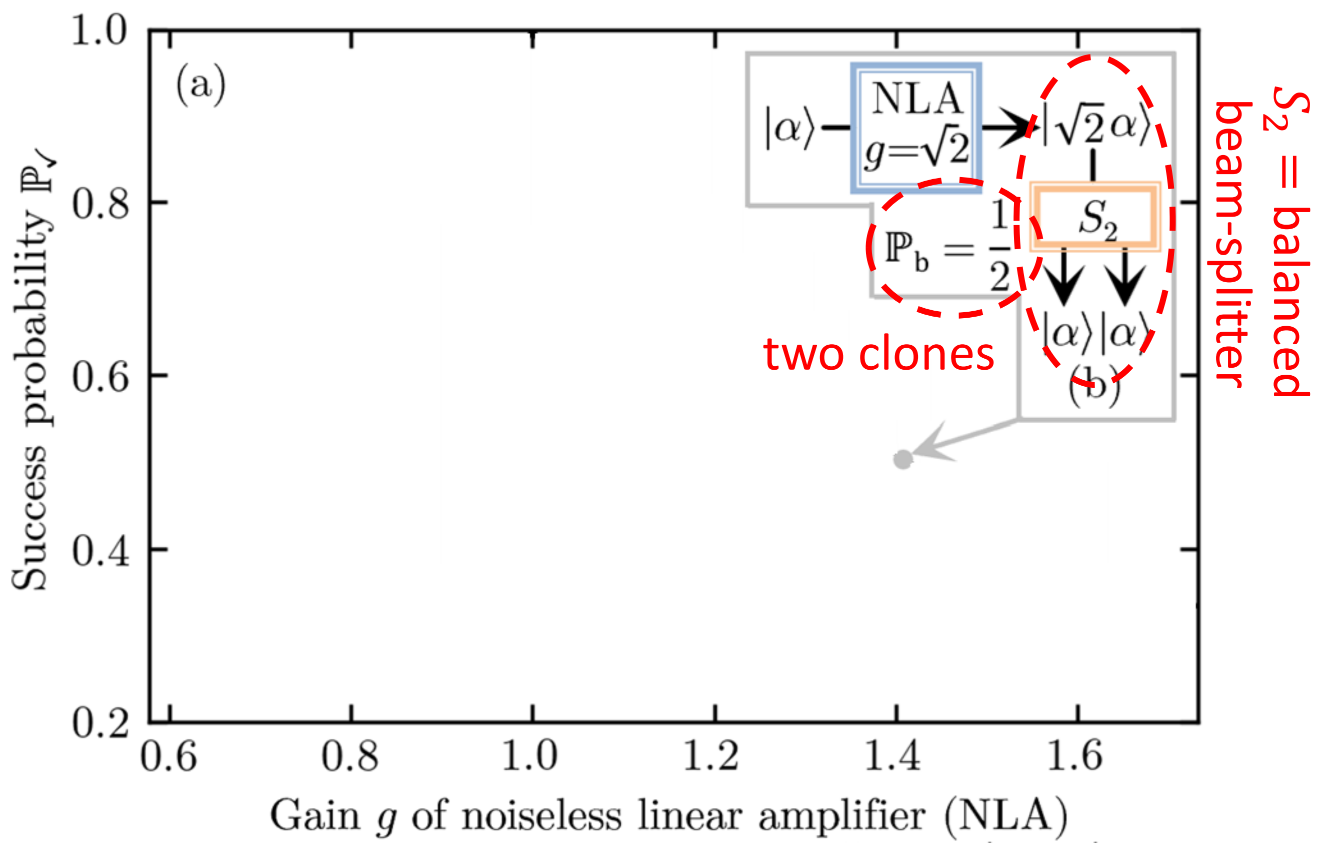

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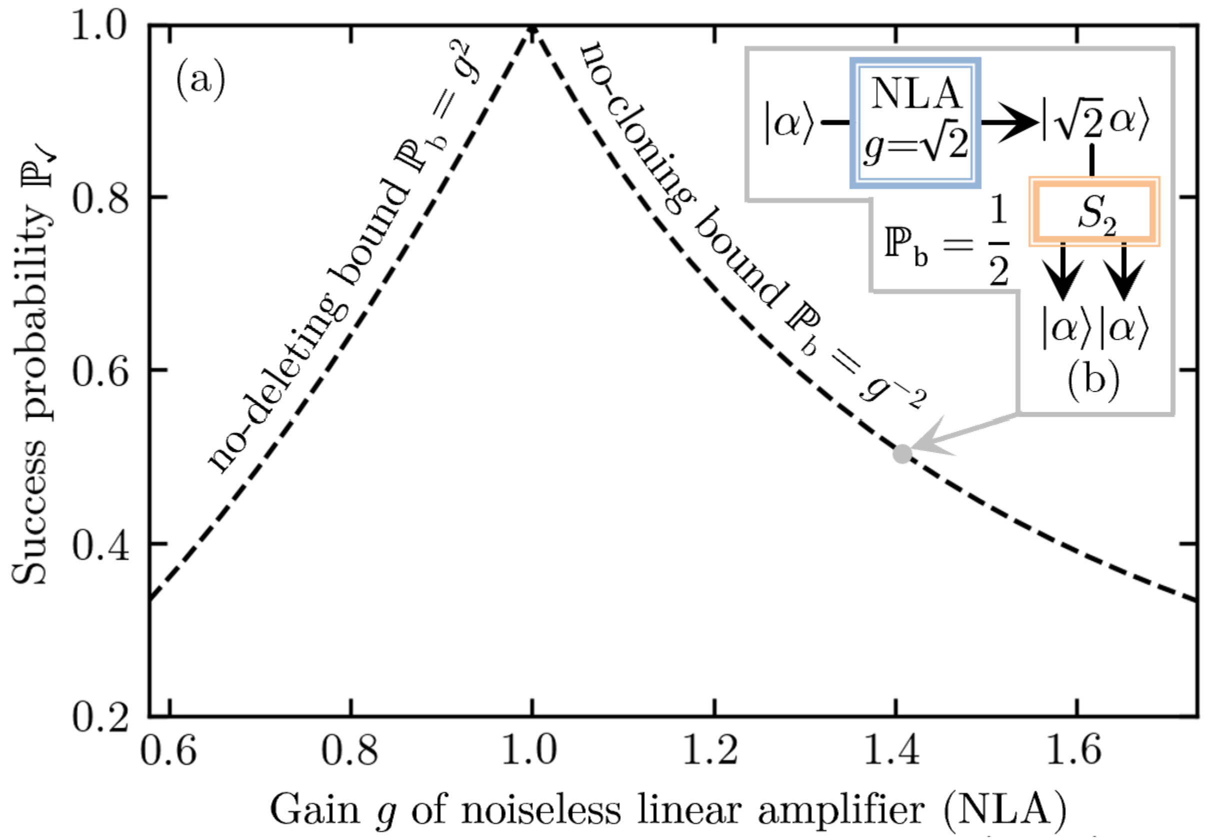
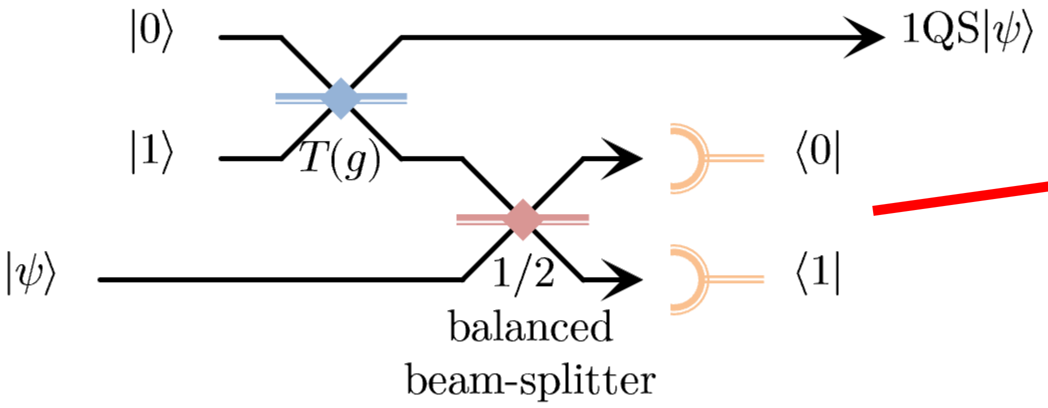
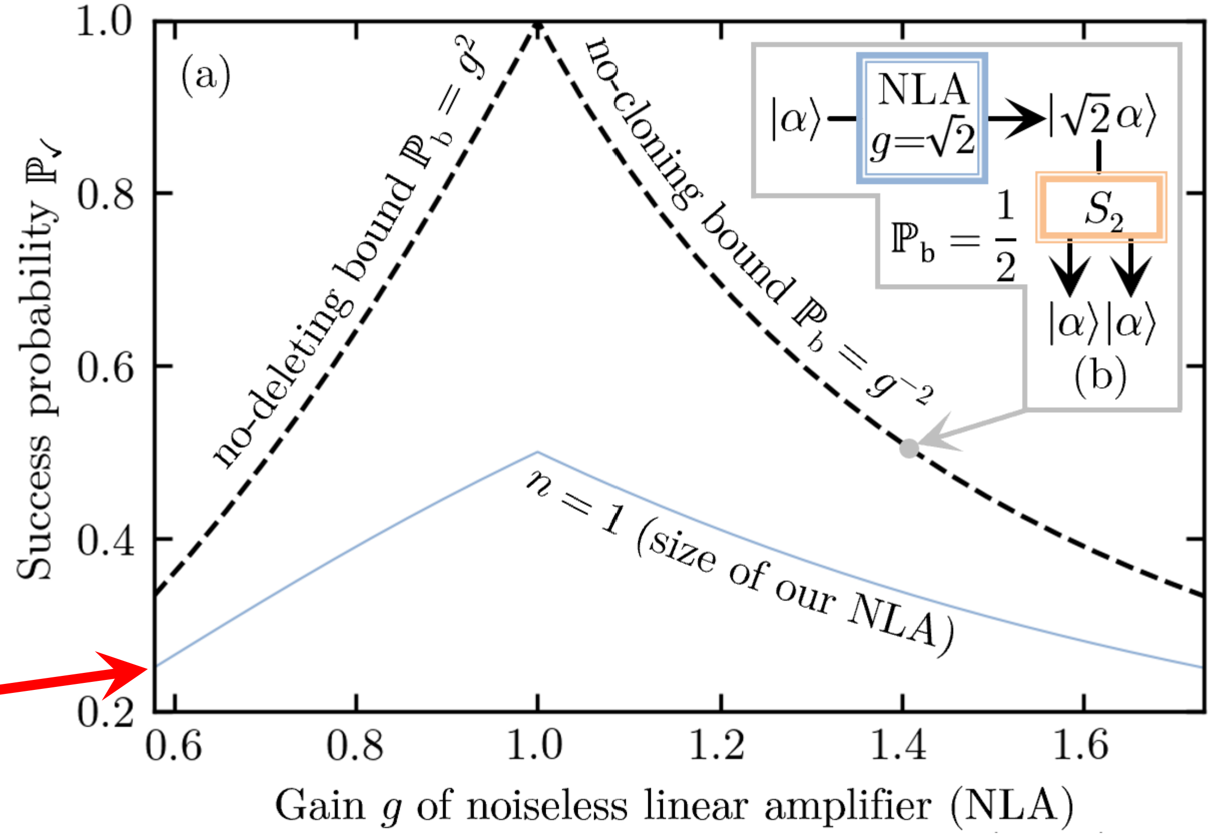


Limitation 2: Success Probability Bounded

Is a maximum success probability noiseless linear amplifier possible?





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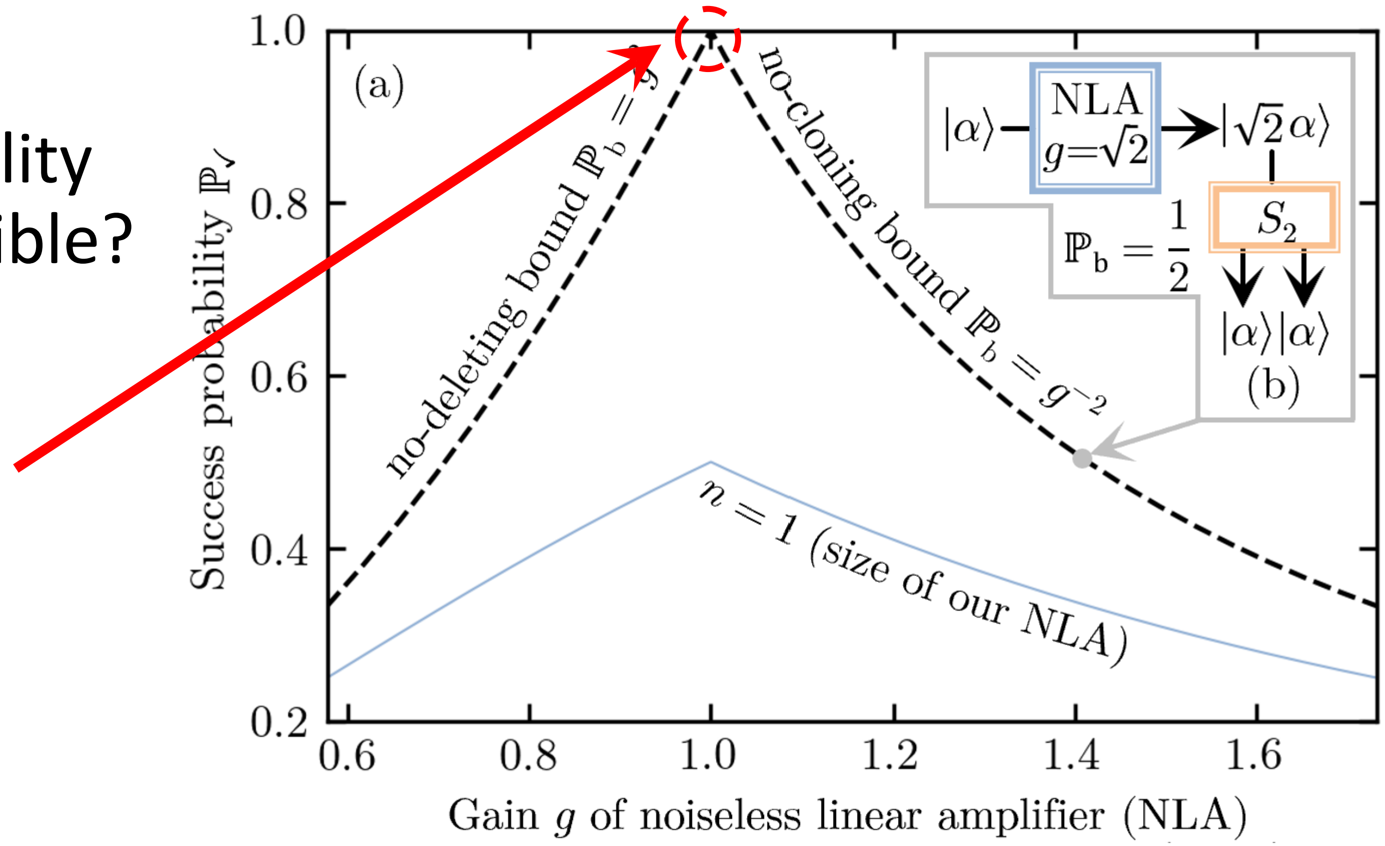
Is a maximum success probability noiseless linear amplifier possible?

Two hints:

1. No gain $g = 1$ the success probability is 100%.

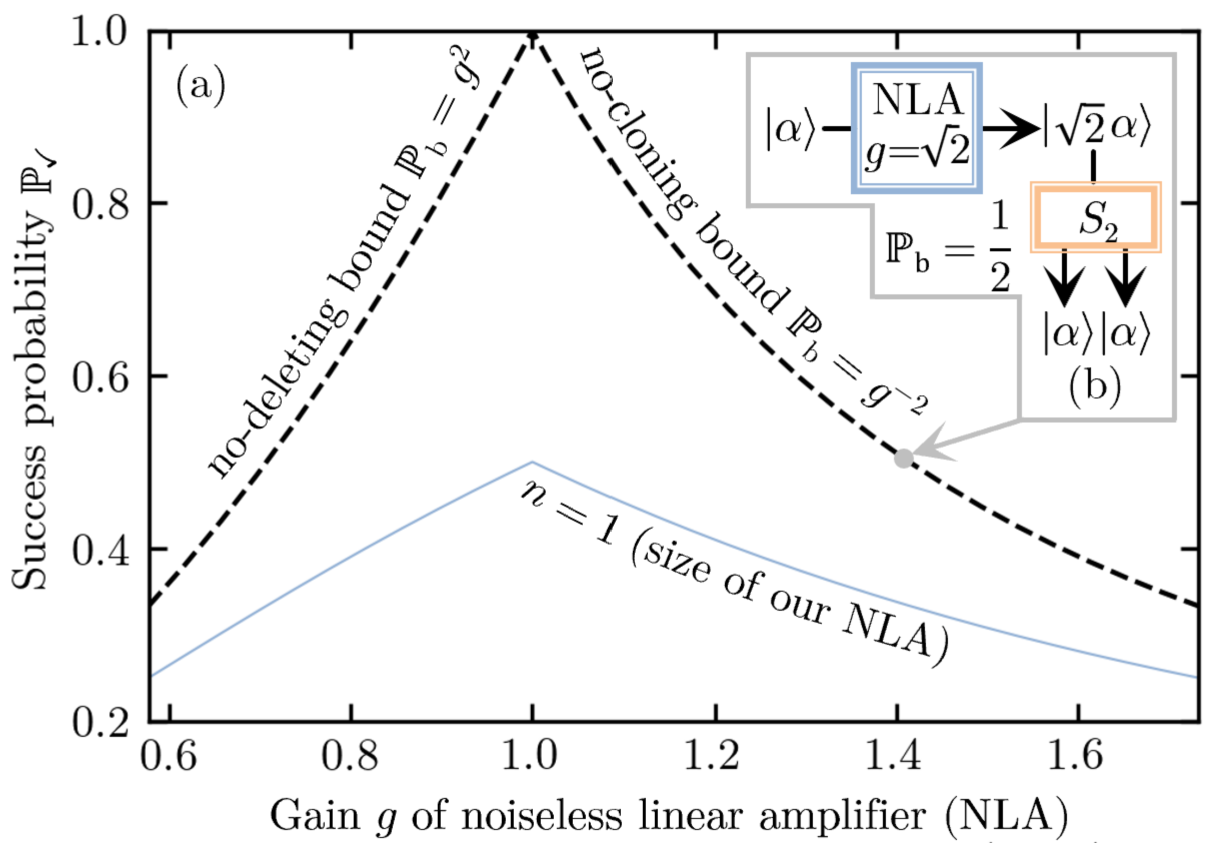


Limitation 2: Success Probability Bounded

Is a maximum success probability noiseless linear amplifier possible?

Two hints:

1. No gain $g = 1$ the success probability is 100%.
2. These amplifiers are also teleporters.



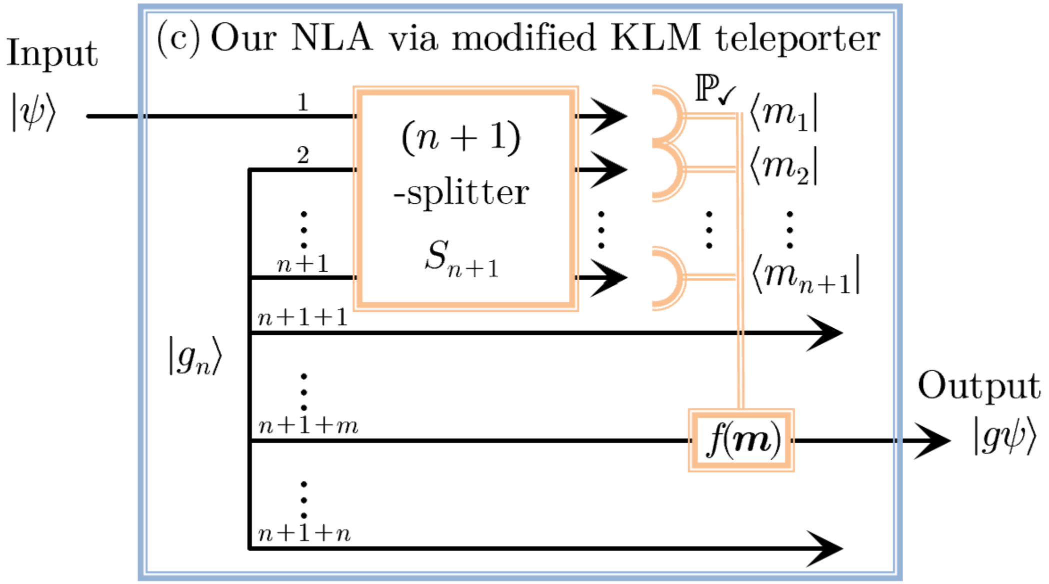
Published: 04 January 2001

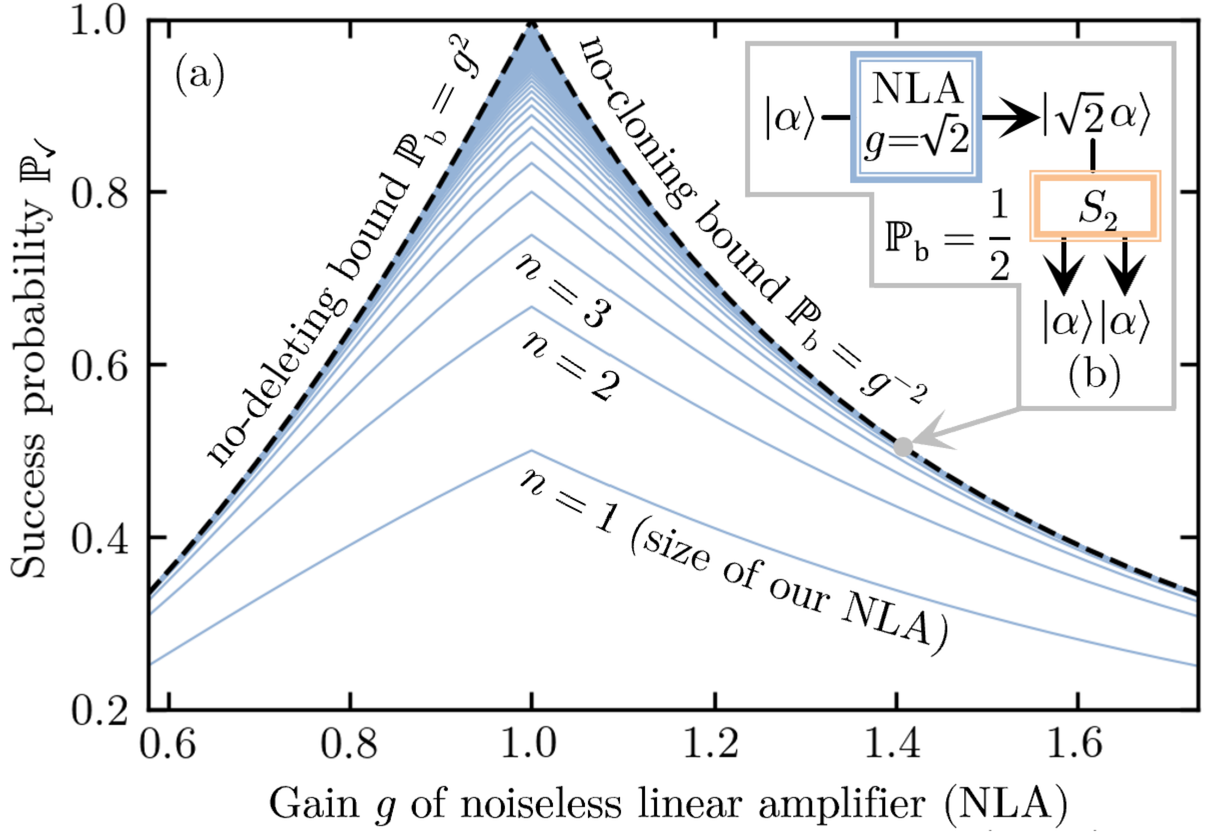
A scheme for efficient quantum computation with linear optics

E. Knill, R. Laflamme & G. J. Milburn

Nature **409**, 46–52 (2001) | Cite this article

Limitation 2: Success Probability Bounded



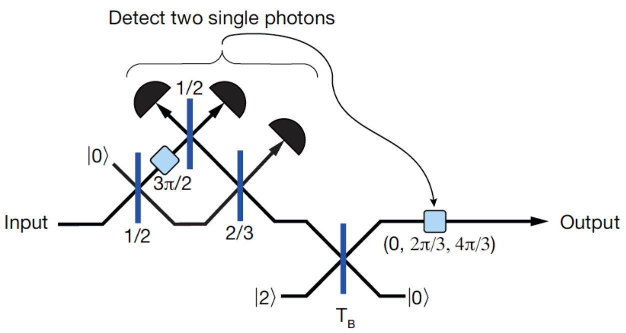


Saturating the Maximum Success Probability Bound for Noiseless Linear Amplification using Linear Optics

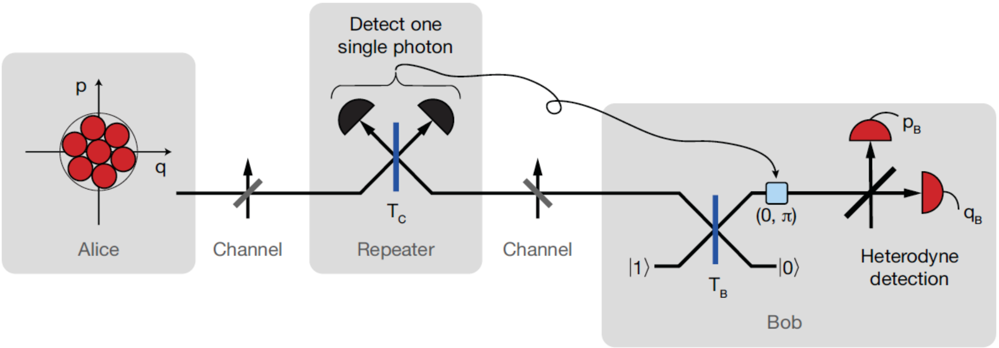
Joshua J. Guanzon,[1,*] Matthew S. Winnel,[1] Deepesh Singh,[1] Austin P. Lund,[2,1] and Timothy C. Ralph[1]

arXiv:2212.04274

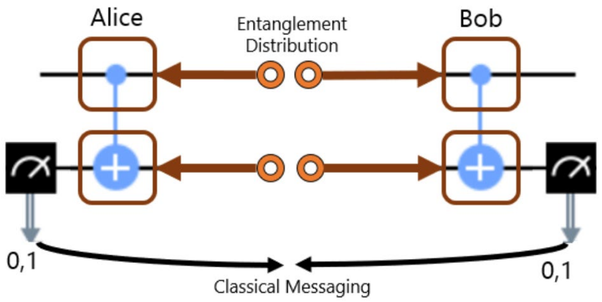
Suggestions for Near-Term Experiments



[1] two-photon quantum-scissor NLA



[2] CV-QKD protocol (using one-photon quantum-scissor NLA) for beating the repeaterless bound

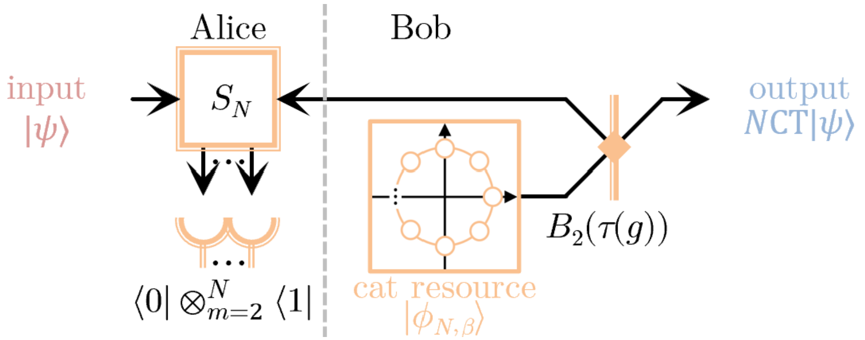


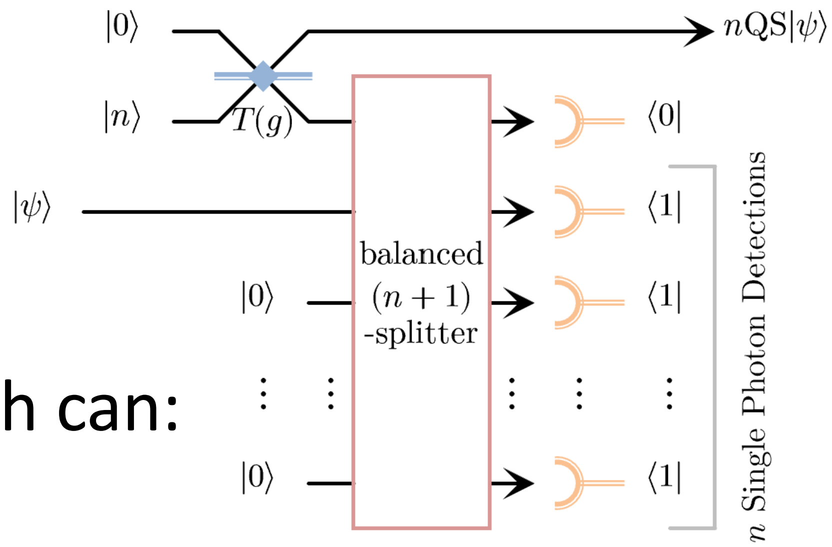
[3] simplest purification protocols

Attend Matthew Winnel's talk, Wednesday 3:15 pm QST 13, for more information about [3]!

[1] Guanzon, J. J., Winnel, M. S., Lund, A. P., & Ralph, T. C. (2022). *Physical Review Letters*, 128(16), 160501.
[2] Winnel, M. S., Guanzon, J. J., Hosseinidehaj, N., & Ralph, T. C. (2021). *arXiv preprint*, arXiv:2105.03586.
[3] Winnel, M. S., Guanzon, J. J., Hosseinidehaj, N., & Ralph, T. C. (2022). *npj Quantum Information* 8, 129.

Summary



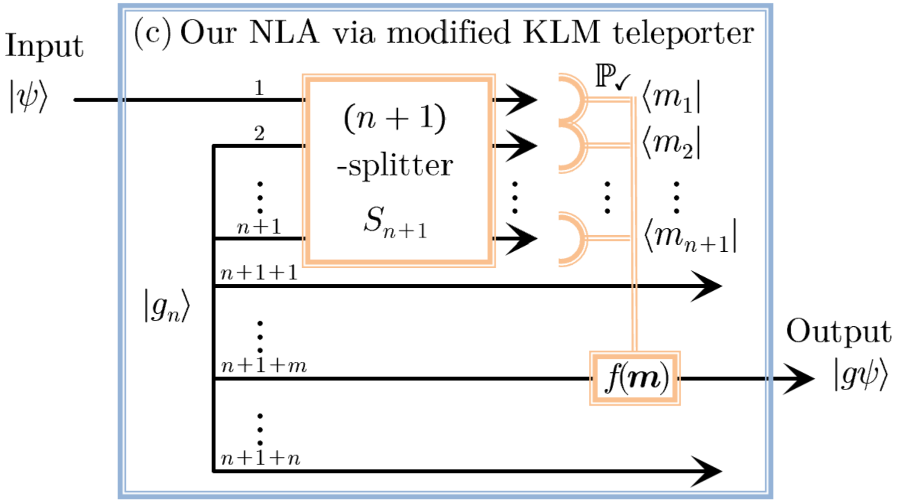


Three quantum amplifiers using linear optics, which can:

[1] Amplify any state containing up to n photons with unity fidelity.

[2] Amplify coherent state superpositions with unity fidelity.

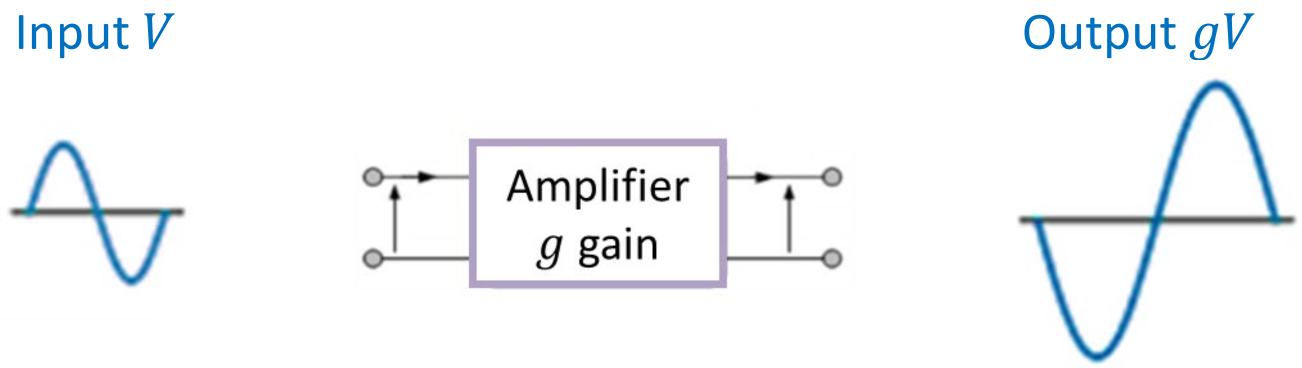
[3] Amplify with maximum success probability.



[1] Guanzon, J. J., Winnel, M. S., Lund, A. P., & Ralph, T. C. (2022). *Physical Review Letters*, 128(16), 160501.
[2] Guanzon, J. J., Winnel, M. S., Lund, A. P., & Ralph, T. C. (2022). *arXiv preprint*, arXiv:2211.08035.
[3] Guanzon, J. J., Winnel, M. S., Singh, D., Lund, A. P., & Ralph, T. C. (2022). *arXiv preprint*, arXiv:2212.04274.

Extra: What is an amplifier?

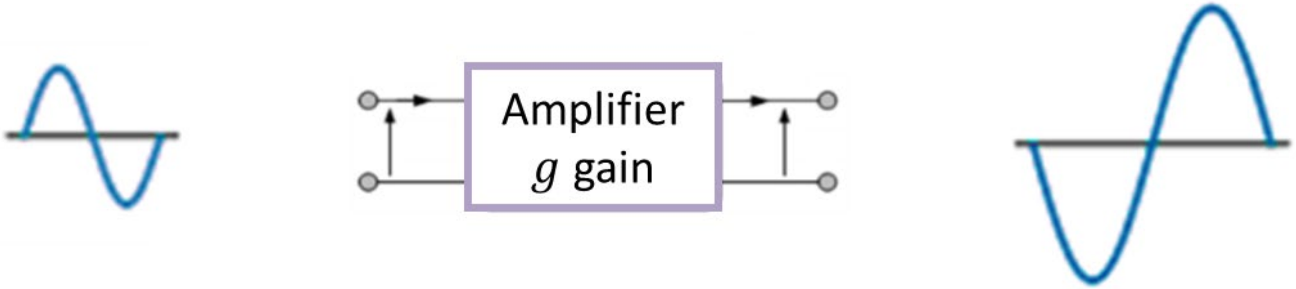
Electronic amplifiers increases the amplitude of voltage/current:



Extra: What is an amplifier?

Electronic amplifiers increases the amplitude of voltage/current:

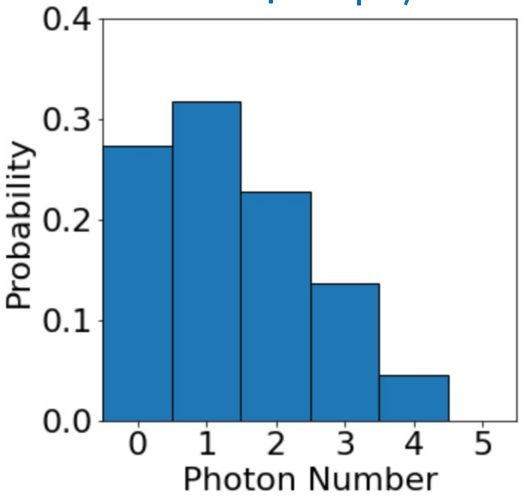
Input V | Output gV



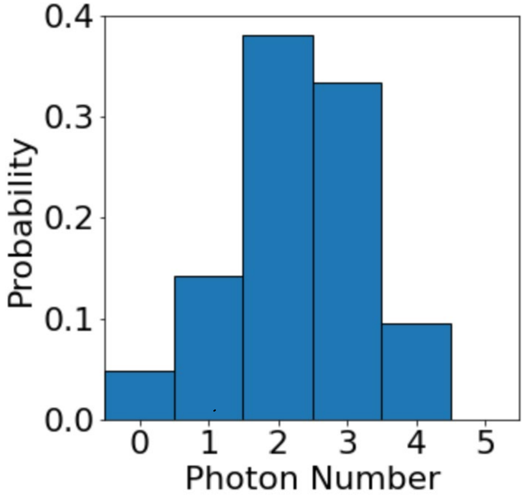
Quantum amplifiers increases the amplitude of quantum states:

(Fock space)

Input $|\alpha\rangle$



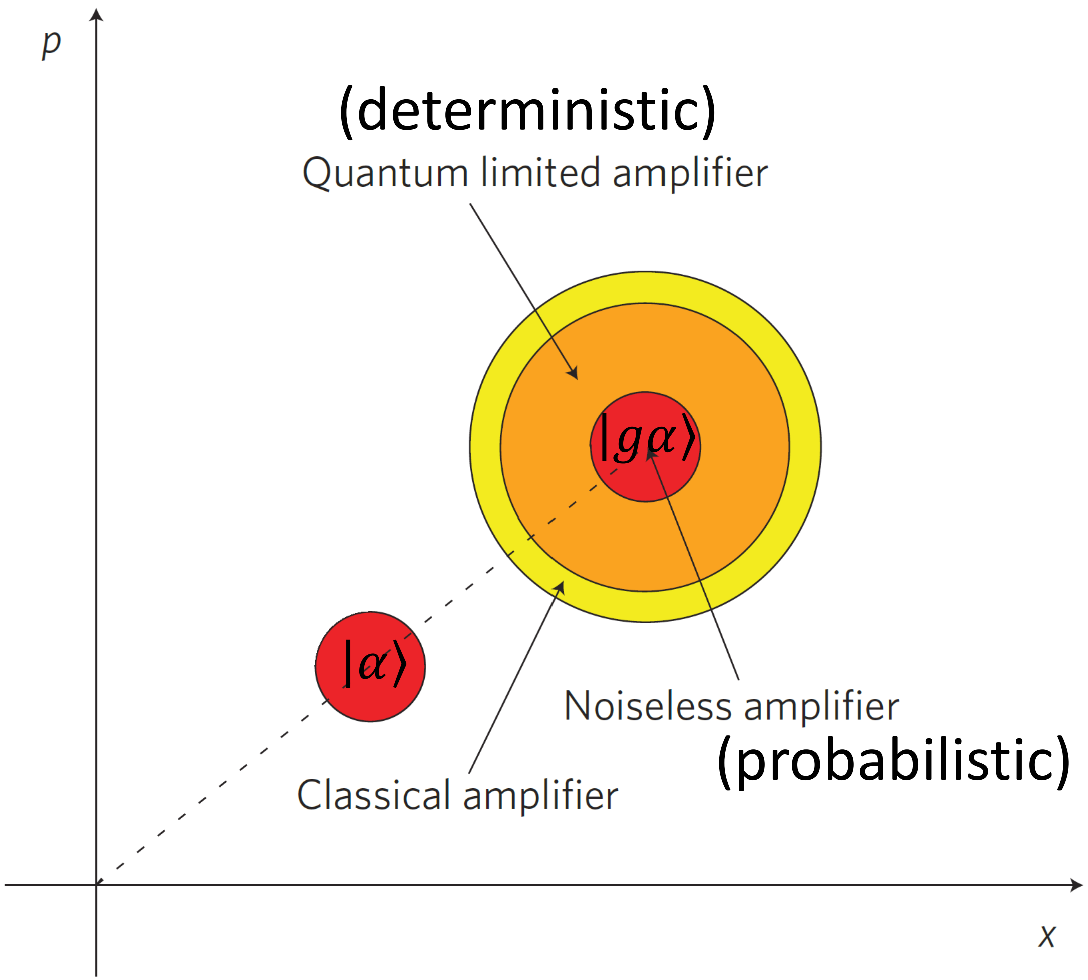
Output $|g\alpha\rangle$



Extra: Noiseless Linear Amplifier

Highest Quality Quantum Amplifier

Linear Amplifier	Extra Noise	Success Probability
Classical	$g^2\delta$	1
Quantum Limited	$g^2\delta + g^2 - 1$	1 ✓
Noiseless	1 ✓	$1/g^2$

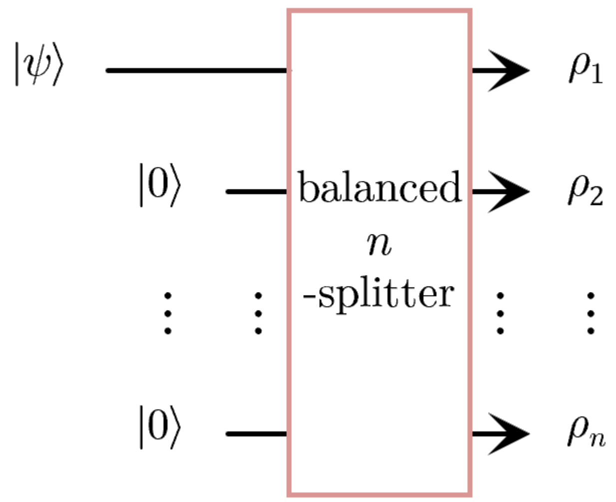


doi.org/10.1038/nphoton.2010.260

Extra: $1QS$ in Parallel

Past Solution to Move Beyond 1 Photon

1. Split up the high photon state $|\psi\rangle$ into n low photon rails.

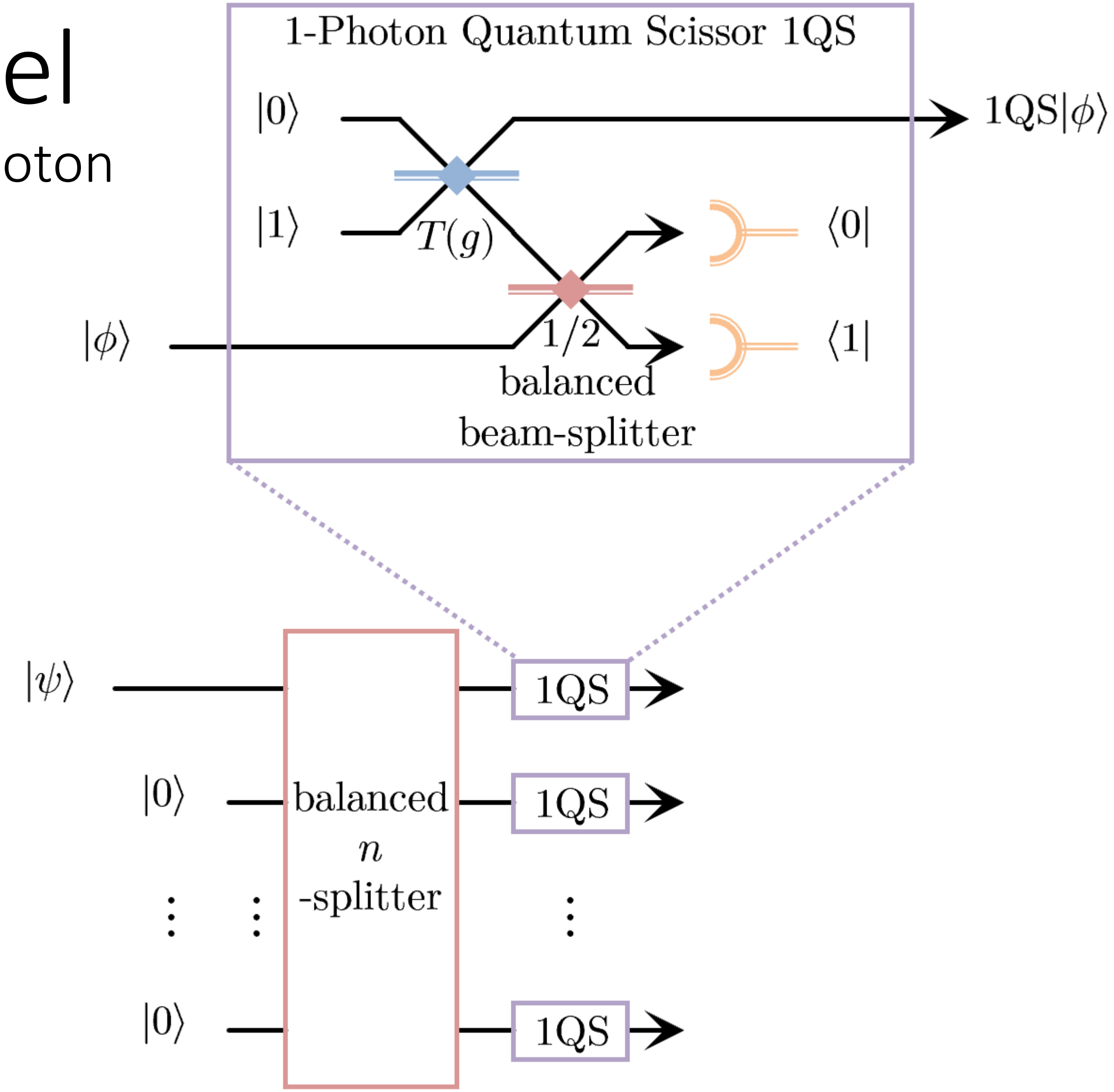


Extra: 1QS in Parallel

Past Solution to Move Beyond 1 Photon

1. Split up the high photon state $|\psi\rangle$ into n low photon rails.

2. Amplify each rail.



Extra: 1QS in Parallel

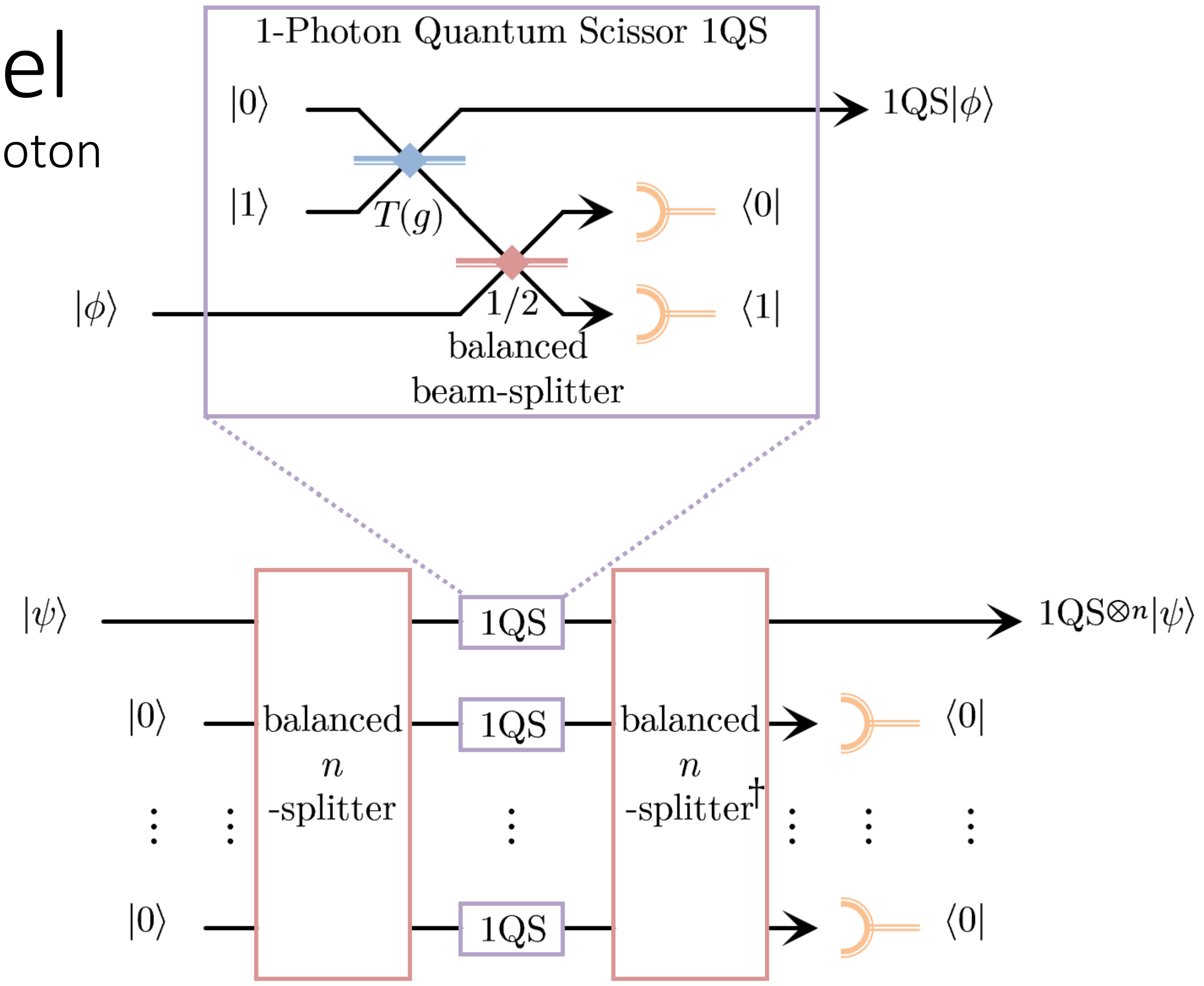
Past Solution to Move Beyond 1 Photon

1. Split up the high photon state $|\psi\rangle$ into n low photon rails.

2. Amplify each rail.

3. Recombined into a complete state $1QS^{\otimes n}|\psi\rangle$.

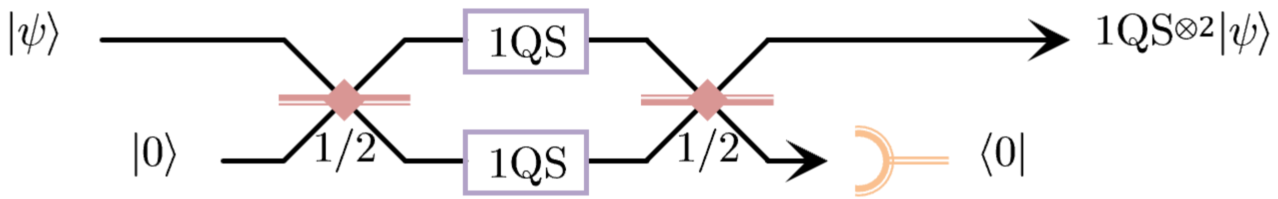
$$\lim_{n\to\infty} 1QS^{\otimes n} = g^{a^\dagger a}$$



Extra: 1QS in Parallel

Past Solution to Move Beyond 1 Photon

For finite 1QS, there is distortion of higher photon states.

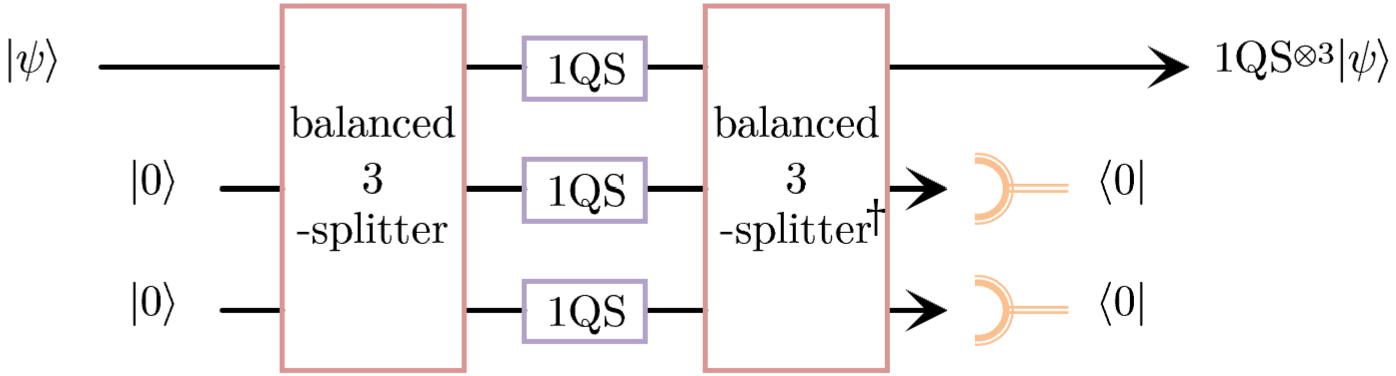


$$1QS^{\otimes 2} \propto |0\rangle\langle 0| + g|1\rangle\langle 1| + \frac{1}{2}g^2|2\rangle\langle 2|$$

Extra: 1QS in Parallel

Past Solution to Move Beyond 1 Photon

For finite 1QS, there is distortion of higher photon states.

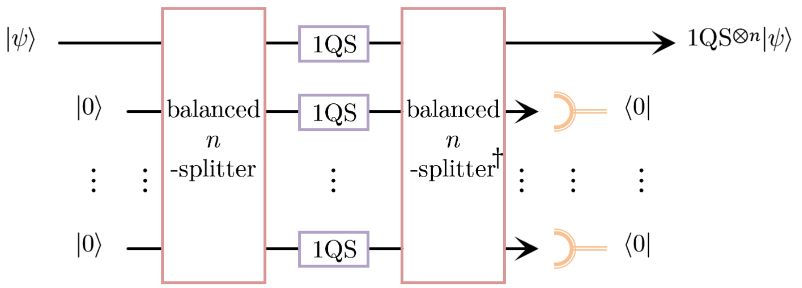


$$1QS^{\otimes 3} \propto |0\rangle\langle 0| + g|1\rangle\langle 1| + \frac{2}{3}g^2|2\rangle\langle 2| + \frac{2}{9}g^3|3\rangle\langle 3|$$

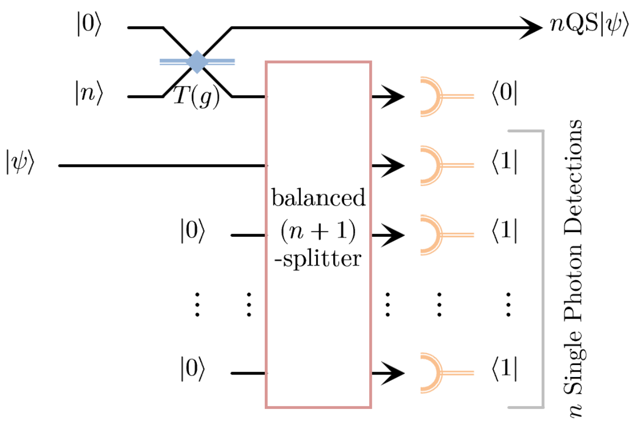
Is there a way to perform ideal NLA $g^{a^\dagger a}$ up to n without distortion?

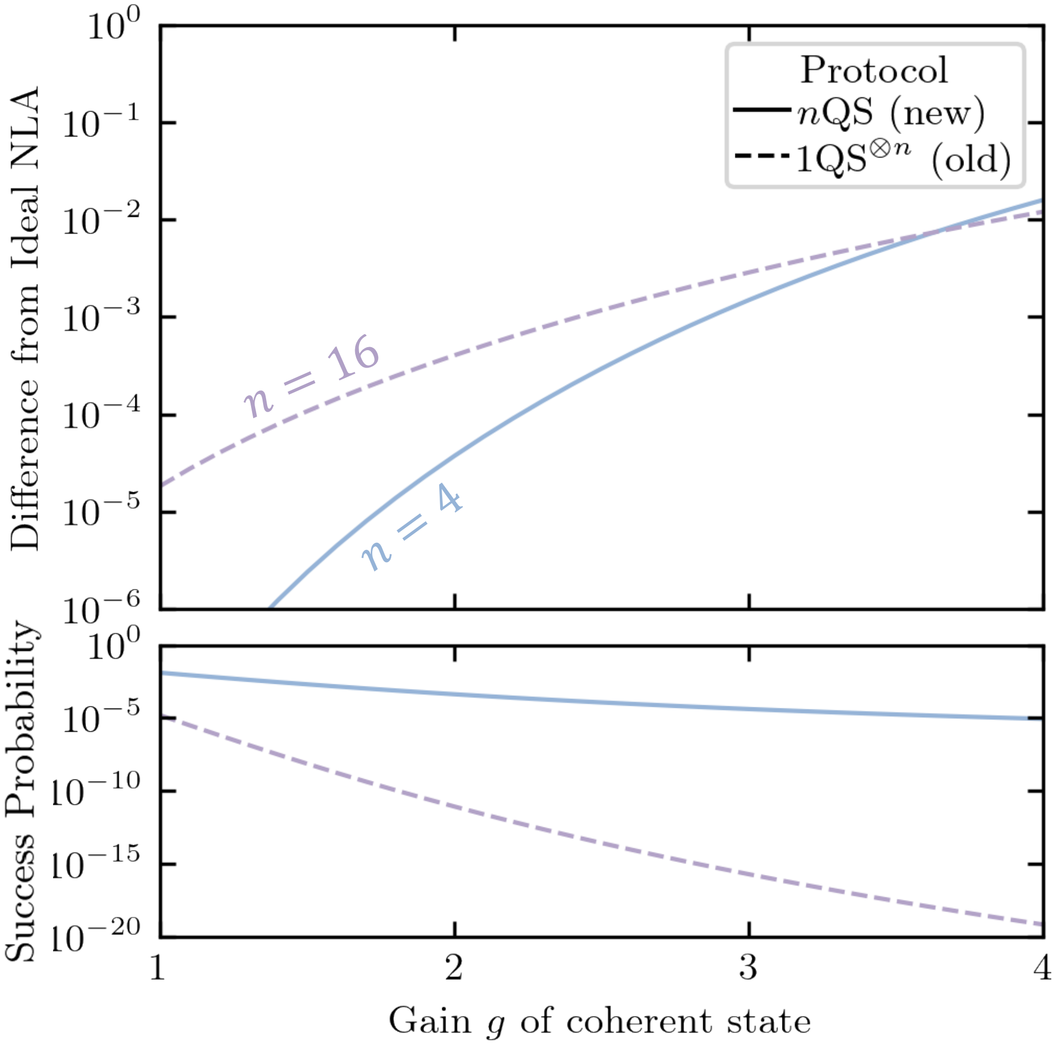
Yes.

Extra: nQS for Amplification



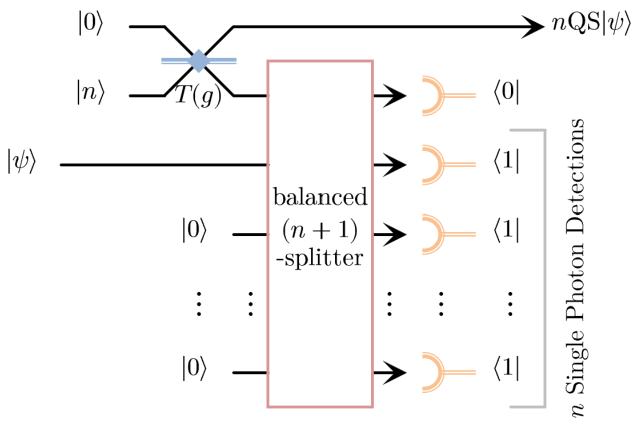
Our new protocol can beat the old one simultaneously in fidelity, success probability and resources.



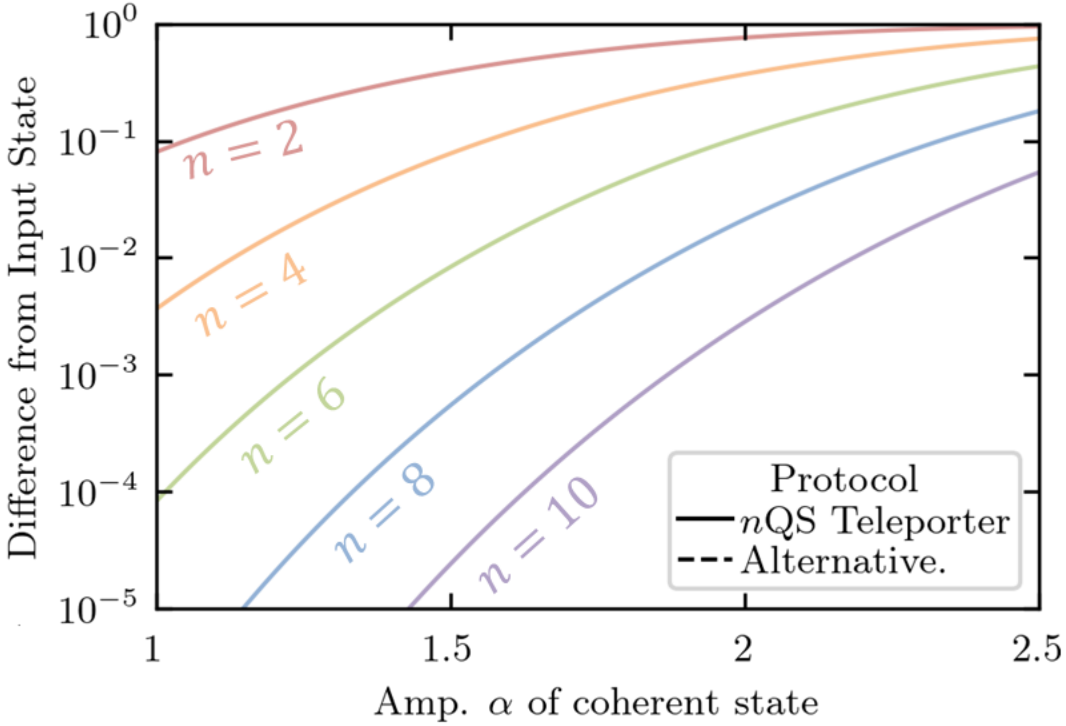


Extra: nQS for Continuous-Variable Teleportation

Solid lines is our nQS with no gain:

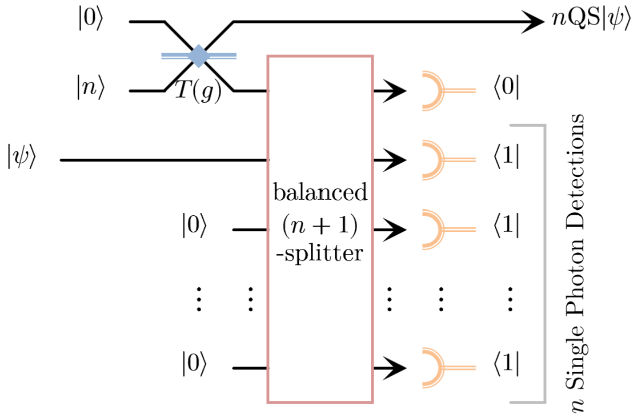


$$nQS(g = 1) \propto \sum_{j=0}^{n} |j\rangle\langle j|$$

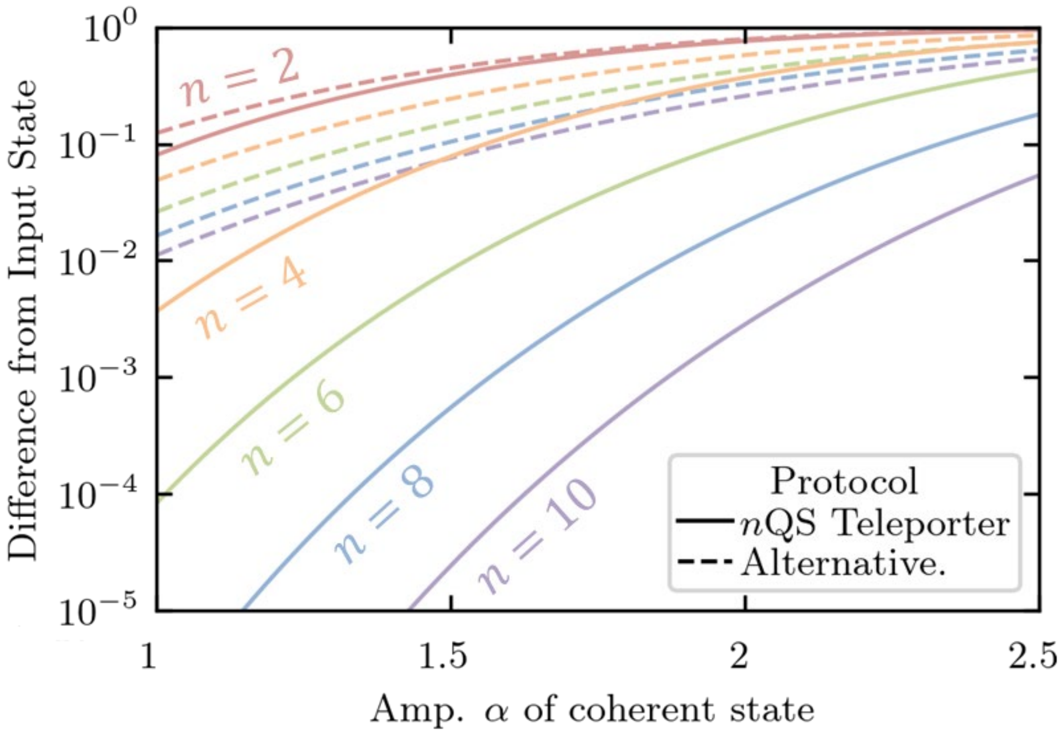


Extra: nQS for Continuous-Variable Teleportation

Solid lines is our nQS with no gain:



$$nQS(g = 1) \propto \sum_{j=0}^{n} |j\rangle\langle j|$$



Dashed lines are an alternative:

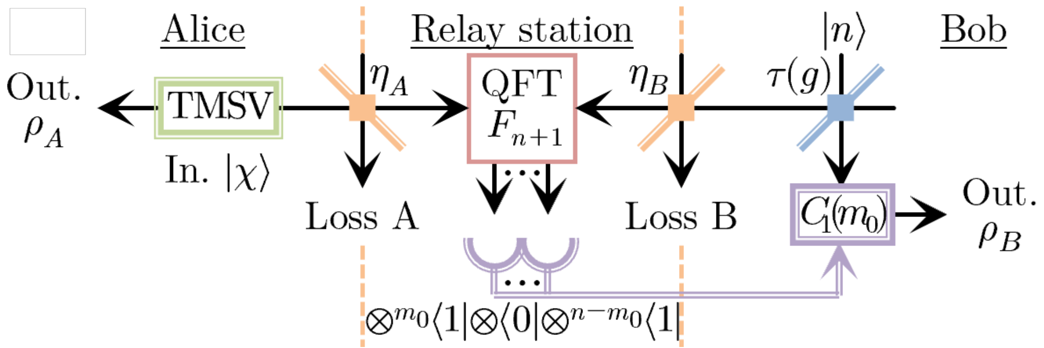
PRL **111,** 050504 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

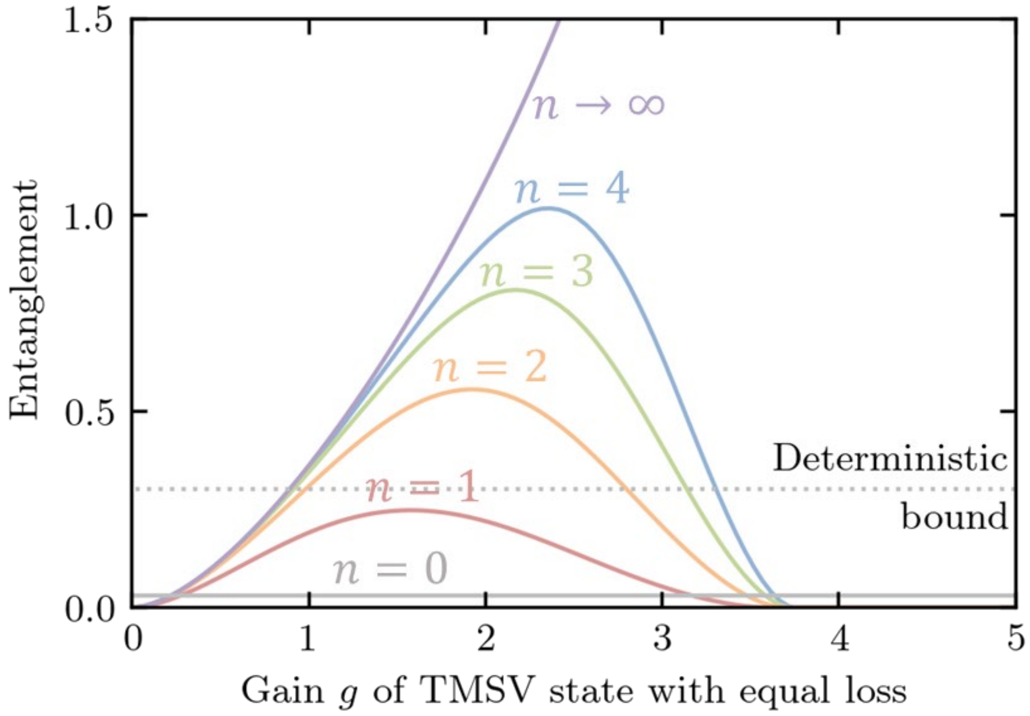
High-Fidelity Teleportation of Continuous-Variable Quantum States Using Delocalized Single Photons

Ulrik L. Andersen[1] and Timothy C. Ralph[2]

Extra: nQS for Entanglement Distillation

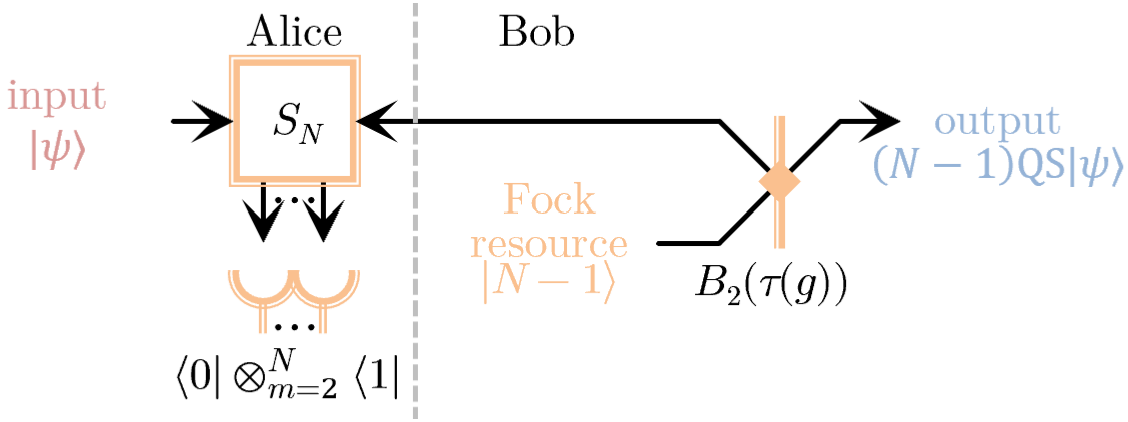
We can distil higher photon entanglement with our protocol.





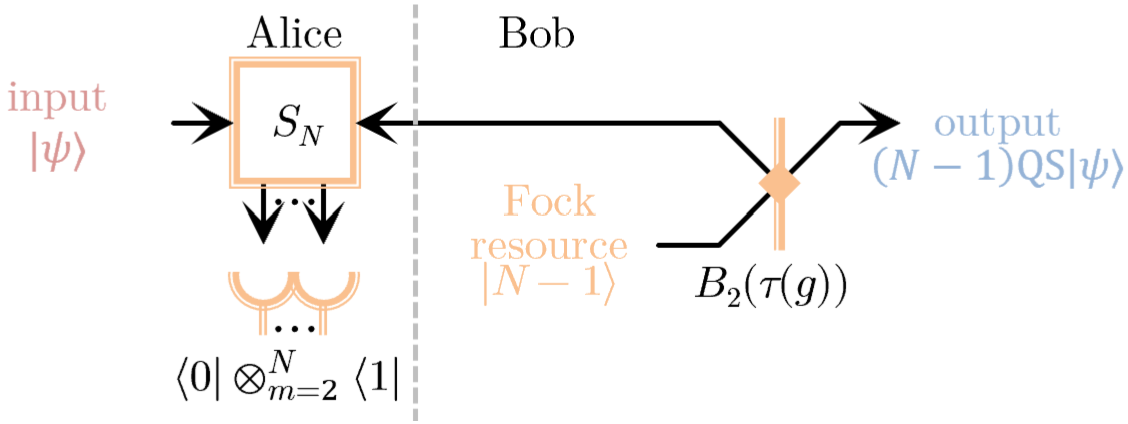
Extra: Amplifier for Coherent States

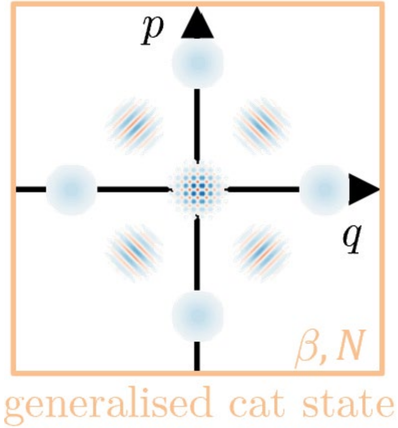
(same n-photon quantum scissor with $n = N - 1$)

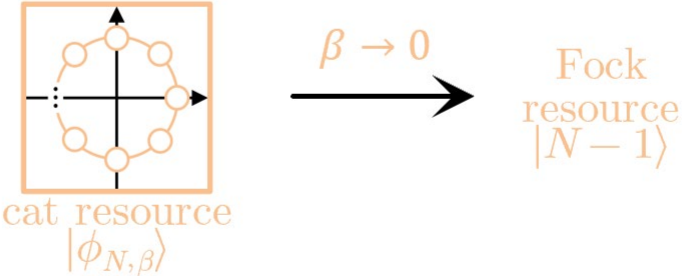


Extra: Amplifier for Coherent States

(same n-photon quantum scissor with $n = N - 1$)

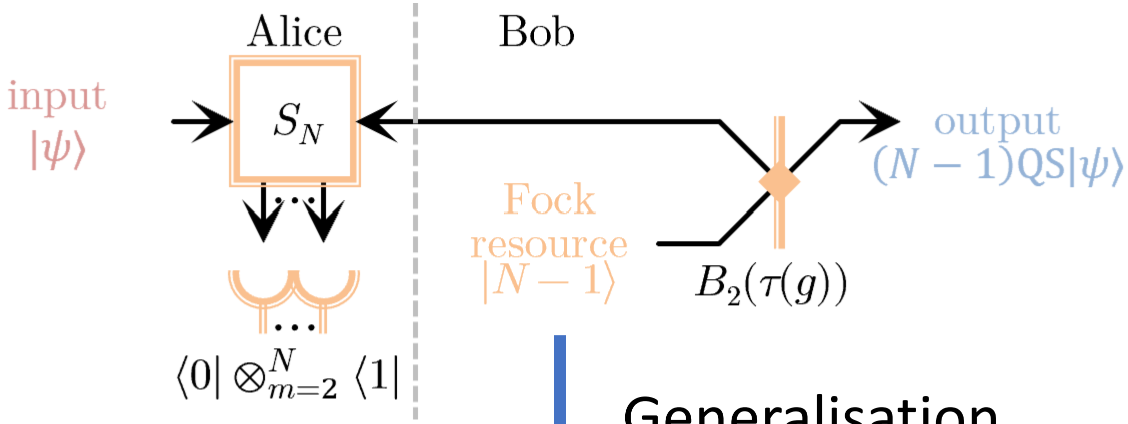




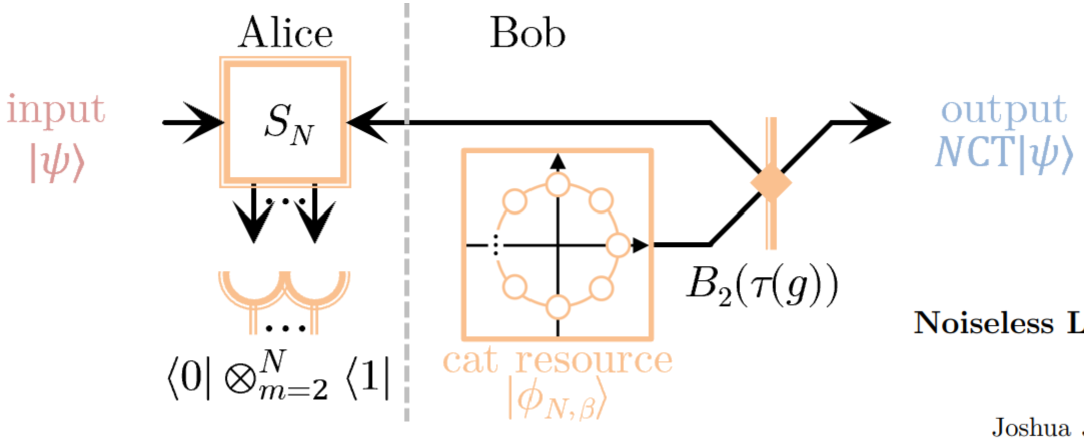


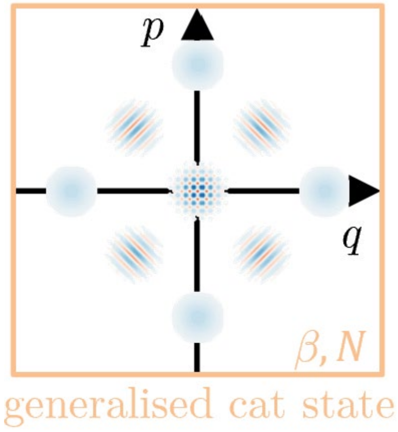
Extra: Amplifier for Coherent States

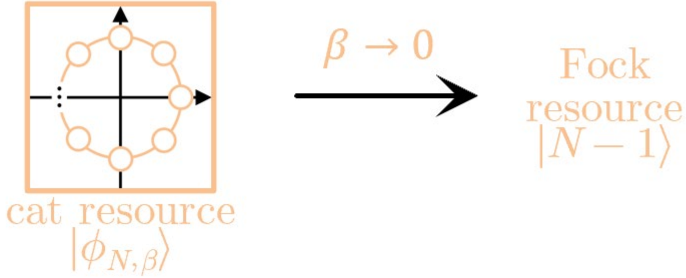
(same n-photon quantum scissor with $n = N - 1$)



Generalisation



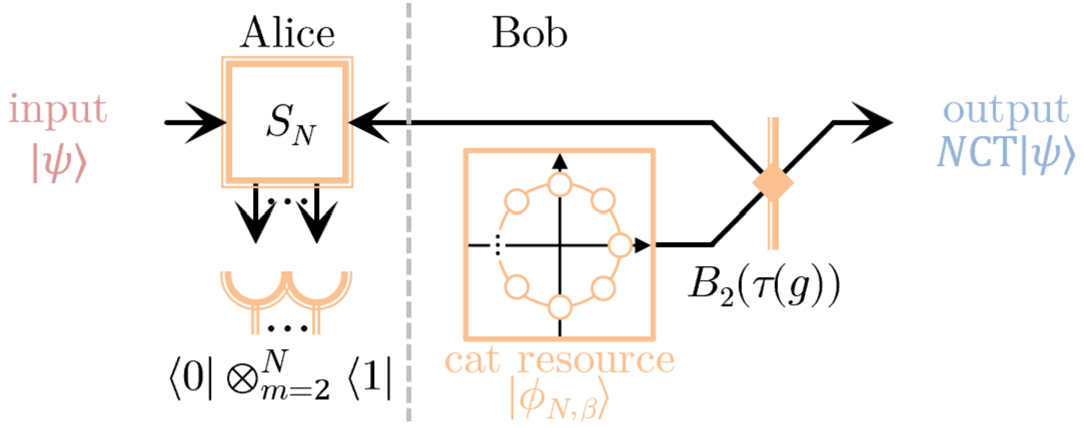




Noiseless Linear Amplification and Loss-Tolerant Quantum Relay using Coherent State Superpositions

Joshua J. Guanzon[1,*] Matthew S. Winnel[1] Austin P. Lund[2,1] and Timothy C. Ralph[1]

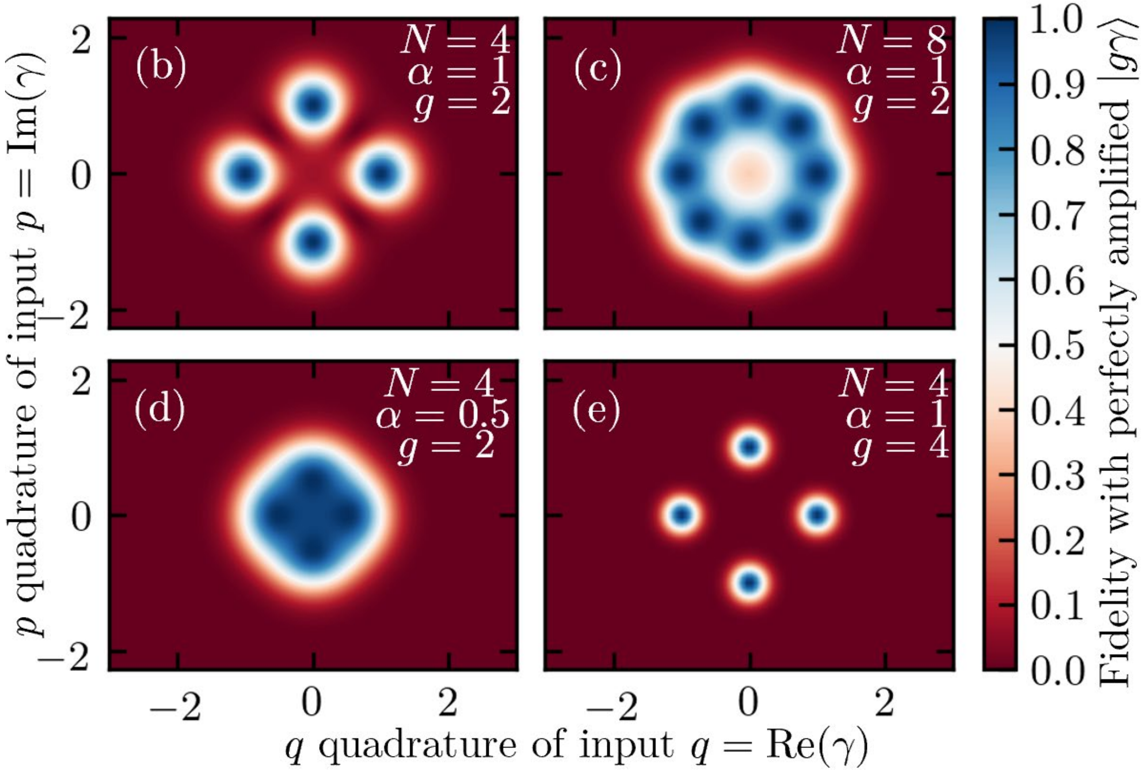
Extra: Amplifier for Coherent States



Which input coherent states

$$|\psi\rangle = |\gamma = q + ip\rangle$$

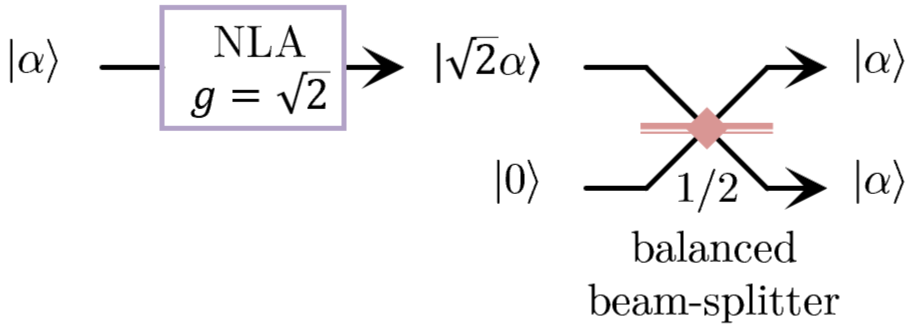
are amplified well by NCT?



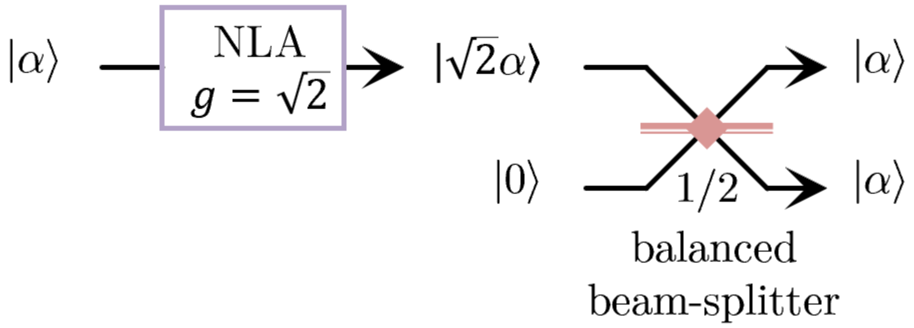
Noiseless Linear Amplification and Loss-Tolerant Quantum Relay using Coherent State Superpositions

Joshua J. Guanzon,[1,*] Matthew S. Winnel,[1] Austin P. Lund,[2,1] and Timothy C. Ralph[1]

Extra: Noiseless + Linear = Probabilistic



Extra: Noiseless + Linear = Probabilistic



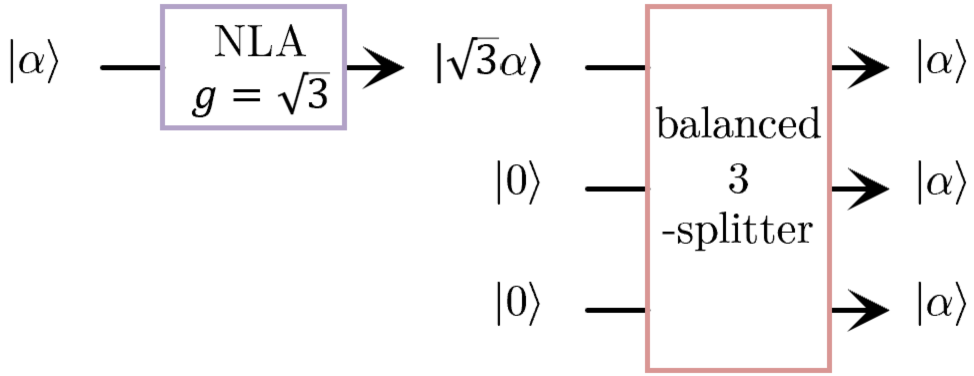
Average Clones Accounting

$$E[N_{\text{clones}}] = N_{\text{clones}|\text{success}} P_{\text{success}}$$
$$= 2 * \frac{1}{2} = 1$$

The no-cloning theorem means an NLA <u>must fail some of the time</u>:

$$P_{\text{success}} = \frac{1}{2}$$

Extra: Noiseless + Linear = Probabilistic



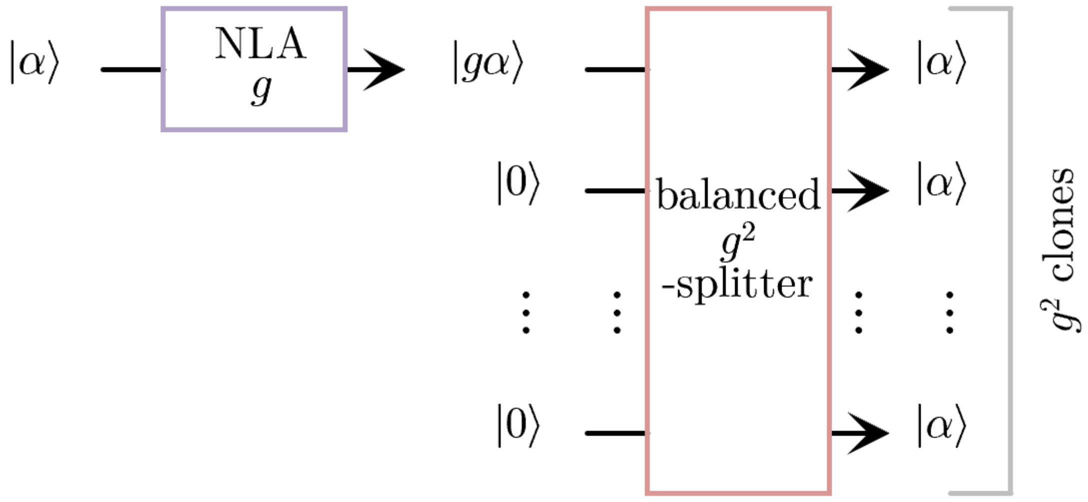
The no-cloning theorem means an NLA <u>must fail some of the time</u>:

$$P_{\text{success}} = \frac{1}{3}$$

Average Clones Accounting

$$E[N_{\text{clones}}] = N_{\text{clones}|\text{success}} P_{\text{success}}$$
$$= 3 * \frac{1}{3} = 1$$

Extra: Noiseless + Linear = Probabilistic



Average Clones Accounting

$$E[N_{\text{clones}}] = N_{\text{clones}|\text{success}} P_{\text{success}}$$
$$= g^2 * \frac{1}{g^2} = 1$$

The no-cloning theorem means an NLA <u>must fail some of the time</u>:

$$P_{\text{success}} = \frac{1}{g^2}$$

Extra: Some papers assuming $1/g^2$…

PHYSICAL REVIEW A **86**, 012327 (2012)

Improving the maximum transmission distance of continuous-variable quantum key distribution using a noiseless amplifier

Rémi Blandino,[1,*] Anthony Leverrier,[2] Marco Barbieri,[1,†] Jean Etesse,[1] Philippe Grangier,[1] and Rosa Tualle-Brouri[1,3]

In Appendix D, we show that the probability of success for a NLA of gain g is upper bounded by $1/g^2$. We can therefore use this bound, keeping in mind that the relevant conclusion that can be taken is only whether the secret key rate is positive

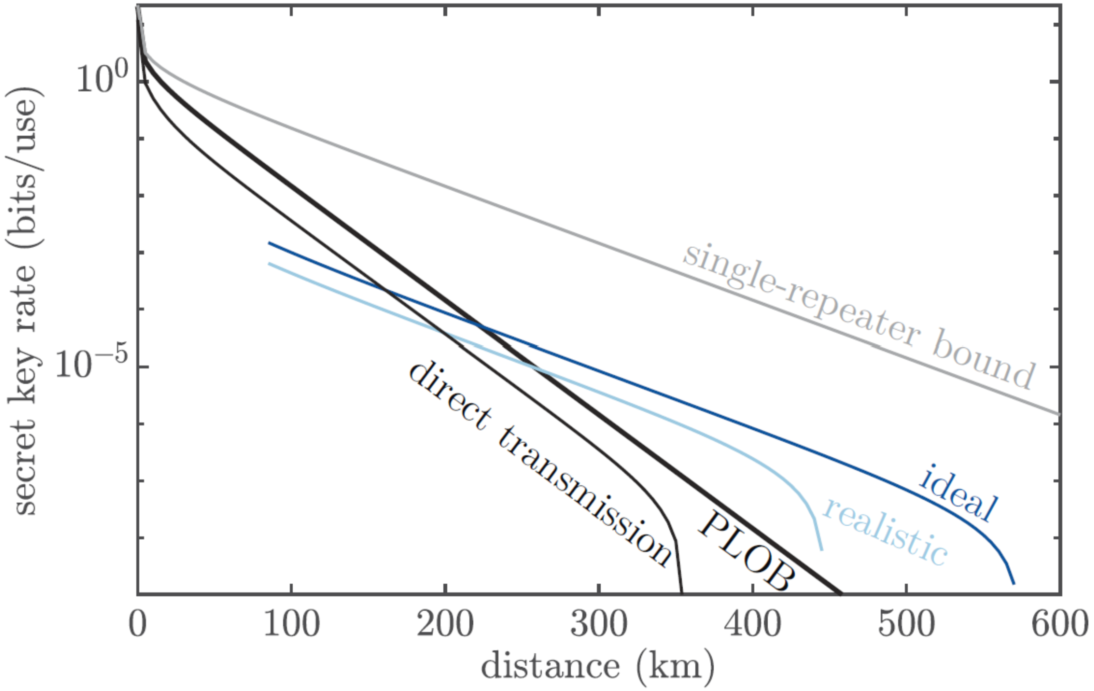
PAPER • OPEN ACCESS

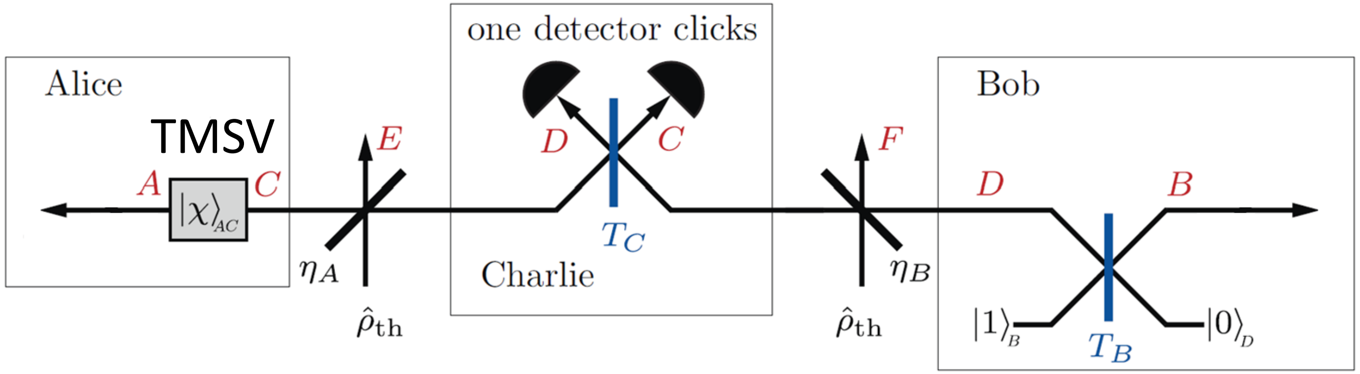
Noiseless linear amplification in quantum target detection using Gaussian states

To cite this article: Athena Karsa *et al* 2022 *Quantum Sci. Technol.* **7** 035026

Taking into account constraints on effective parameters given by equation (6), figures 2 and 3 plot the performance of the TMSV state with NLA relative to that of a coherent state with NLA. Note that the full, exact forms of the QCB have been employed in the computation, that is, without any assumptions as to the relative magnitude of parameter values. Further, the plots have been generated assuming a maximum theoretical probability of success, given by $P_{\mathrm{succ}}^{\mathrm{NLA}} = 1/g^2$ to model the absolute limits of NLA performance.

Extra: Simple Repeater - Rate





Overcoming the repeaterless bound in continuous-variable quantum communication without quantum memories

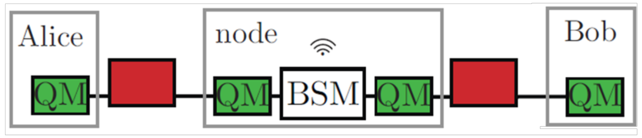
Matthew S. Winnel,[1,*] Joshua J. Guanzon,[1] Nedasadat Hosseinidehaj,[1] and Timothy C. Ralph[1]

[1]*Centre for Quantum Computation and Communication Technology, School of Mathematics and Physics, University of Queensland, St Lucia, Queensland 4072, Australia*

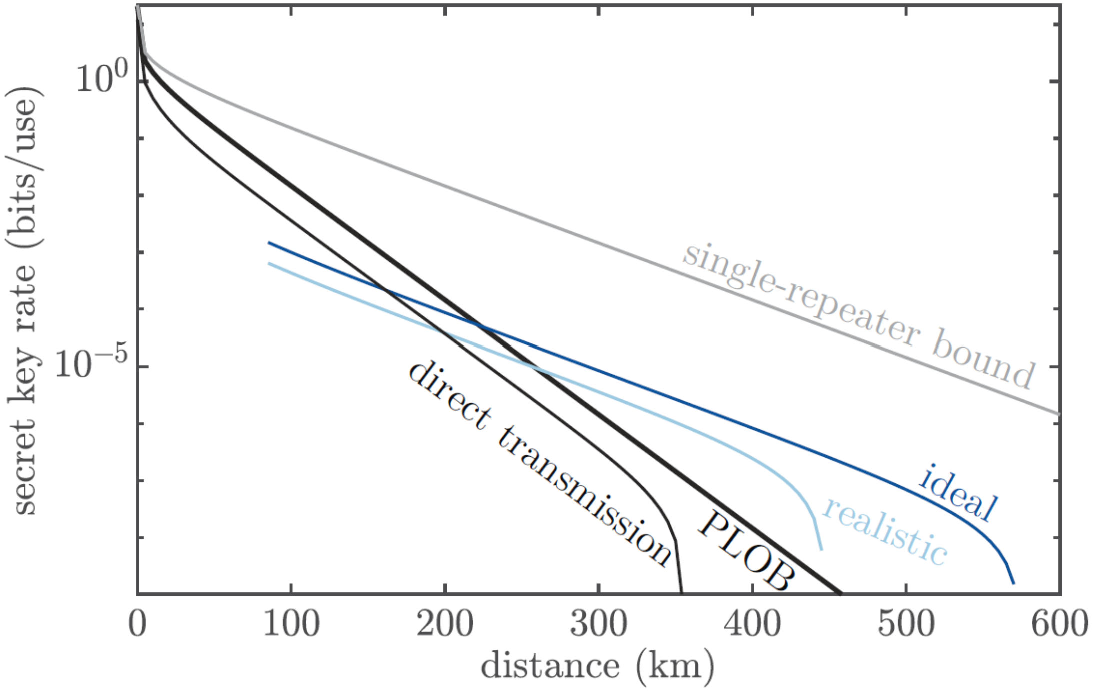
(Dated: November 8, 2021)

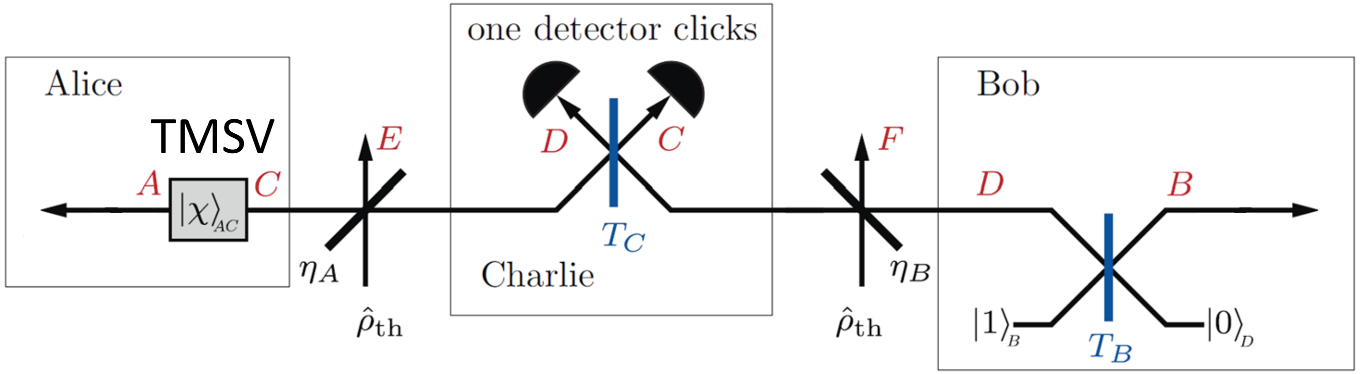
arxiv.org/abs/2105.03586

Extra: Simple Repeater - Rate



Equivalent rate scaling





- Combines quantum distillation and entanglement swapping steps = no quantum memories required!

- Collaborating with DTU for experimental implementation.

Overcoming the repeaterless bound in continuous-variable quantum communication without quantum memories

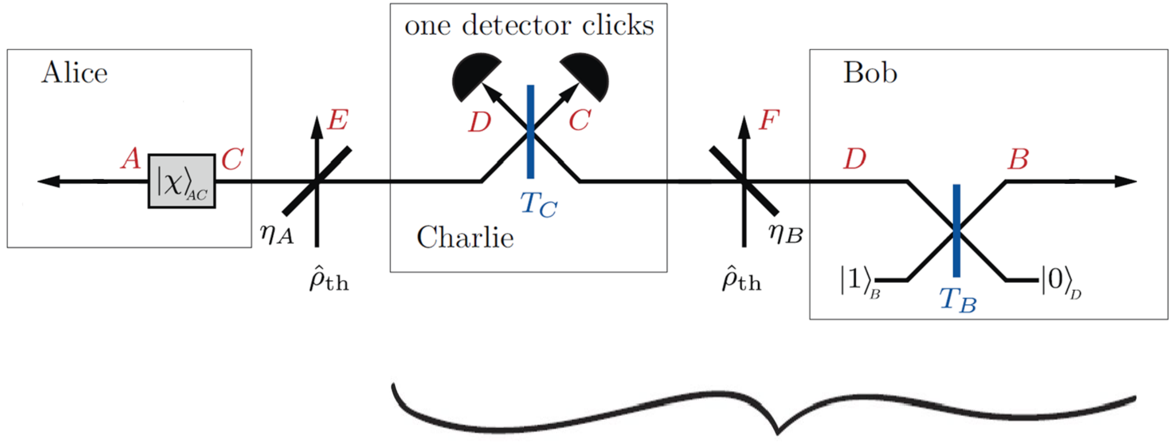
Matthew S. Winnel,[1,*] Joshua J. Guanzon,[1] Nedasadat Hosseinidehaj,[1] and Timothy C. Ralph[1]

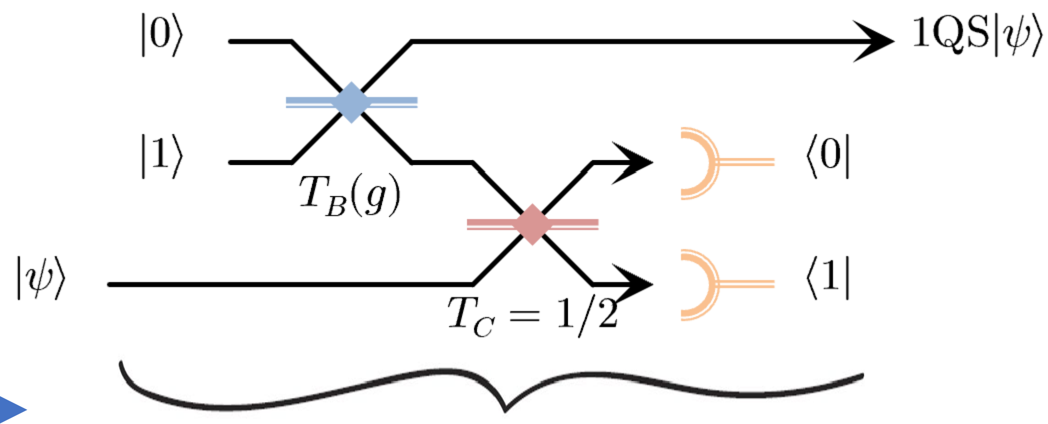
[1]*Centre for Quantum Computation and Communication Technology, School of Mathematics and Physics, University of Queensland, St Lucia, Queensland 4072, Australia*

(Dated: November 8, 2021)

arxiv.org/abs/2105.03586

Extra: Simple Repeater - Amplifier Component



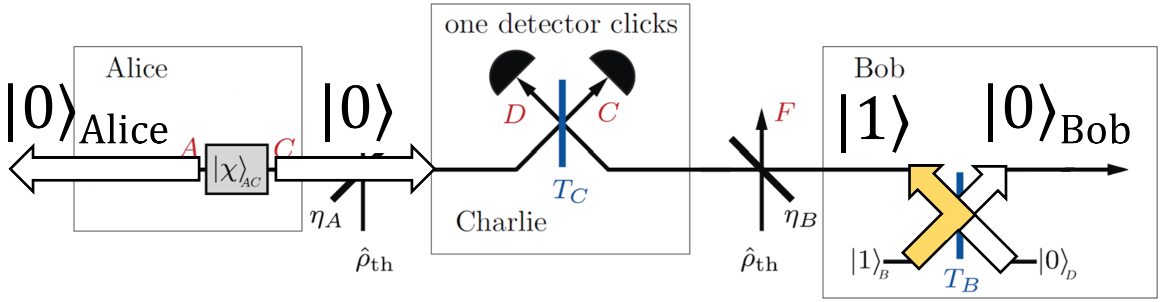


(noiseless linear) amplifier component ↔ 1-Photon Quantum Scissor 1QS

arxiv.org/abs/2105.03586

Extra: Simple Repeater - How it Works

Charlie received photon from Bob



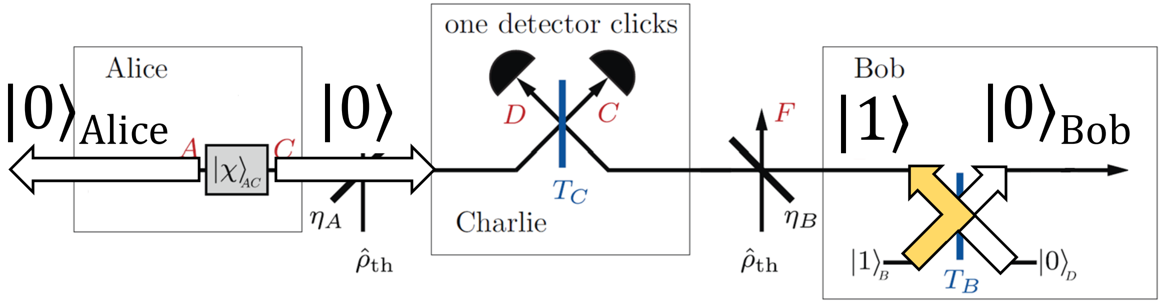
success cases

$$c_{00}|0\rangle_{\text{Alice}}|0\rangle_{\text{Bob}}$$

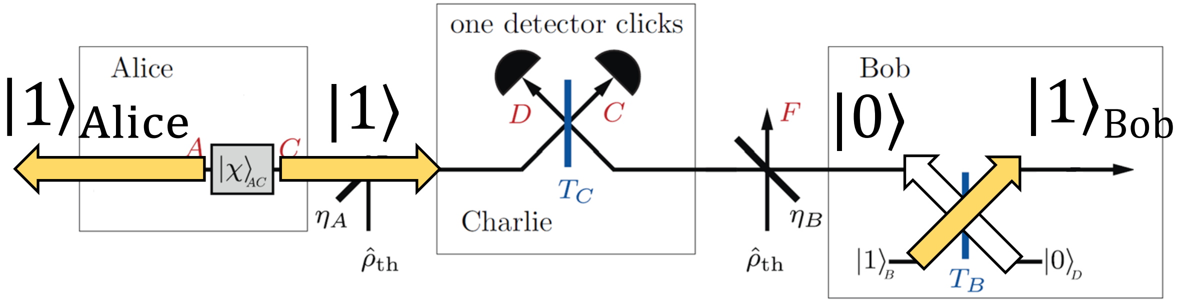
arxiv.org/abs/2105.03586

Extra: Simple Repeater - How it Works

Charlie received photon from Bob



Charlie received photon from Alice



success cases

$$c_{00}|0\rangle_{\text{Alice}}|0\rangle_{\text{Bob}}$$

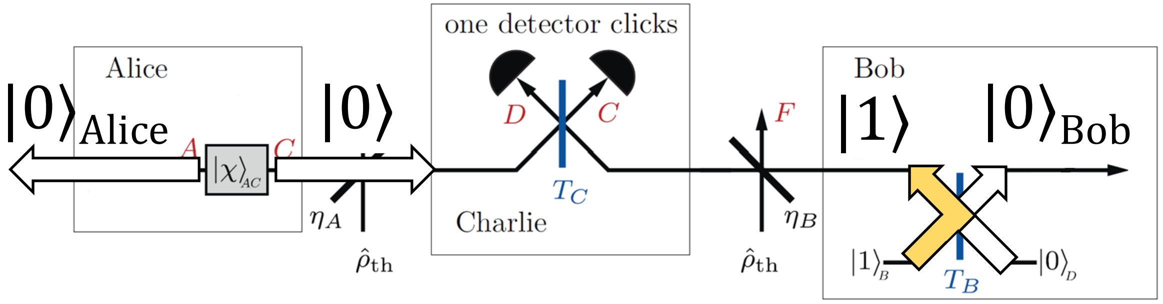
$$+c_{11}|1\rangle_{\text{Alice}}|1\rangle_{\text{Bob}}$$

(Note $|2\rangle_{\text{Alice}}|2\rangle_{\text{Bob}}$ is not possible because Bob only has one photon.)

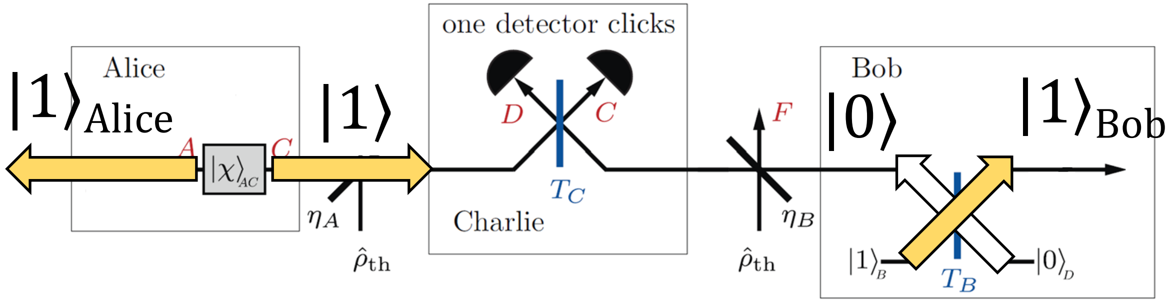
arxiv.org/abs/2105.03586

Extra: Simple Repeater - How it Works

Charlie received photon from Bob



Charlie received photon from Alice



success cases

$c_{00}|0\rangle_{Alice}|0\rangle_{Bob}$

$+c_{11}|1\rangle_{Alice}|1\rangle_{Bob}$

error cases

(lost photons)

$+c_{10}|1\rangle_{Alice}|0\rangle_{Bob}$

(dark counts)

$+c_{01}|0\rangle_{Alice}|1\rangle_{Bob}$

$c_{ij} =$
$fij\ (\chi, TB, TC, \eta_A, \eta_B)$
so we can optimise.

(Note $|2\rangle_{Alice}|2\rangle_{Bob}$ is not possible because Bob only has one photon.)

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