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AUSTRALIAN INSTITUTE OF
PHYSICS CONGRESS

Quantum control in foundational experiments, or *HV in XXI*

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School of Mathematical and Physical Sciences



MACQUARIE
University

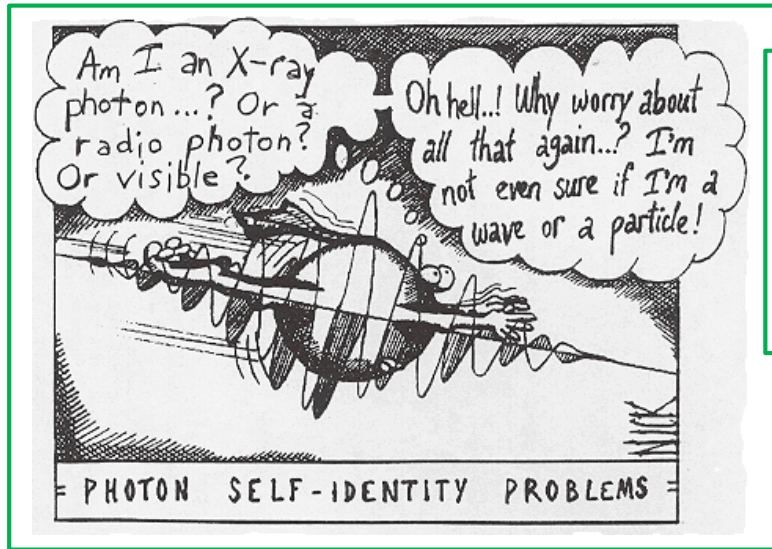
דניאל טרנו

Shenzhen Institute of Quantum
Science and Engineering

南方科技大学

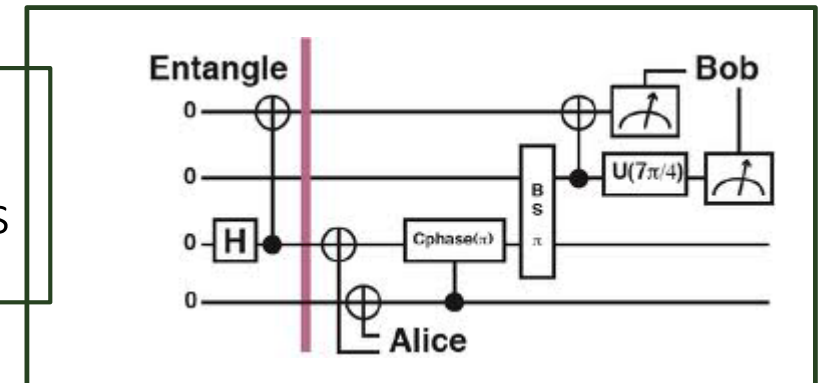


two parts of the story



Wave particle duality
Complementarity
Delayed choice experiments
and their HV interpretation

Gates and quantum control
in service of quantum foundations



references & promo

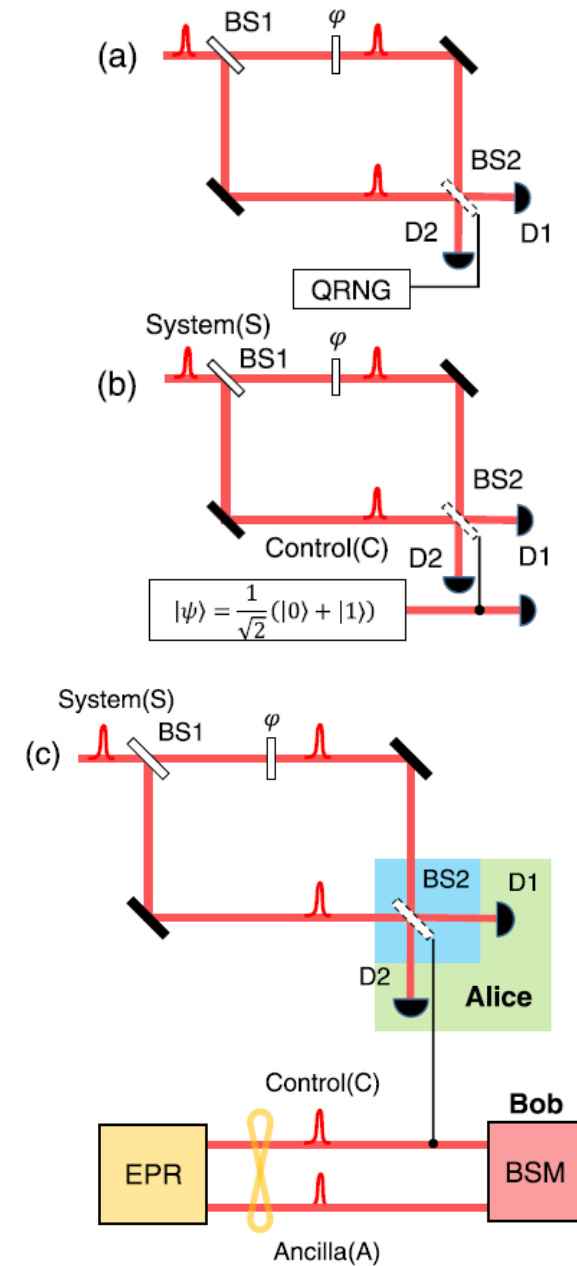
- Ma, Kofler, & Zeilinger,
Delayed-choice gedanken experiments and their realizations,
Rev. Mod. Phys. **88**, 015005 (2016)

- Ionicioiu and Terno, Phys. Rev. Lett. **107**, 230406 (2011).
- Ionicioiu, Mann, & Terno, Phys. Rev. Lett. **114**, 060405 (2015)

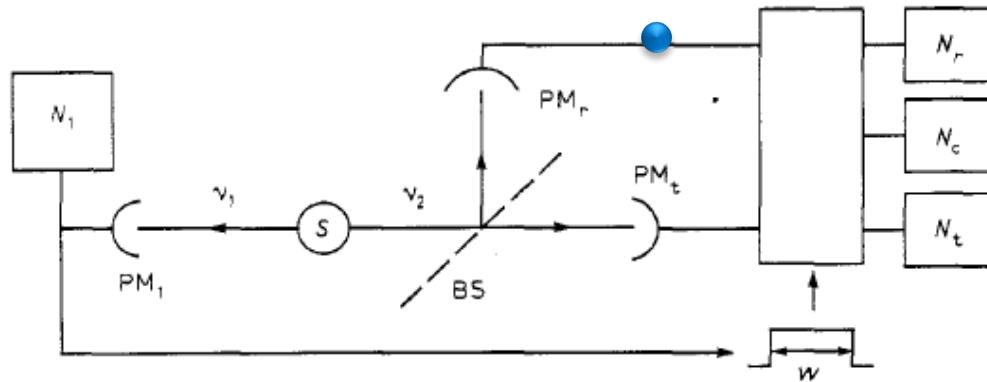
- Brandenburger & Yanofsky,
A classification of hidden-variable properties, J. Phys. A **41**, 425302 (2008)

- Gisin & Gisin, Phys. Lett. A **297**, 279 (2002).
- Branciard, Gisin, & Pironio, Phys. Rev. Lett. **104**, 170401 (2010).

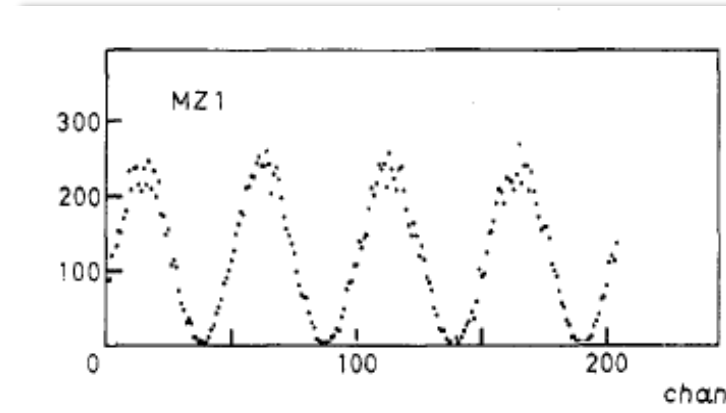
- Ionicioiu, Jennewein, Mann, and Terno, Nature Comm. **5**, 3997 (2014)
- Wang, Terno, Brukner, Zhu, Ma, Phys. Rev. A **106**, 053715 (2022)



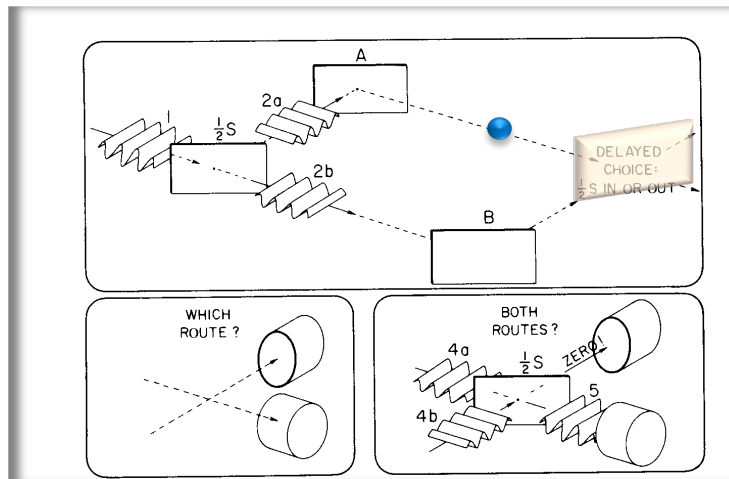
wave-particle duality



Single photons **behave as particles**
 Single photons **behave as waves**



P. Grangier, G. Roger and A. Aspect Europhys. Lett. **1**, 173 (1986)



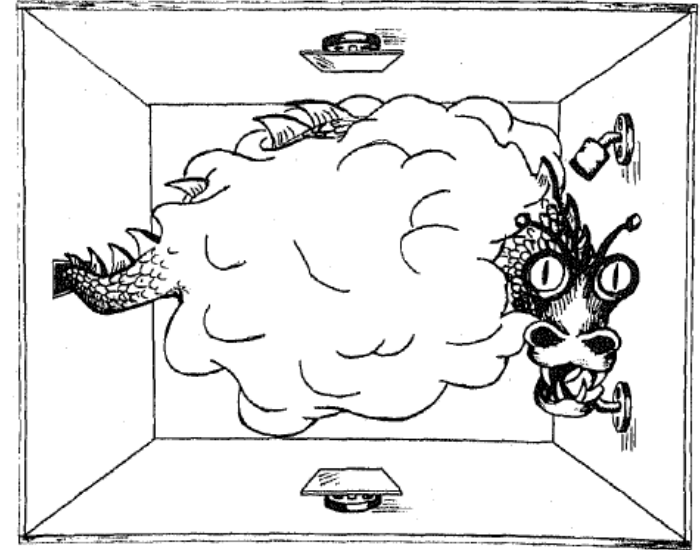
DEFINITIONS

- Particles*: no interference,
 - ▶ single path
- Waves*: interference,
 - ▶ both paths

complementarity

... the information provided by different experimental procedures that in principle cannot, because of the physical character of the needed apparatus, be performed simultaneously, cannot be represented by any mathematically allowed quantum state of the system. The elements of information obtainable from incompatible measurements are said to be *complementary*.

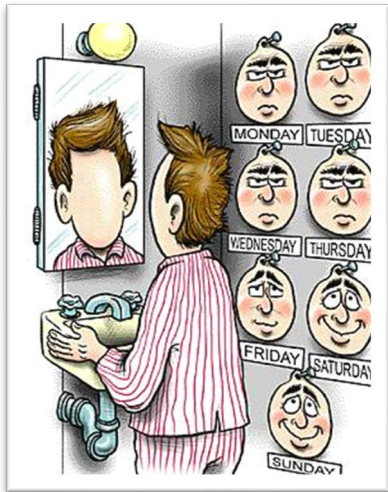
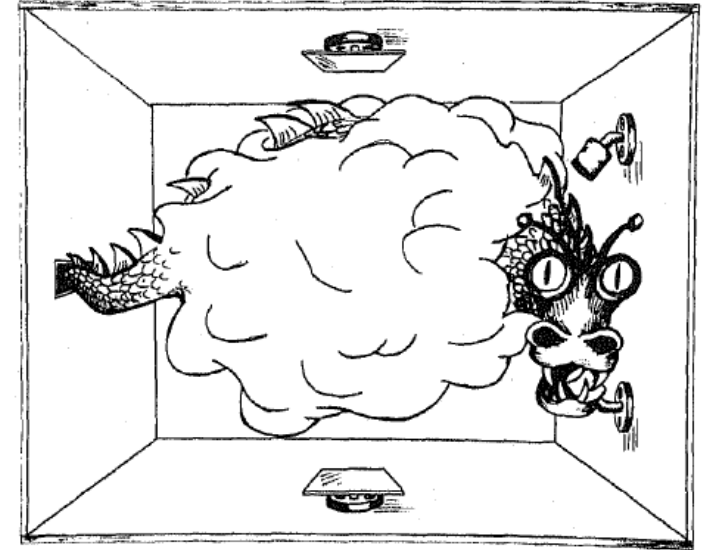
Stapp, in *Compendium of Quantum Physics*



complementarity

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Stapp, in *Compendium of Quantum Physics*



conspiracy

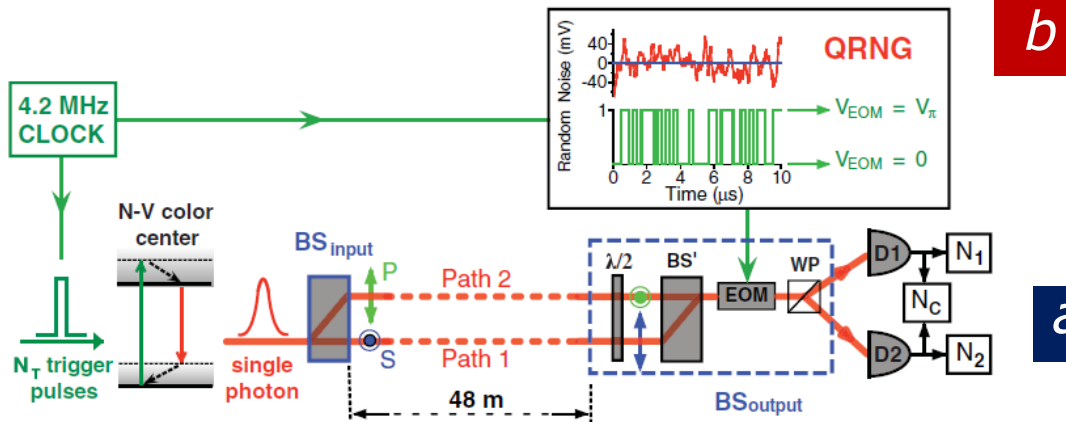
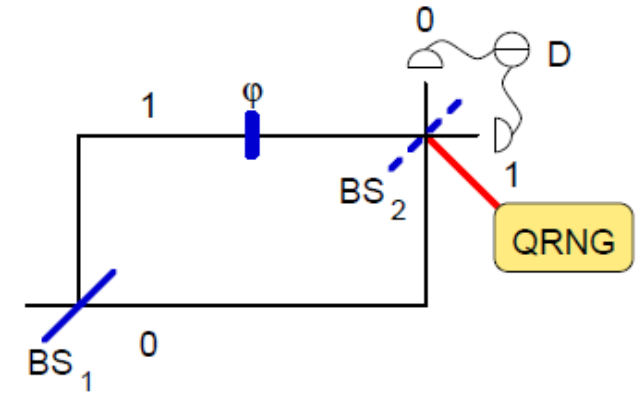
The photon could know in advance of entering the apparatus whether the latter has been set up in the "wave" configuration with BS_2 in place or the "particle" one (BS_2 removed) and adjust accordingly.

delayed choice

+ QRNG

Open interferometer [particle] $n(a) = (\frac{1}{2}, \frac{1}{2})$

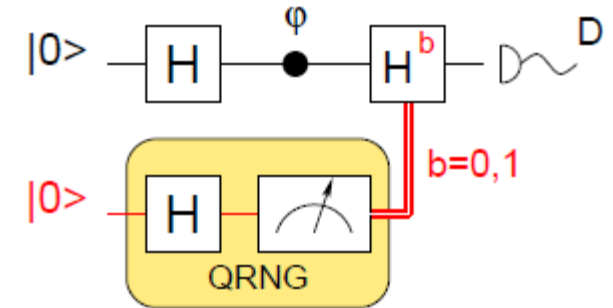
Closed interferometer [wave] $n(a) = (\cos^2 \frac{\varphi}{2}, \sin^2 \frac{\varphi}{2})$



Jacques *et al.*,
Science **315**, 966 (2007)

b

a



*Spacelike separation between
the source and the RNG*



control the world without attracting attention

HV M. Born, *Quantenmechanik der Stoßvorgänge*,
Z. Physik **38**, 803 (1926).

Purpose: reproduce observed statistics and
maintain classical concepts

Construction: HV exist, control the world, but are unknown

$$p(a,b... | A,B...) := \sum_{\lambda} p(a,b... | A,B...; \lambda) p(\lambda)$$

Adequacy: reproduce observed statistics

$$p(a,b... | A,B...) \equiv q(a,b... | A,B...)$$

Measurements
and settings:

$A, A'; B, B'$

Outcomes

$a, a'; b, b'$

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Measurements
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Outcomes

$a, a'; b, b'$

Counter-HV action

- ◇ consider a set-up
- ◇ make a QM prediction
- ◇ make a HV prediction
- ◇ compare
 - get a contradiction
- ◇ make an experiment

Counter-counter-HV action

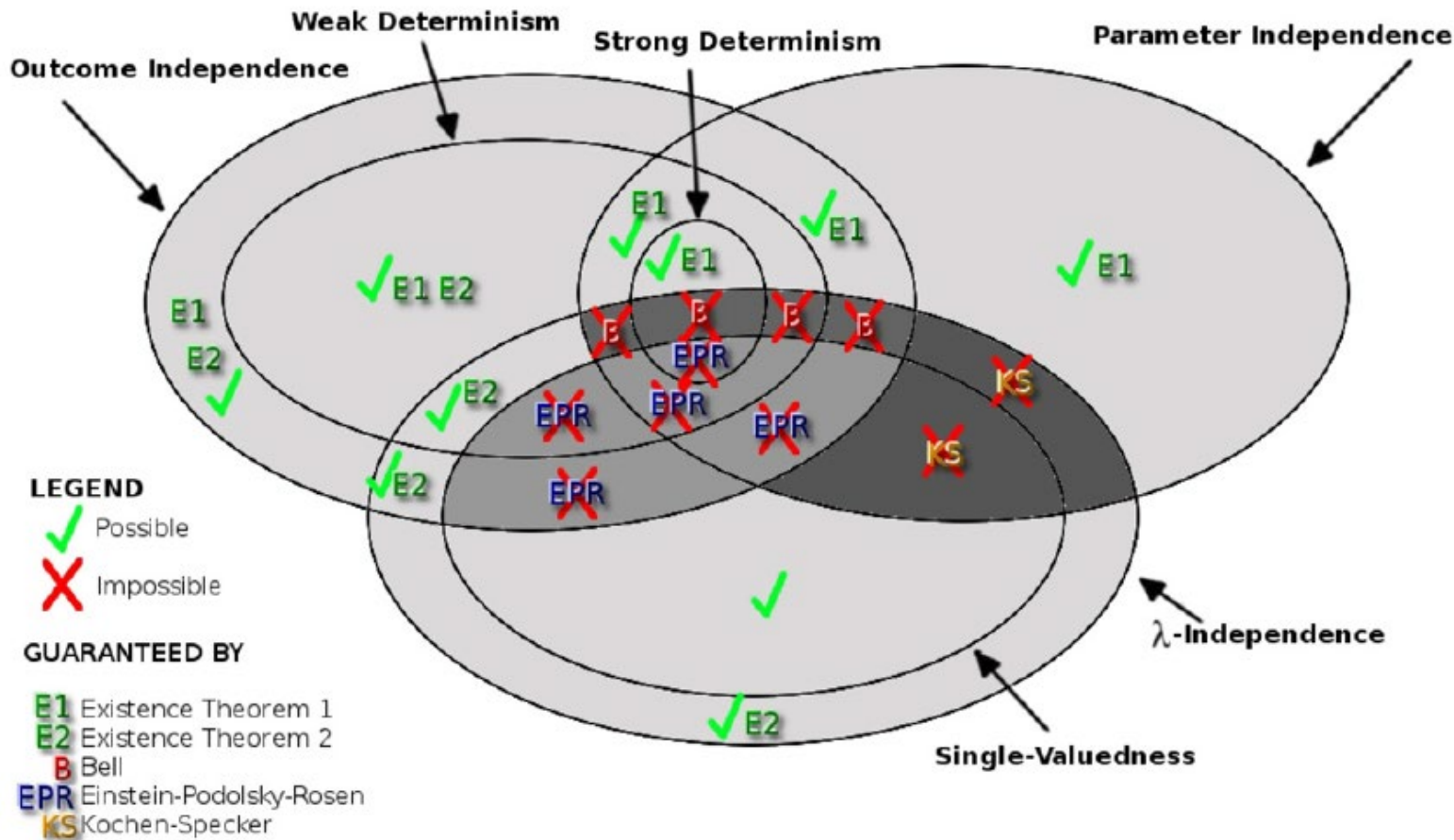
- find a loophole
- introduce conspiratorial correlations



how to...

HIDDEN VARIABLES

Brandenburger & Yanofsky
JPA **41** 425302 (2008)



□ **Determinism:** once hidden variables are defined, there are no residual randomness [several flavors]

□ **Parameter independence:** the outcome of any measurement depends only on the HV and the set-up of this measurement

$$p(a | A, B, C, \dots, \lambda) = p(a | A, \lambda)$$

□ **HV (λ -)independence:** determination of the hidden variable is independent of the choice of measurement

$$p(\lambda | A, B, \dots) = p(\lambda | A', B', \dots)$$

outcome independence + parameter independence
= Bell's locality

HV theories: objectivity [definiteness]

- What is the basis for assertion of wave-particle duality?
- Can we detect “it” first and decide what was it later?
- Is space-like separation necessary?
- What if the controlling devices are quantum?

*Extensions
& questions*

HV theories: objectivity [definiteness]

- What is the basis for assertion of wave-particle duality?
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Extensions & questions

HV Conspiracy

- A hidden variable $\lambda=p,w$ set at production/before splitting

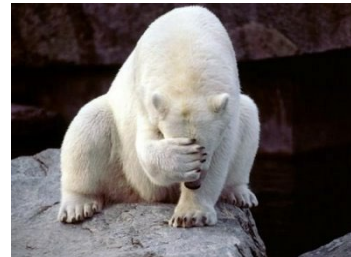
$$p(a | b = 1, \lambda = w) = (\cos^2 \frac{\phi}{2}, \sin^2 \frac{\phi}{2})$$



$$p(a | b = 0, \lambda = p) = (\frac{1}{2}, \frac{1}{2})$$

$$p(a | b = 0, \lambda = w) = (x, 1 - x)$$

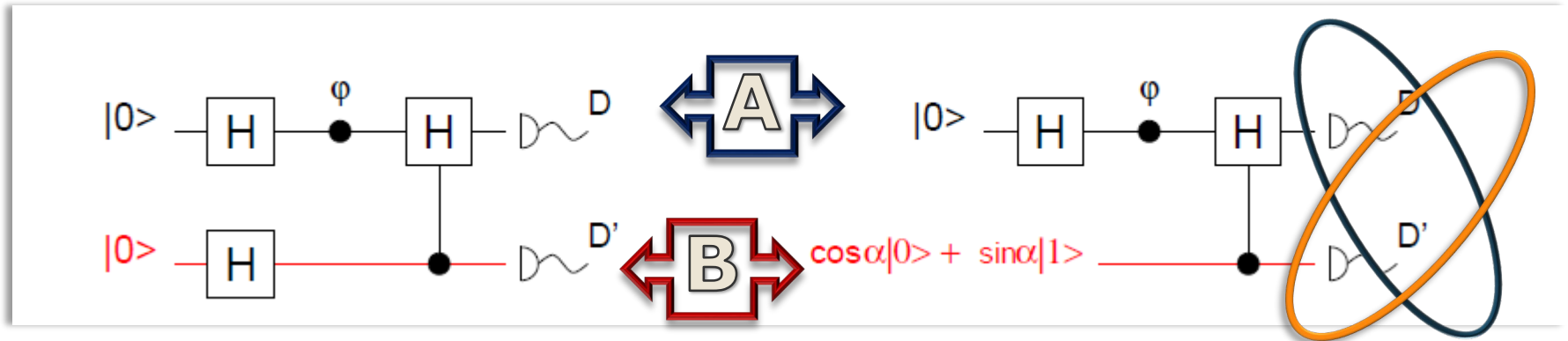
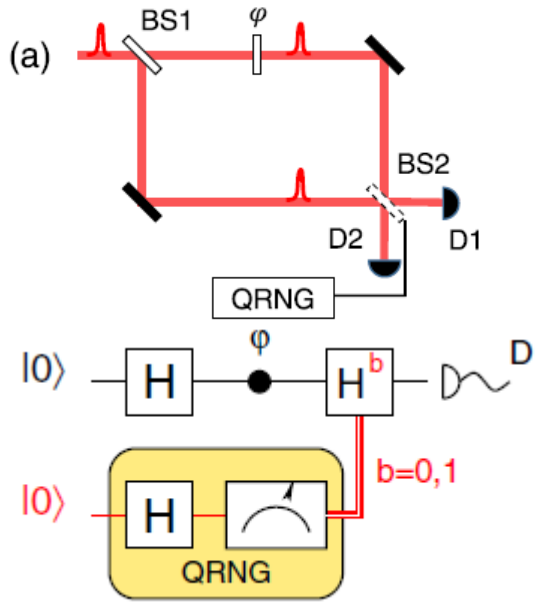
$$p(a | b = 1, \lambda = p) = (y, 1 - y)$$



- Emission probability with $\lambda=p,w$

$$p(\lambda) = (f, 1 - f)$$

quantum control



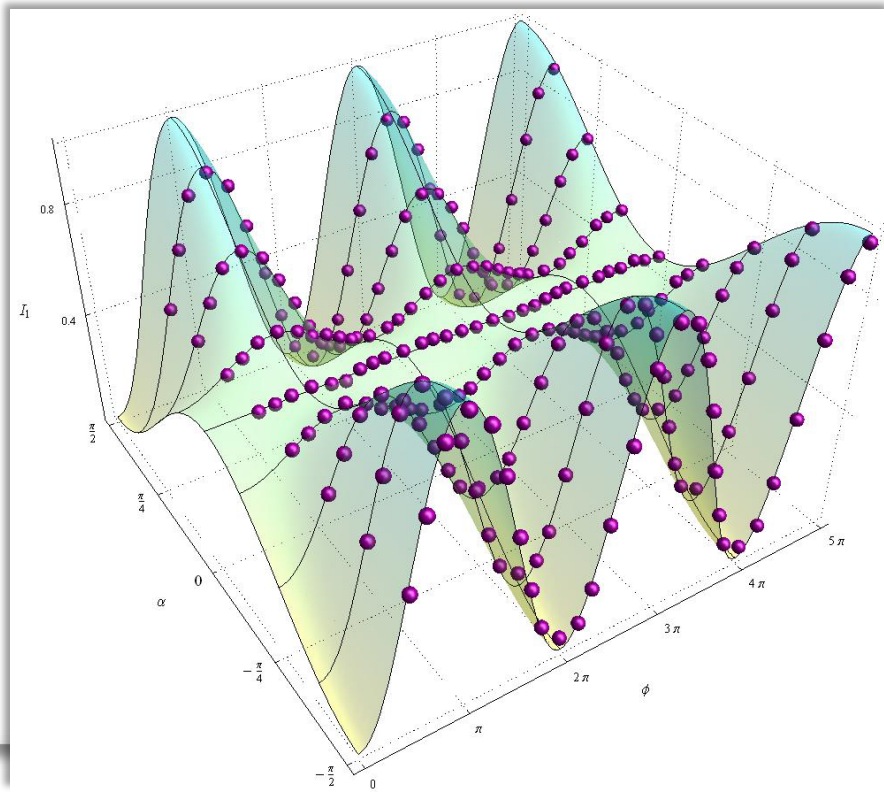
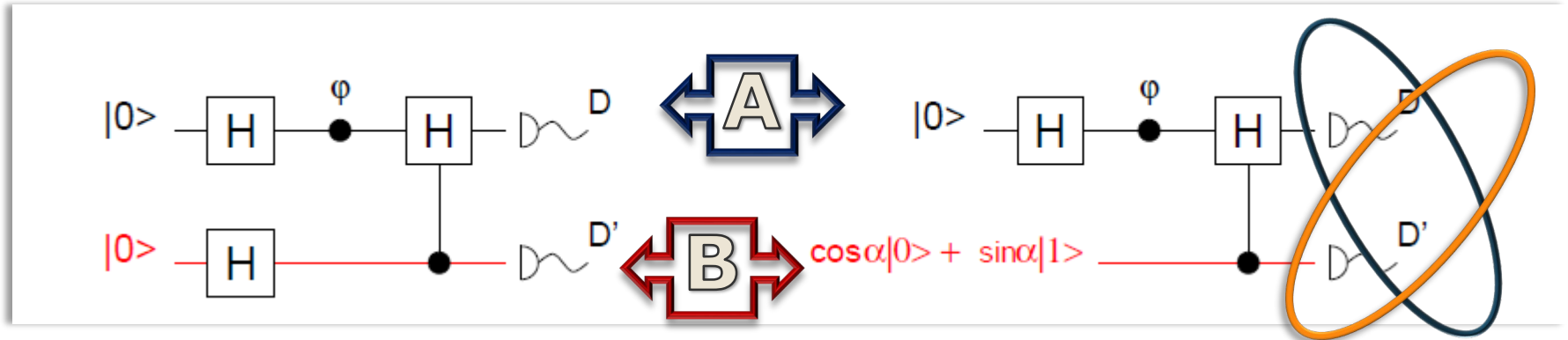
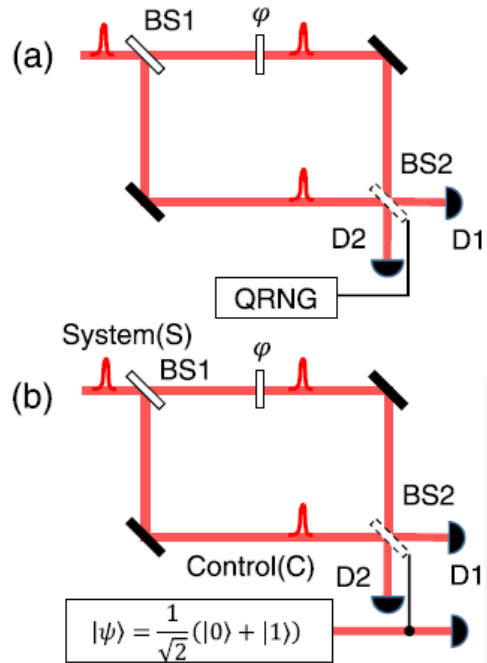
State after the gates [before the detectors]

$$|\psi_f\rangle = \cos \alpha |\psi_p\rangle |0\rangle + \sin \alpha |\psi_w\rangle |1\rangle$$

$$|\psi_p\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\phi} |1\rangle)$$

$$|\psi_w\rangle = \frac{1}{\sqrt{2}} e^{i\phi/2} (\cos \frac{\phi}{2} |0\rangle - i \sin \frac{\phi}{2} |1\rangle)$$

quantum control



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Tang *et al.*,
Nature Phot. **6**, 602 (2012)

countering HV

HV model maintains

- ◇ objectivity
- ◇ determinism
- ◇ ?

HV model must be adequate

$$q(a,b) = \sum_{\lambda=p,w} p(a,b,\lambda)$$

countering HV

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$$q(a,b) = \sum_{\lambda=p,w} p(a,b,\lambda)$$

HV in WDC

adequacy

$$q(a,b) = n(a,b) = \sum_{\lambda} p(a|b,\lambda)p(\lambda|b)n(b)$$

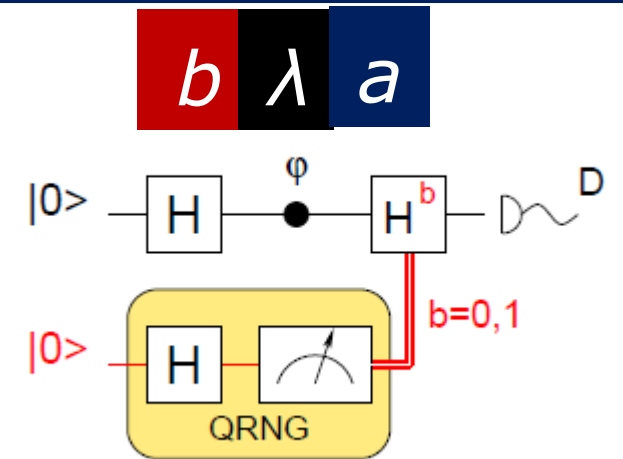
is possible if

$$p(\lambda|b) = \delta_{\lambda p}\delta_{b0} + \delta_{\lambda w}\delta_{b1}$$

[violation of parameter independence]

[countered by the spacelike separation]

[counter-countered by the conspiracy: settings of QRNG with λ]



countering HV

HV model maintains

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HV in WDC

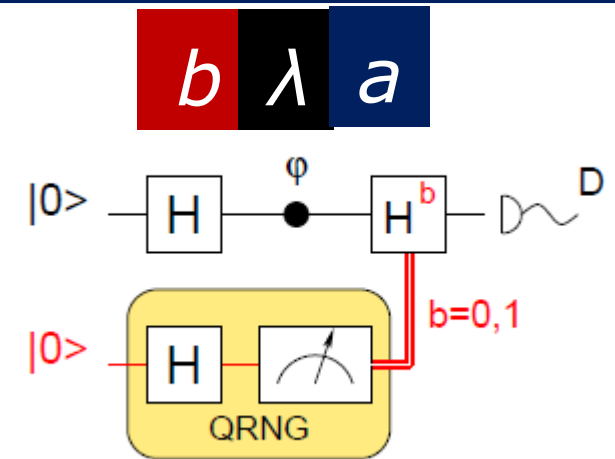
adequacy is possible if $q(a,b) = n(a,b) = \sum_{\lambda} p(a|b,\lambda)p(\lambda|b)n(b)$

$$p(\lambda|b) = \delta_{\lambda p} \delta_{b0} + \delta_{\lambda w} \delta_{b1}$$

[violation of parameter independence]

[countered by the spacelike separation]

[counter-countered by the conspiracy: settings of QRNG with λ]

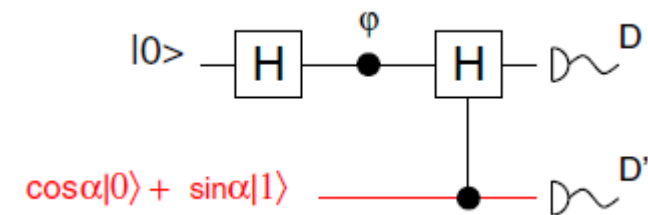


HV in QDC adequacy is possible if either

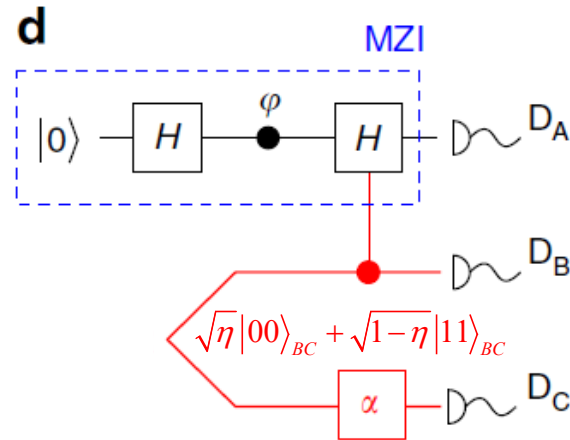
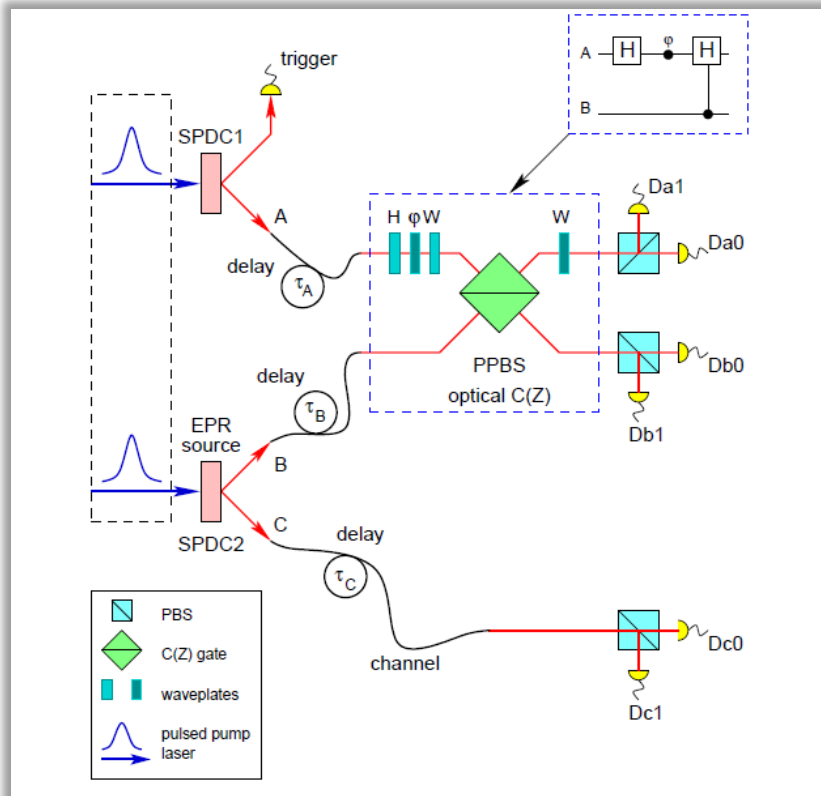
[objectivity is lost] $p(a|b,\lambda) = p(a|b)$

[conspiracy: emission is governed by the gate settings]

$$f = \cos \alpha$$



countering HV with entanglement (1)

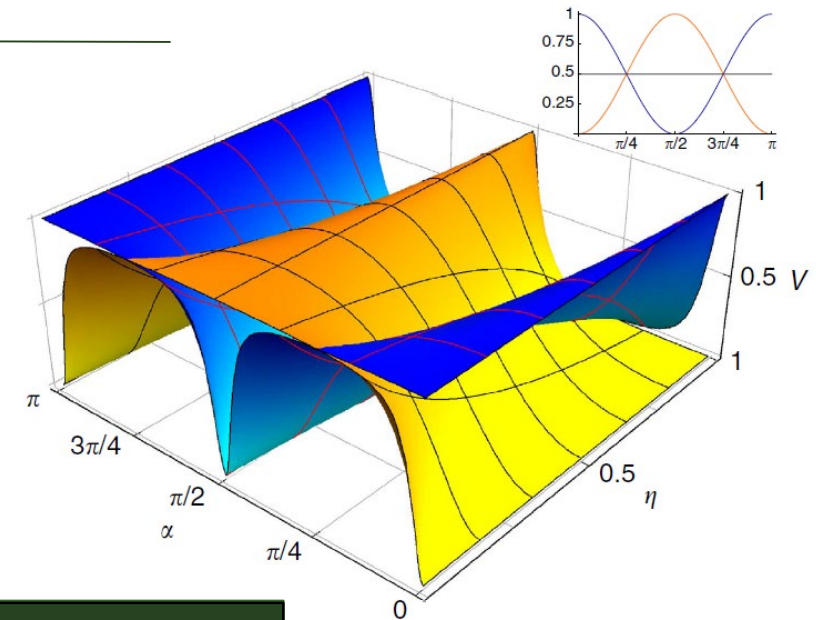
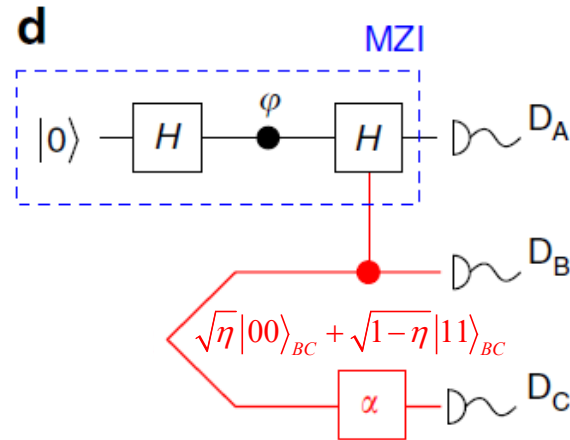
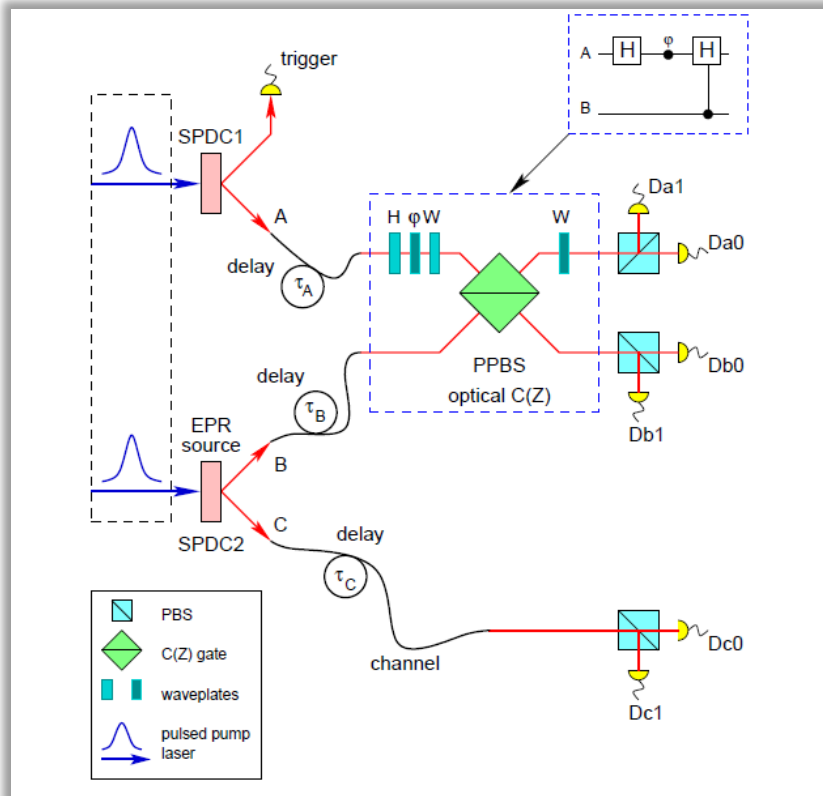


three assumptions

- (i) **Determinism:** HV determines all the outcomes (3 flavours)
- (ii) **Λ -independence:** HV are not influenced by the settings
- (iv) **Local (λ) independence:** the space of HV has a product structure and

$$p(\lambda_S, \lambda_B) = p_S(\lambda_S)p_B(\lambda_B)$$

countering HV with entanglement (1)



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countering HV with entanglement (2)

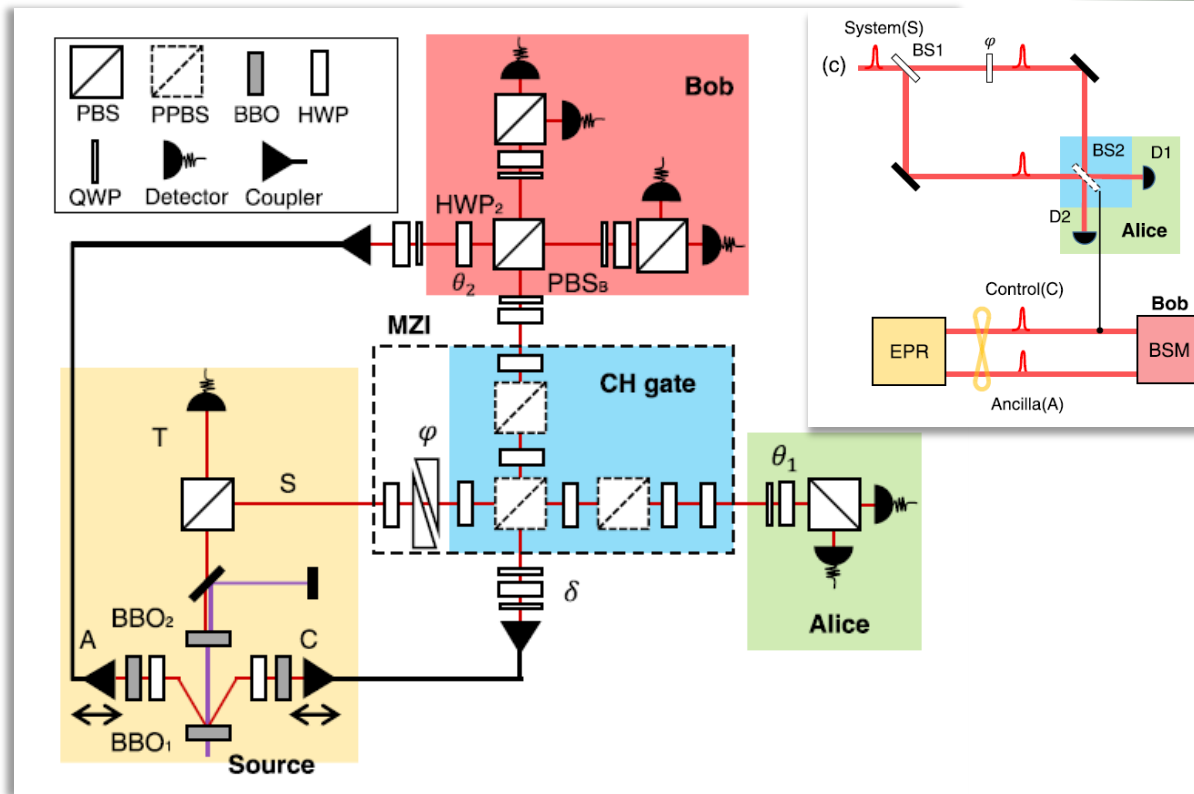


FIG. 2. Experimental setup. Entangled photons C and A are generated in the $|\psi^+\rangle$ state from a β -barium borate crystal (BBO1). From BBO2 we detect photon T and herald the existence of photon S . We then send photon S through the polarization Mach-Zehnder interferometer. Alice performs polarization measurements on photon S with two waveplates and a polarizing beam splitter, which project the polarization of photon S along θ_1 . Photons C and A are sent to Bob, who employs a Bell-state analyzer to project these two photons onto a coherent superposition of $|\phi^-\rangle$ and $|\psi^+\rangle$. HWP₂ tunes the superposition of the bipartite states of photons C and A

❑ High-quality Bell state measurement
[but not a good refutation of HV]

❑ Adequacy of (a/the) HV model(s) is achieved by the loss of objectivity:
the state of BS₂ is undefined

countering HV with entanglement (2): details

The state of three photons:

$$\begin{aligned}
 |\psi^f\rangle &= \frac{1}{2} [|p\rangle_S (|\phi^-\rangle_{CA} - i|\psi^+\rangle_{CA}) + e^{i\pi/4} |w\rangle_S (|\phi^-\rangle_{CA} + i|\psi^+\rangle_{CA})] \\
 &= \frac{1}{\sqrt{2}} \left[\frac{e^{i(\pi/4)} |w\rangle_S + |p\rangle_S}{\sqrt{2}} |\phi^-\rangle_{CA} + i \frac{e^{i(\pi/4)} |w\rangle_S - |p\rangle_S}{\sqrt{2}} |\psi^+\rangle_{CA} \right] \\
 &= \frac{1}{2} [(e^{i(\theta+\pi/4)} |w\rangle + e^{-i\theta} |p\rangle)_S (\cos\theta |\phi^-\rangle + \sin\theta |\psi^+\rangle)_{CA} + (e^{i(\theta-\pi/4)} |w\rangle + ie^{-i\theta} |p\rangle)_S (\sin\theta |\phi^-\rangle - \cos\theta |\psi^+\rangle)_{CA}]
 \end{aligned}$$

$$|p\rangle = \frac{1}{\sqrt{2}} (|H\rangle - e^{i\varphi} |V\rangle)$$

$$|w\rangle = \frac{1}{\sqrt{2}} e^{i\varphi/2} (\cos\frac{\varphi}{2} |V\rangle - i \sin\frac{\varphi}{2} |H\rangle)$$

Superpositions in w/p of S are created by projecting onto the Bell states; the Bell state superposition w & p

countering HV with entanglement (2): details

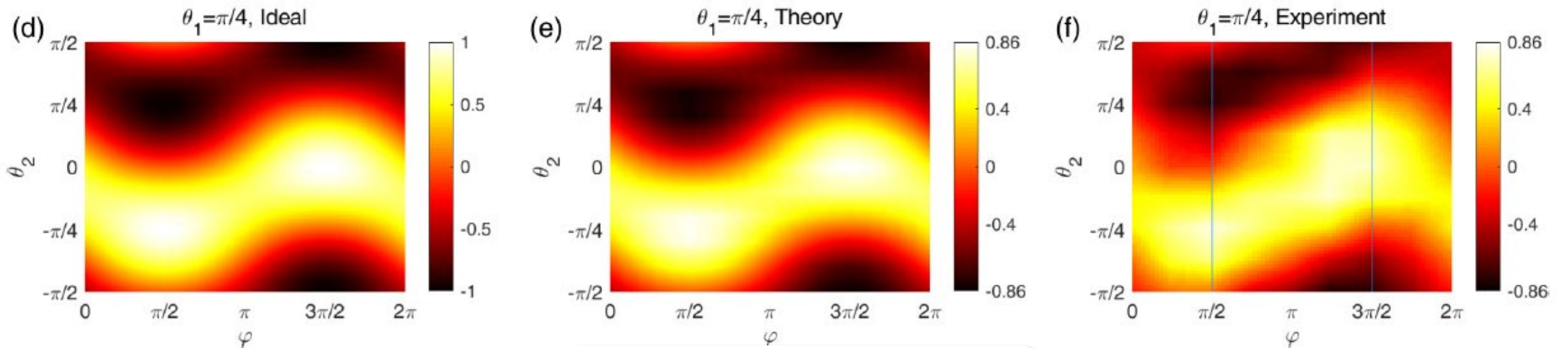
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 &= \frac{1}{2} [(e^{i(\theta+\pi/4)} |w\rangle + e^{-i\theta} |p\rangle)_S (\cos\theta |\phi^-\rangle + \sin\theta |\psi^+\rangle)_{CA} + (e^{i(\theta-\pi/4)} |w\rangle + ie^{-i\theta} |p\rangle)_S (\sin\theta |\phi^-\rangle - \cos\theta |\psi^+\rangle)_{CA}]
 \end{aligned}$$

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Superpositions in w/p of S are created by projecting onto the Bell states; the Bell state superposition w & p



S|CA correlation function



Thomas Jennewein
Rob Mann



Radu Ionicioiu



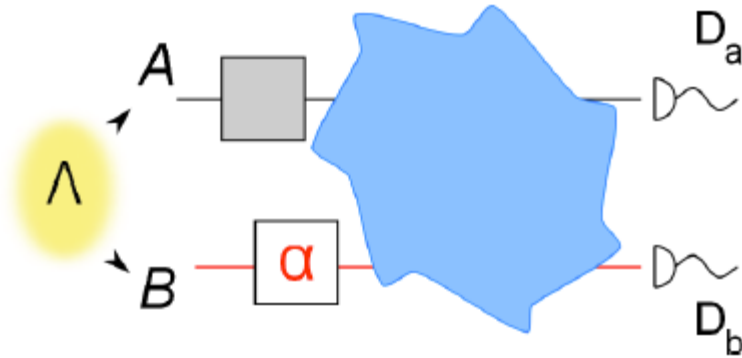
Časlav Brukner



Ma Xiaosong
Wang Kai
Zhu Xining



a bonus feature: HV & three incompatible assumptions



(*) Adequacy

Empirical statistics

$$e(a, b) = (xe_p, (1-x)e_w, x(1-e_p), (1-x)(1-e_w))$$

$$e(b) = (x, 1-x) \quad \blacktriangleleft \text{controller}$$

two types of stats \blacktriangleright $\bar{e}_p(a) = (e_p, 1 - e_p), \quad \bar{e}_w(a) = (e_w, 1 - e_w)$

$$e(a, b) = p(a, b) = \sum_{\Lambda} p(a, b, \Lambda) = \sum_{\Lambda} p(a, b|\Lambda) p(\Lambda)$$

HV settings

The system is definitely one or another

$$p(a|b = 1, \lambda = w) = \bar{e}_w(a)$$

$$p(a|b = 0, \lambda = p) = \bar{e}_p(a)$$

(O) Objectivity

$$\lambda = \lambda(\Lambda)$$

HV theory is (weakly) deterministic

$$p(a, b|\Lambda) = \chi_{ab}(\Lambda)$$

(i) Determinism

00

01

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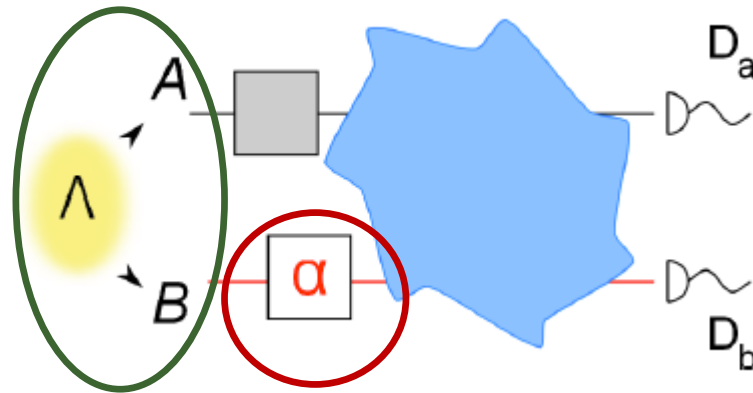
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Boundaries of the regions
depend on the settings

HV settings

Is λ -independent

(ii) Independence



$p(\lambda)$ is independent of the settings[#]

IMT, Phys. Rev. Lett. **114**,
060405 (2015)

Adequacy | three assumptions

Objectivity: two types of statistics e_p, e_w

Determinism: HV determines all the outcomes

λ -independence: single HV that is not influenced by the settings

HV (i-ii) simulate QM

if we don't ask too much

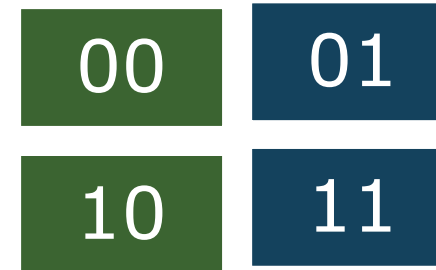
- Assume a uniform distribution $p(\Lambda)$
(ii) is satisfied
- Follow the weak determinism: let \mathcal{L}_{ij}
to depend on the measurement parameters

$$p(i, j | \alpha, \dots, \Lambda) = 1 \quad \forall \Lambda \in \mathcal{L}_{ij}$$

(i) is built in

- Basically cheat:

$$q_{ij} \equiv p_{ij} = \sum_{\Lambda} p(i, j | \alpha; \Lambda) p(\Lambda)$$
$$= \sum_{\Lambda \in \mathcal{L}_{ij}} 1 \quad (\text{adequacy}) \text{ is satisfied}$$



Boundaries of the regions depend on the settings

Follows Bell, not-so-famous-paper, 1964

a contradiction

LOGIC

Stage 1: find a unique non-trivial solution to (i)+(ii)+(O)

Ignoring how it arises from Λ

Exists (very special), but

$$p_s(\lambda | b) = \delta_{\lambda p} \delta_{b0} + \delta_{\lambda w} \delta_{b1} = p_s(b | \lambda)$$

$$p_s(a, b, \lambda) = e(a, b) p_s(b | \lambda)$$

00	01
10	11
p	w

Stage 2: derive a contradiction

By checking how the boundaries shift (long) OR

$$p_s(\lambda) = [x(\alpha), 1 - x(\alpha)] \equiv p[\lambda(\Lambda)]$$