Fundamental limits of quantum error mitigation

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NISQ devices

NISQ = Noisy Intermediate-Scale Quantum

Focus on manipulating O(10) – O(100) qubits.

Let's NOT dive into a complicated error correction and see if we can do **anything useful** with it.

Obstacle: noise, of course!!

Error mitigation: run a given NISQ device many times + classical postprocessing

- Trade space resource with time resource (<u>sampling cost</u>)

Zoo of error-mitigation protocols

Various error-mitigation protocols proposed so far.

- Probabilistic error cancellation [Buscemi et al., ’13] [Temme et al., PRL ’17]
- Noise extrapolation [Temme et al., PRL ’17] [Li, Benjamin, PRX ’17]
- Virtual distillation [Koczor, PRX ’21] [Huggins, PRX ’21]
- Symmetry verification [Bonet-Monroig et al., PRA ’18]
- …

What is the ultimate performance shared by all error-mitigation protocols?

General theory for quantum error mitigation?

General quantum error mitigation

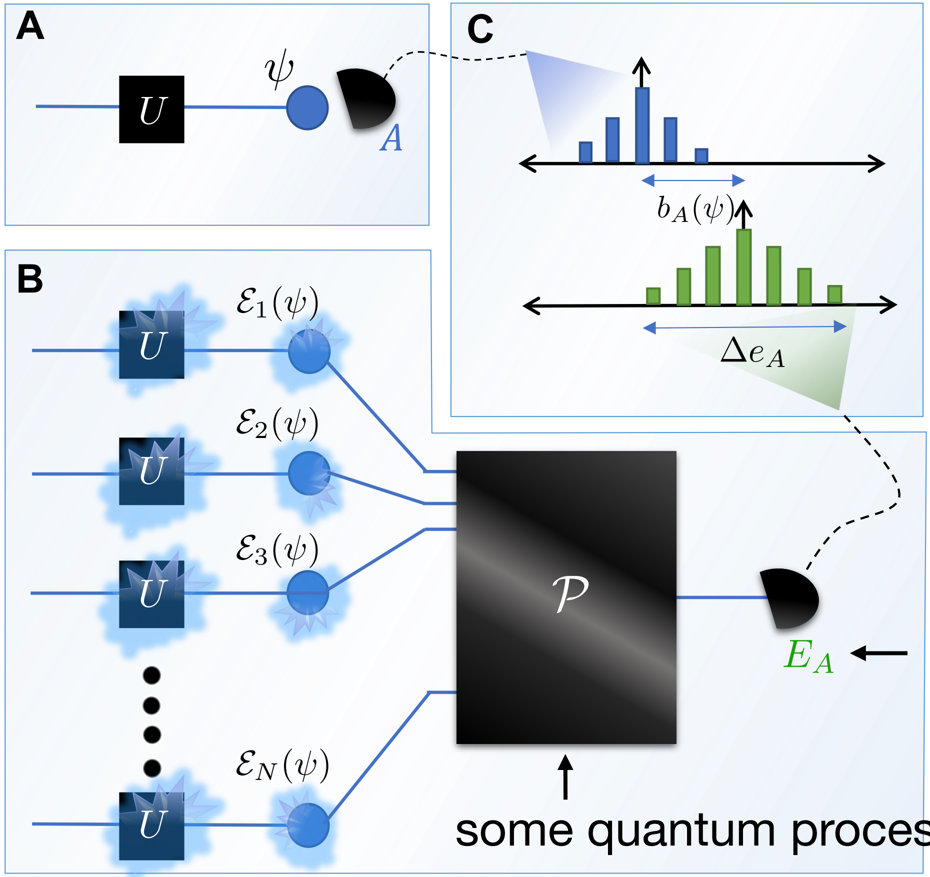
Ideal:

$\psi \longrightarrow \mathrm{Tr}(A\psi)$

Real:

$\mathcal{E}_1(\psi), \ldots, \mathcal{E}_N(\psi)$

$\longrightarrow \mathrm{Tr}(A\psi)$



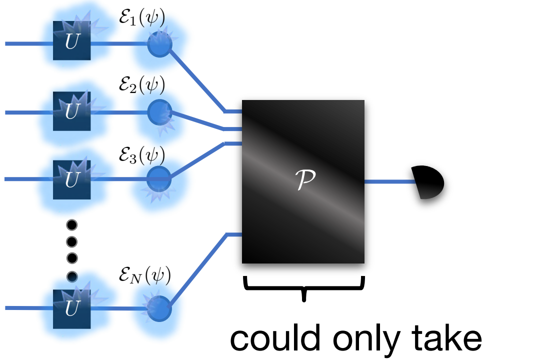
$b_A(\psi)$: bias

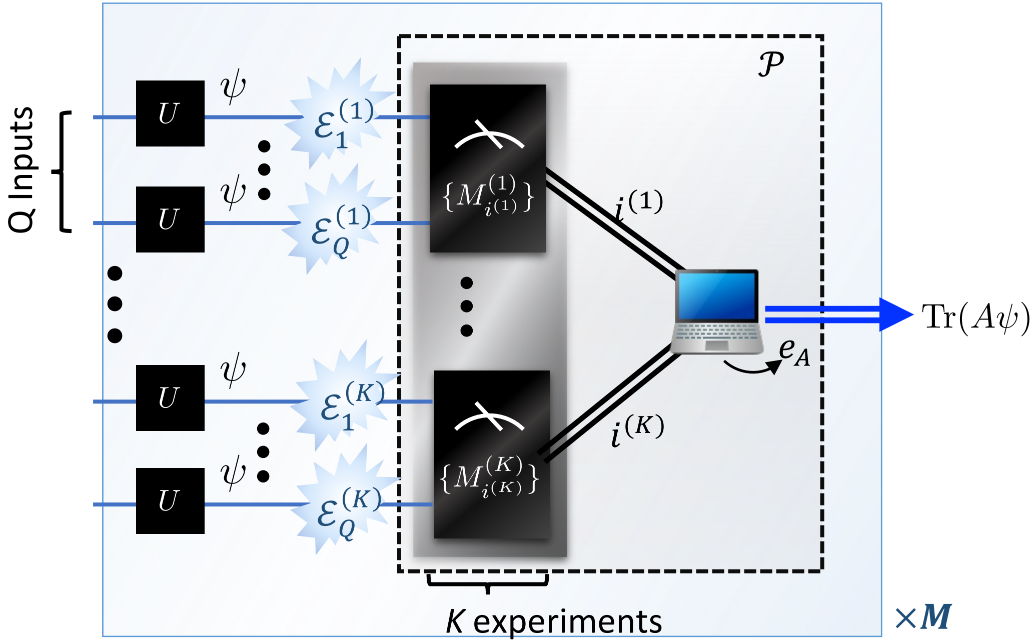
Δe_A : spread

$E_A \longleftarrow$ estimate of $\mathrm{Tr}(A\psi)$ for each round

$\uparrow$ some quantum process

General quantum error mitigation



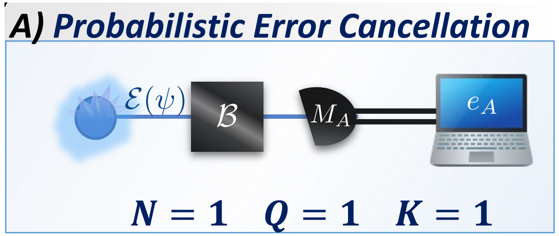


(Q,K)-error mitigation

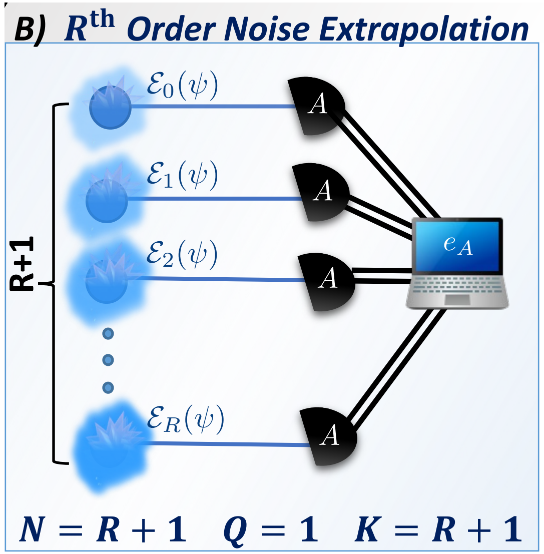
Repeat *M* rounds to get estimate $e_A(i^{(1)},\ldots,i^{(K)}) = e_{A,m}$ for each *m*=1,…,*M*

$\bar{E}_A^{(M)} = \frac{1}{M}\sum_{m=1}^{M} e_{A,m}$: estimate of $\mathrm{Tr}(A\psi)$

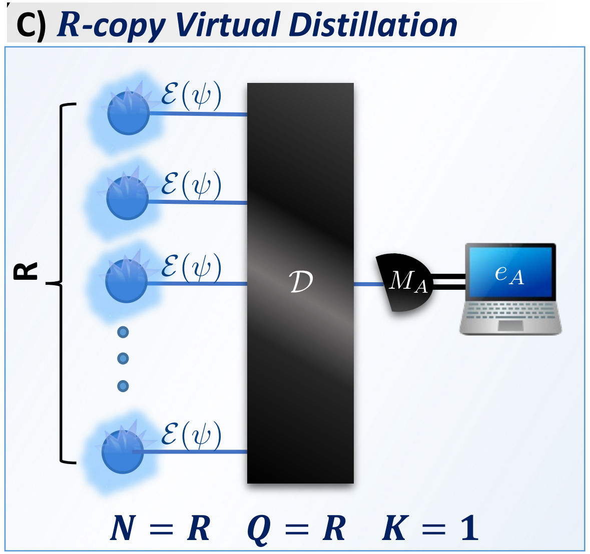
Examples



[Temme et al., PRL '17]



[Temme et al., PRL '17] [Li, Benjamin, PRX '17]



[Koczor, PRX '21] [Huggins, PRX '21]

Quantifying performance

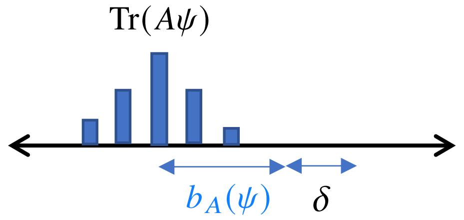
- Bias $b_A(\psi) = \langle E_A \rangle - \mathrm{Tr}(A\psi)$ E_A : random variable for each estimate
- Statistical Error $|\bar{E}_A^{(M)} - \langle E_A \rangle|$ $\bar{E}_A^{(M)} = \frac{1}{M}\sum_{m=1}^{M} e_{A,m}$: estimate of $\mathrm{Tr}(A\psi)$

How many round number *M* ensures the statistical error δ ?

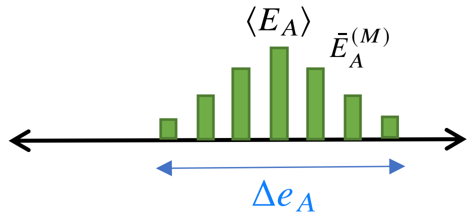
- Hoeffding inequality

M samples ensure error $|\bar{E}_A^{(M)} - \langle E_A \rangle| < \delta$ with probability $1-\varepsilon$ when

$$M > \frac{2\,\Delta e_A^2\,\log(2/\varepsilon)}{\delta^2}$$



- Maximum bias $b_{\max} = \max_{-\mathbb{I}/2 \le A \le \mathbb{I}/2} \max_{\psi} b_A(\psi)$
- Maximum spread $\Delta e_{\max} = \max_{-\mathbb{I}/2 \le A \le \mathbb{I}/2} \Delta e_A$



Lower bounds for maximum spread

What is the unavoidable maximum spread for a given maximum bias?

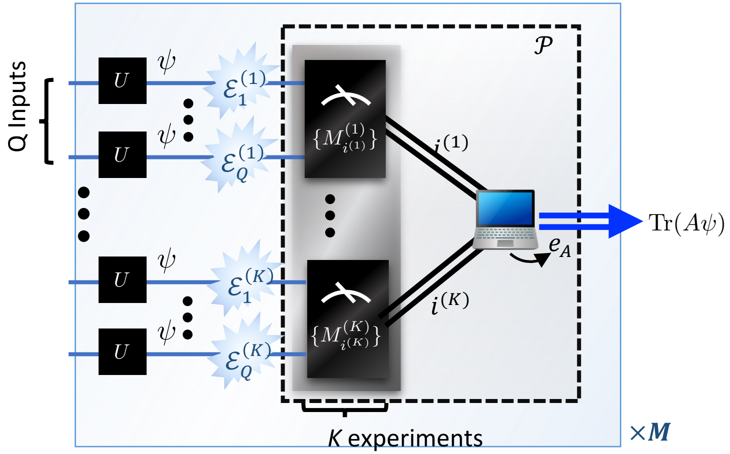
Maximum bias $b_{\max} = \max_{-\mathbb{I}/2 \le A \le \mathbb{I}/2} \max_{\psi} b_A(\psi)$

Maximum spread $\Delta e_{\max} = \max_{-\mathbb{I}/2 \le A \le \mathbb{I}/2} \Delta e_A$

For an arbitrary (*Q*,*K*)-mitigation protocol,

$$\Delta e_{\max} \ge \max_{\psi,\phi} \frac{D_{\mathrm{tr}}(\psi,\phi) - 2b_{\max}}{D_{\mathrm{LM}}\left(\tilde{\psi}_Q^{(K)}, \tilde{\phi}_Q^{(K)}\right)}$$

$$\tilde{\psi}_Q^{(K)} := \otimes_{k=1}^{K} \otimes_{q=1}^{Q} \left[\mathcal{E}_q^{(k)}(\psi)\right] \quad \tilde{\phi}_Q^{(K)} := \otimes_{k=1}^{K} \otimes_{q=1}^{Q} \left[\mathcal{E}_q^{(k)}(\phi)\right]$$



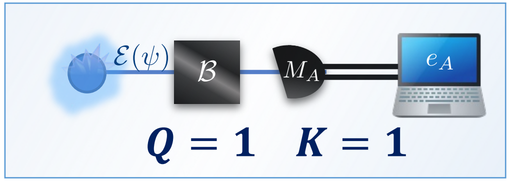
$D_{\mathrm{tr}}(\rho,\sigma)$: trace distance (= distinguishability using any measurement)

$D_{\mathrm{LM}}(\rho,\sigma)$: local distinguishability (= distinguishability using local measurements)

Connecting error-mitigation performance with information-theoretic measures.

Application: protocol benchmarking

Probabilistic error cancellation: (1,1)-protocol. (Q=K=1)



$\mathcal{E}^{-1} = \sum_j c_j \mathcal{B}_j,\ c_j \in \mathbb{R}$ $\qquad \gamma := \sum_j |c_j|$

$\gamma_{\rm opt}$: minimum γ over all feasible $\{\mathcal{B}_j\}_j$

$\Delta e_{\max}^{\rm PEC} = \gamma_{\rm opt}$ characterizes the sampling cost for this protocol. [Takagi, PRR '21] [Regula, Takagi, Gu, Quantum '21]

e.g.) Local dephasing noise $\mathcal{Z}_\epsilon^{\otimes n}$

$$\mathcal{Z}_\epsilon(\rho) = (1-\epsilon)\rho + \epsilon Z\rho Z$$

$$\Delta e_{\max}^{\rm PEC} = \gamma_{\rm opt} = \frac{1}{(1-2\epsilon)^n}$$

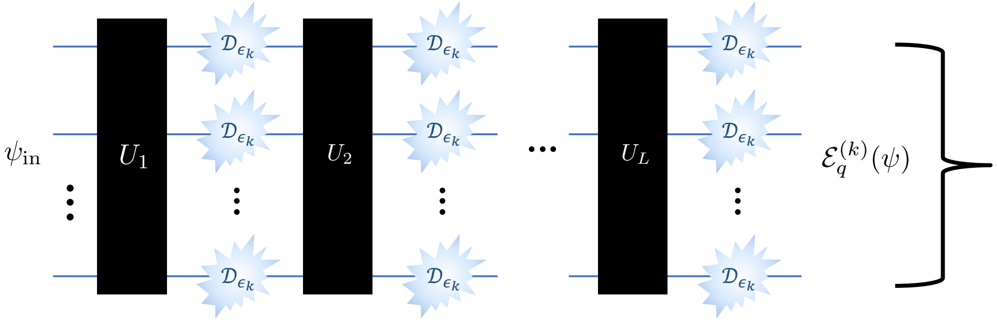
Our bound implies that any unbiased (1,1)-protocol satisfies

$$\Delta e_{\max} \geq \max_{\psi,\phi} \frac{D_{\rm tr}(\psi,\phi)}{D_{\rm tr}(\mathcal{Z}_\epsilon^{\otimes n}(\psi), \mathcal{Z}_\epsilon^{\otimes n}(\phi))} \geq \frac{1}{(1-2\epsilon)^n}$$

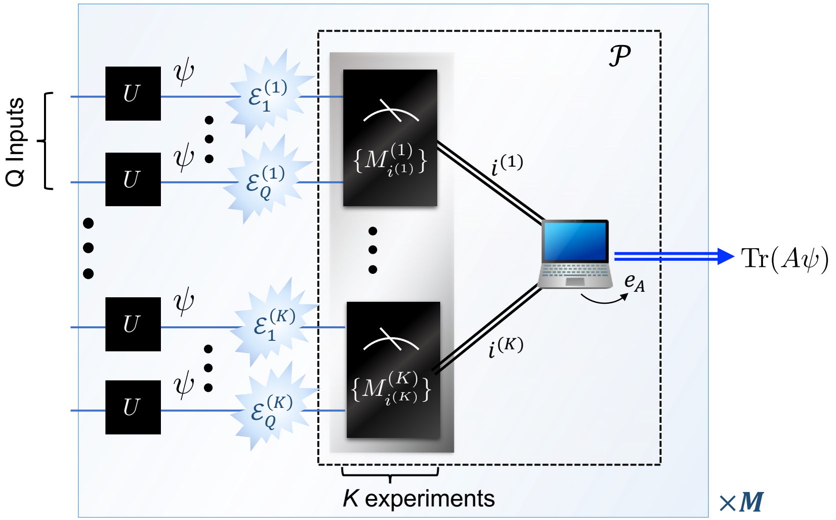
Lower bound achieved!

Probabilistic error cancellation is optimal for this noise.

Application: error-mitigating layered circuits



$\mathcal{D}_{\epsilon_k}(\rho) = (1-\epsilon_k)\rho + \epsilon_k \mathbb{I}/2$: local depolarizing noise



For an arbitrary (*Q*,*K*)-mitigation protocol,

$$\Delta e_{\max} \geq \frac{1-2b_{\max}}{\sqrt{2\ln 2}\sqrt{nQK}} \left(\frac{1}{1-\epsilon_{\min}}\right)^L$$

$\epsilon_{\min} := \min_k \epsilon_k$

Exponential error spread w.r.t. circuit depth *L*!

Summary

- Introduced a general framework for quantum error mitigation.
- Established a general lower bound for the maximum spread.
- Showed the optimality of probabilistic error cancellation for the local dephasing noise in terms of the maximum spread.
- Showed that mitigating errors in layered circuits requires exponentially large maximum spread.
- Sampling lower bounds? See our follow-up work "Universal sample lower bounds for quantum error mitigation", arXiv:2208.09178