Quantum Chaos and Universal Trotterisation Behaviours in Quantum Simulations

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Resource optimization will be crucial to utilize the full computational power of quantum computers for digital quantum simulations in real-world applications. In particular, the digitization method used has a profound impact on qubit and gate numbers required for a given application. Here, we analyse the performance of Trotterisation, a state-of-the-art approach to digitisation, across a range of experimentally achievable physical models and system sizes.

It was recently shown that the Trotter approximation can exhibit a sharp breakdown threshold in Trotter step size [Heyl et al., Sci Adv. (2019), Sieberer et al., npj QI (2019)]. Here, we extend and generalise these results by demonstrating additional pre-threshold performance behaviours and the universality of these behaviours (incl. the threshold). We also provide conclusive proof that this breakdown results from the onset of digitisation-induced quantum chaos beyond this critical step size. Analysing the detailed system dynamics (e.g. Fig. 1 (a)-(b)), we show that the same threshold is shared by various dynamical quantities and that the systems show dynamical signatures of quantum chaos beyond the threshold. We support the dynamical observations using rigorous random matrix theory techniques to analyse the static properties of the digitised time evolution operators. We combine these RMT techniques with statistical goodness of fit tests (e.g. Fig. 1 (c)) to provide an objective and quantitative metric for the onset of quantum chaos beyond the threshold.

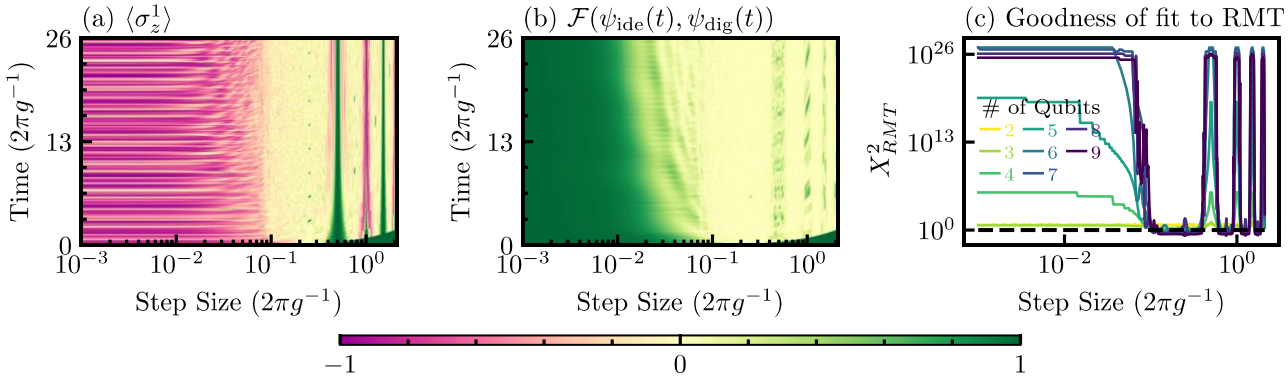

Figure 1. *Example results for the Heisenberg Model. The same clear threshold is observed in (a) expectation values, (b) simulation fidelities, and (c) statistical analyses of the eigenvectors of the Trotterised unitary operators (X^2_{RMT}). In addition to the signatures of quantum chaos beyond the threshold (except in certain regions of regular dynamics), the dynamical quantities also show two distinct pre-threshold regions. X^2_{RMT} provides conclusive, rigorous, and initial state independent evidence for the conclusions drawn from the dynamical observations.*