Compression of QFT States using Entropic Quantities From Wavelet-Compressed QFT States

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Quantum field theory (QFT) describes nature using continuous fields, but physical properties of QFT are revealed in terms of measurements of observables at a finite resolution. We describe a multi-scale representation of free scalar bosonic and Ising model fermionic QFTs using Daubechies wavelets [1]. Making use of the orthogonality and self similarity of the wavelet basis functions, we demonstrate some well known relations such as scale dependent subsystem entanglement entropy and renormalisation of correlations in the ground state. The multi-scale basis allows for an exact holographic mapping from a boundary QFT to a bulk representation in one higher spatial dimension [2]. Conformal field theories (CFTs) in $(1+1)$D have a bulk dual where the scaling of mutual information falls off exponentially with a distance given by the 3D anti-de Sitter (AdS3) metric. With the Daubechies wavelet family, the radius of curvature depends on the chosen integer index, essentially forming a tunable bulk geometry [3].

We find new applications of the wavelet transform as a compressed representation of ground states of QFTs. One is the identification of quantum phase transitions via fidelity overlap [4], which has potential to benefit experiments by allowing the relevant entropic information to be obtained from observables on only a few renormalized degrees of freedom. Another is a test of the conjecture [5] that the entanglement of purification in conformal field theories CFTs is equal to the minimal-area cross-section of the entanglement wedge. Entanglement of purification is notoriously difficult to calculate because the minimization must be performed over an extensive number of parameters, but the compressed representation allows approximation using only a few coarse-grained degrees of freedom.

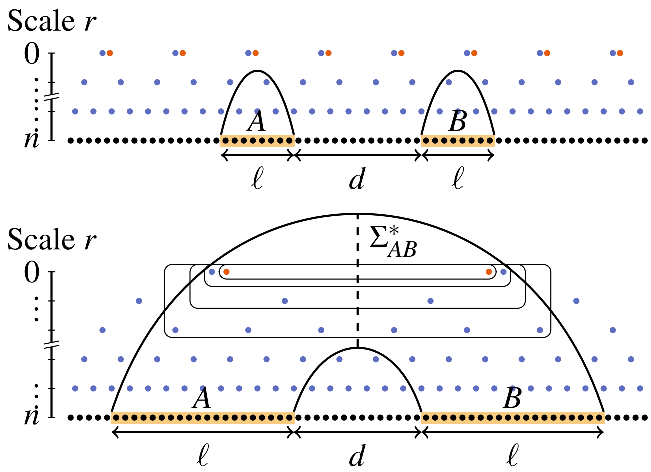

Figure: Physical degrees of freedom of a $(1+1)$D CFT at the finest scale are represented by scale modes (black), and form the boundary in the holographic picture. Coarser wavelet modes (blue) and the coarsest scale modes (orange) form the bulk description and are related to the boundary by a unitary wavelet transformation. When the ratio of size ℓ to separation d of subregions A and B is small (top) then the mutual information is zero and there is no entanglement wedge. When this ratio is large (bottom) there is an entanglement wedge and the cross-section Σ^*_{AB} takes a well-defined value. The ellipses define subsystems of coarse modes. If these accurately capture the mutual information between A and B, then we speak of compressed representations of ρ_{AB}.

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