Heisenberg-Limited Laser Coherence: No Trade-off with Sub-Poissonianity

AIP Congress, December 2022

Lucas Ostrowski ∘ Travis Baker ∘ Nariman Saadatmand ∘ Howard Wiseman

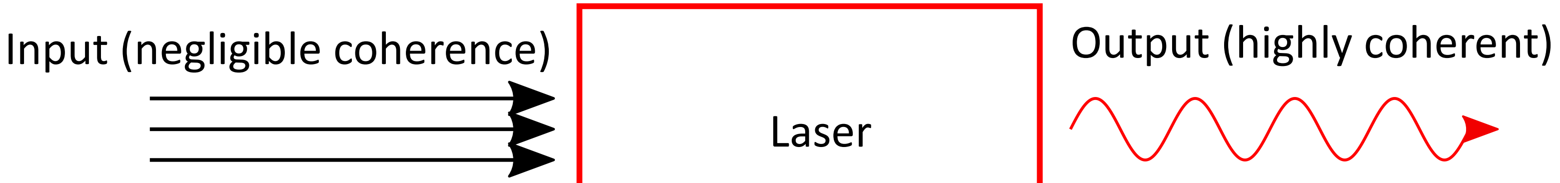
Griffith University
Queensland, Australia

Australian Government
Australian Research Council

Laser Coherence

- Coherence: A property of the field to act as a classical phase reference.

$$\mathfrak{C} = \text{The mean number of photons in the most populated mode.}$$



$$[\hat{b}(t), \hat{b}^\dagger(s)] = \delta(t-s)$$

- For a beam of light with time-stationary statistics, typically:

$$\mathfrak{C} \approx \mathcal{N} \times \tau_{\mathrm{coh}}$$

Photon Flux ($\mathcal{N}$) — Coherence time (τ_{coh})

Interpretation: Number of photons emitted into the beam that are mutually coherent.

Laser Coherence: Some History

- How does a laser actually produce a highly coherent beam?
  - By storing a large mean number of excitations, μ, within the device.
- But what does this imply for the coherence of the beam?

Schawlow and Townes establish a connection between $\mathfrak{C}$ and μ in a classic 1958 paper.

$$\tau_{\mathrm{coh}} = O(\mu/\kappa)$$

$\kappa =$ Bare cavity linewidth

With perfect output coupling, $\mathcal{N} = \kappa\mu$, a standard quantum limit is implied:

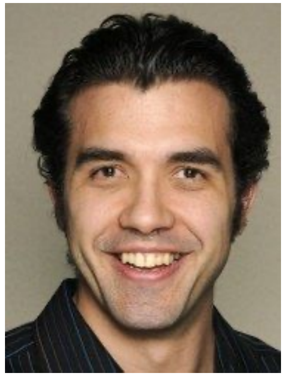
$$\mathfrak{C}_{\mathrm{SQL}}^{\mathrm{ideal}} \approx \mathcal{N}\tau_{\mathrm{coh}} = \Theta(\mu^2)$$

The Heisenberg Limit for Laser Coherence (Baker et al. 2020)*

- Quantum theory sets the ultimate limits to the performance of a device in terms of a particular resource, such as energy or power.
- The ***standard quantum limit*** (SQL) arises from “standard” assumptions about how the device must operate.
- The ***Heisenberg limit*** is the ultimate limit imposed by quantum theory.
- A ***quantum enhancement*** exists when the Heisenberg limit scales better than the SQL, in terms of the resource.

Baker et al. (2020): a quadratic quantum enhancement exists in the production of laser coherence:

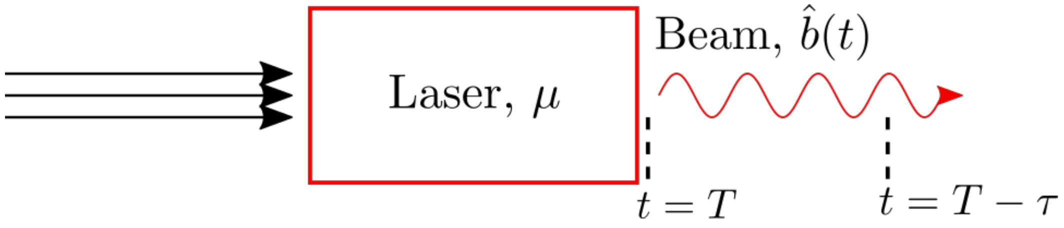
$$\mathfrak{C}_{\mathrm{HL}}^{\mathrm{ideal}} = \Theta(\mu^4)$$



*T. Baker, S. Saadatmand, D. Berry and H. Wiseman, The Heisenberg Limit for Laser Coherence, Nat. Phys. **17**, 179 (2020).

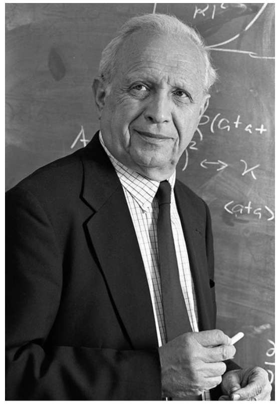
Generalising the Upper Bound for $\mathfrak{C}$

Four conditions placed on the laser and its beam:



1. **<u>One-dimensional beam.</u>**
   - The beam propagates away from the laser in a particular direction, at a constant speed, and has a single transverse mode and a single polarisation.
2. **<u>Endogenous phase.</u>**
   - Beam coherence proceeds only from the laser. That is, a phase shift imposed on the laser state at some time T will lead, in the future, to the same phase shift on the beam emitted after time T, as well as on the laser state.
3. **<u>Stationarity.</u>**
   - The statistics of the laser and beam have a long-time limit that is unique and invariant under time translation. In particular, $\langle \hat{n}_c \rangle$ has a unique limit, μ.
4. **<u>Passably Ideal Glauber (1), (2) coherence.</u>**
   - The stationary first- and second-order Glauber coherence functions *passably-approximate* that of an ideal laser beam.

Generalising the Upper Bound for $\mathfrak{C}$



Condition 4: Passably Ideal Glauber (1), (2) coherence.

- First- and second-order Glauber coherence functions,

$$G^{(1)}(s,t) = \langle \hat{b}^\dagger(s)\hat{b}(t)\rangle, \qquad G^{(2)}(s,s',t',t) = \langle \hat{b}^\dagger(s)\hat{b}^\dagger(s')\hat{b}(t')\hat{b}(t)\rangle.$$

- Consider the state of an "ideal" laser beam according to 1960s laser theory (Scully and Lamb, Lax and Louisell),

eigenstate of $\hat{b}(t)$: $\quad |\beta(t)\rangle = |\sqrt{\mathcal{N}}e^{i\sqrt{\ell}W(t)}\rangle.$

$W(t)$ (Wiener process)

$\ell \equiv 4\mathcal{N}/\mathfrak{C}$ (Laser linewidth)

- First- and second-order Glauber coherence functions of the laser *passably* approximate that of an ideal laser beam (limit $\mu \to \infty$):

$$|g^{(1)}_{\text{laser}}(s,t) - g^{(1)}_{\text{ideal}}(s,t)| = O(1)$$

$$|g^{(2)}_{\text{laser}}(s,s',t',t) - g^{(2)}_{\text{ideal}}(s,s',t',t)| = O(\mathfrak{C}^{-1/2})$$

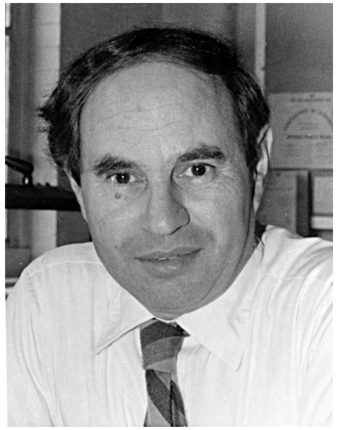
Conditions 1-3 + Condition 4 leads to the Heisenberg Limit:

$$\mathfrak{C} = O(\mu^4)$$

Generalising the Upper Bound for $\mathfrak{C}$

Why?

- Experimental implementation likely requires a more robust upper bound.
- Study of photon statistics of Heisenberg-limited lasers is now permitted.
  - Relaxed condition on $g^{(2)}$ encompasses beams that exhibit sub-Poissonian photon statistics.
  - Specifically, $Q_S < 1$, where the Mandel-Q parameter of the beam is:



$$Q_S := \frac{\langle(\Delta\hat{n}_S)^2\rangle - \langle\hat{n}_S\rangle}{\langle\hat{n}_S\rangle} \in (-1, \infty), \qquad \text{with} \qquad \hat{n}_S = \int_0^S dt \hat{b}^\dagger(t)\hat{b}(t)$$

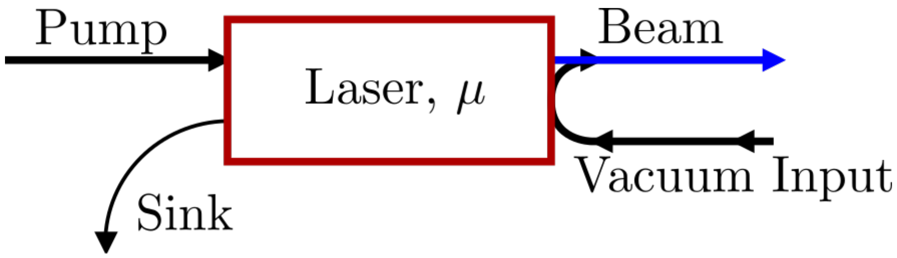
Does a trade-off exist between coherence and sub-Poissonianity in a Heisenberg-limited laser beam?

Sub-Poissonian Laser Models with Heisenberg-Limited Coherence

Does a trade-off exist between coherence and sub-Poissonianity in a Heisenberg-limited laser beam?

- We consider two families of laser models that produce sub-Poissonian beams, which are generalisations of the original HL model put forth in Baker et al. (2020).

λ-Family

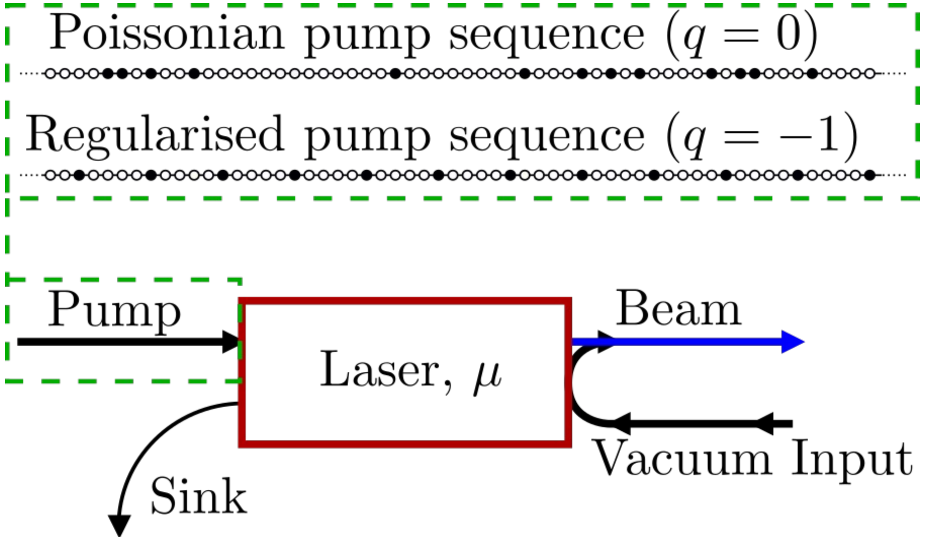


$$\dot{\rho} = \mathcal{D}[\hat{G}^{(\lambda)}]\rho + \mathcal{D}[\hat{L}^{(\lambda)}]\rho$$

$$\langle n|\hat{G}^{(\lambda)}|n-1\rangle = r_n^{\lambda}, \quad \langle n-1|\hat{L}^{(\lambda)}|n\rangle = r_n^{\lambda-1},$$

$$r_n := \frac{\sin^2\left(\pi\frac{n+1}{D+1}\right)}{\sin^2\left(\pi\frac{n}{D+1}\right)}.$$

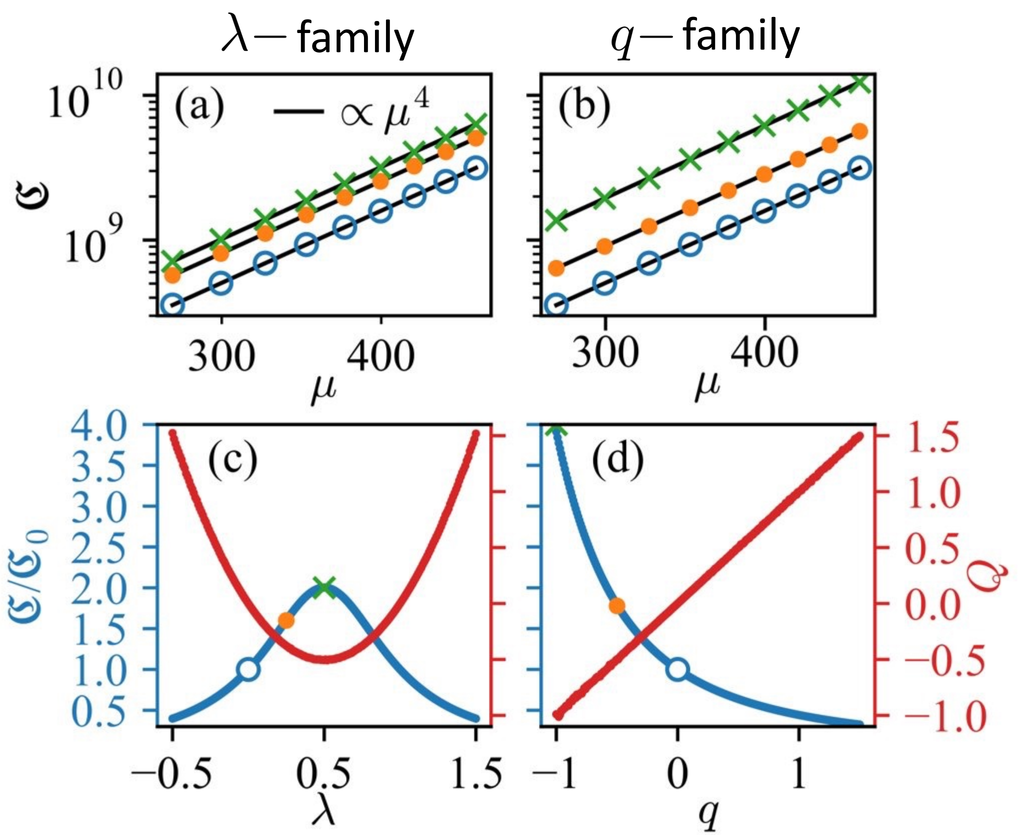
q-Family



$\langle n|\hat{G}|n-1\rangle \equiv 1$, non-Markovian pump with Mandel-Q parameter $q \in (-1, \infty)$.

Heisenberg-Limited Coherence and Sub-Poissonianity: No Trade-off!

Have Your Cake and Eat it Too!
Sub-Poissonian, Heisenberg-Limited lasers



$\mathfrak{C}_0 = \mathfrak{C}_{\lambda=0} = \mathfrak{C}_{q=0}$ (Poissonian limit).

- (a) and (b) show that Heisenberg-limited coherence is achieved for all choices of parameter, with $\mathfrak{C} = \Theta(\mu^4)$.
- (c) and (d) show $\mathfrak{C}$ is maximised for exact choices of parameter that minimise $Q \equiv Q_{S\to\infty}$.
- Synergistic relationship persists even in extreme case, where $Q \to -1$.

These results show "win-win" relationship (no tradeoff) between coherence and sub-Poissonianity in Heisenberg-limited lasers.

Thank You For Your Attention!

Summary

- $\mathfrak{C} =$ The mean number of photons in the most populated mode.
- The Heisenberg limit is $\mathfrak{C}_{\rm HL}^{\rm ideal} = \Theta(\mu^4)$, quadratically larger than $\mathfrak{C}_{\rm SQL}^{\rm ideal} = \Theta(\mu^2)$.
- We have generalised this upper bound for laser coherence to beams that can exhibit significant sub-Poissonian photon statistics.
- HL coherence and sub-Poissonianity: a trade-off?
- Sub-Poissonian, Heisenberg-limited laser models have been developed.
- With these models, we have demonstrated a "win-win" relationship between coherence and sub-Poissonianity *for Heisenberg-limited lasers*.
    - Perfect correlation between an increase in $\mathfrak{C}$ and decrease in $Q_{S\to\infty}$.

For details, see Arxiv: 2208.14081 and Arxiv: 2208.14082