Causal Mediation in Quantum Causal Models

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In recent years, significant advances have been made in understanding quantum phenomena and resources through the lens of causal models. Bell nonlocality, for example, can be understood as the impossibility to explain quantum correlations within a classical theory of causality [1], while maintaining relativistic causal structure. More recently, it has been shown that both Bell nonlocality and Kochen-Specker contextuality cannot be explained with a *faithful* classical causal model, without fine-tuned parameters [2-4]. More recently, novel forms of quantum violations of classical causal assumptions have been revealed by considering more general causal structures in network configurations.

The program of quantum causal models (QCMs) [5] aims to extend the classical framework to accommodate quantum correlations. For a Bell scenario – with two or more parties connected by a common cause – one of the insights gained is that within a QCM it is possible to maintain a version of the Principle of Common Cause – requiring that correlations be explained causally – while rejecting the classical idea that full specification of a common cause factorises the probabilities of its effects. This is replaced by a requirement that the operators representing *quantum channels* factorise instead [6].

Here we analyse a prepare-and-measure scenario in quantum causal models, with a motivation to elucidate Spekkens' notion of contextuality [7]. We first discuss how the ontological models framework underlying that formalism can be understood as a particular type of classical causal model, and compare it to the quantum description of the same causal structure. We show that, in analogy with the case of Bell scenarios, quantum causal models reject a classical assumption that has been called "lambda-mediation" – the assumption that a complete specification of intermediate causes factorises the probabilities of the end nodes connected via a causal chain. We argue that this allows QCMs to maintain the spirit of Spekkens' notion of noncontextuality, when suitably reformulated. Along the way, we display various interesting analogies between Bell scenarios and sequential scenarios, opening questions for further research.

[1] J. Pearl, Causality (Cambridge University Press, 2000).

[2] C. J. Wood and R. W. Spekkens, *New J. Phys.* **17**, 033002 (2015).

[3] E. G. Cavalcanti, Phys. Rev. X **8**, 021018 (2018).

[4] J.C. Pearl and E.G. Cavalcanti, Quantum **5**, 518 (2021).

[5] F. Costa and S. Shrapnel, New J. Phys. 18, 063032 (2016).

[6] E. G. Cavalcanti and R. Lal, J. Phys. A 47, 424018 (2014).

[7] R. W. Spekkens, Phys. Rev. A 71, 052108 (2005).