No Tradeoff between Coherence and Sub-Poissonianity in Heisenberg-Limited Lasers

L. A. Ostrowski, T. J. Baker, S. N. Saadatmand and H. M. Wiseman

Centre for Quantum Dynamics, Griffith University, Yuggera Country, Brisbane, Queensland 4111, Australia.

Phase fluctuations in a CW laser beam may be quantified with the dimensionless quantity coined as *laser coherence*, denoted by $\mathfrak{C}$. This is defined generally as the maximally populated spatial mode of a bosonic beam, which, for a laser exhibiting 'ideal' properties (that is, its beam can be described as a coherent state undergoing pure phase diffusion), can be expressed as

$$\mathfrak{C} = \frac{4\mathscr{N}}{\ell}. \tag{1}$$

Here, $\mathscr{N}$ is the photon flux from the laser and ℓ represents its linewidth. Applying the Schawlow-Townes formula would say that the limit for this quantity is $\mathfrak{C} = O(\mu^2)$, where μ is the total number of coherent excitations stored inside the laser cavity. However, recent work [1] has rigorously proven that the ultimate limit imposed by quantum mechanics, or *Heisenberg limit*, is $\mathfrak{C} = O(\mu^4)$, demonstrating a quadratic enhancement over the former historic limit. Models for lasers which could attain a value of $\mathfrak{C}$ that surpass the standard Schawlow-Townes μ^2 scaling have also been proposed. These require the coupling between the laser cavity and its environment to be highly nonlinear, a feat which is thought to be achievable on the platform of circuit-QED.

In this work, we generalize the proof of the Heisenberg limit for $\mathfrak{C}$, such that it encompasses beams that can be highly sub-Poissonian (quantified by the Mandel-Q parameter). For such beams, we show that $\mathfrak{C} = O(\mu^4)$ remains as the Heisenberg limit. Additionally, we introduce two new families of laser models that, under specific parameter values, can attain both Heisenberg-limited scaling for the coherence, $\mathfrak{C} = \Theta(\mu^4)$, and exhibit sub-Poissonian photon statistics ($Q < 0$) in the beam. Interestingly, we demonstrate that for both families of laser models, there is a 'win-win' relationship between $\mathfrak{C}$ and Q—the former is maximized for the exact parameter values that minimize the latter. Moreover, the maximum coherence attained here can surpass that of the model from Ref. [1] by a factor of at least 4. These results mark a generalization of the findings from studies of 'standard' (linear-loss) lasers, where we have extended the notion that a tradeoff between $\mathfrak{C}$ and Q does not necessarily exist, even for lasers with a phase diffusion rate that is far smaller than any model previously studied.

[1] T. J. Baker, S. N. Saadatmand, D. W. Berry and H. M. Wiseman, "The Heisenberg Limit for Laser Coherence", *Nat. Phys.* **17**, 179 (2020).