THE UNIVERSITY OF MELBOURNE
IBM Quantum Network Hub
at the University of Melbourne

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GREG WHITE
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From Many-Body to Many-Time Physics

The Spacetime Structure of Quantum Mechanics

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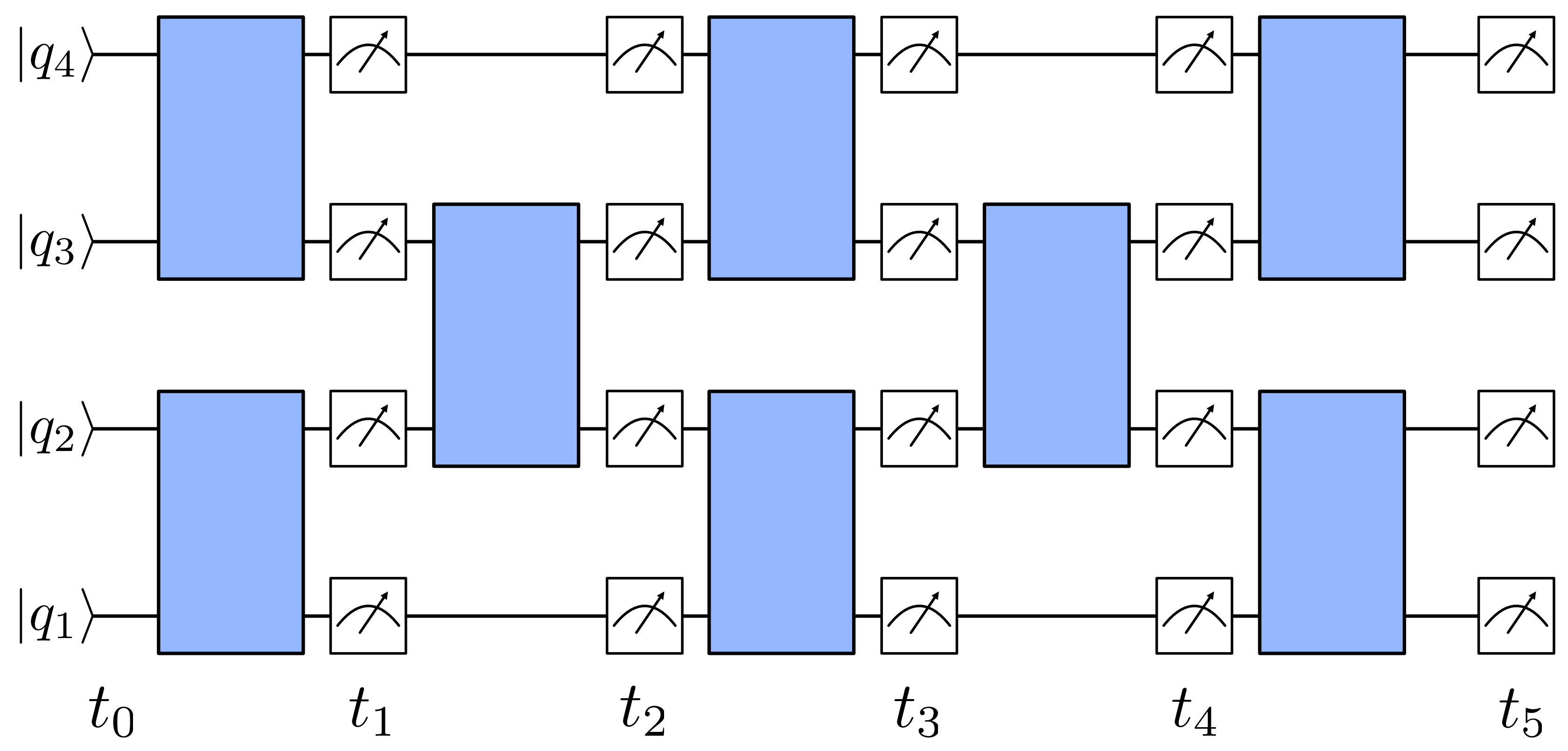
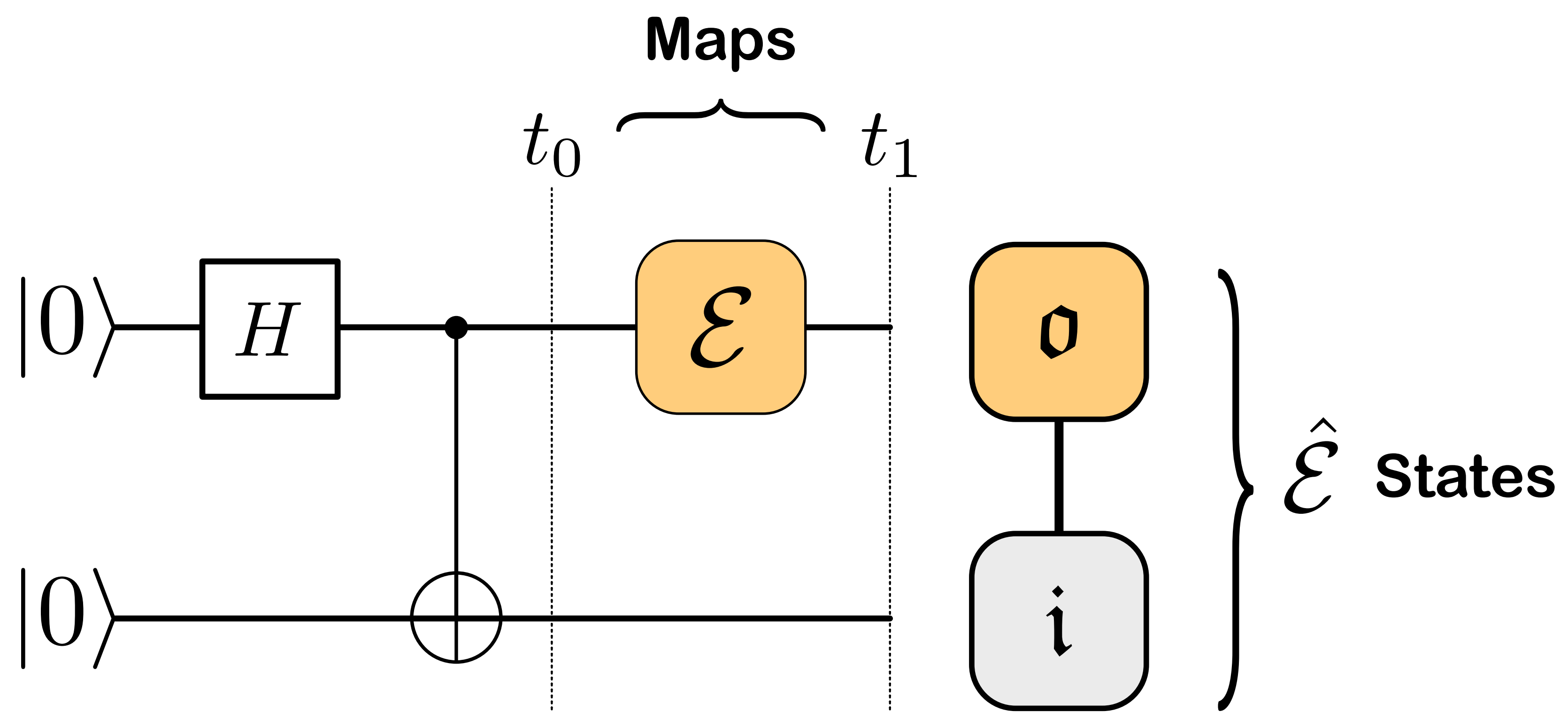


- Quantum theory has a spacetime structure with curious causal properties

Simple example, CJI:



Chiribella et al. Phys. Rev. Letters **101**, 180501 (2008)
Hardy, Phil. Trans. R. Soc. A **370**, 3385 (2012)
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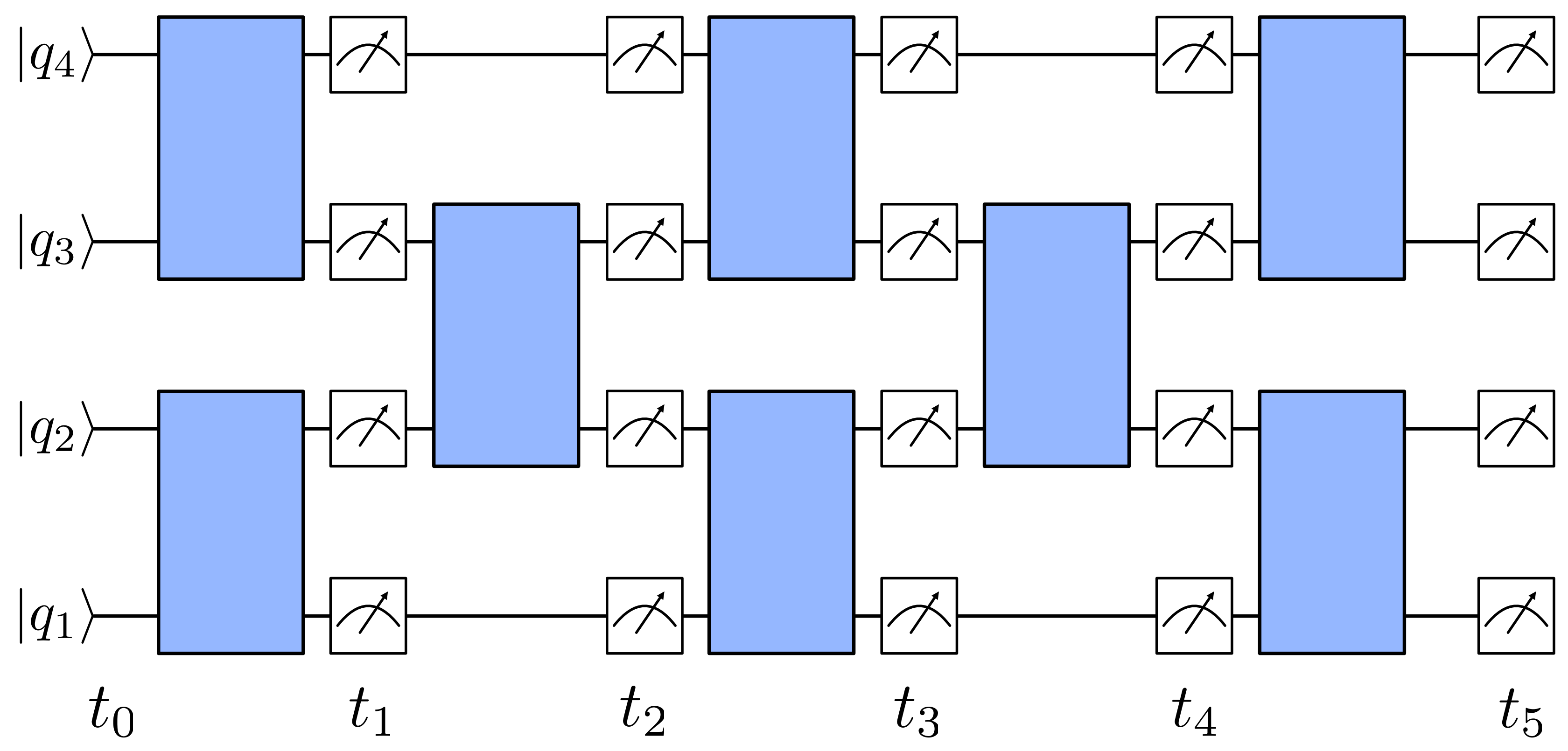
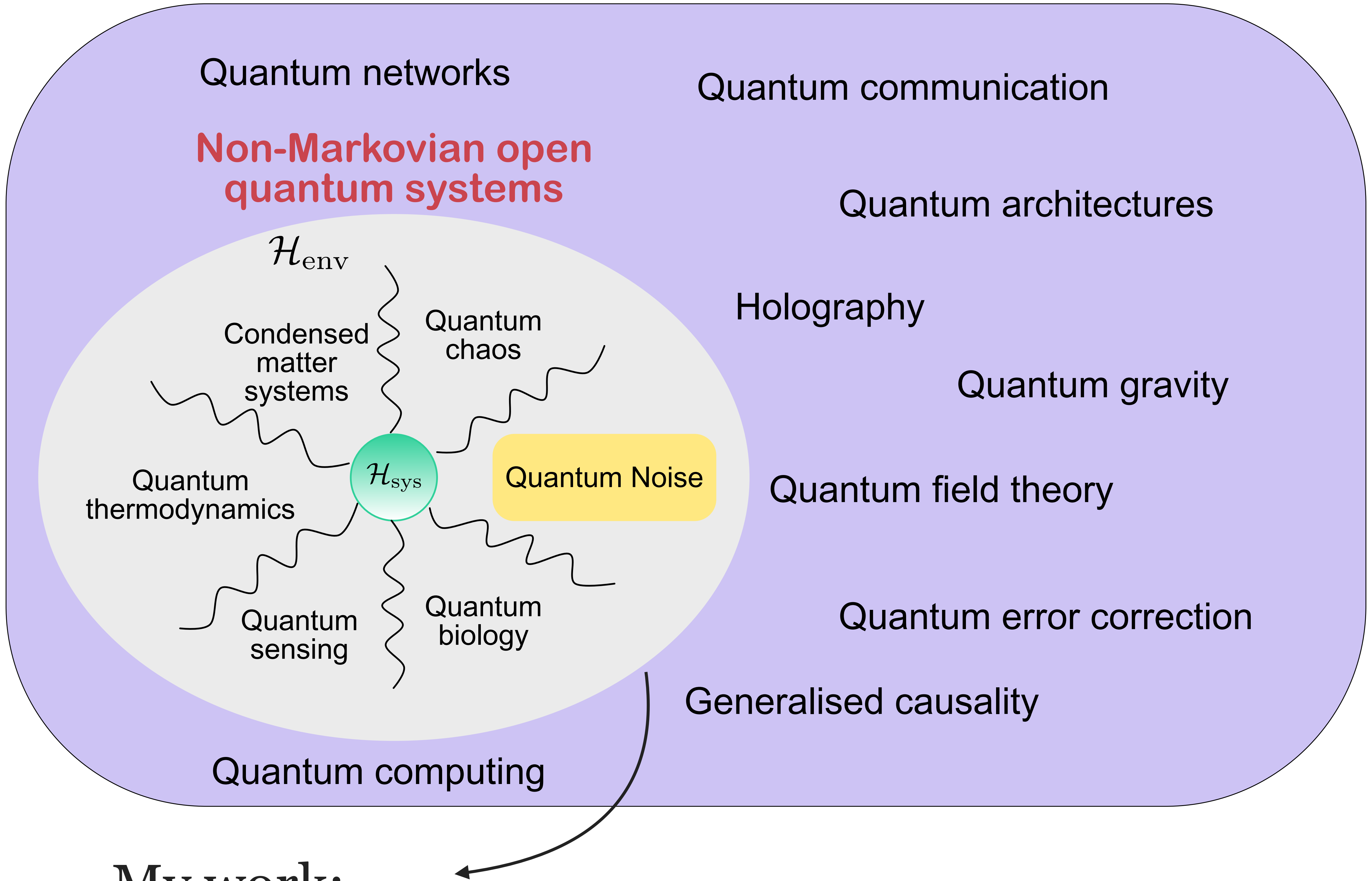
Cotler et al. J. High Energy Phys. **2018**, 93 (2018)

The Spacetime Structure of Quantum Mechanics



- Quantum theory has a spacetime structure with curious causal properties
- Particularly emerges in the context of open quantum systems



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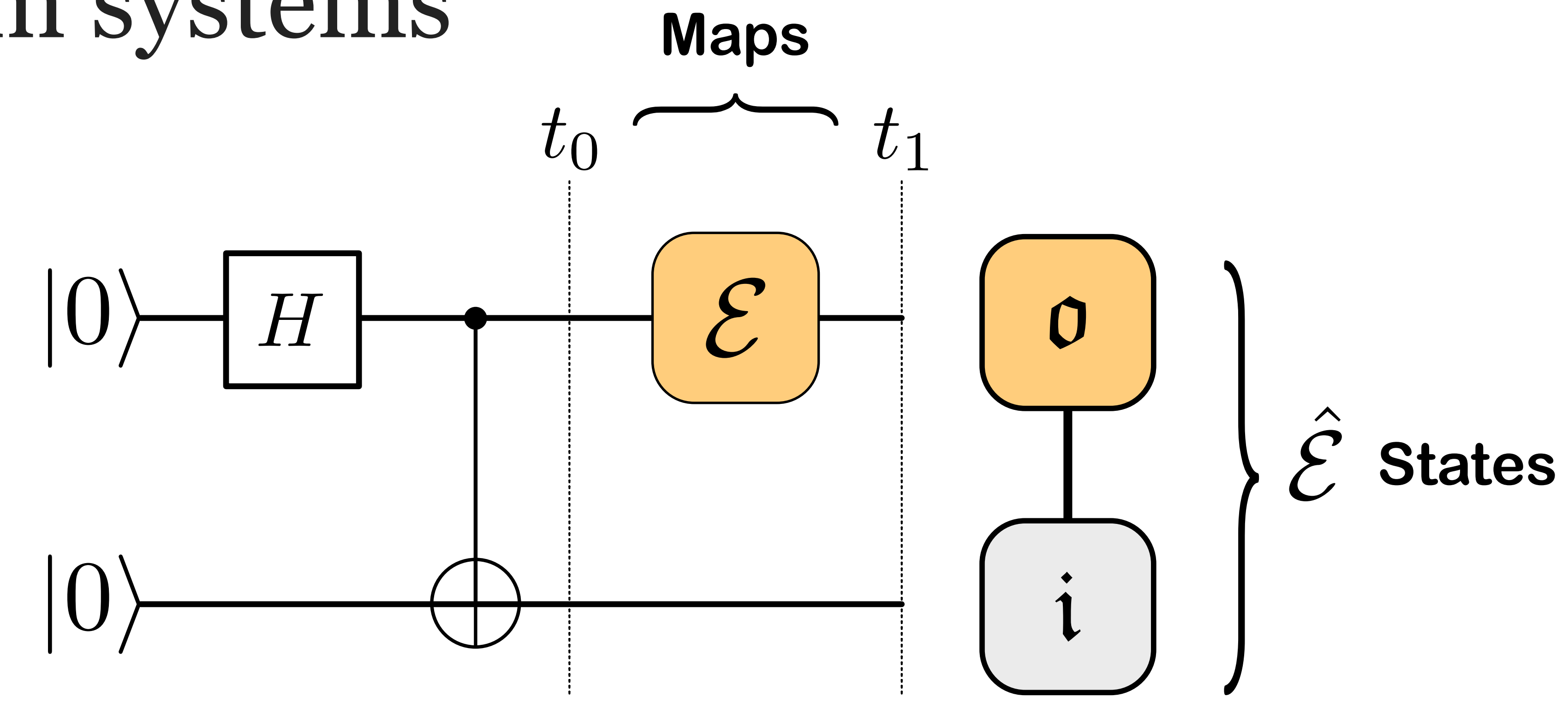

<u>My work:</u>

What generates spatiotemporal correlations?

What is their structure?

How can we access/measure/manipulate them?

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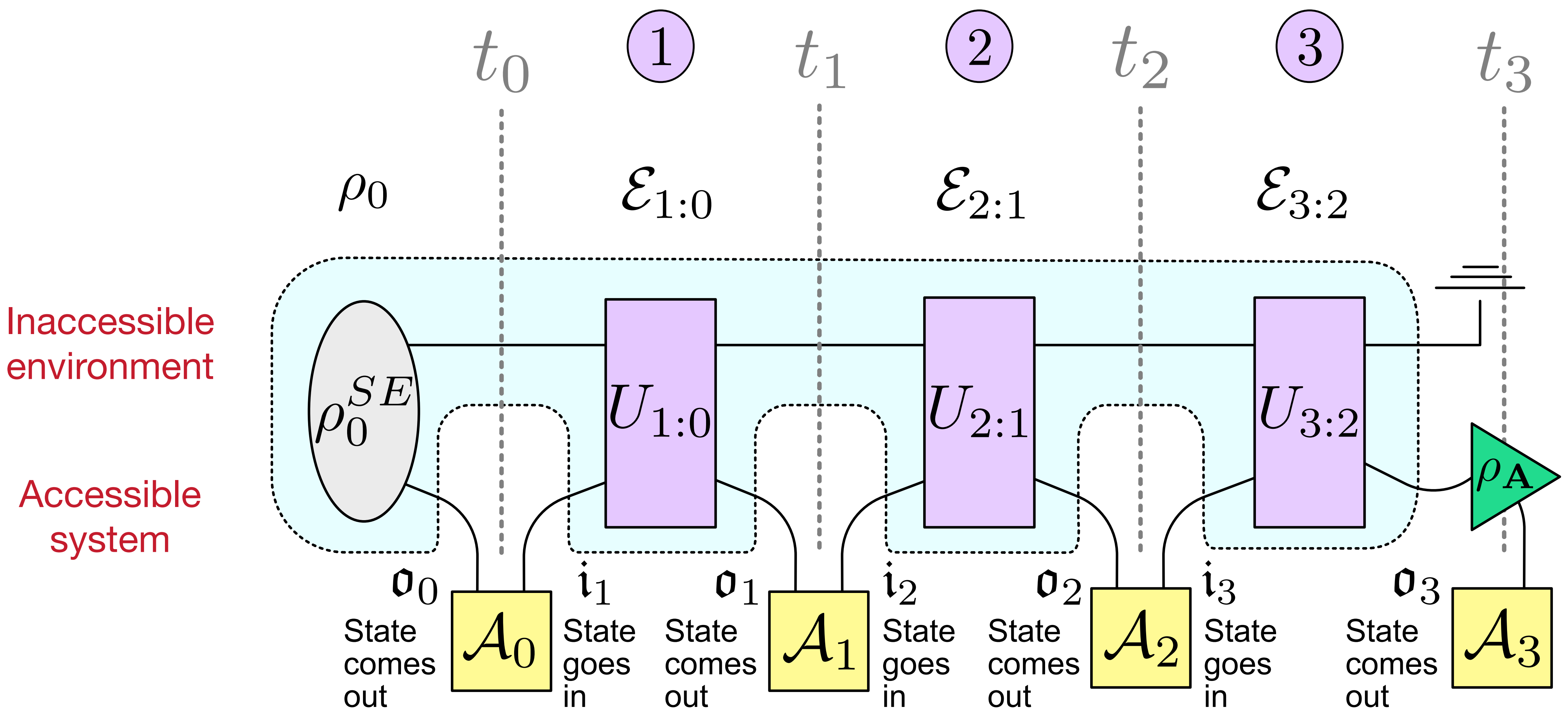
Process Tensors



- Introduce the process tensor

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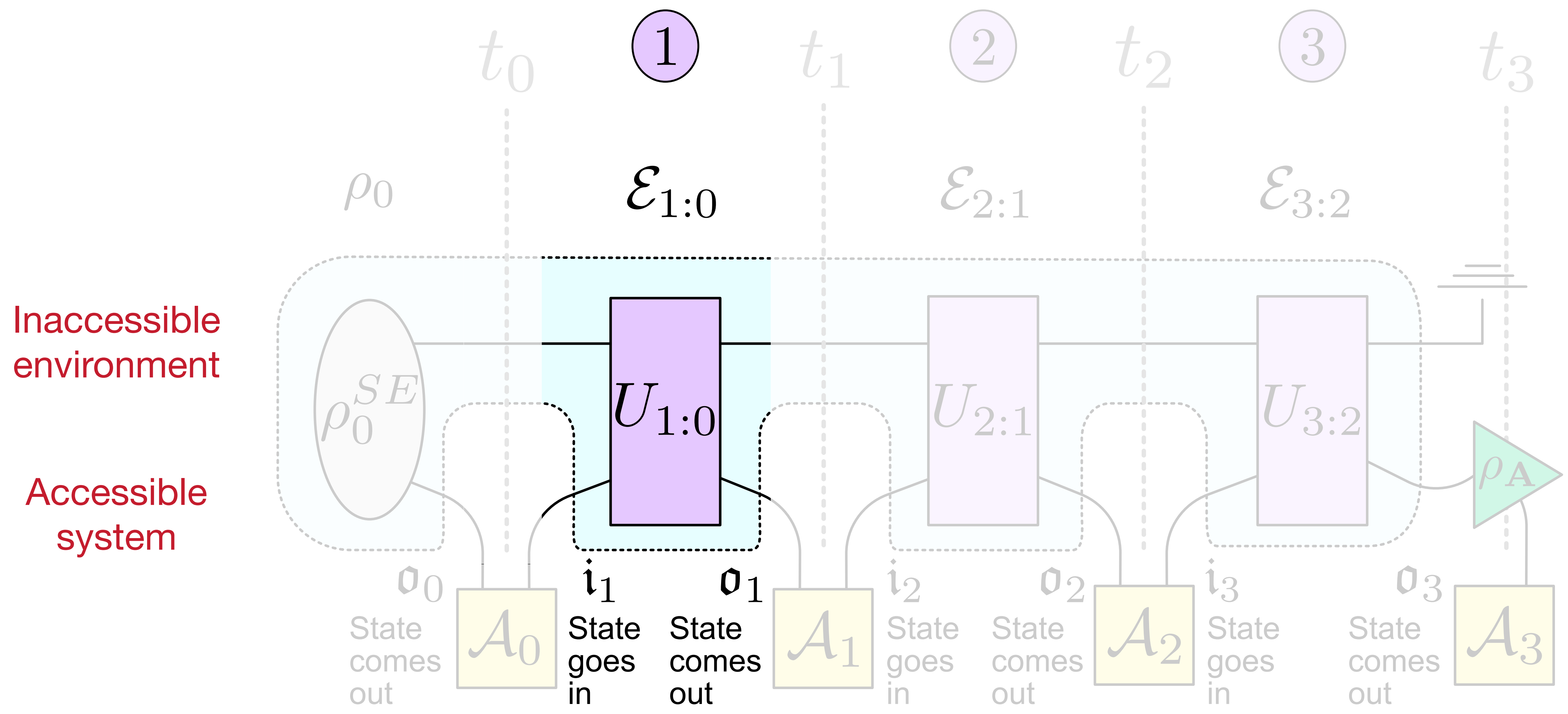
Pollock et al. Phys. Rev. A **97**, 012127 (2018)

Process Tensors



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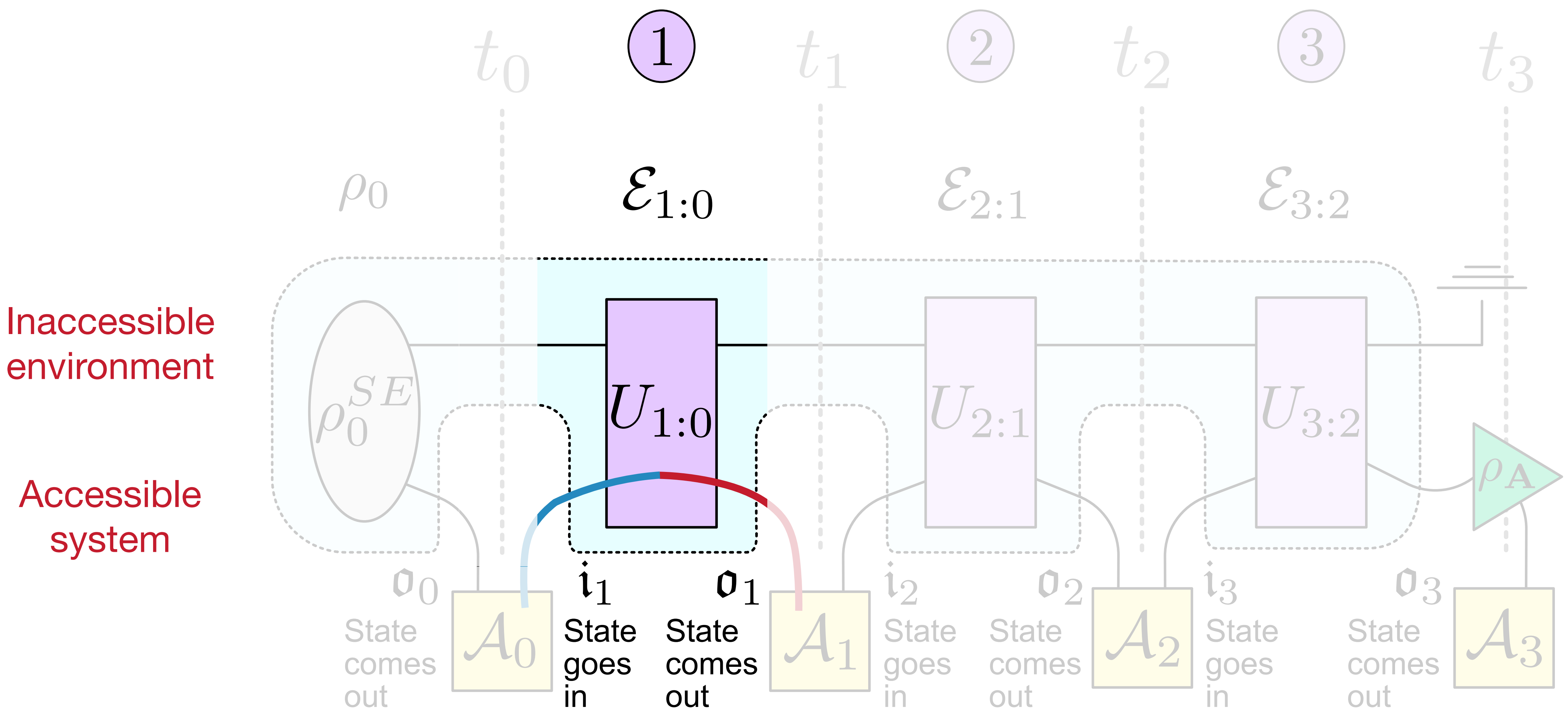

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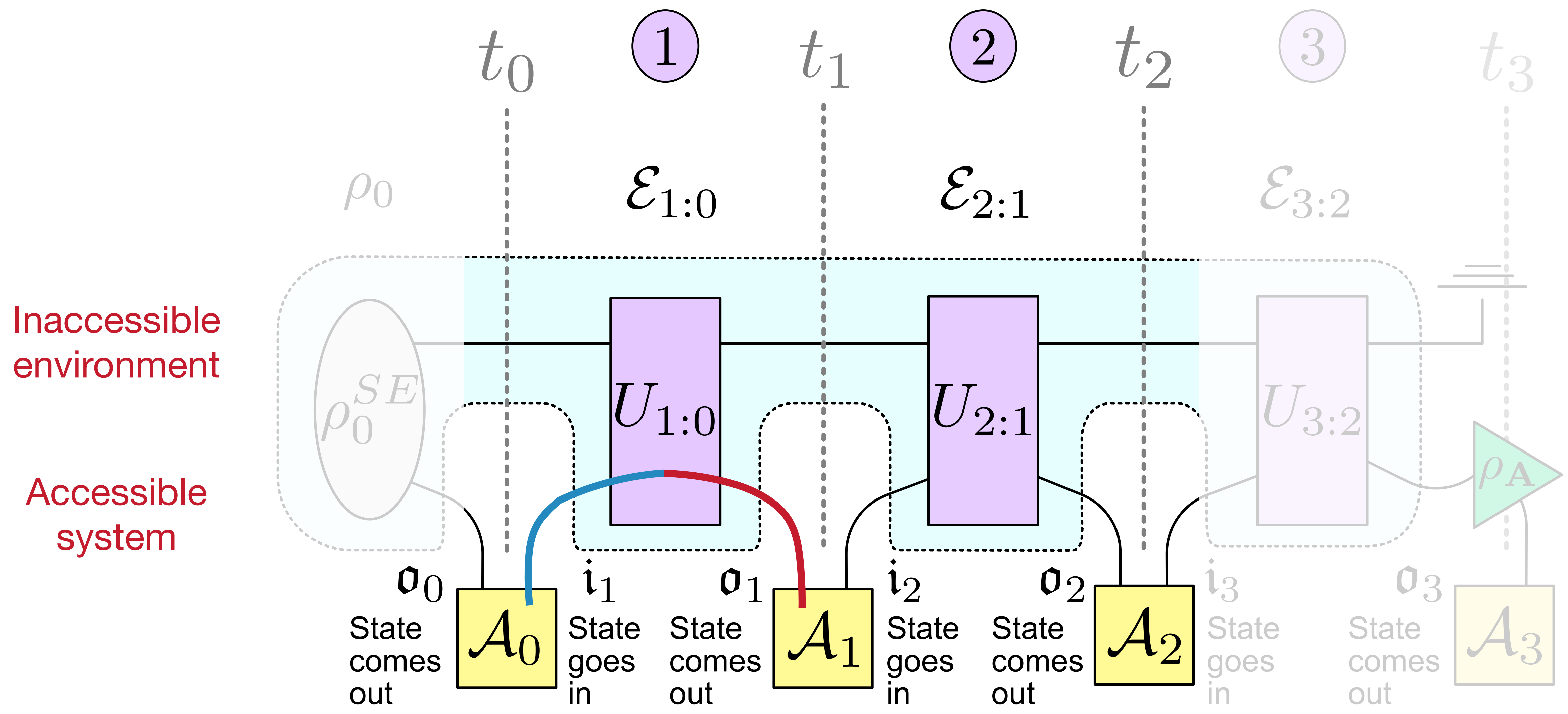

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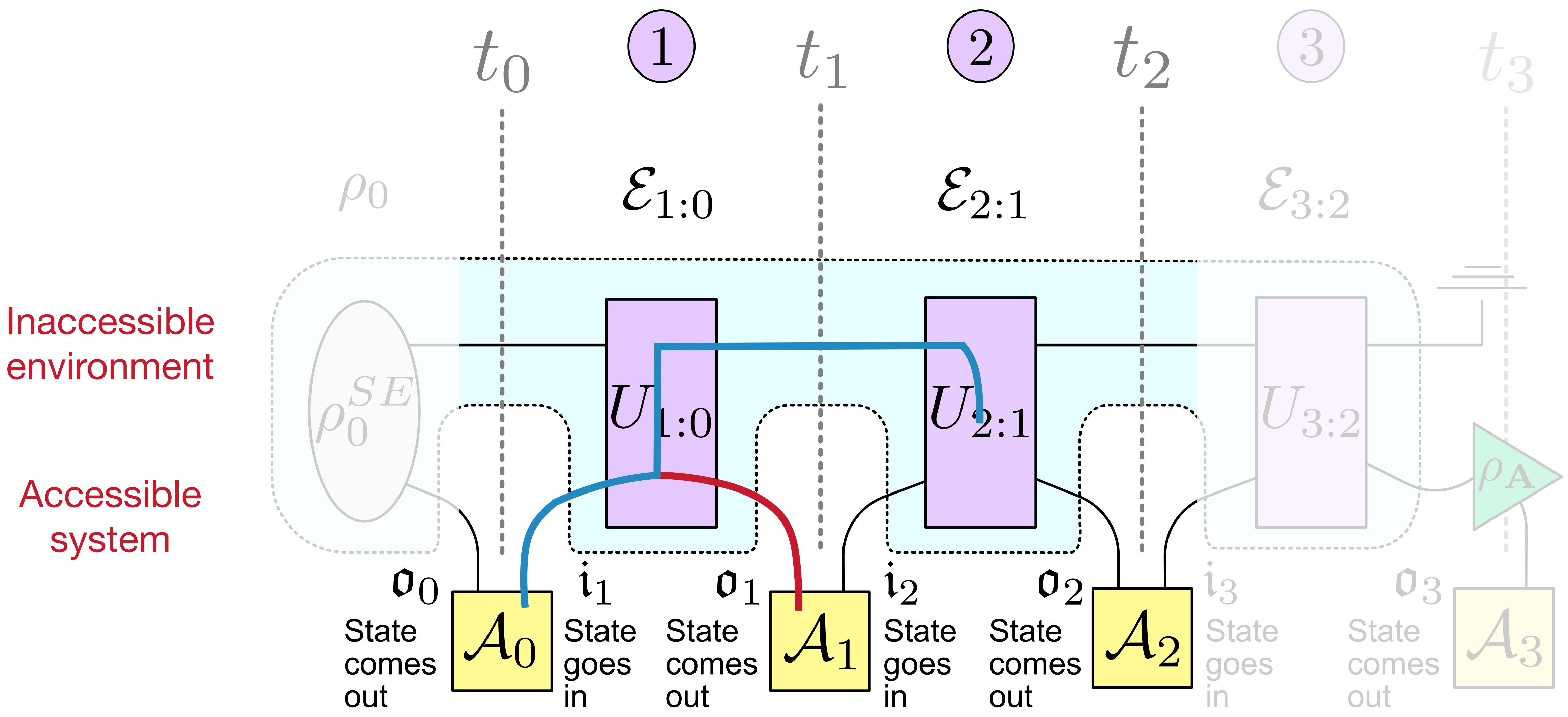

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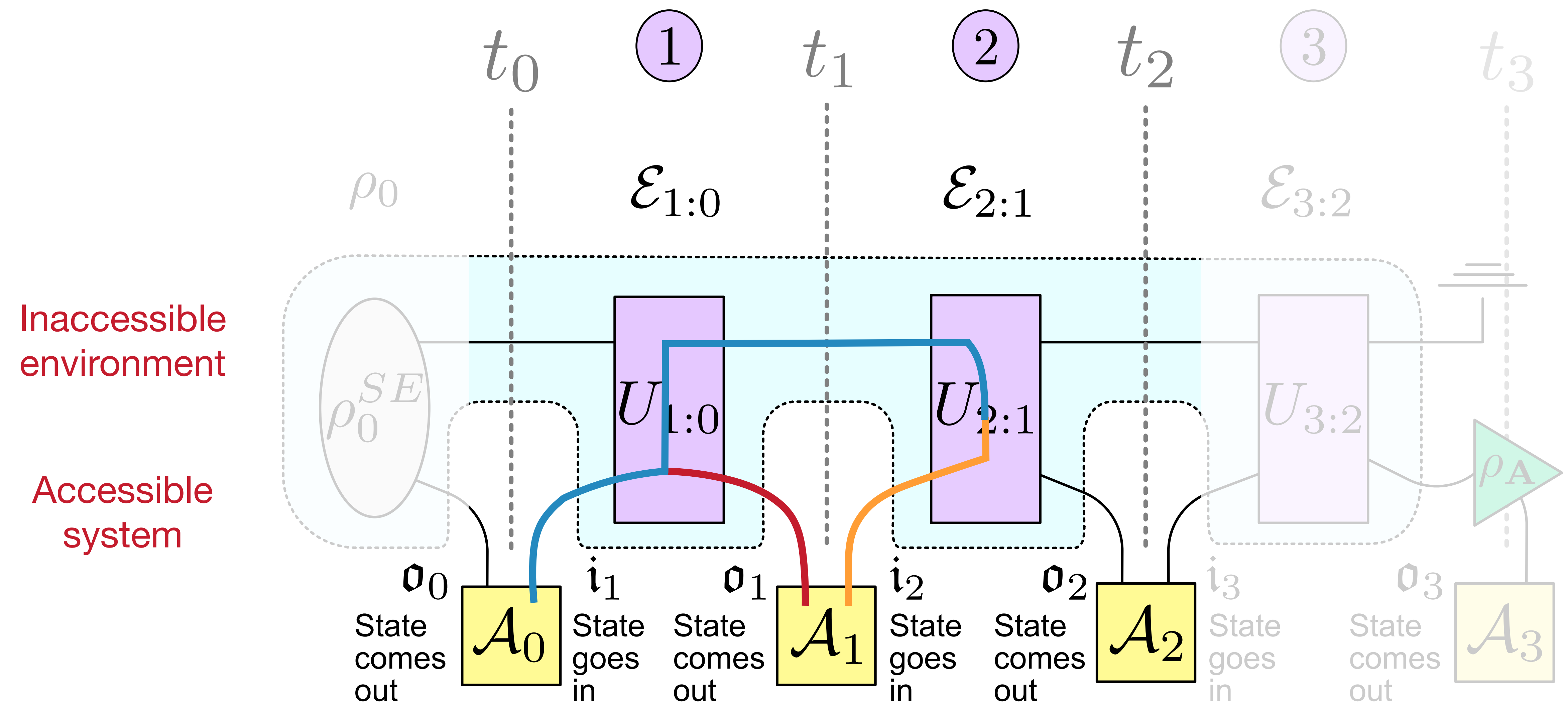

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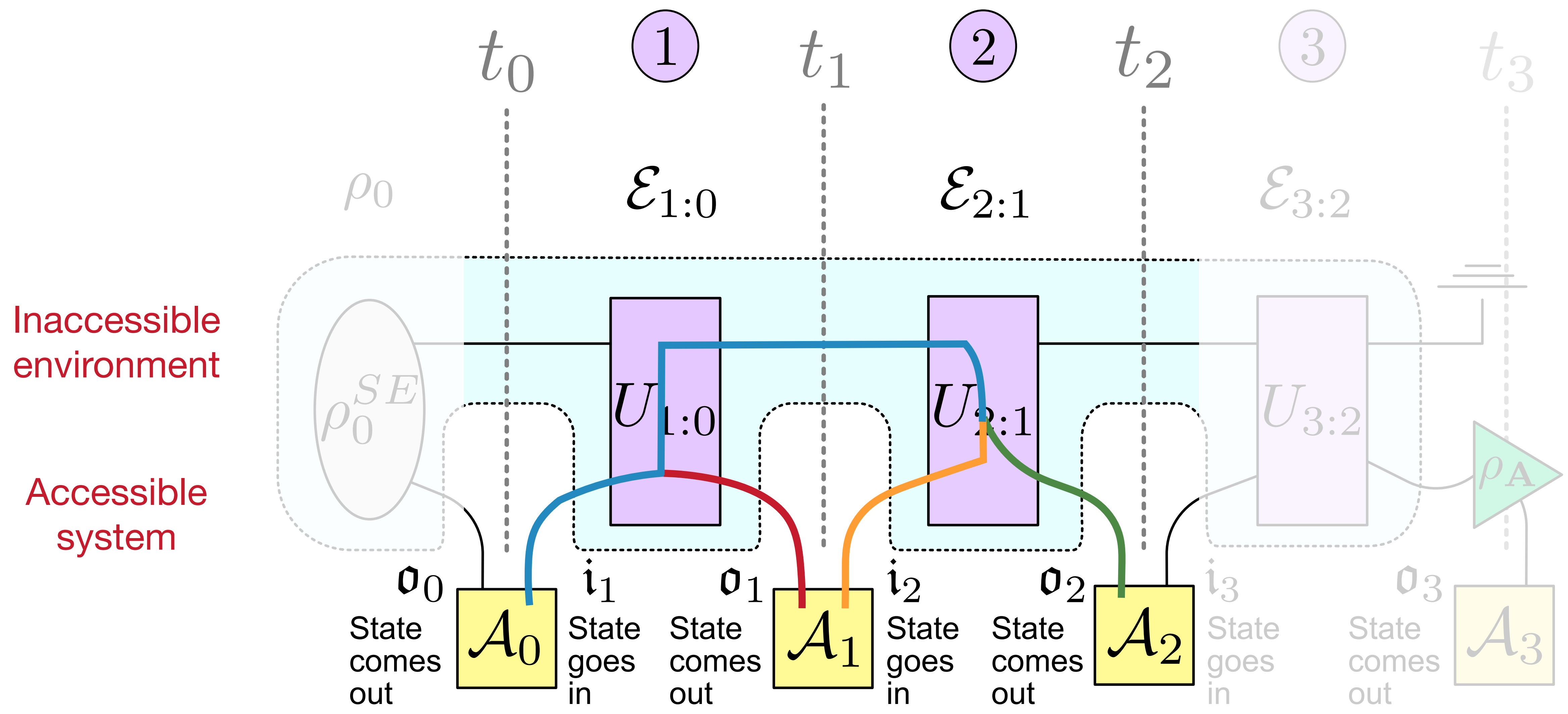

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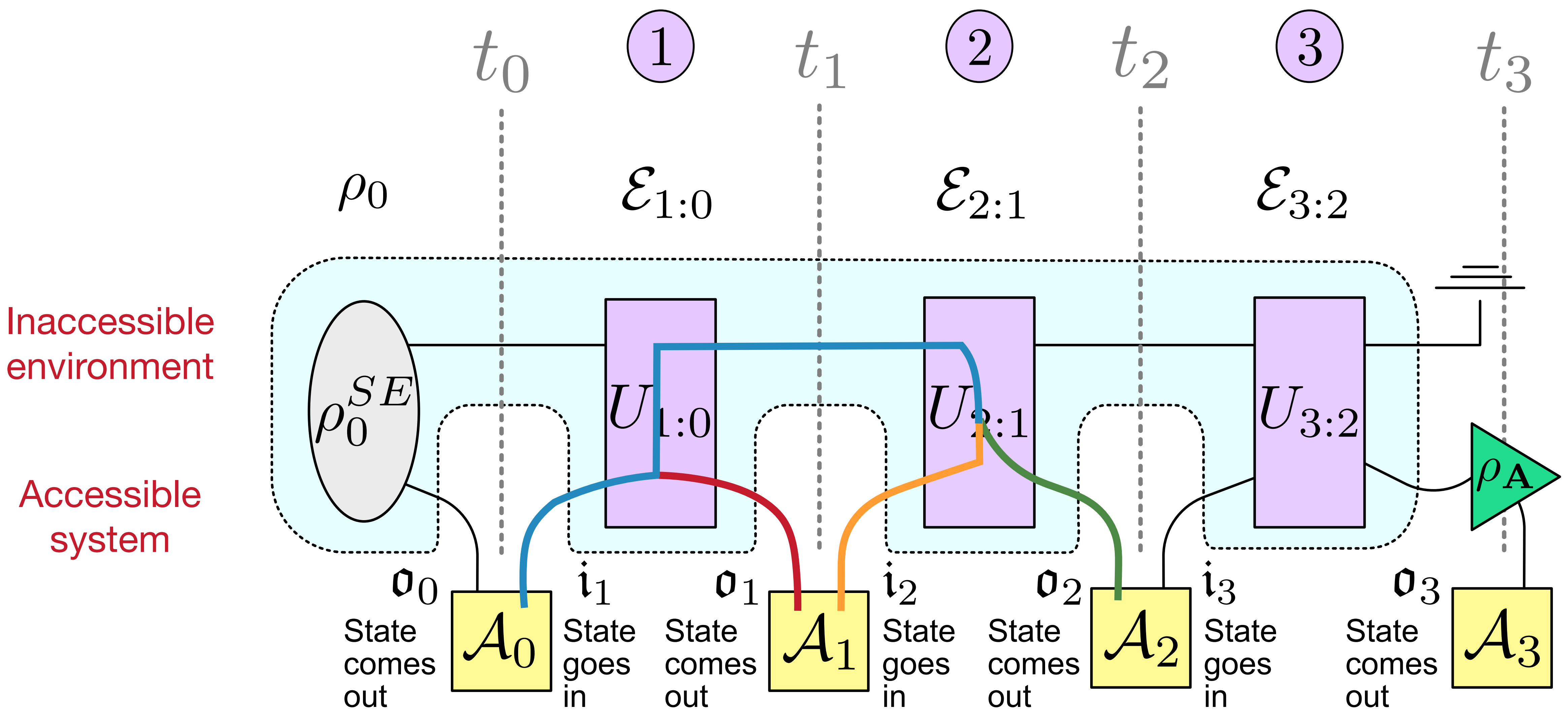

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- Sequences of control operations are observables of the process

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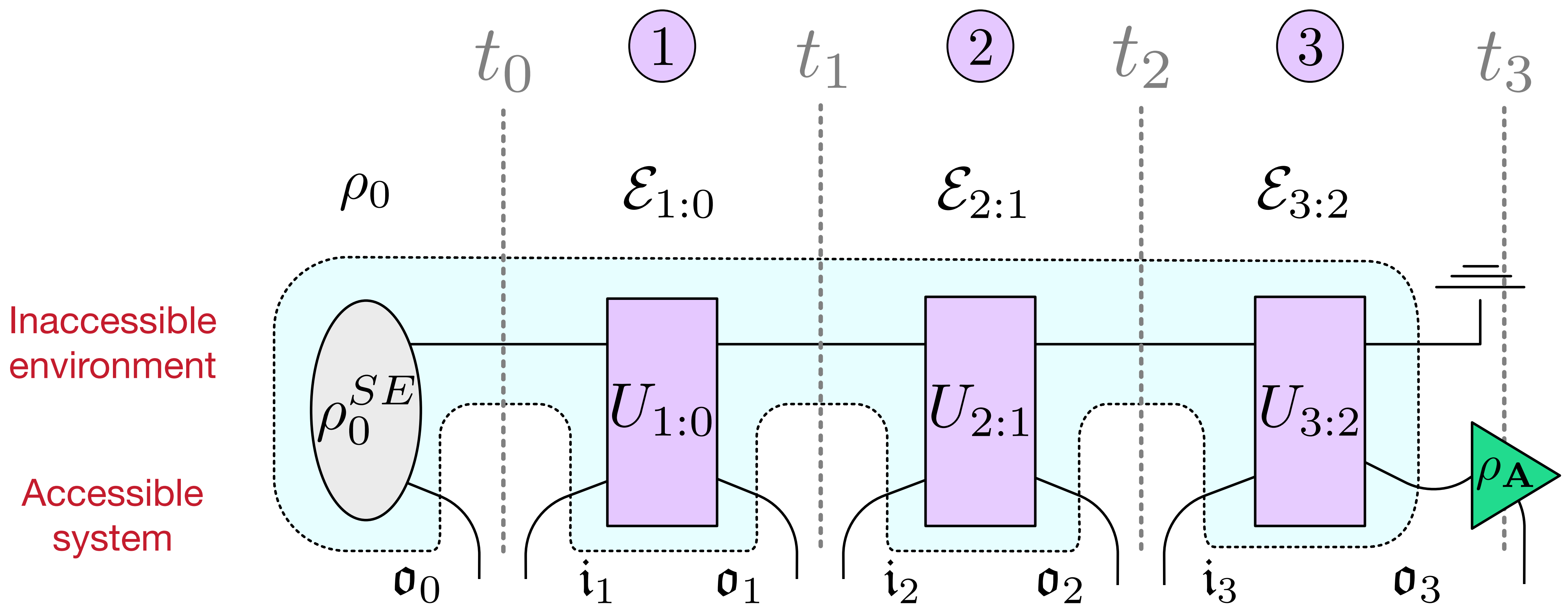

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Controls are events we can choose, process exists independently

Controls: deterministic, classical stochastic, quantum stochastic

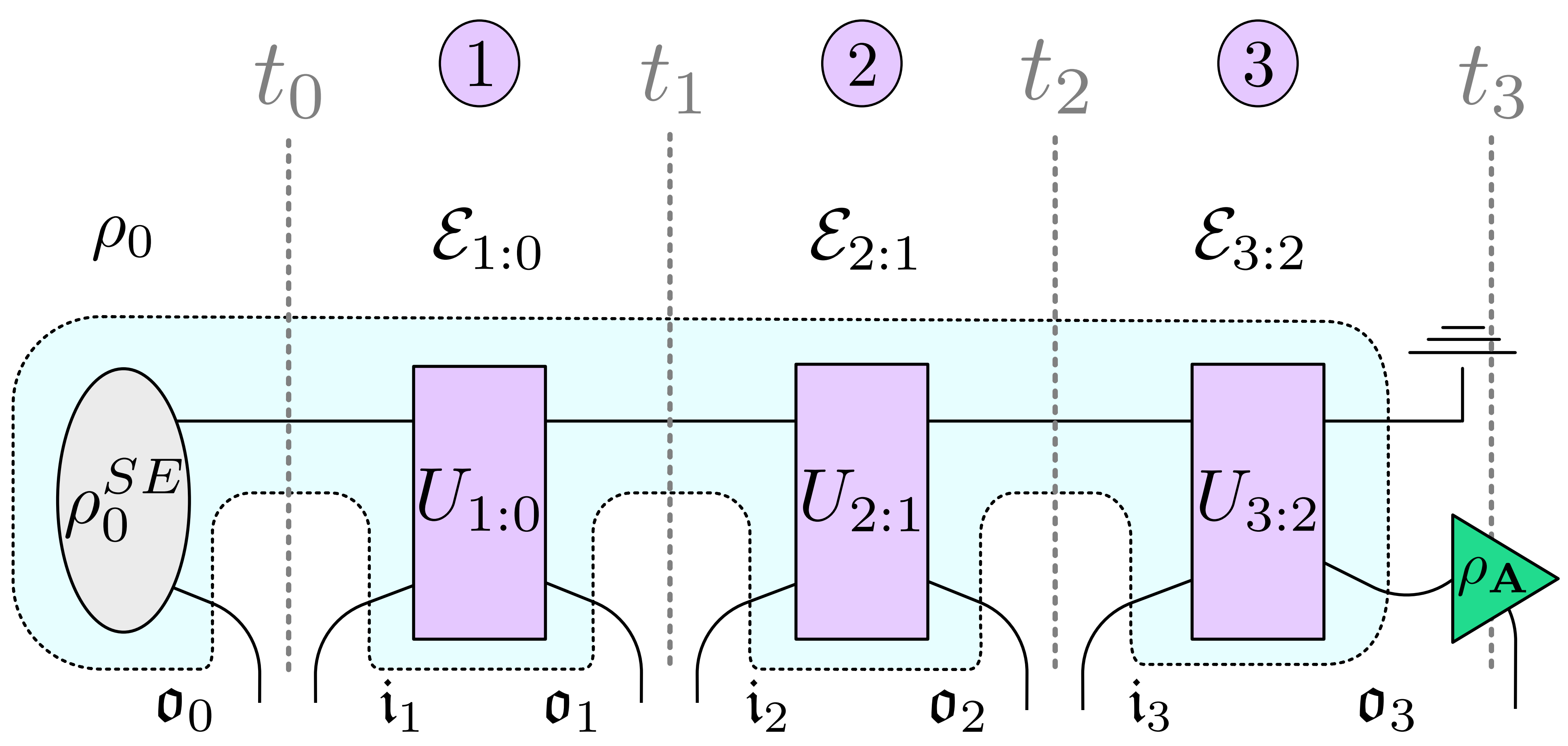
Instruments can click with outcome x_j at time t_j

$$\Pr(x_j, t_j; x_{j-1}, t_{j-1}; \cdots; x_0, t_0 | \mathbf{A}_{j:0}).$$

We now have a description of quantum stochastic processes

Milz et al. Quantum 4, 255 (2020)

Process Tensors



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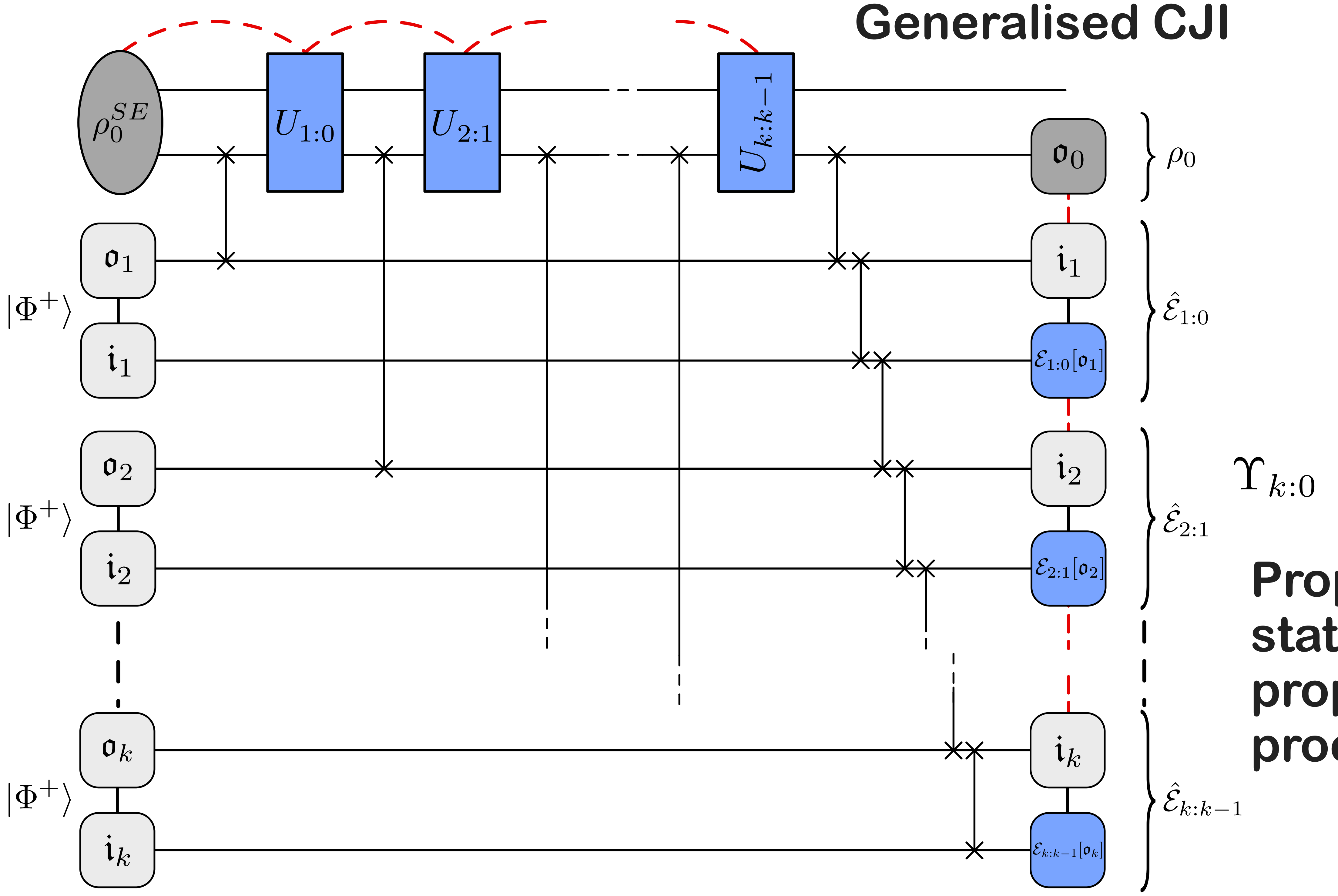
Generalised CJI



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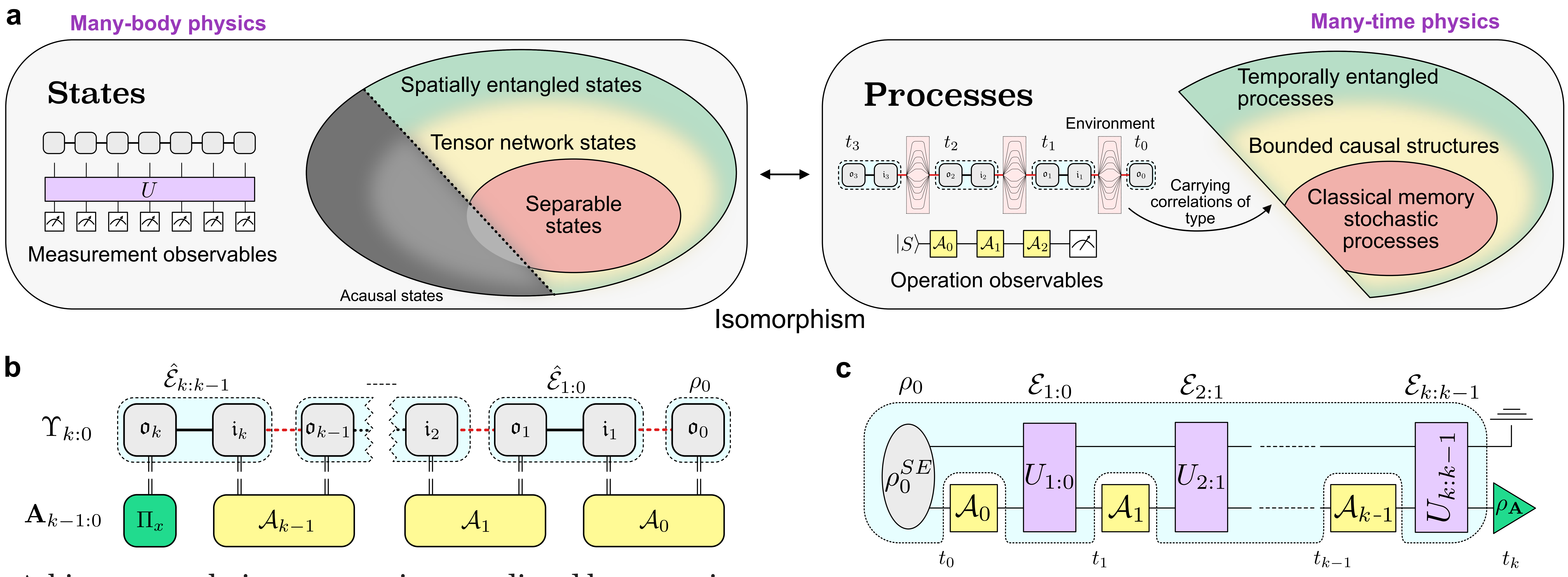
Milz et al. Quantum 4, 255 (2020)

Properties of the state are properties of the process!

Many-Time Physics



- Arbitrary correlations across time, mediated by an environment
- Formally structured in the same way as spatial many-body correlations
- What's missing is a way to study these correlations in a practical setting for arbitrary open dynamics

Aharonov et al. Phys. Rev. A **79**, 052110 (2009)
Costa et al. Phys. Rev. A 98, 012328 (2018)
Ried et al. Nature Physics **11**, 414-420 (2015)
Chiribella et al. Phys. Rev. A **80**, 022339 (2009)
Costa & Shrapnel, New Journal of Physics **18**, 063032 (2016)
Pollock et al. Phys. Rev. A **97**, 012127 (2018)
Milz et al. SciPost Phys. 10, 141 (2021)
White et al. arXiv:2107.13934 (2021)

Process Tensor Tomography

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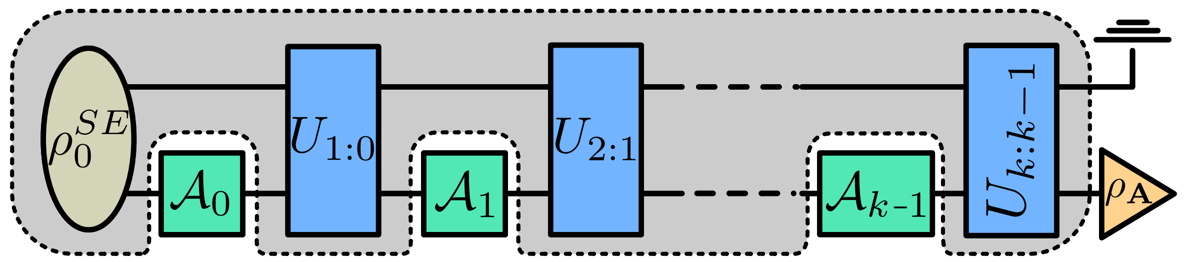
$$\mathbf{A}_{k-1:0} = \bigotimes_{j=0}^{k-1} \mathcal{A}_j$$



- Estimate non-Markovian dynamics
- Uniquely constrain on a complete basis

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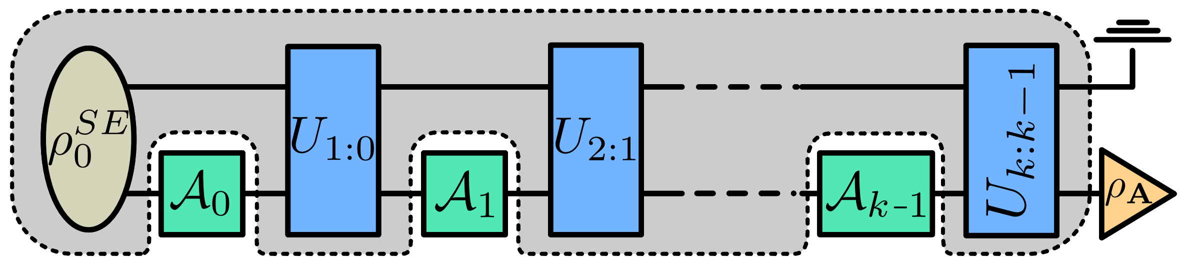

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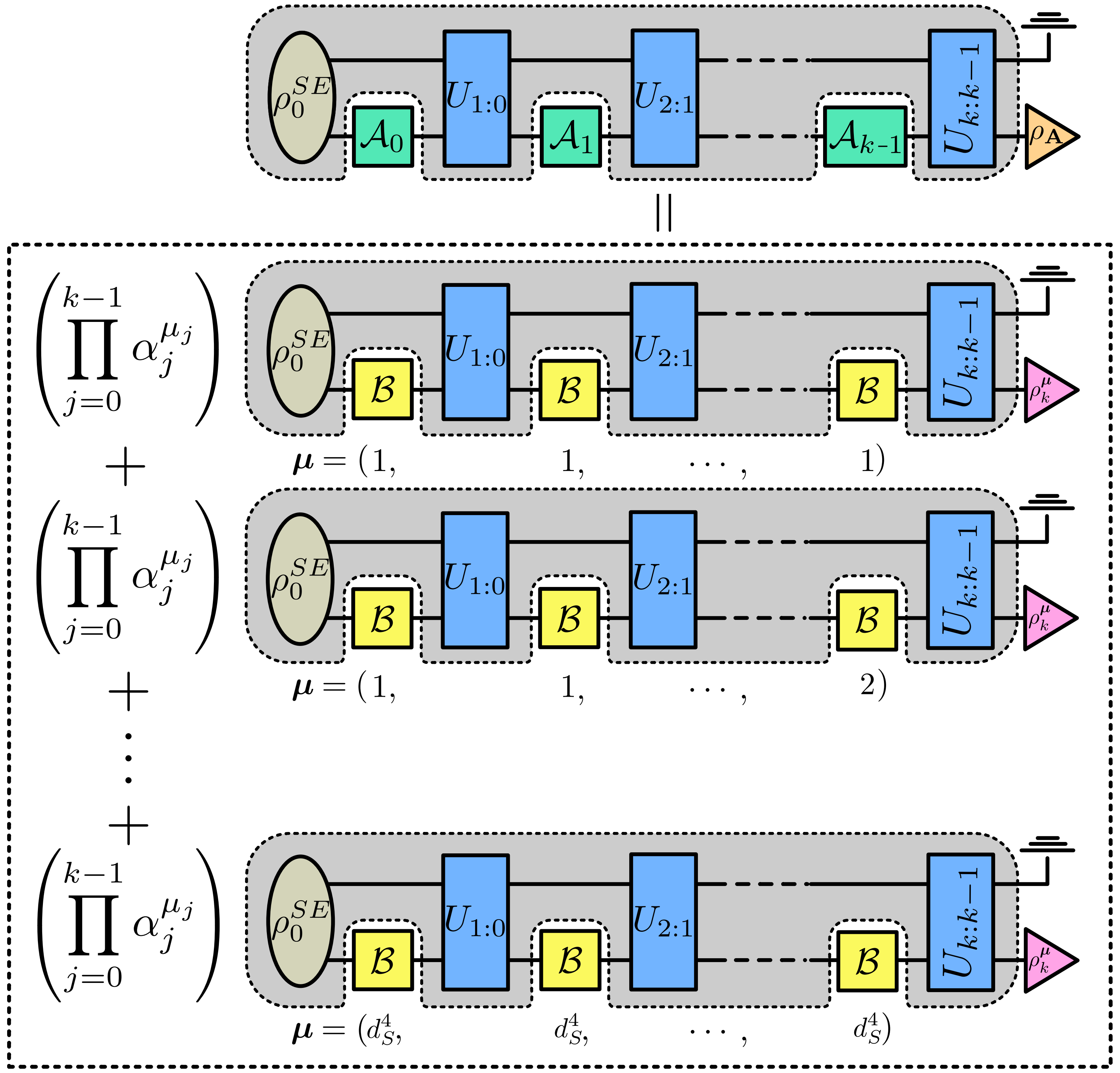

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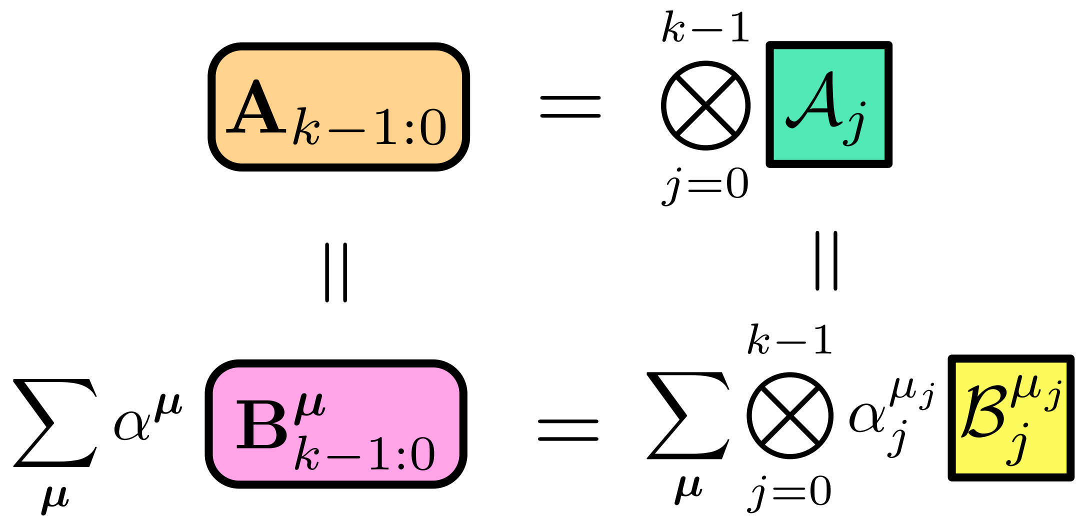

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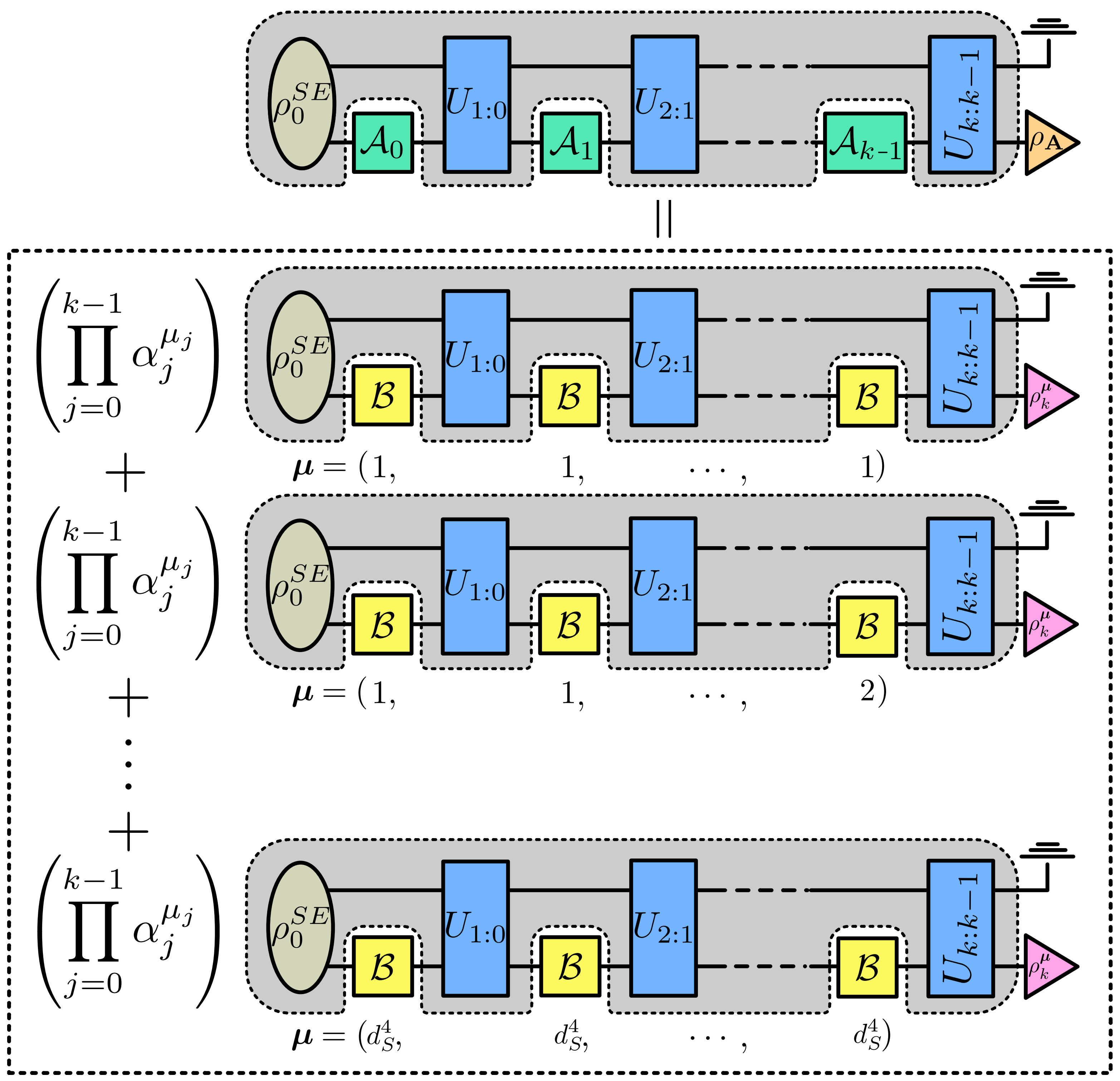

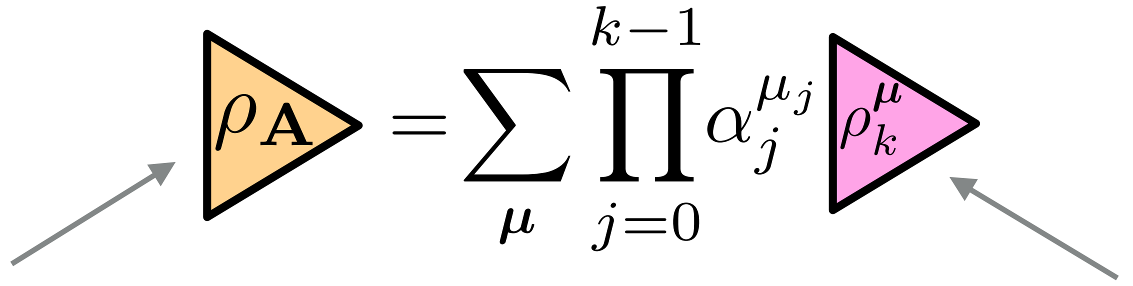

Nature Communications **11**, 6301 (2020)

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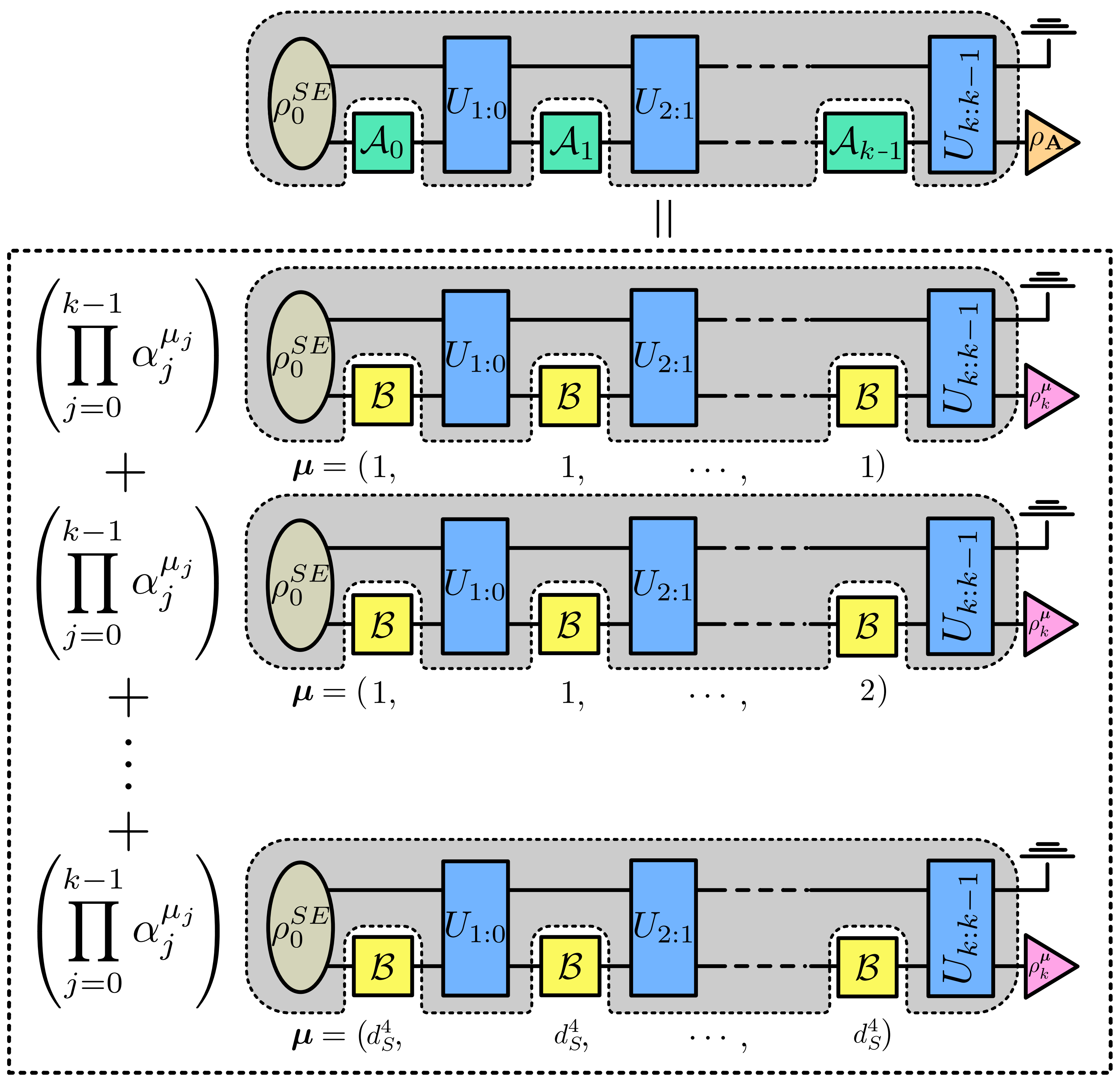
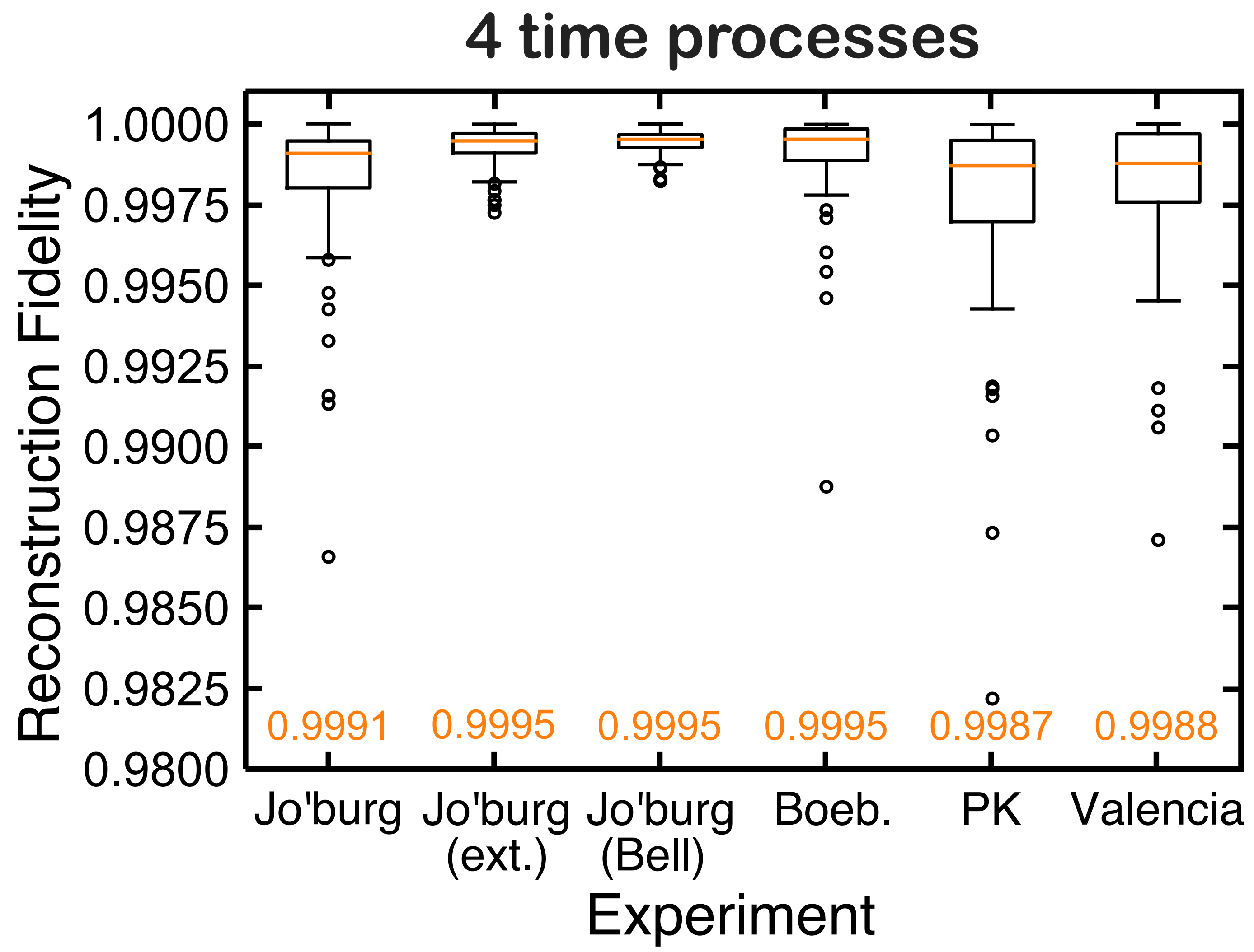
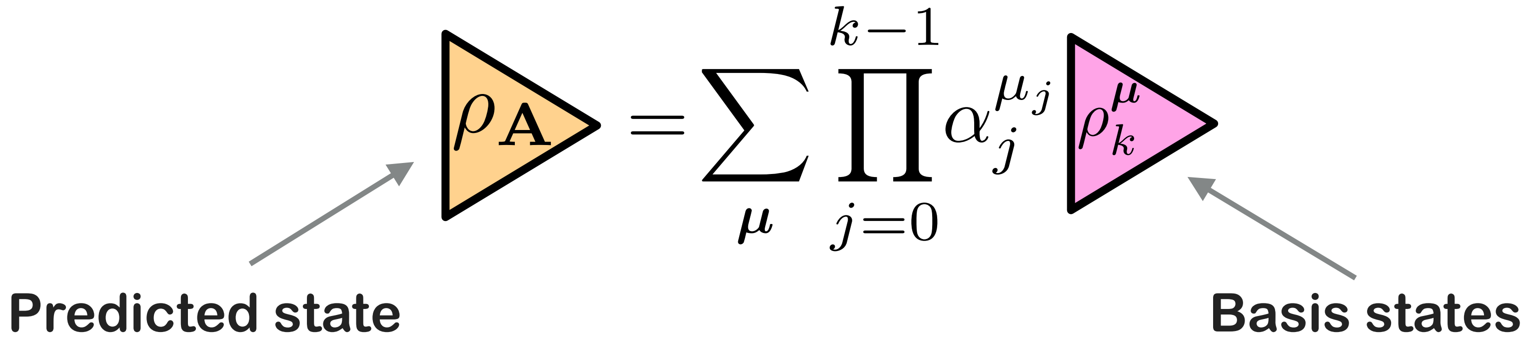
- Use the process tensor to predict random sequences
- Compare how close the two are: 'reconstruction fidelity'







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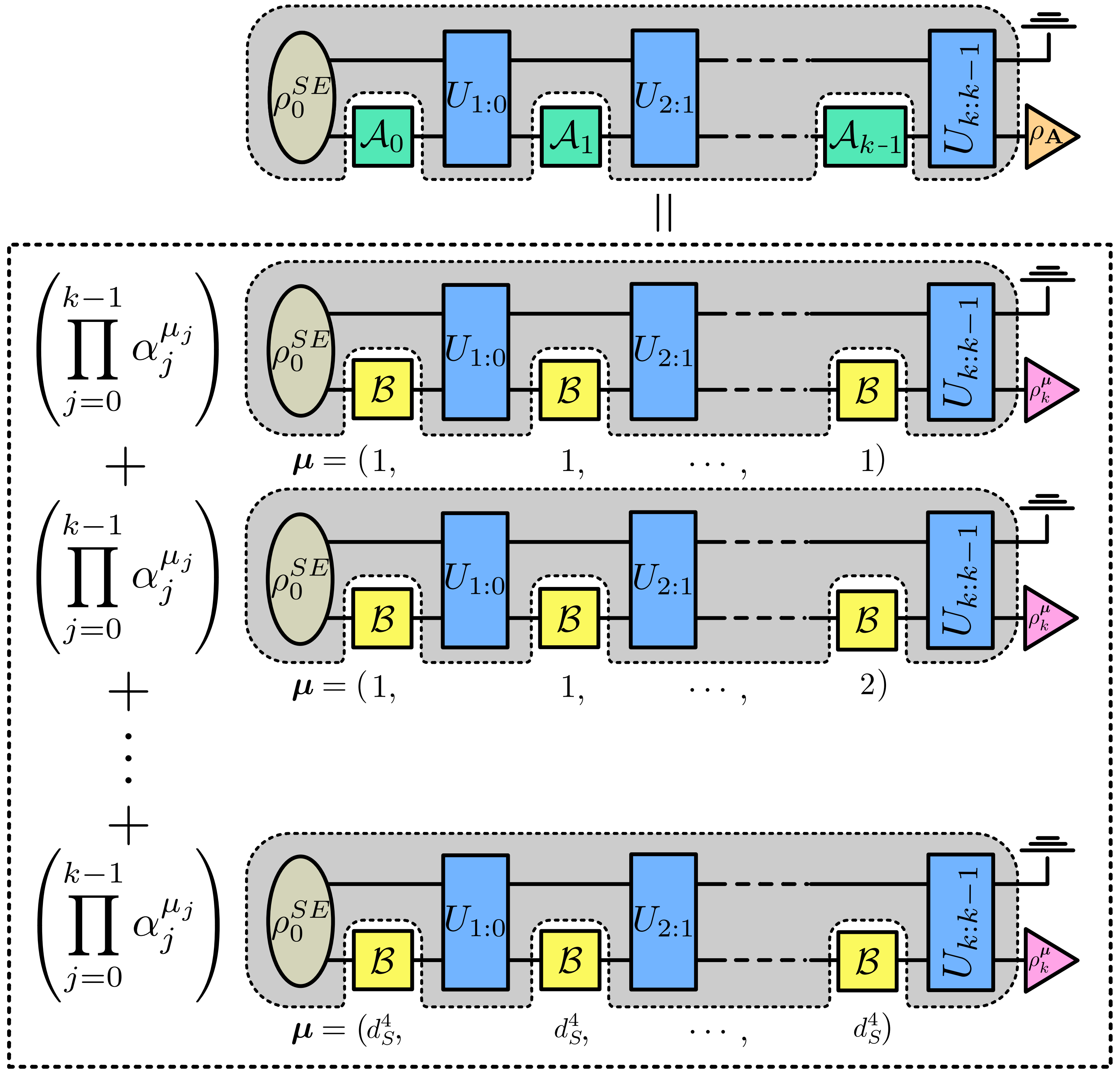
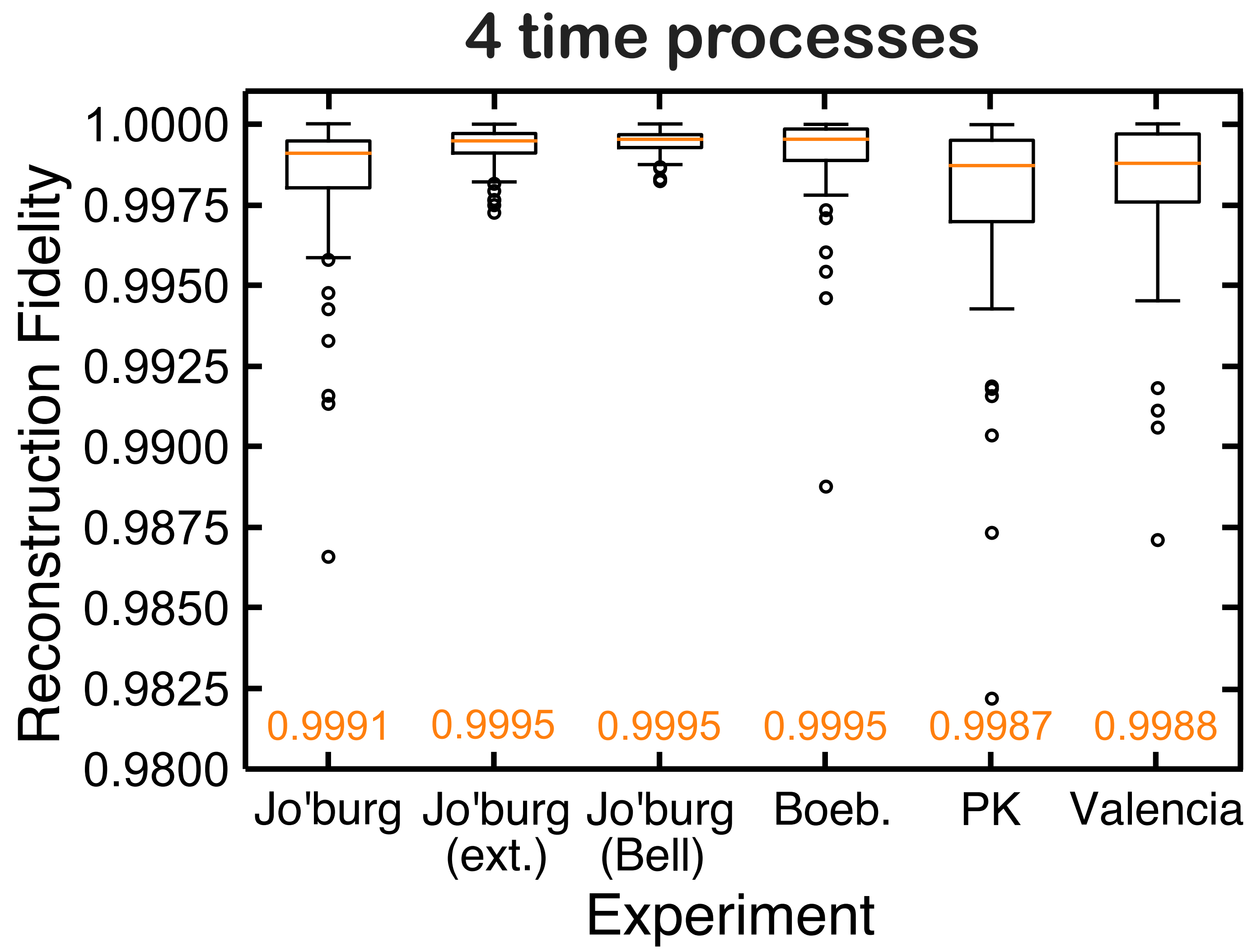


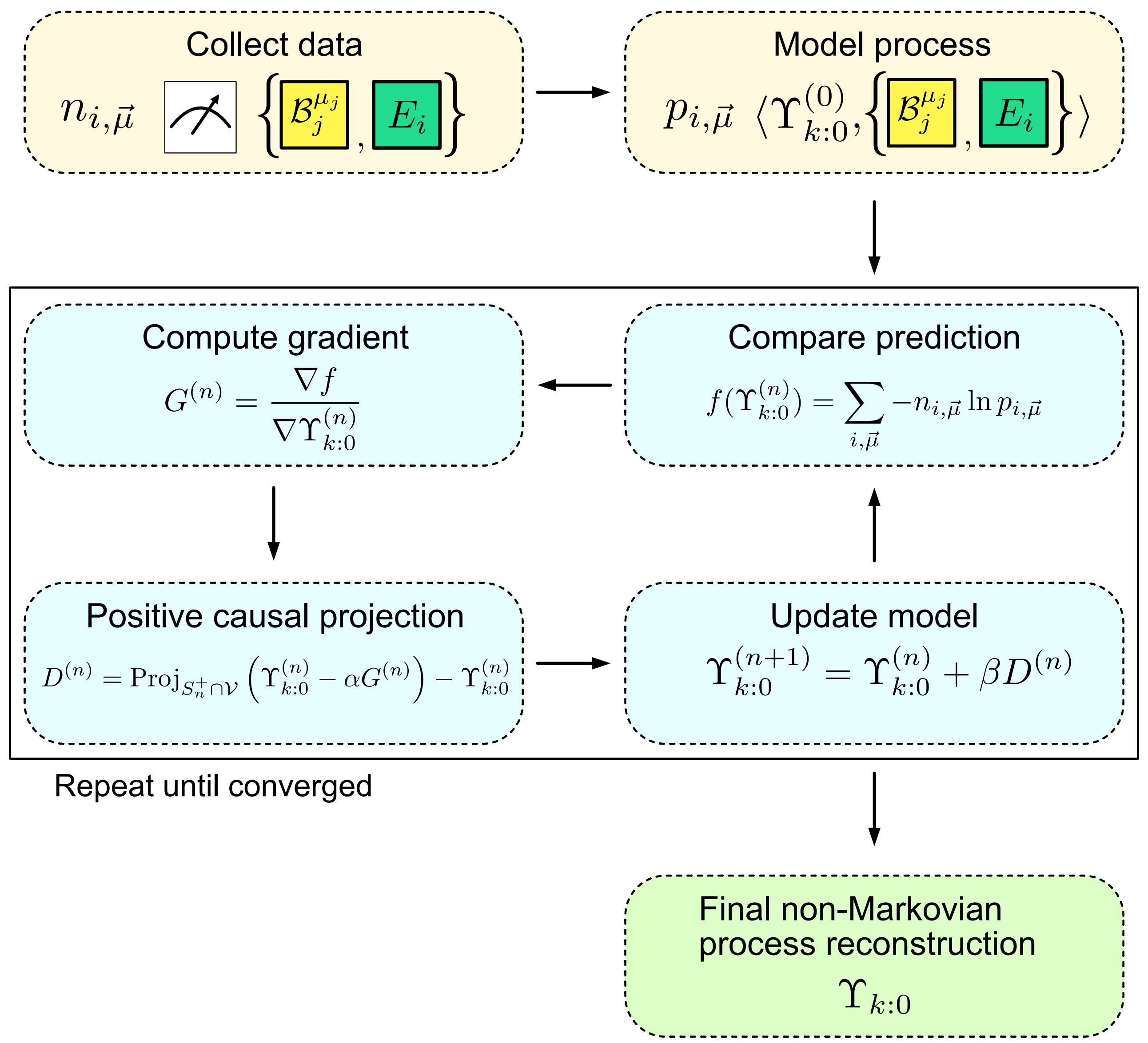
- Estimate non-Markovian dynamics
- Uniquely constrain on a complete basis
- Use the process tensor to predict random sequences
- Compare how close the two are: ‘reconstruction fidelity’
- Generalised Born rule connects process tensor to reality
- Fit a physical, maximum likelihood model

$$\rho_A = \sum_{\boldsymbol{\mu}} \prod_{j=0}^{k-1} \alpha_j^{\mu_j} \rho_k^{\boldsymbol{\mu}}$$

Predicted state — Basis states

$$p_{i,\vec{\mu}} = \text{Tr}\left[(\Pi_i \otimes \mathcal{B}_k^{\mu_{k-1}\,\text{T}} \otimes \cdots \otimes \mathcal{B}_0^{\mu_0\,\text{T}}) \Upsilon_{k:0} \right]$$





Nature Communications **11**, 6301 (2020)
PRX Quantum **3**, 020344 (2022)

Example: Detecting Temporal Entanglement Structures

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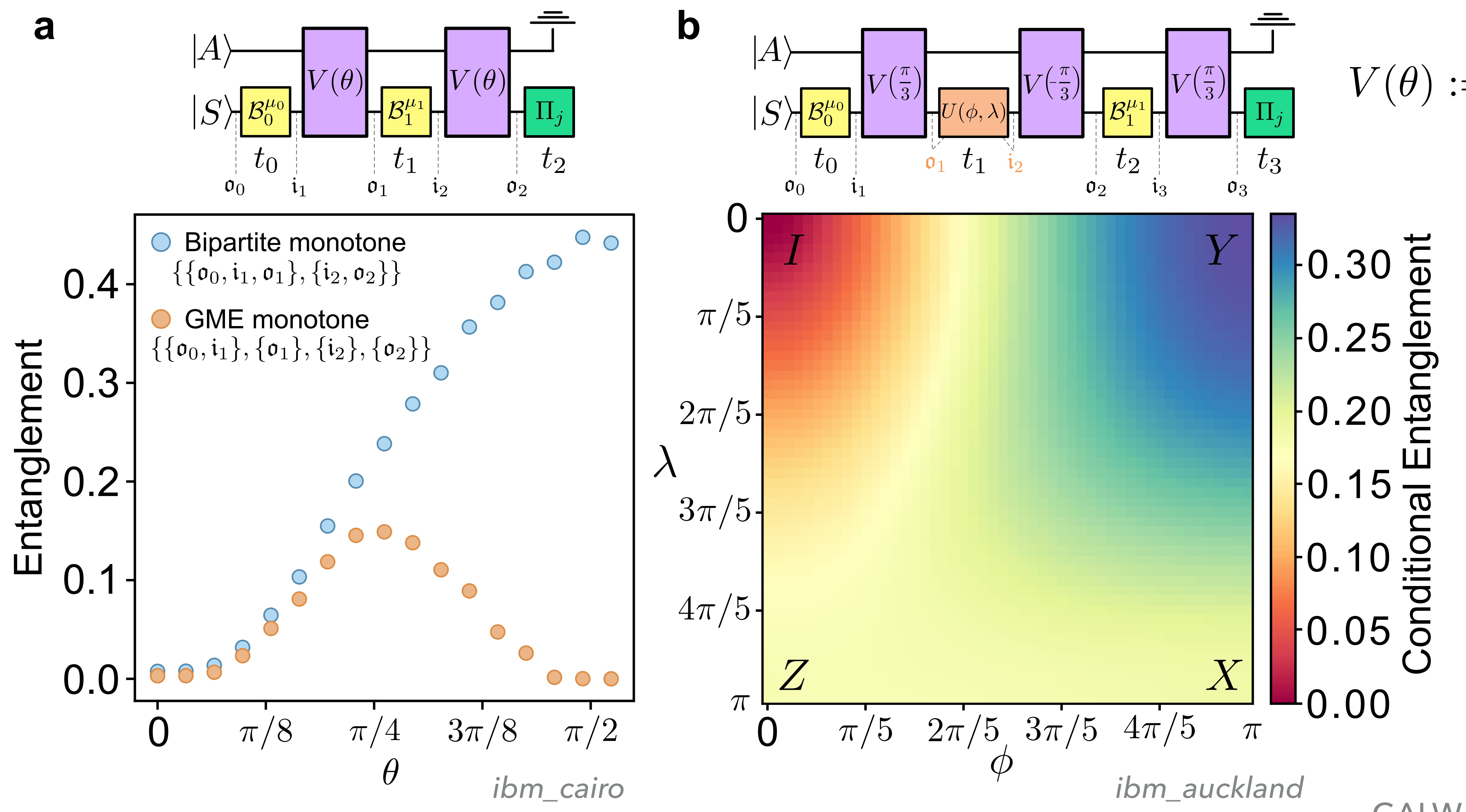

$$V(\theta) := \exp\left[-\frac{i\theta}{2}(XX+YY+ZZ)\right]$$

Unitary sequences can detect temporal entanglement even though they're deterministic!

GALW, Pollock, Hollenberg, Hill, Modi arXiv:2107.13934

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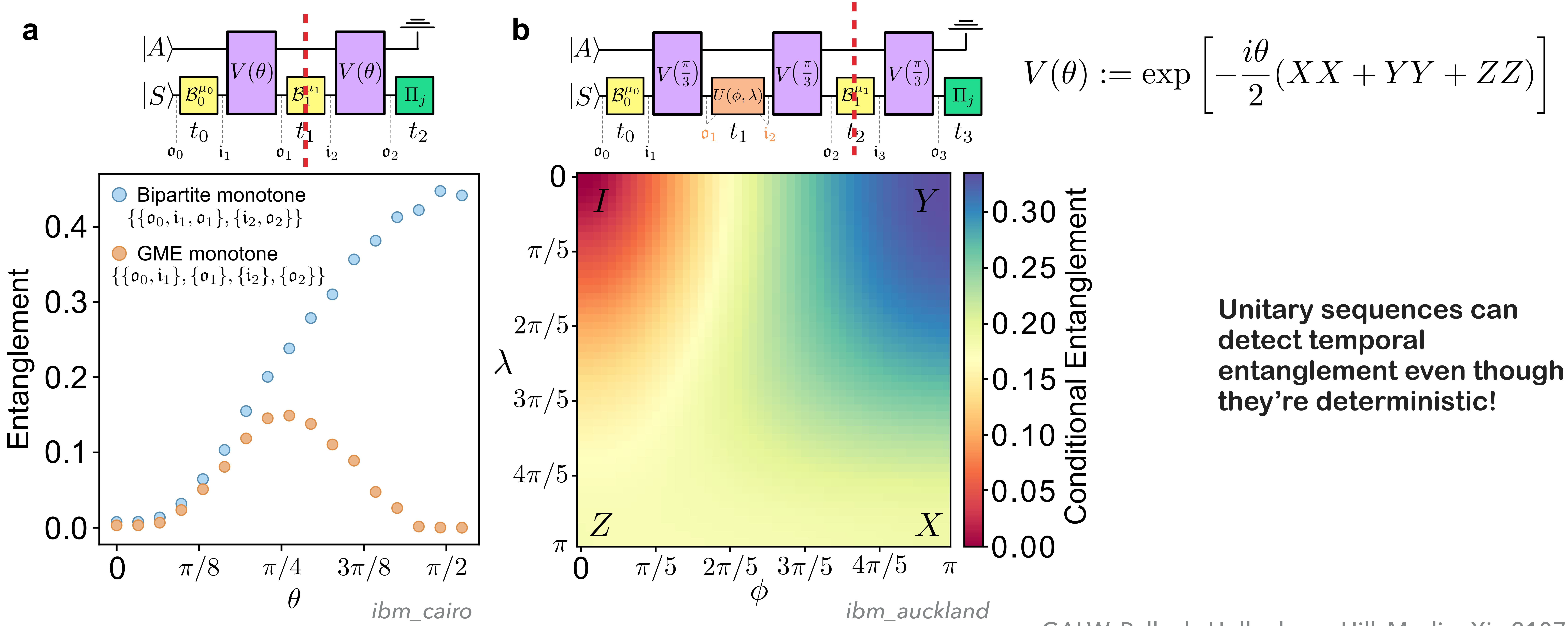

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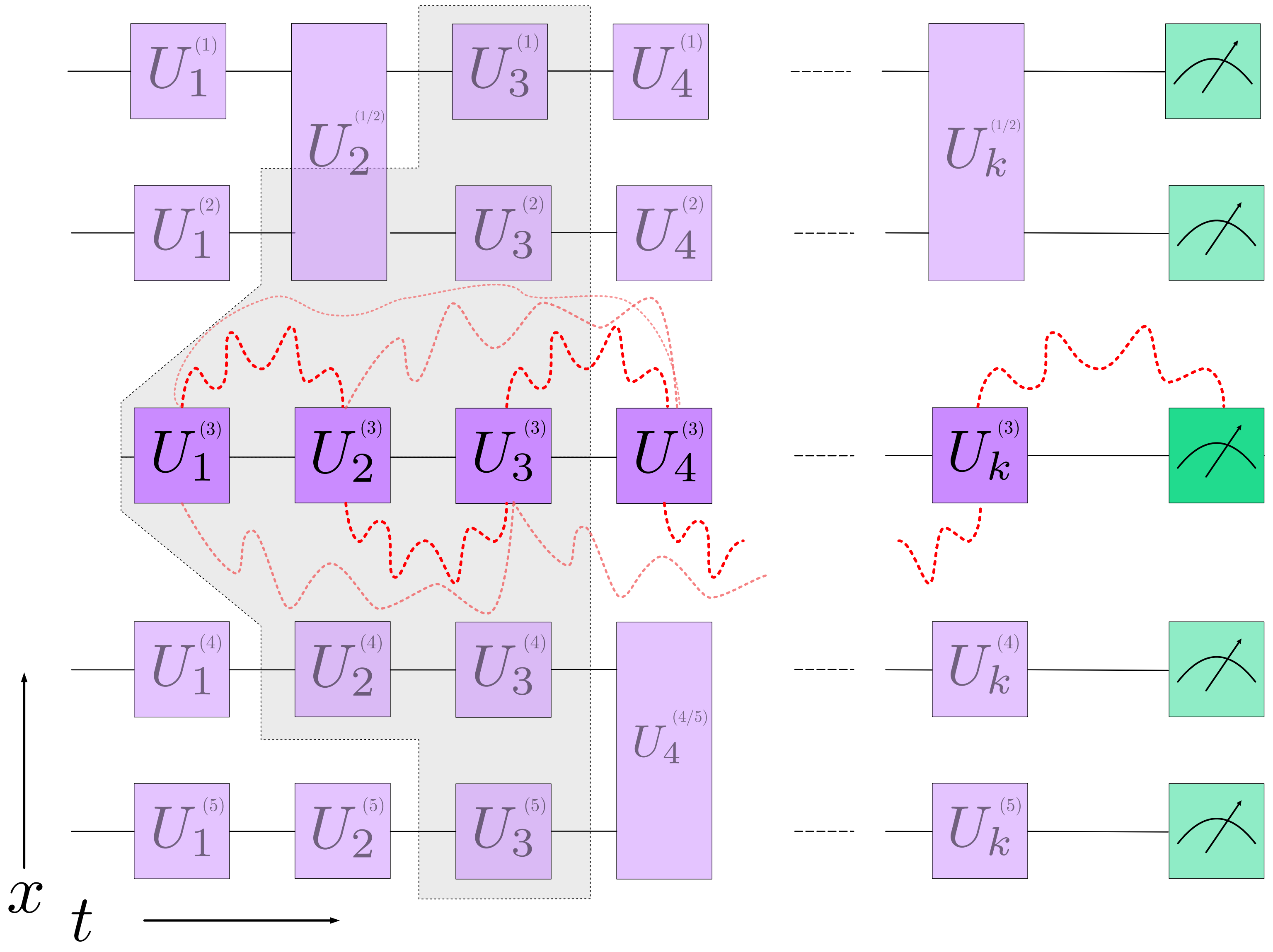
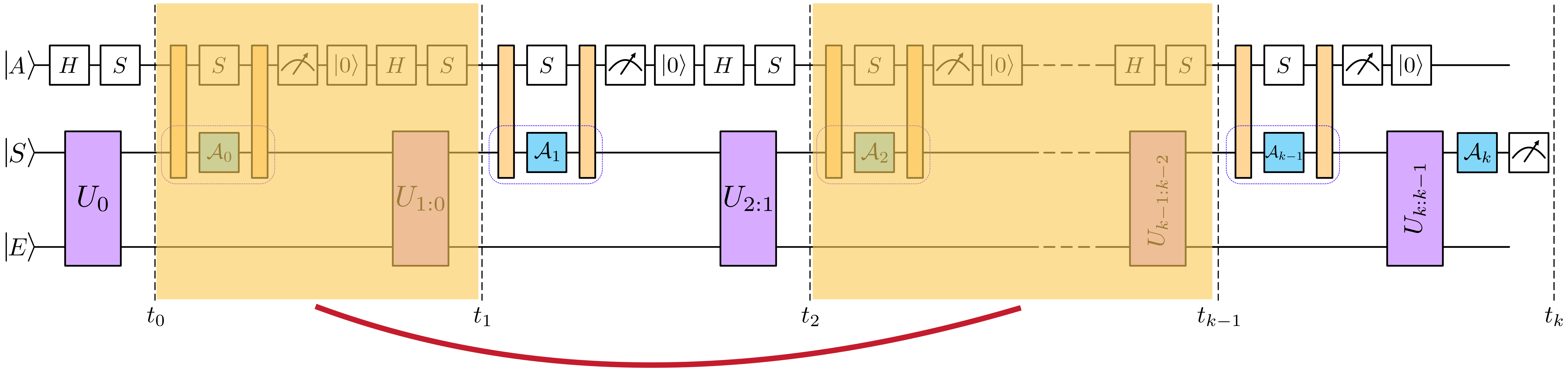
Diagnosing Correlated Noise





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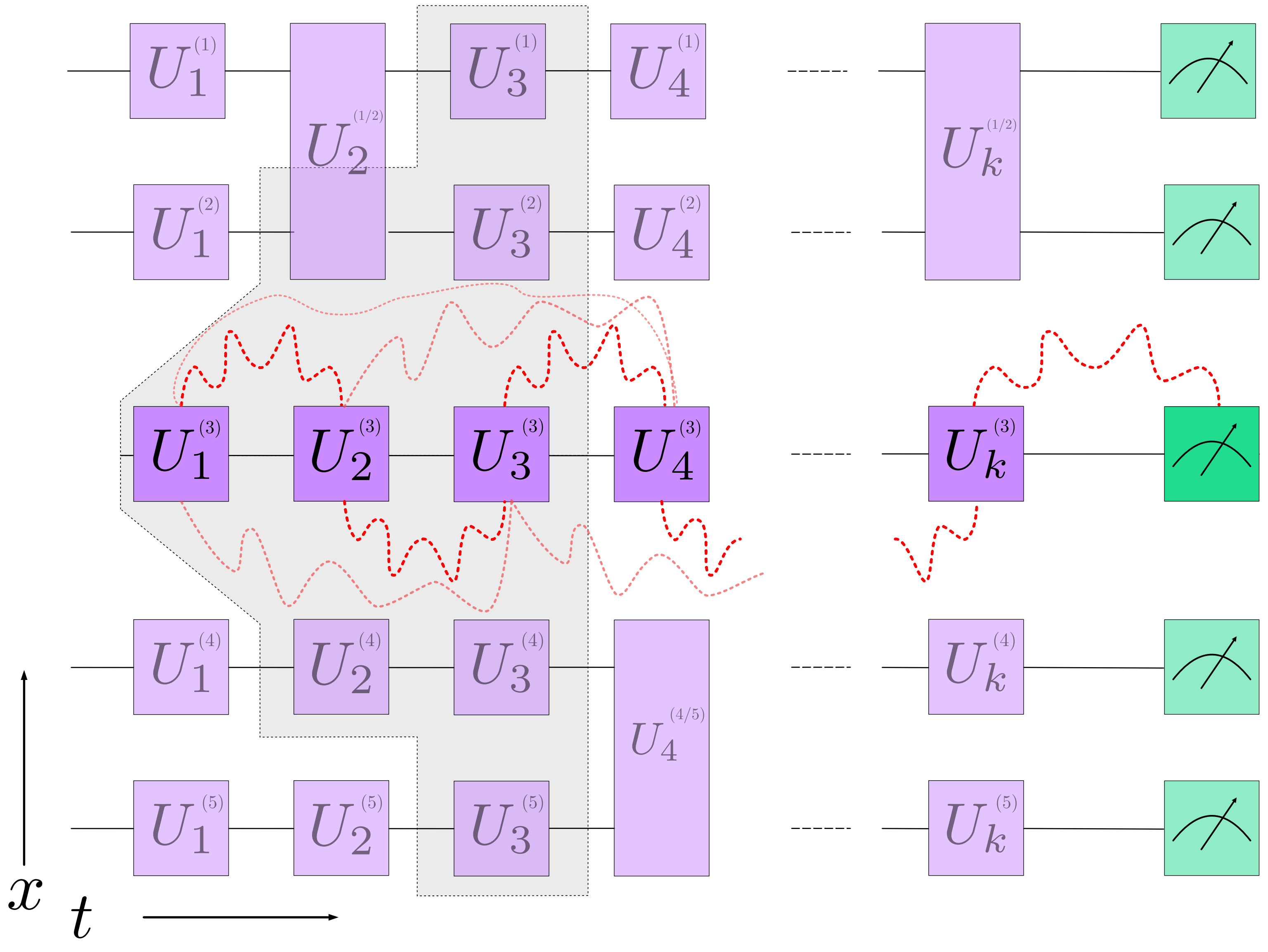
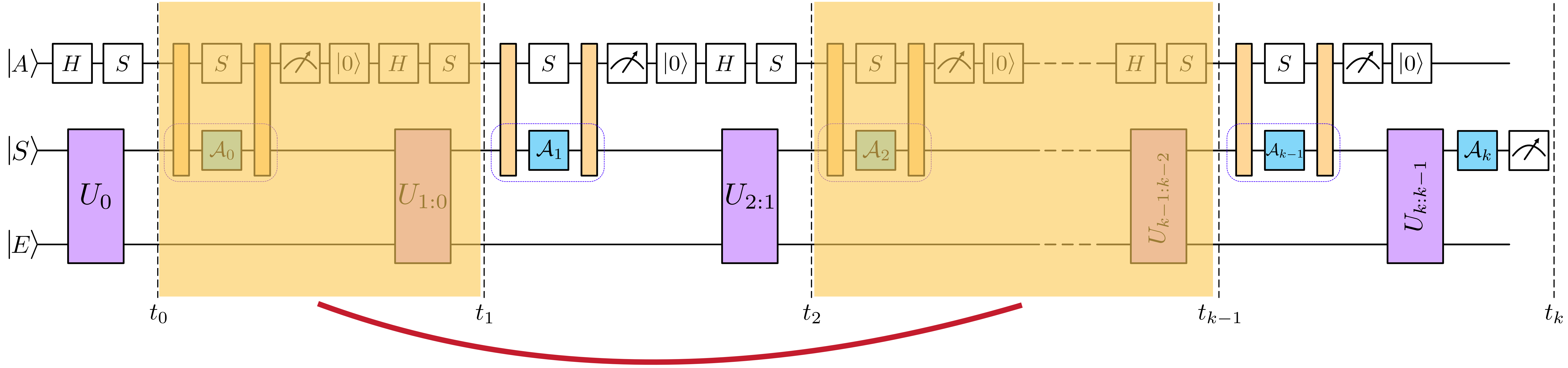
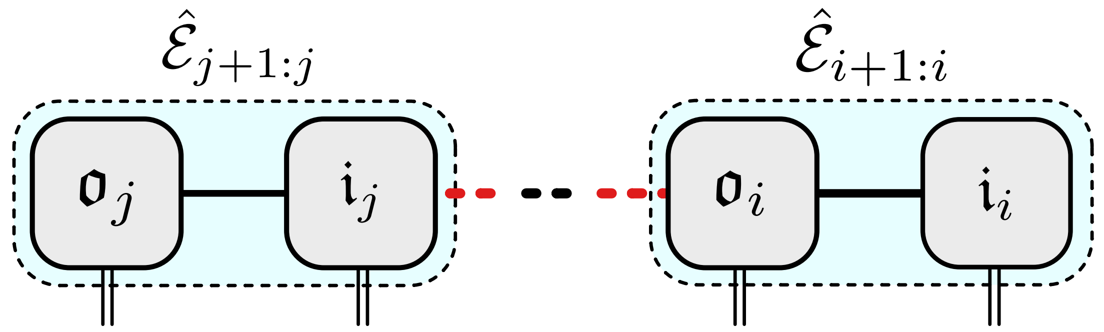
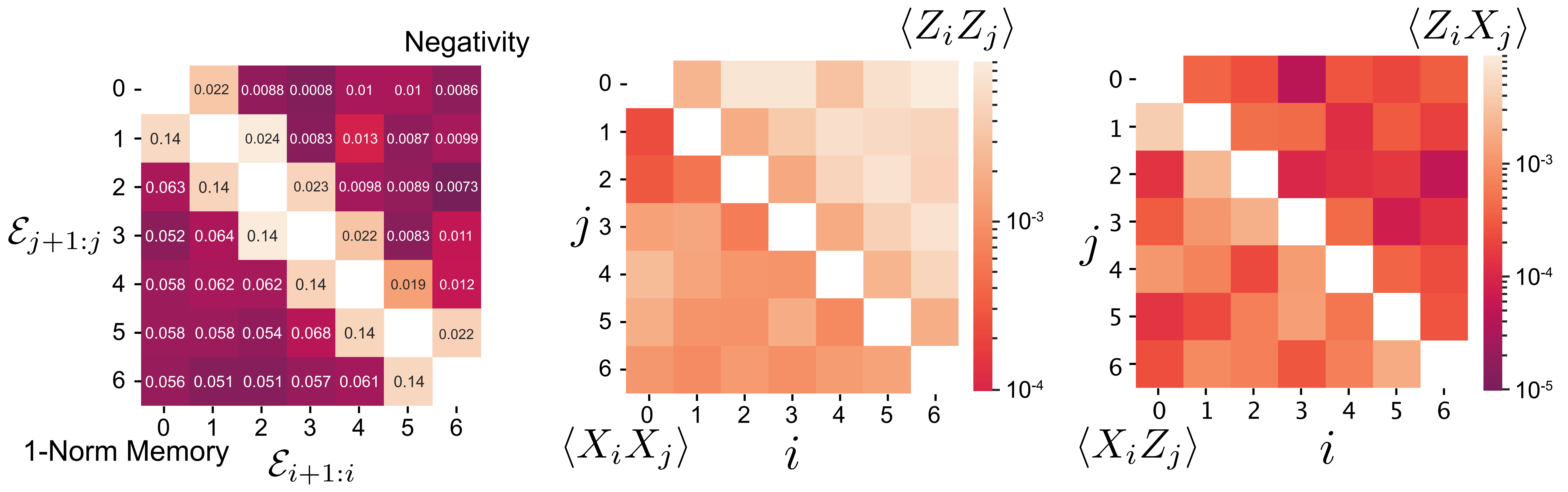




- Employ classical shadows to get the 2-map/4-body process marginals





Real noise is genuinely complex across time

GALW, Pollock, Hollenberg, Hill, Modi arXiv:2107.13934

Open Simulation: Learning Large non-Markovian Processes

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- Applied to simulating OQS, everything you can learn about a process
- Sampling complexity beyond classical simulability
- Using randomised control operations, we reconstructed the matrix product operator (MPO) representation of a 20-step process

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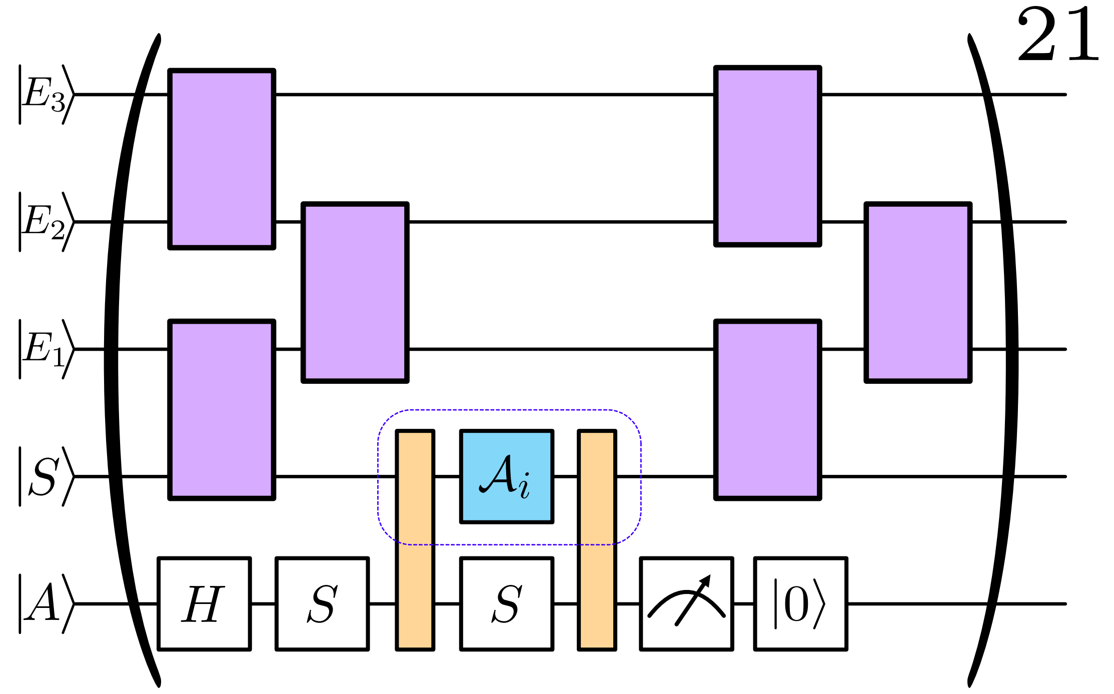

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  Aloisio et al. arXiv:2209.10870
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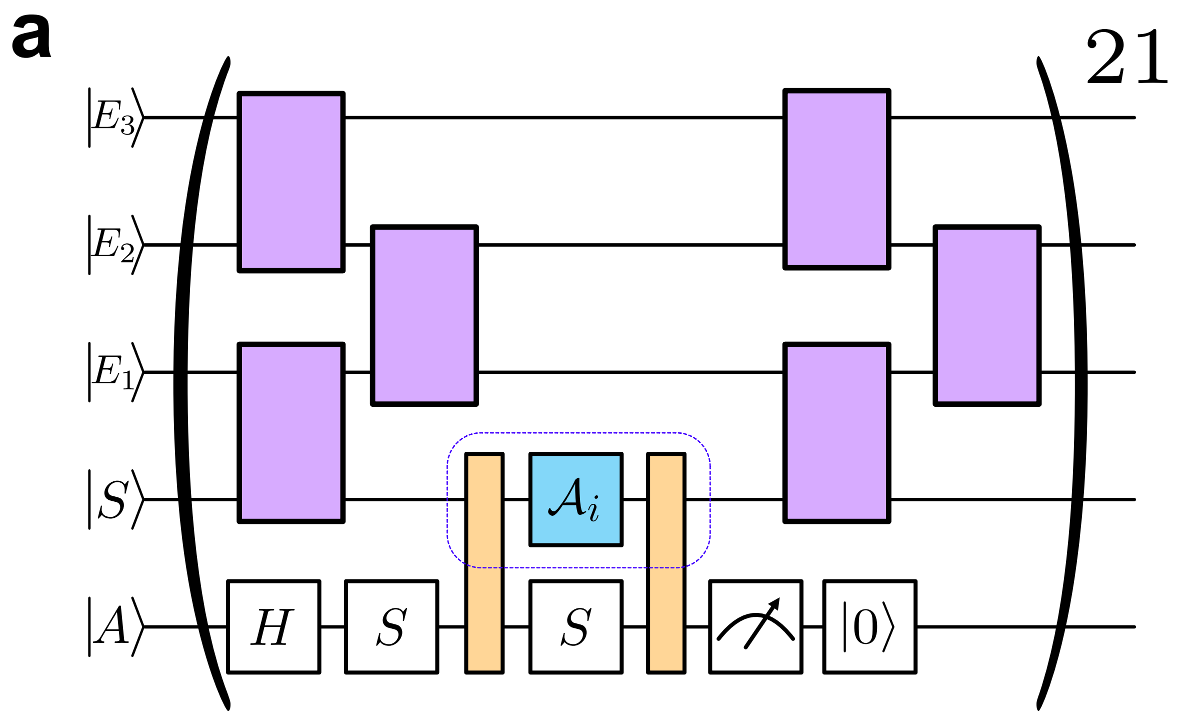

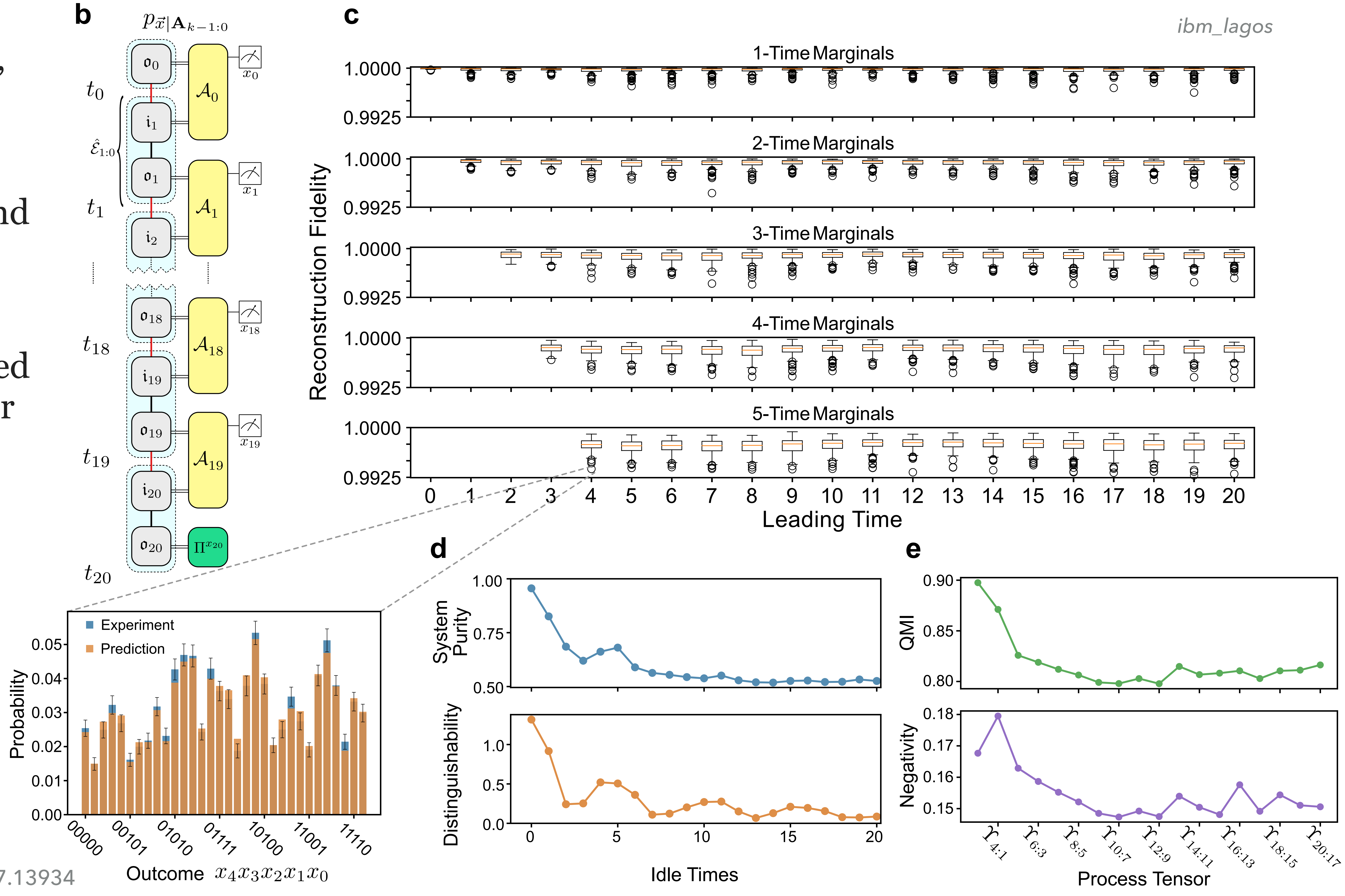

GALW, Pollock, Hollenberg, Hill, Modi arXiv:2107.13934

Summary

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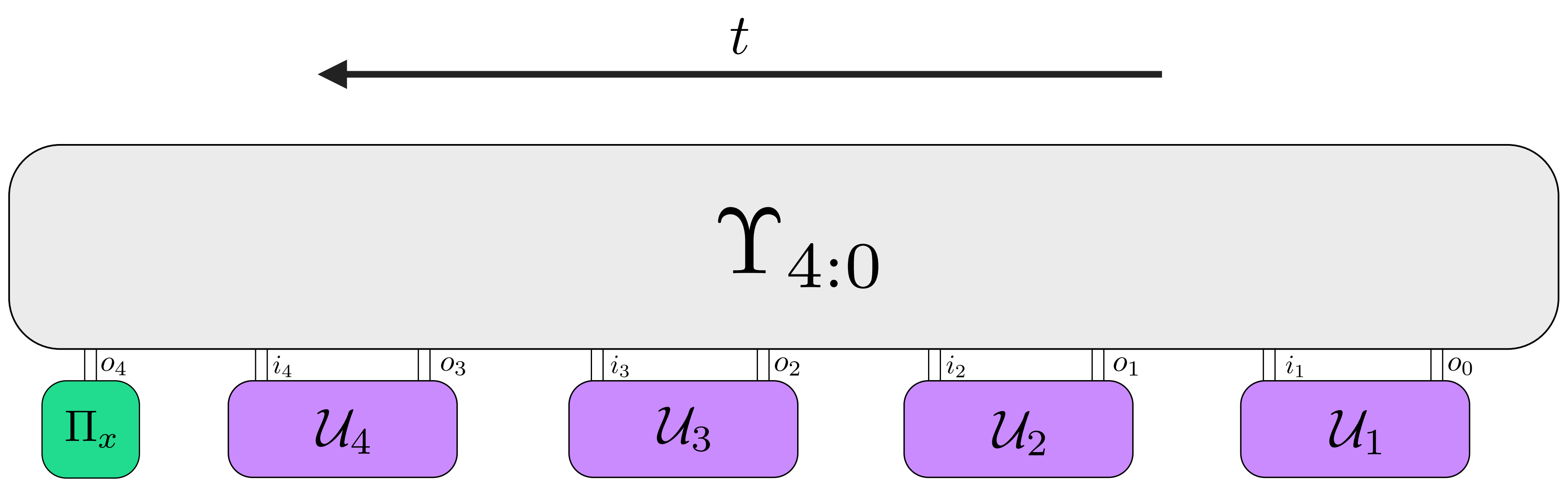
- Quantum stochastic processes are the most general description of open quantum dynamics
- Generalised CJI shows how many-time physics is as rich as many-body physics
- We develop ‘process tensor tomography’ and its variants to capture different aspects of the many-time physics
- We make it scalable, deal with device limitations, and self-consistent
- Estimate contains operational meanings about dynamics
- Demonstrated tomography and applications on IBM Quantum devices

Thanks for Listening!
Any Questions?

Demonstration – Nature Communications 11 (1), 6301 (2020)
Generalised QPT – PRX Quantum **3**, 020344 (2022)
Many-time physics – arXiv:2107.13934
Complexity of OQS – arXiv:2209.10870
Filtering crosstalk with shadows – arXiv:2210.15333

Conditional Markov Order

Conditional Markov Order



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$3 \times 10^k = 30\,000$

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- Markov order: classically, averaging over all but previous l time steps — same statistics at the present
- No generic extension to quantum
- Instead, (approximate) *conditional* Markov order

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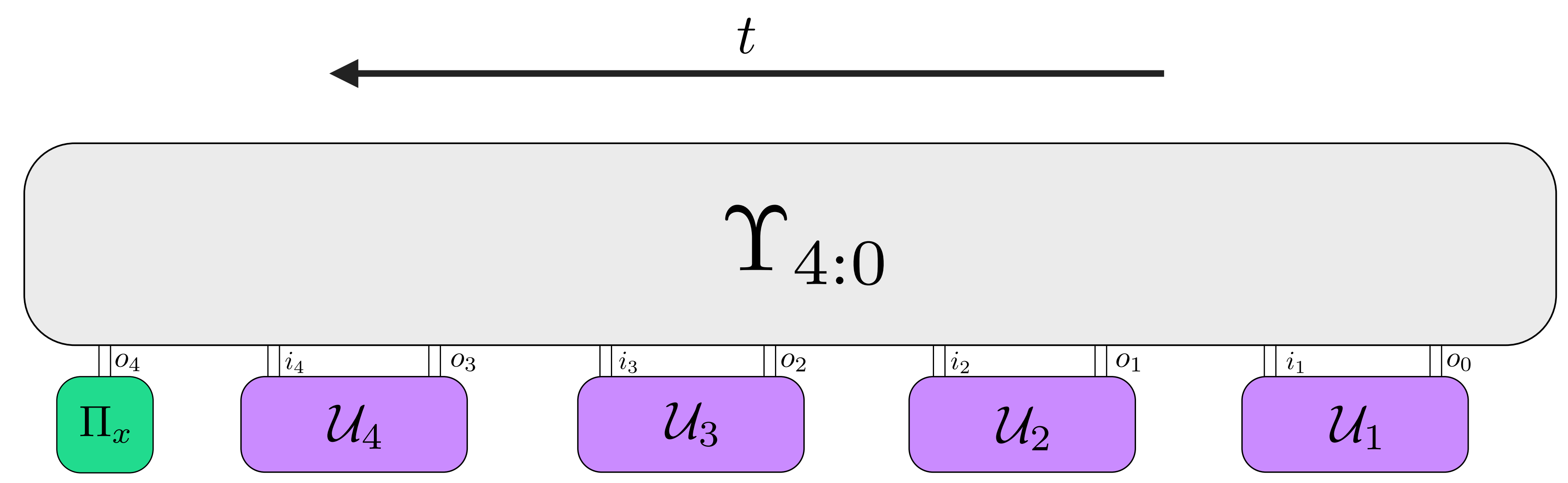

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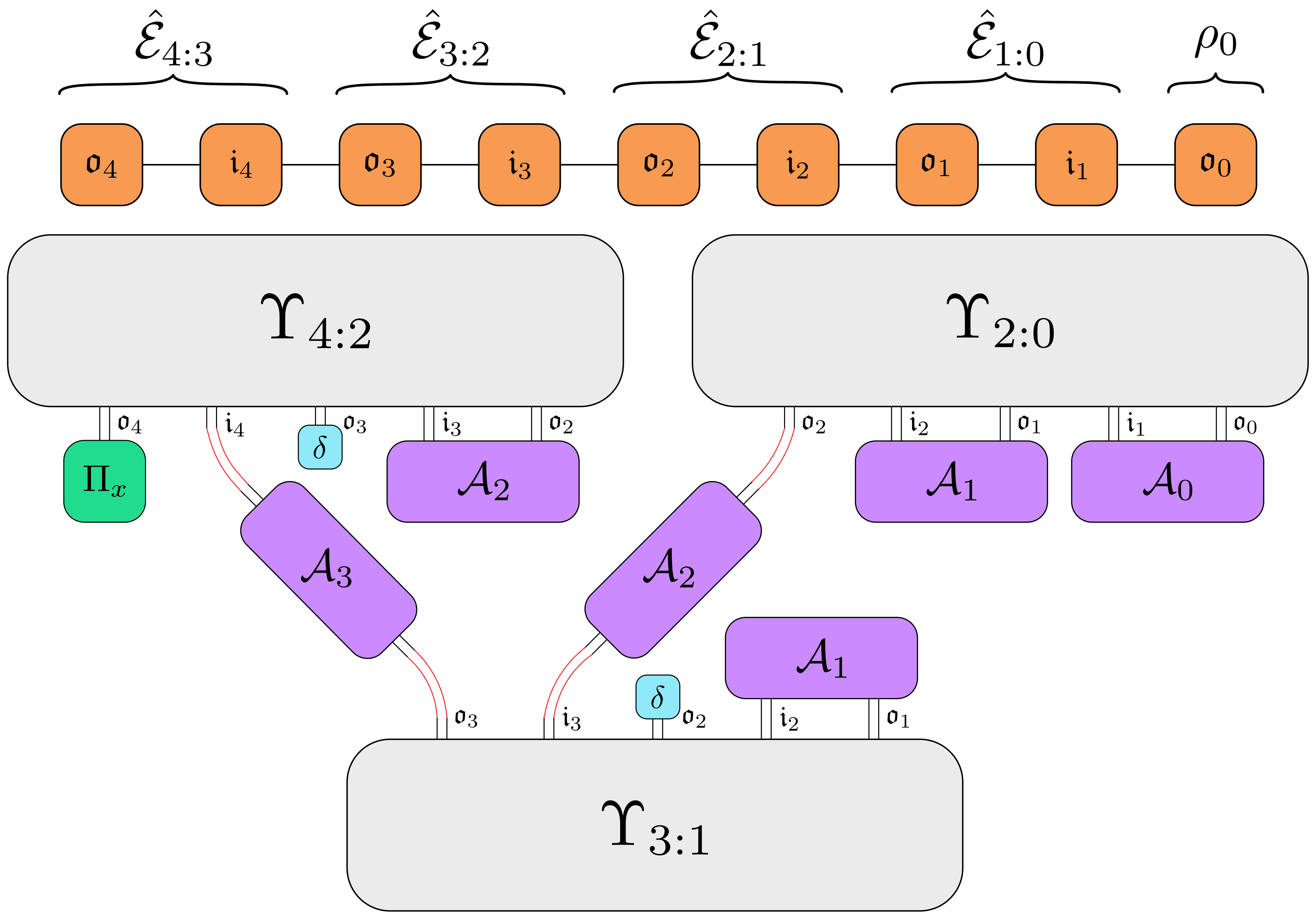

$k = 4,\ \ell = 2$

$(k - \ell + 1) \times 3 \times 10^\ell = 900$

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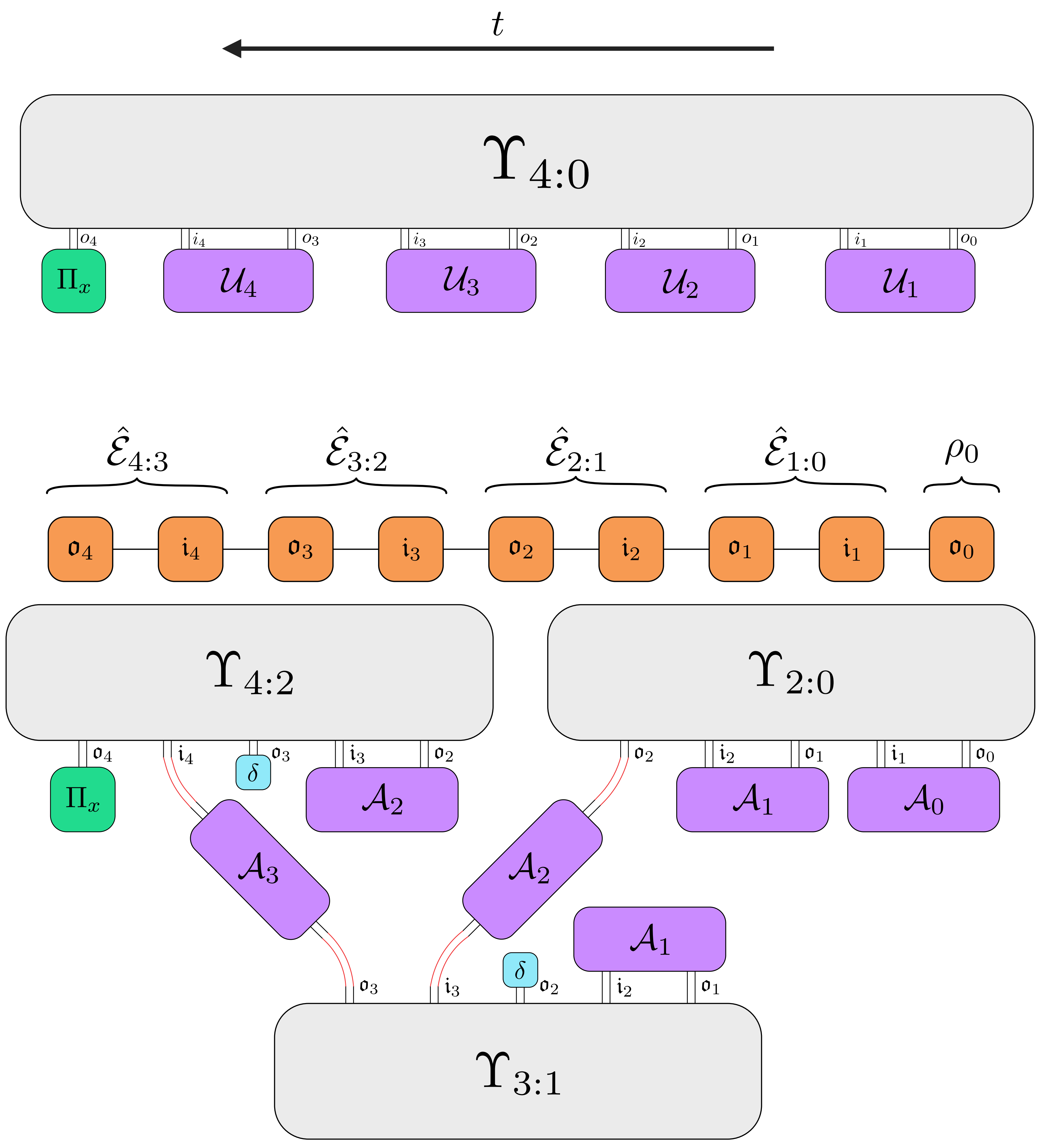
Conditional Markov Order



$k = 4$

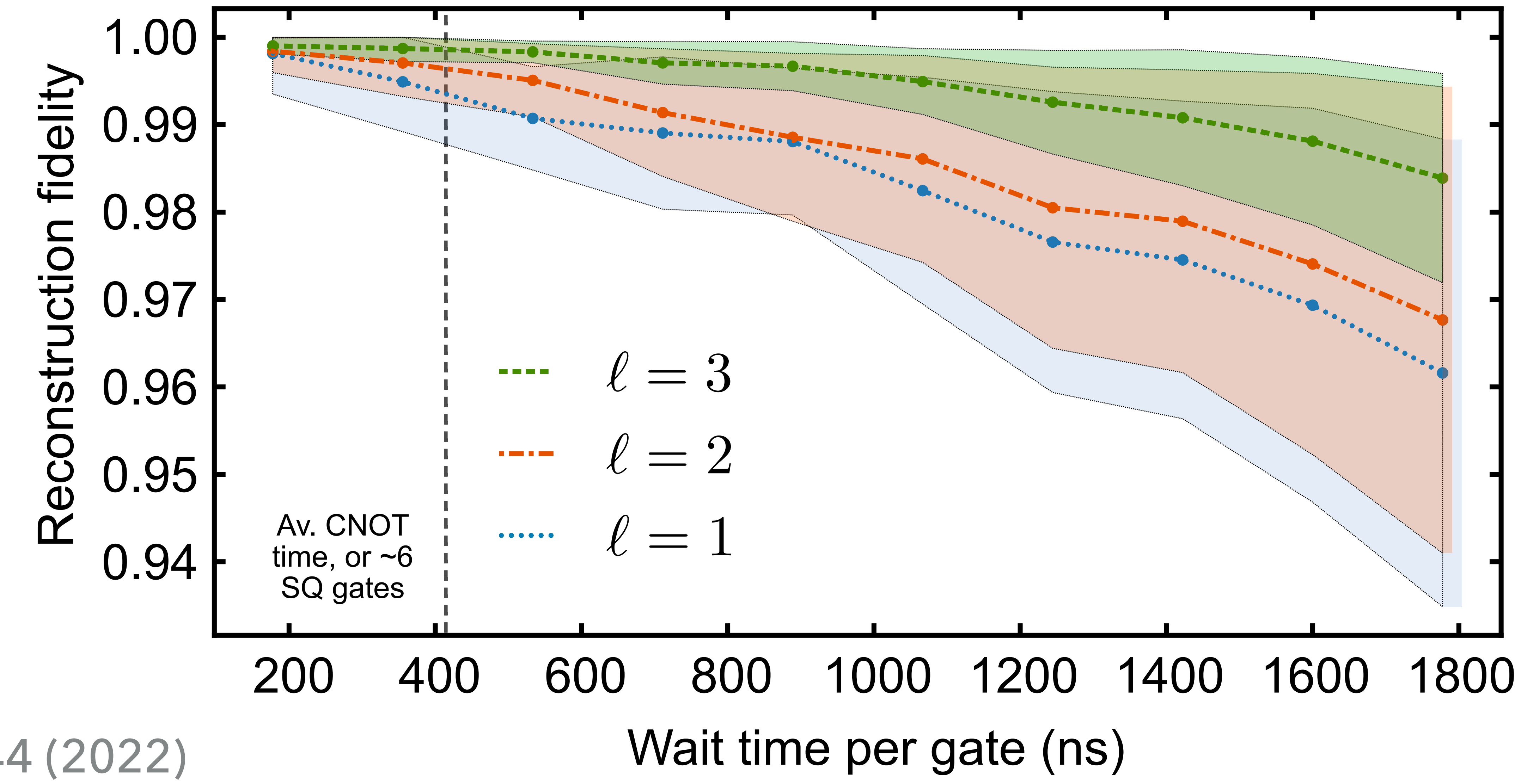
$3 \times 10^k = 30\,000$

$k = 4, \ \ell = 2$

$(k - \ell + 1) \times 3 \times 10^\ell = 900$

- Full characterisation scales exponentially poorly
- Markov order: classically, averaging over all but previous l time steps — same statistics at the present
- No generic extension to quantum
- Instead, (approximate) *conditional* Markov order

ibmq_guadalupe



PRX Quantum **3**, 020344 (2022)

Application: Noise Reduction

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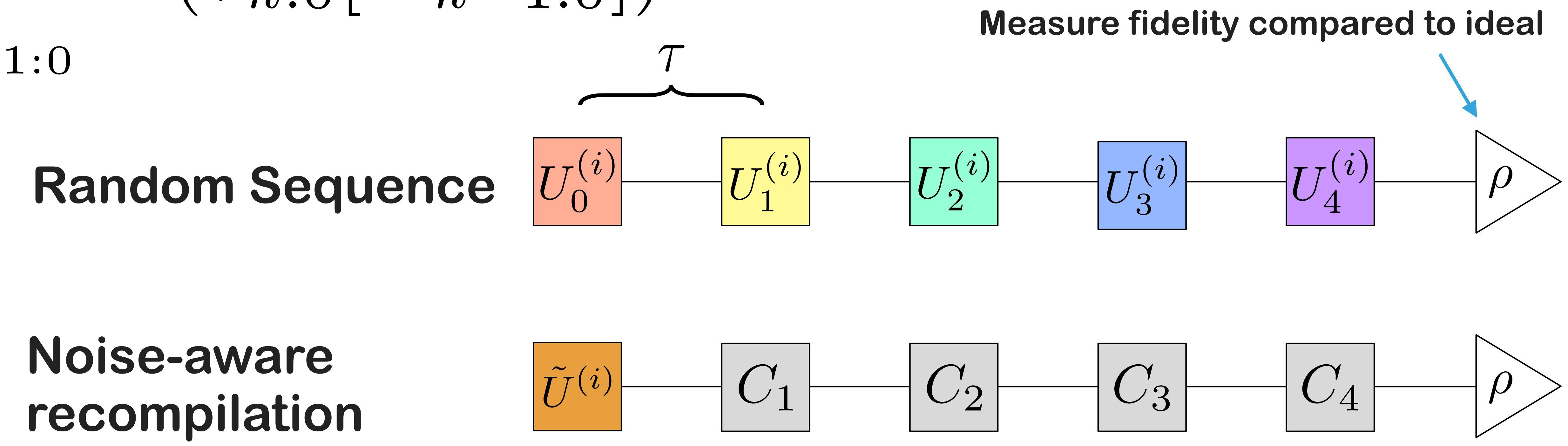

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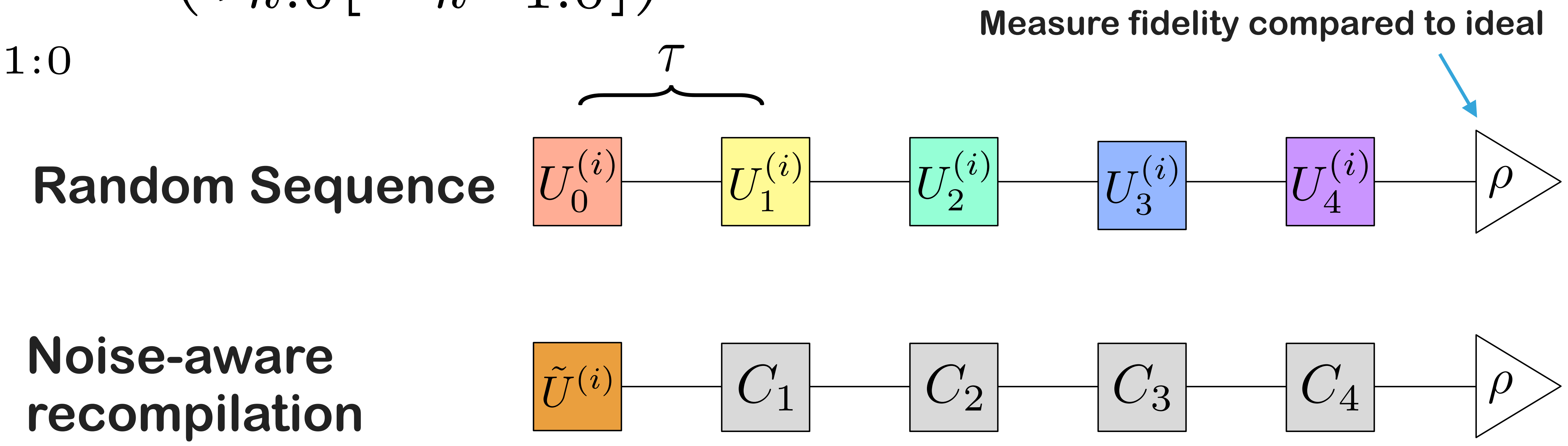

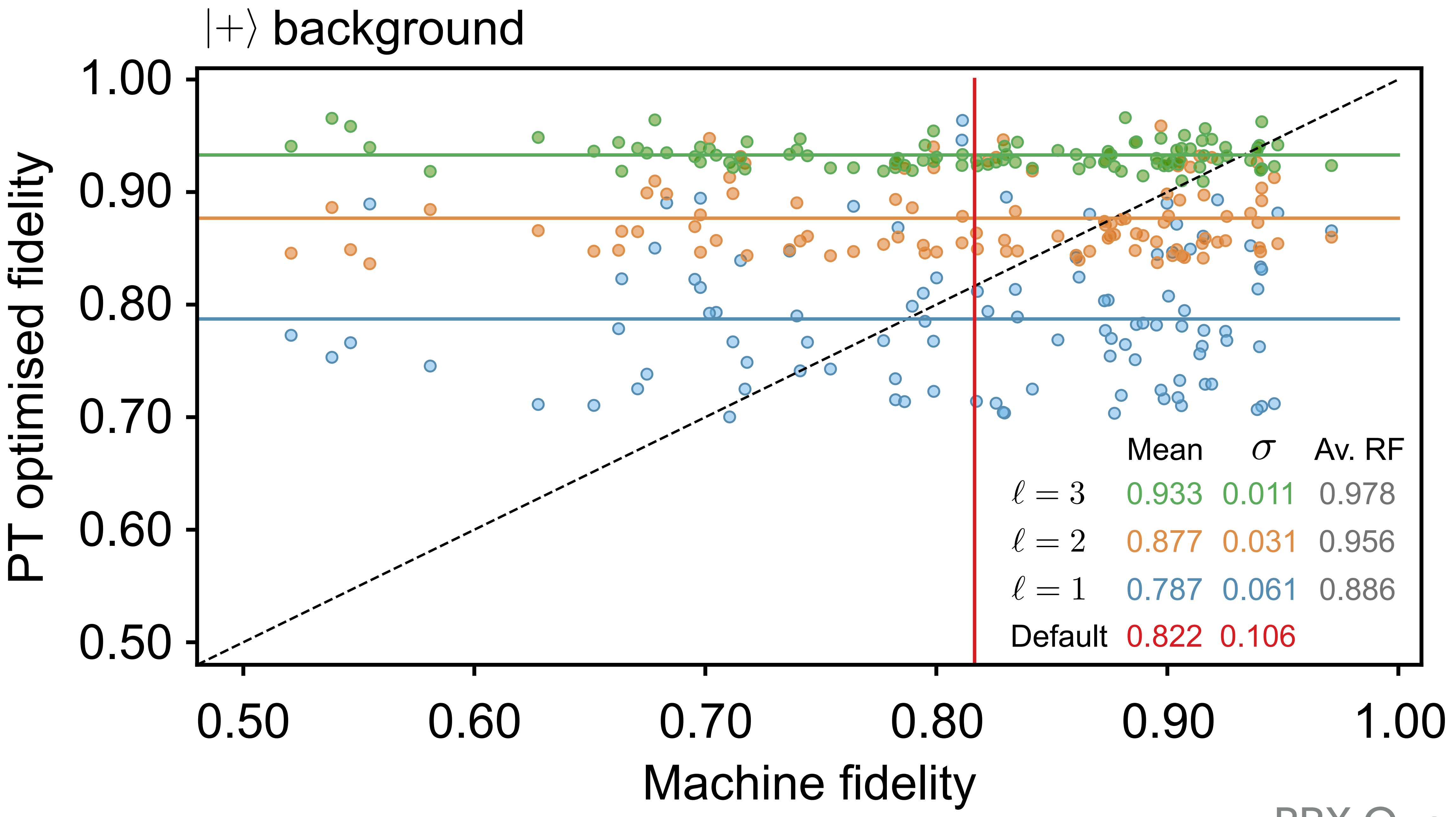

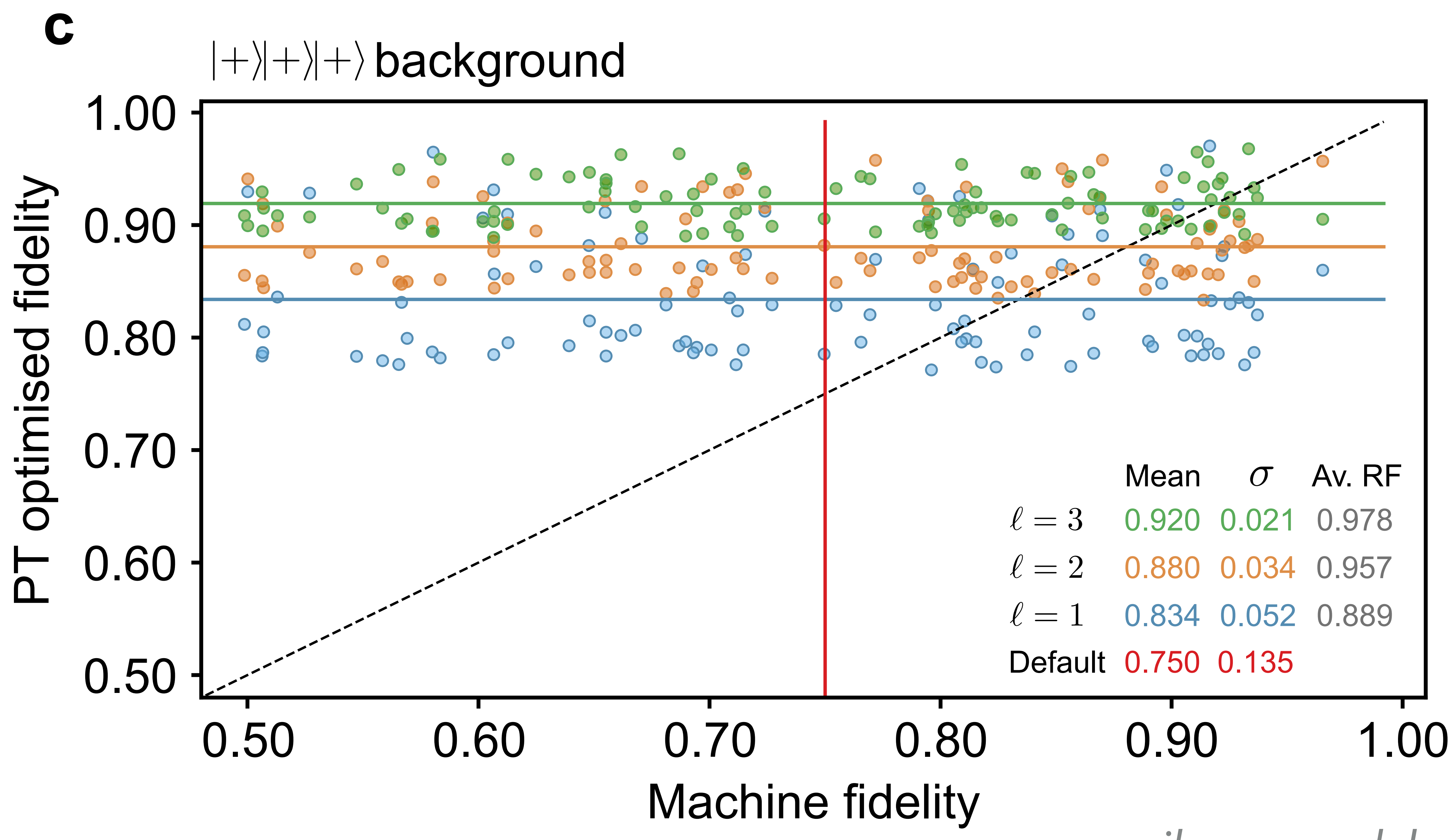

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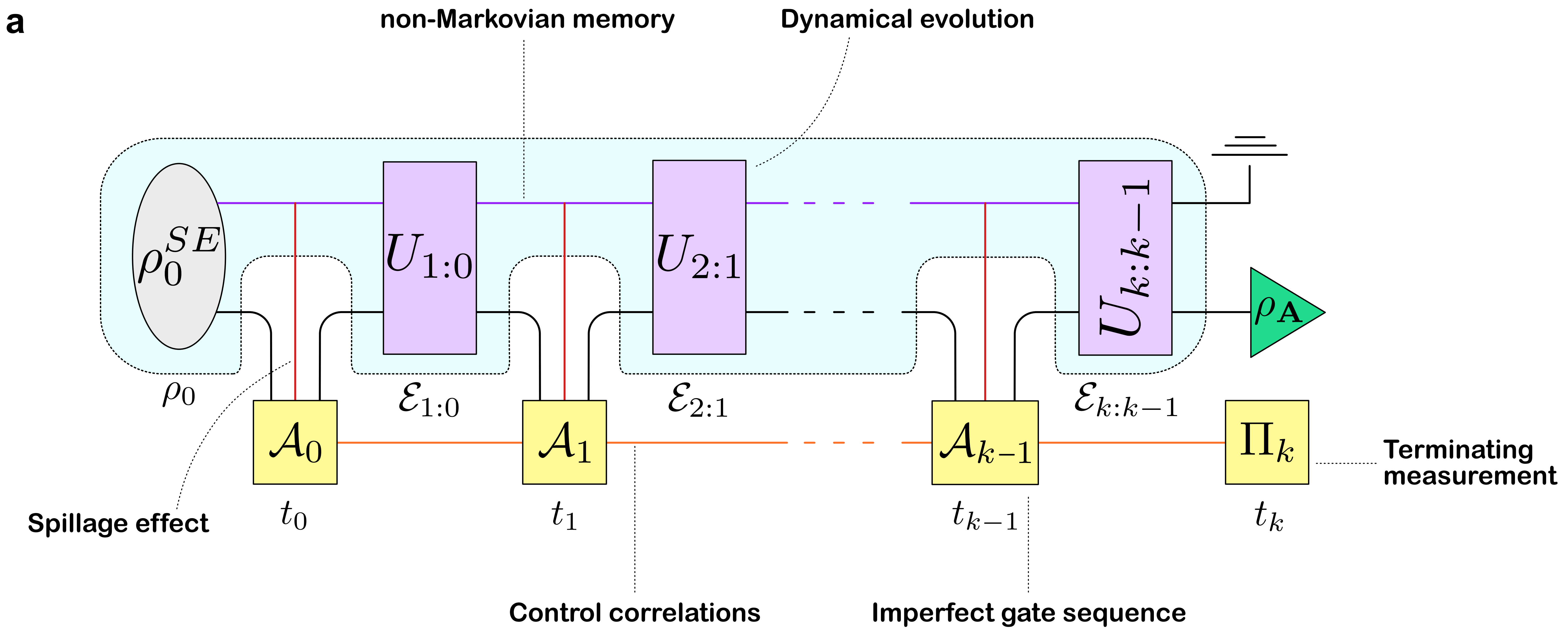
ibmq_guadalupe

Scalability and Self-Consistency

- PTT generically expensive and relies on known controls
- Universally model control process, and interplay of control and process
- Use locally purified tensor networks to model both process and control
- Estimate[1] the process tensors and controls

[1]Torlai et al. arXiv:2006.02424



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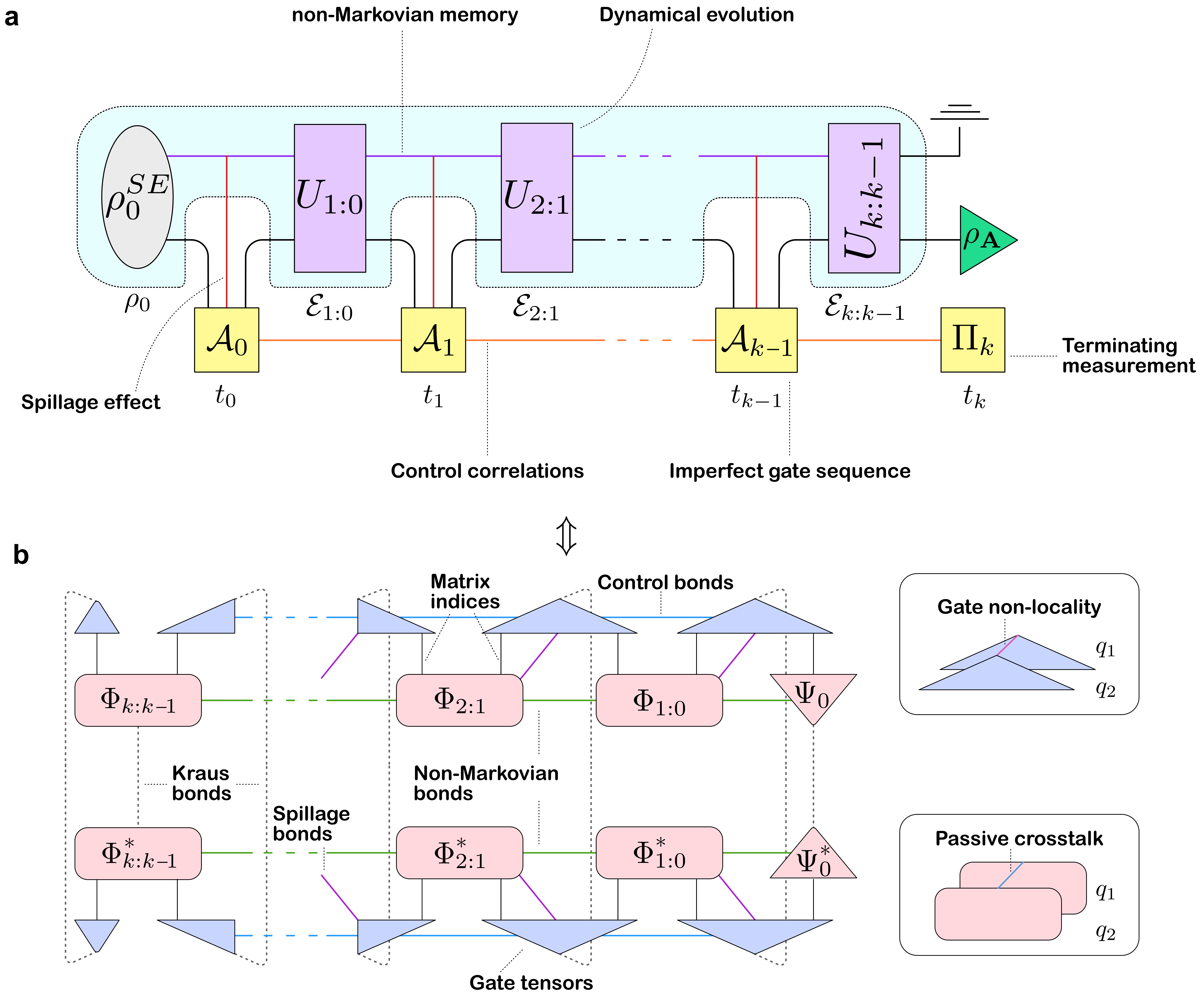


Simultaneously model, e.g.:

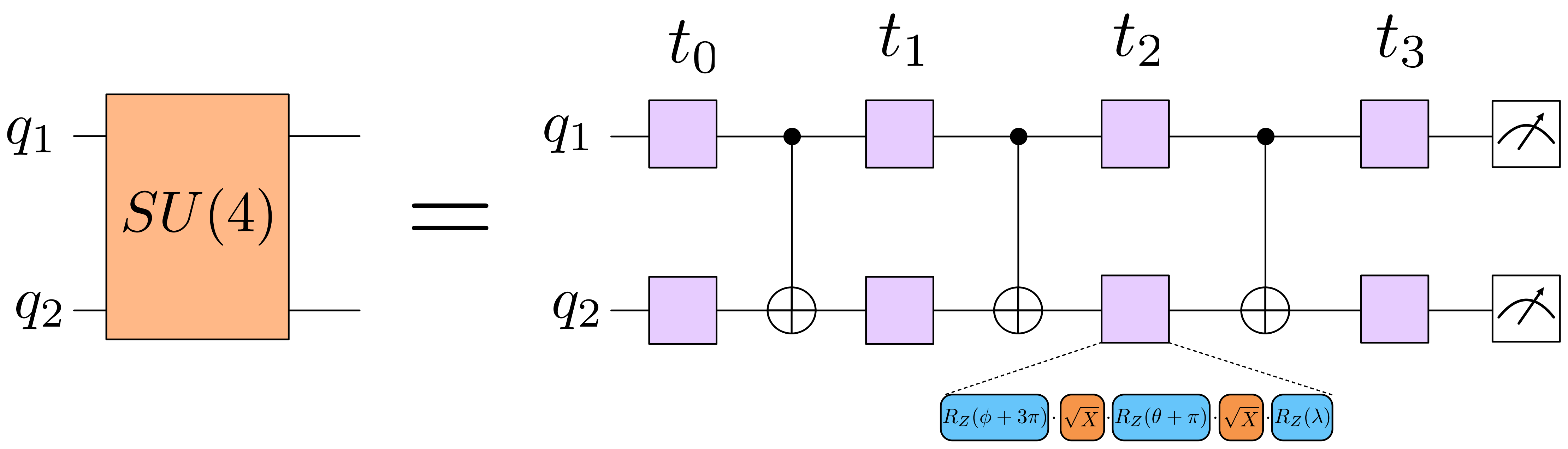
- **imperfect gates**
- **frequency collisions**
- **gate degradation**
- **correlated coherent error**
- **SPAM**
- **correlated instruments — "testers"**
- **(Please suggest more!)**

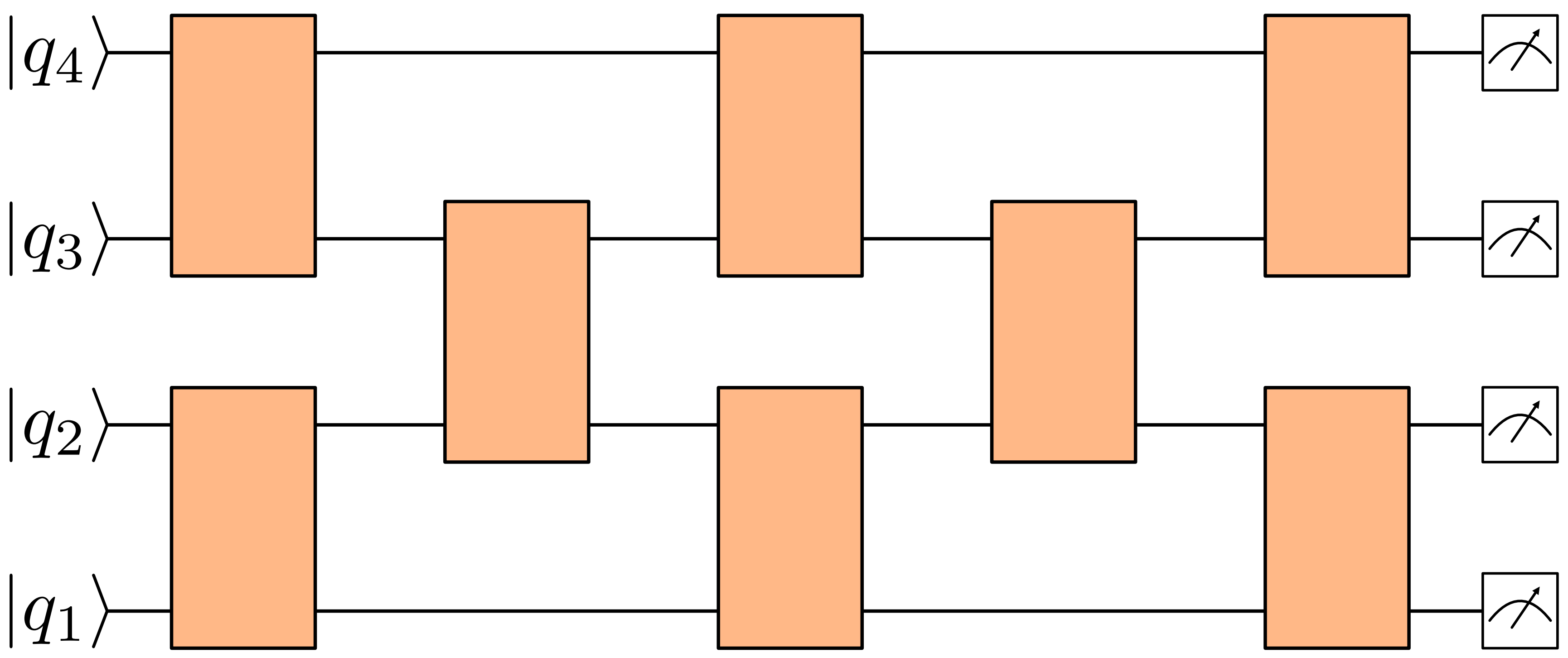
Concrete Example

$$U(\theta, \phi, \lambda) = R_Z(\phi + 3\pi) \cdot \sqrt{X} \cdot R_Z(\theta + \pi) \cdot \sqrt{X} \cdot R_Z(\lambda)$$

- Design around IBM hardware
- e.g. SU4 gate

$$\Upsilon_{k:0} \quad \{\mathbf{A}_{k-1:0}\} \quad \{\Pi_x\}$$





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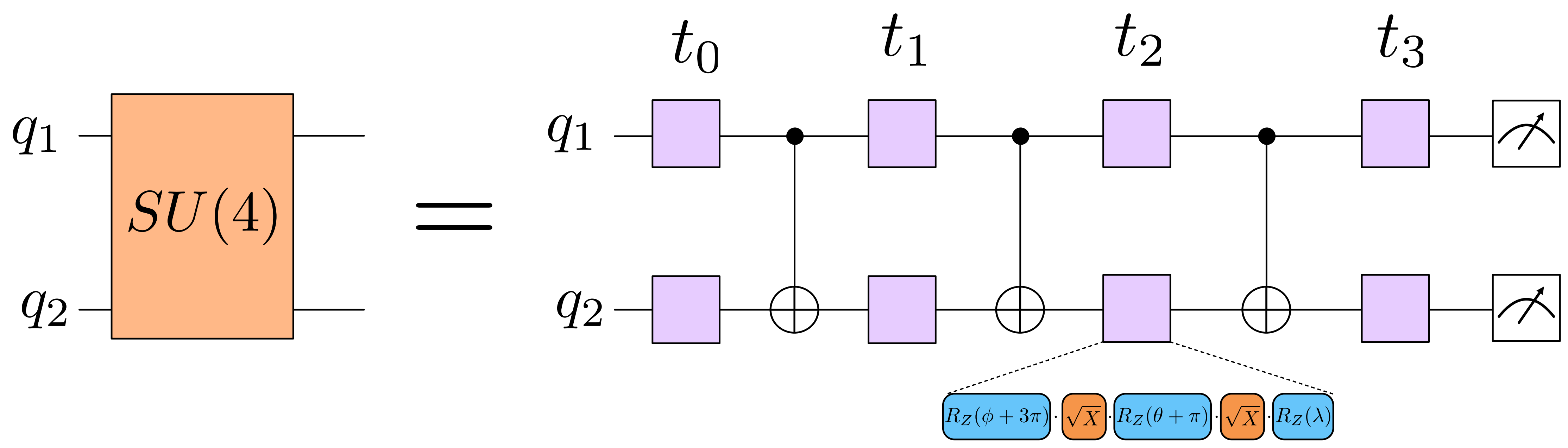

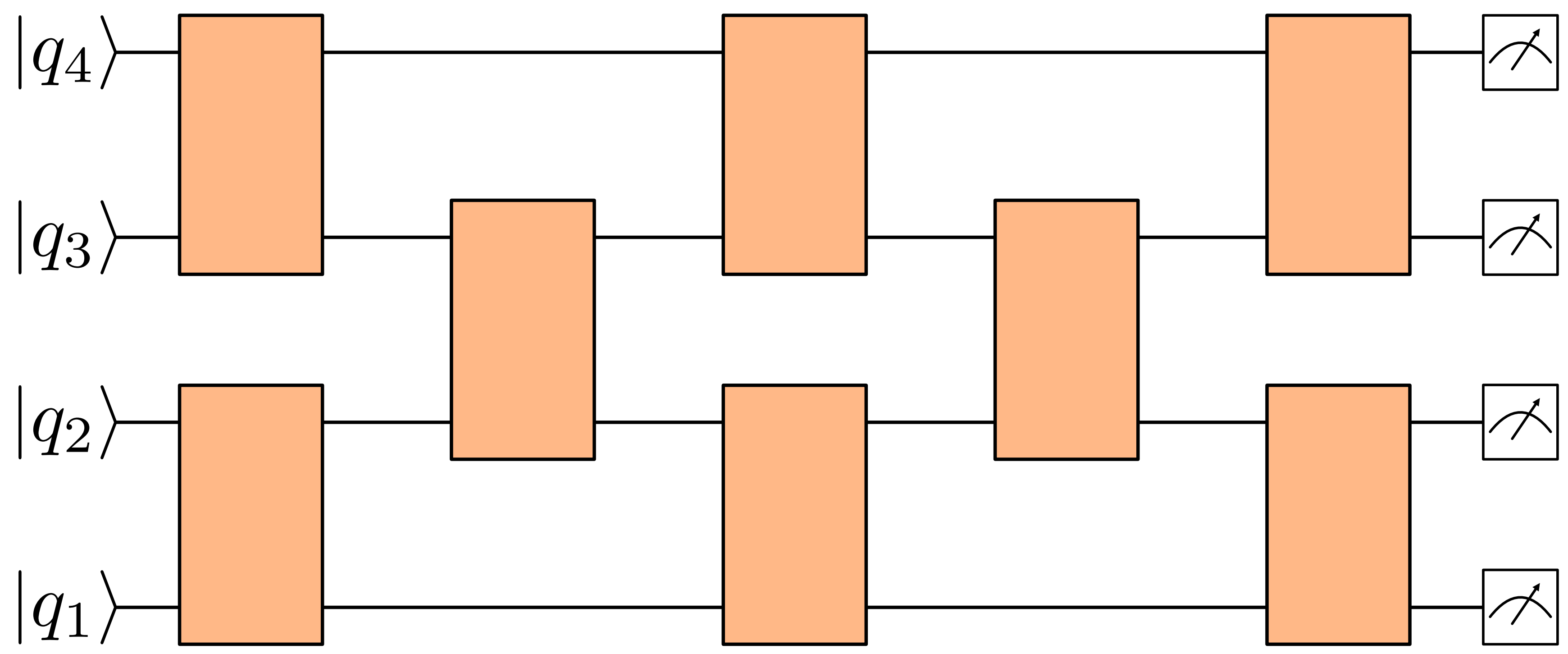

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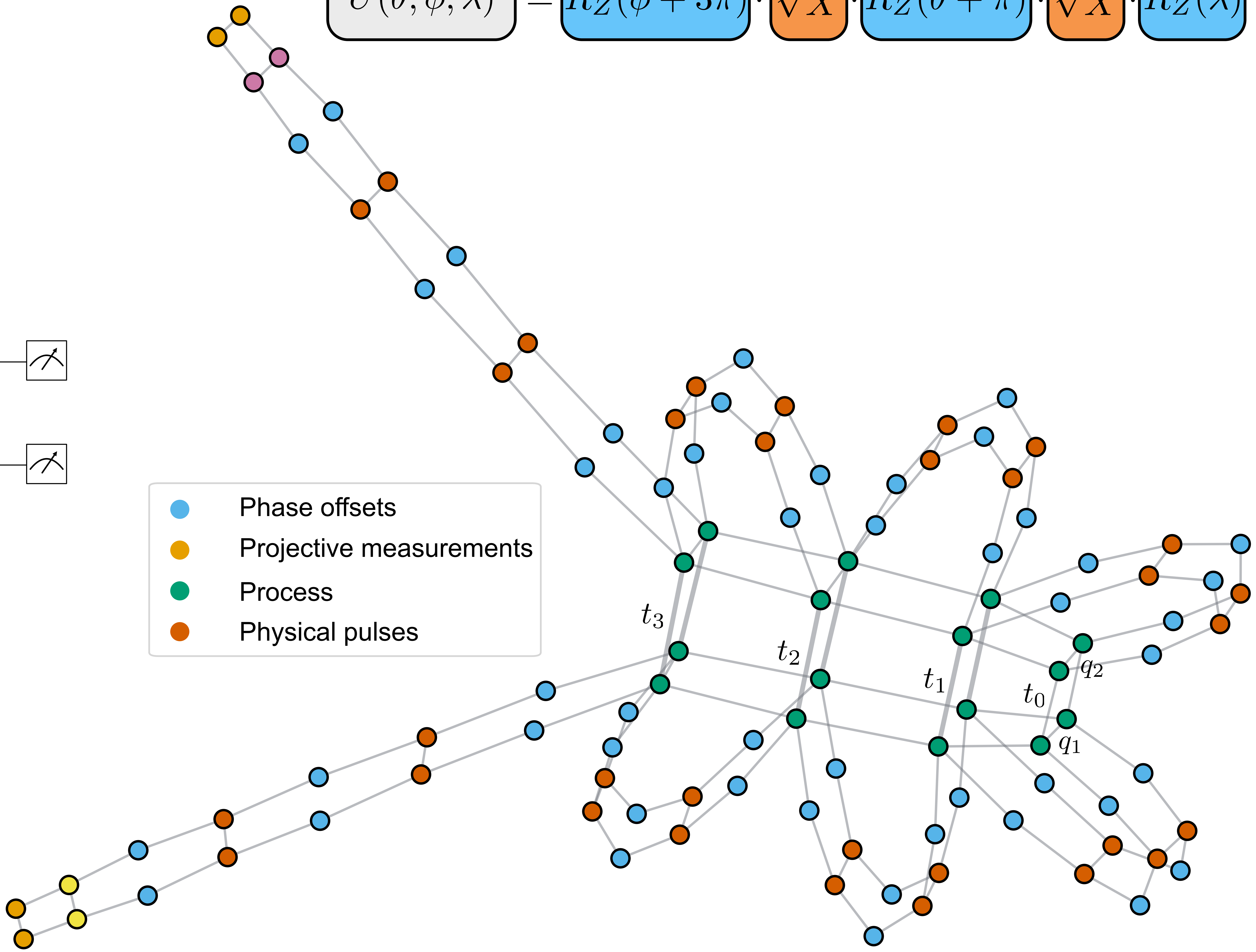
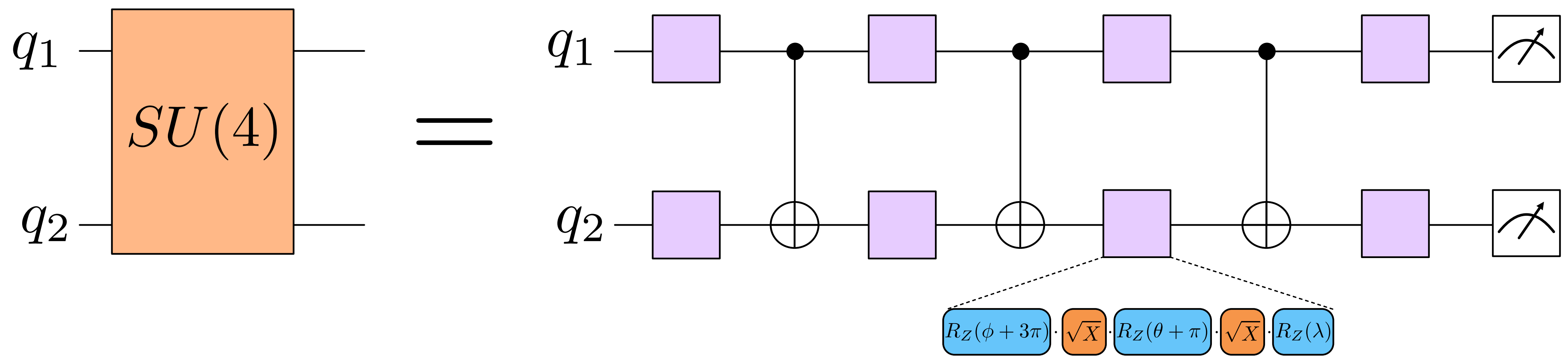
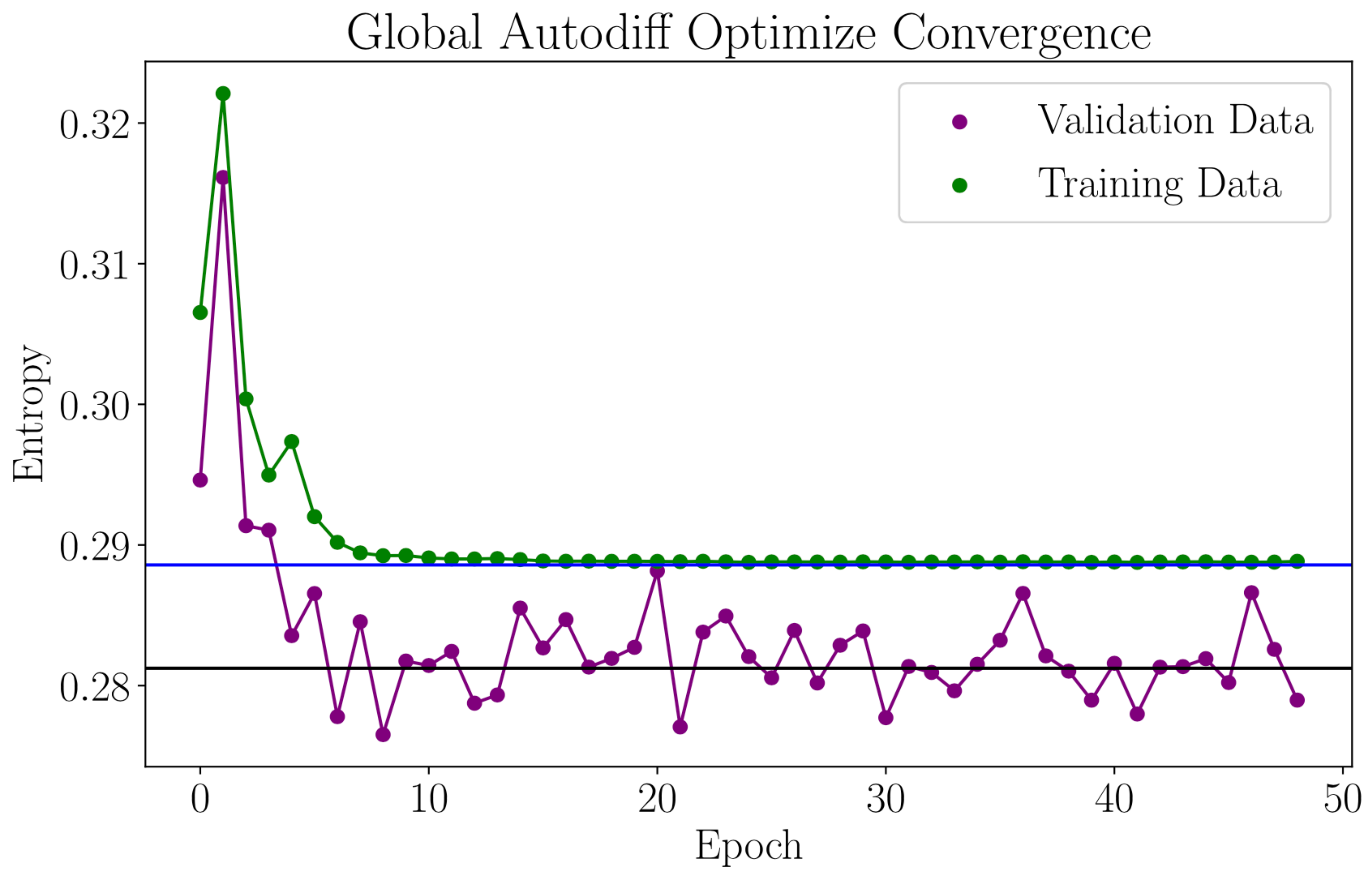


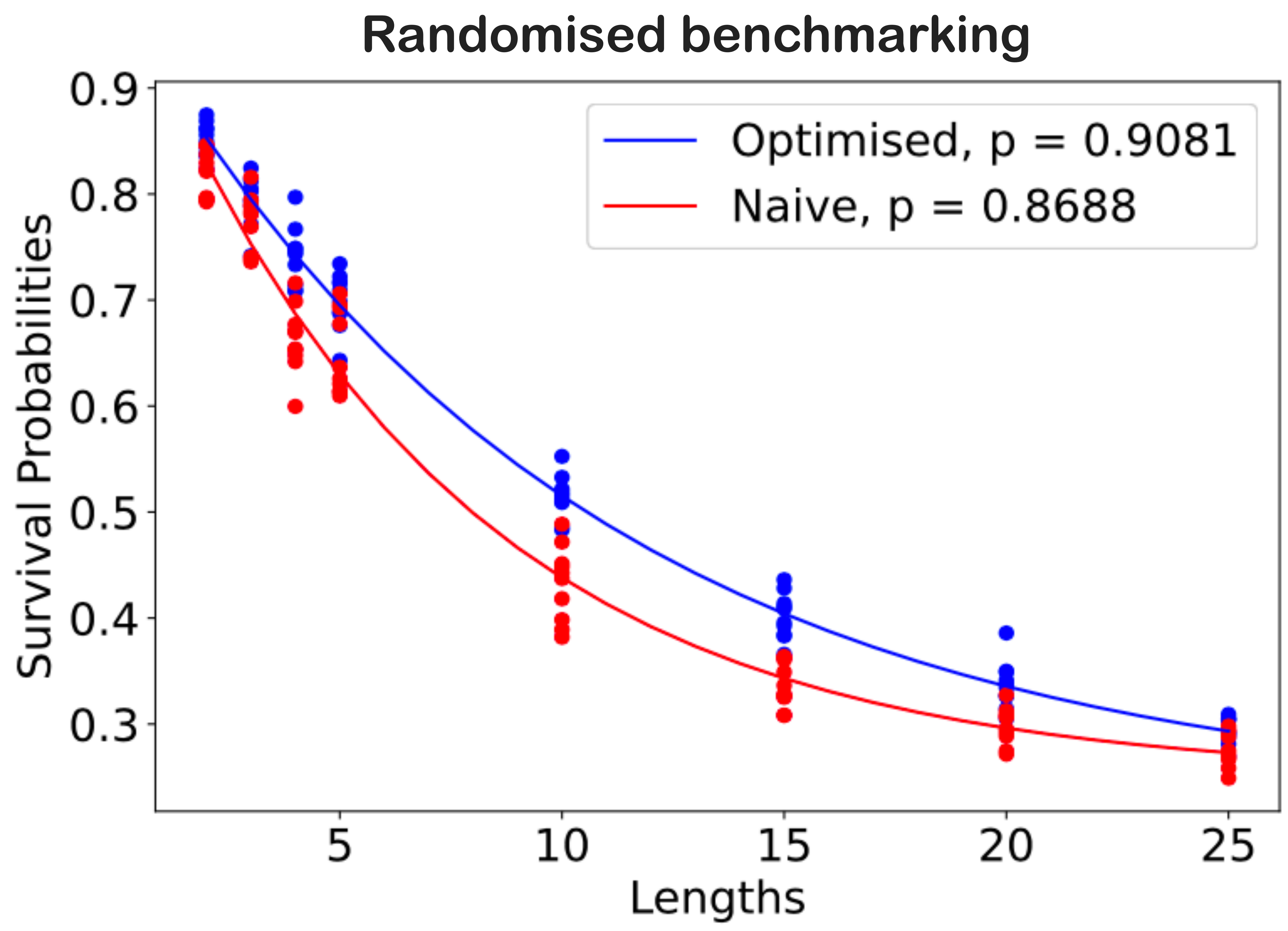
Example Results

One example from a bunch of sets of IBM experiments

2 qubits, 3 steps, ibmq_guadalupe



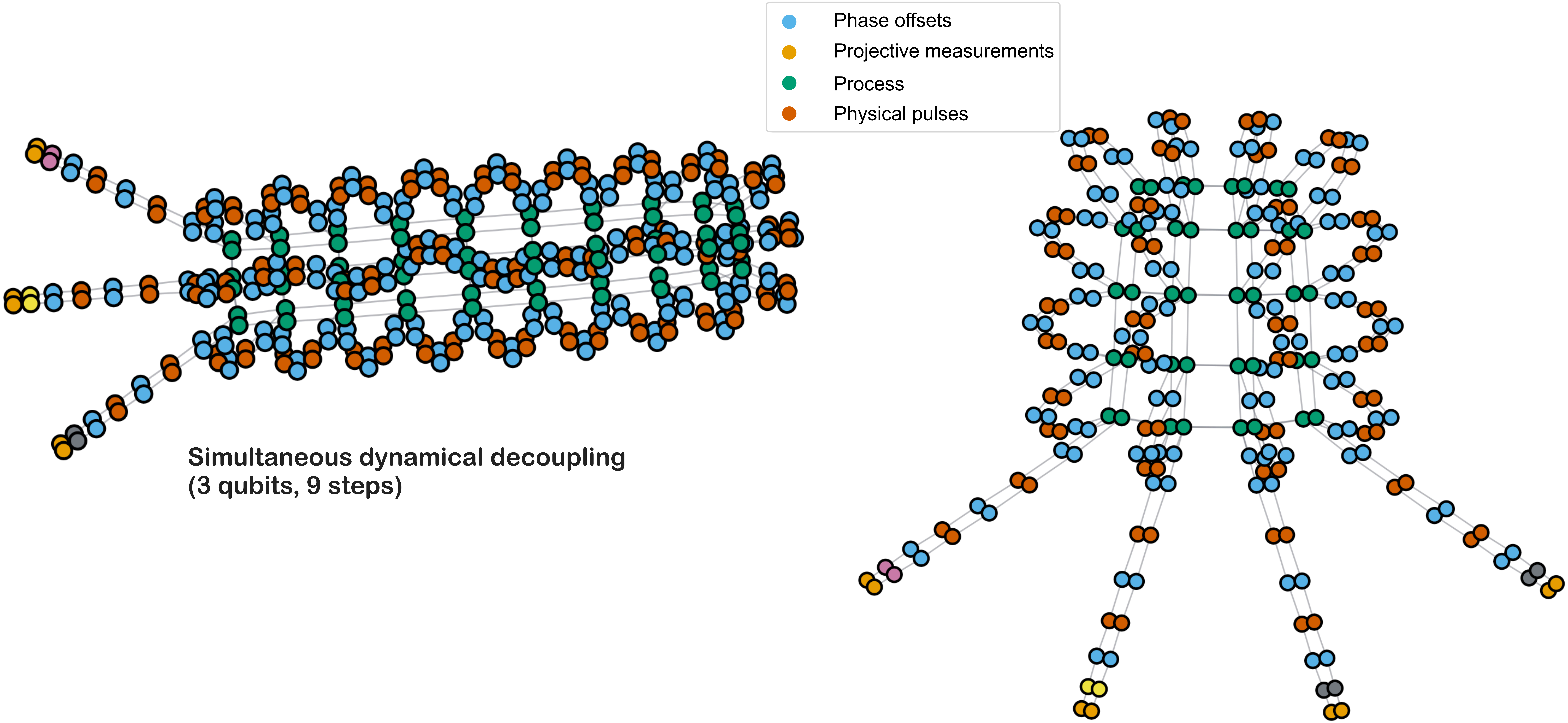




Absolute predictive difference: `minmax=(1.8775463e-06, 0.027240574), mean=0.006607343`

Predictive fidelity: `minmax=(0.9977334332133879, 0.9999972379788054), mean=0.999675008101766`

Can Also Go Bigger and Better



Simultaneous dynamical decoupling
(3 qubits, 9 steps)

Parallel two-qubit gates (4 qubits, 3 steps)

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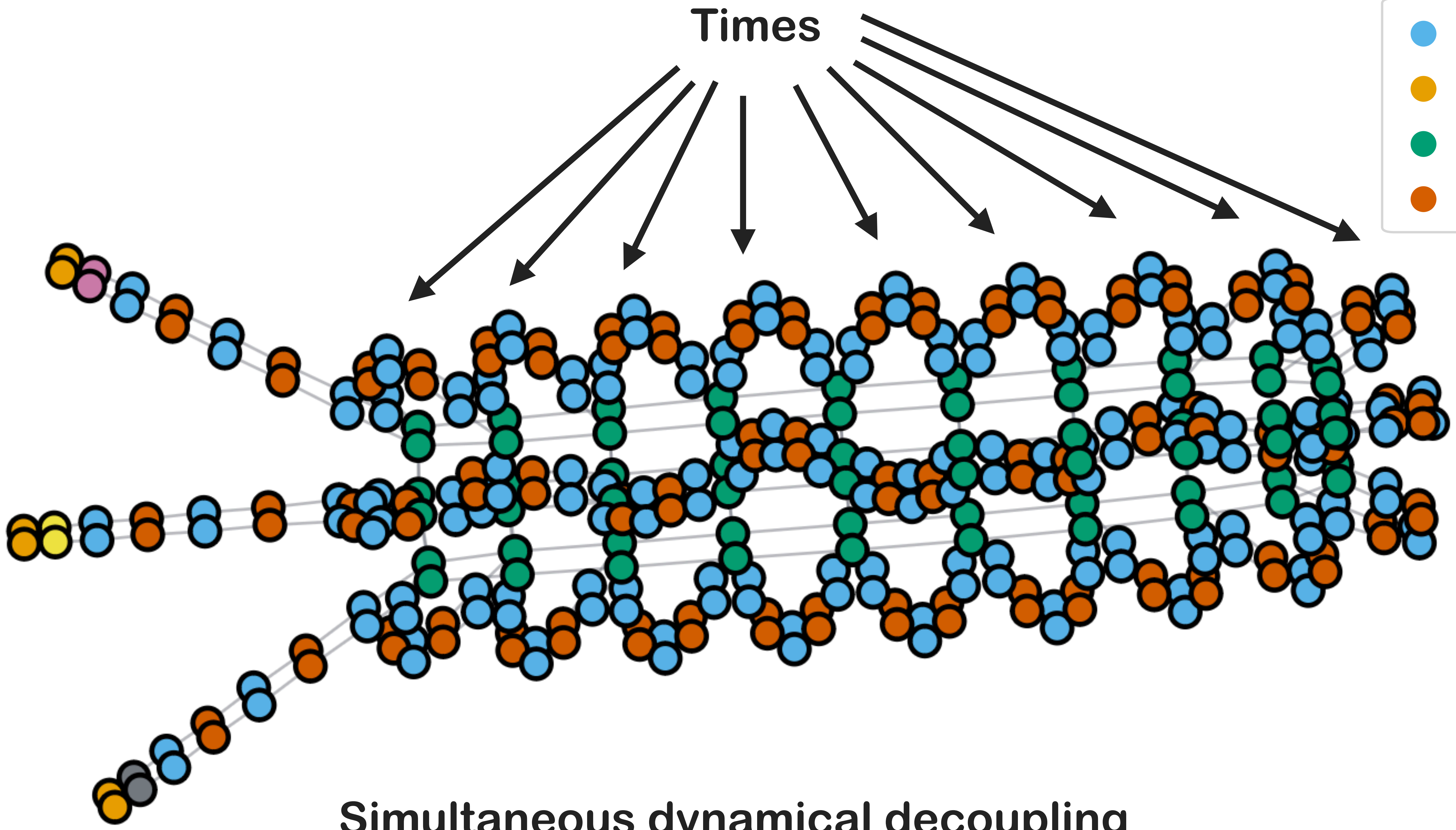

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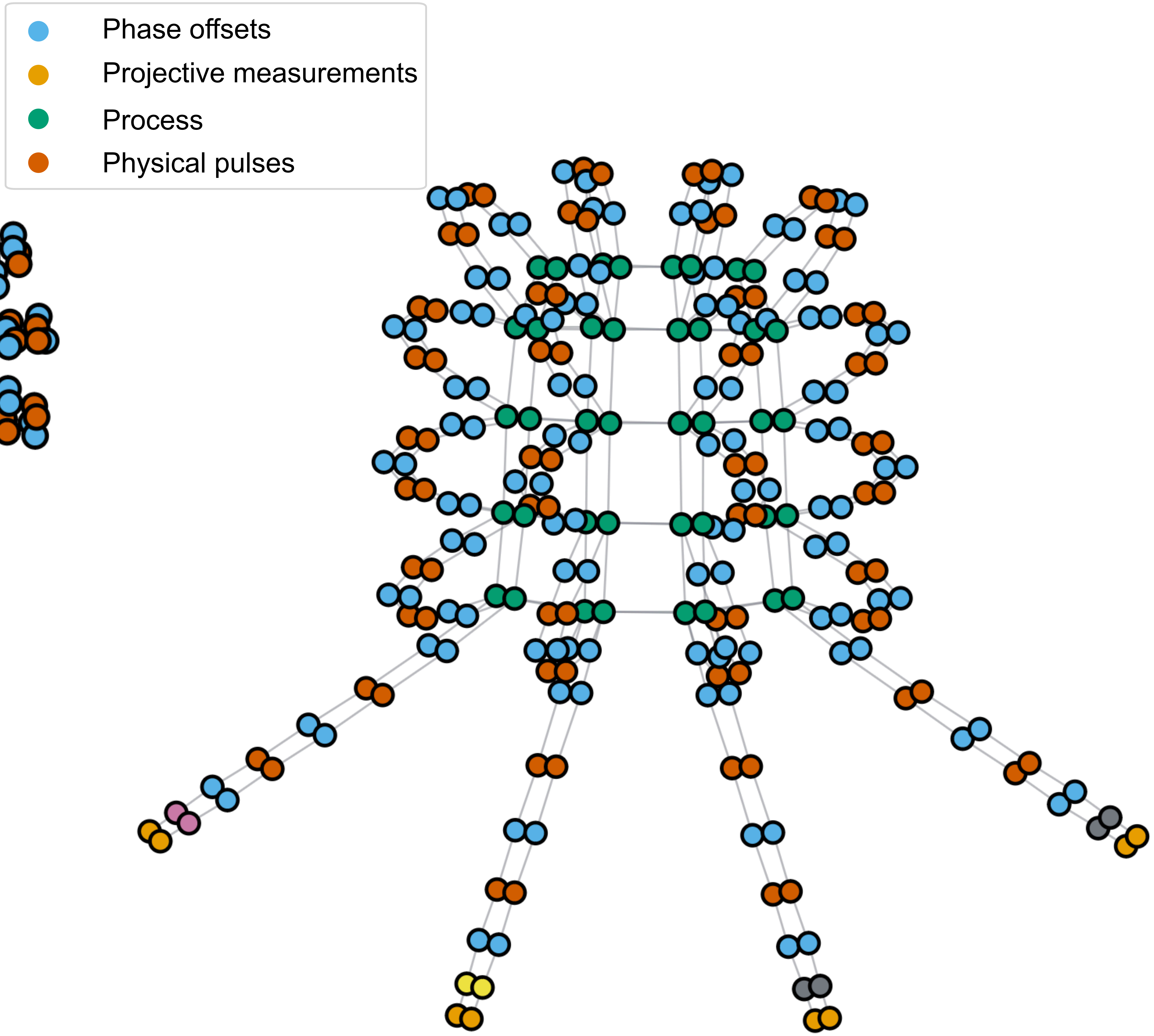

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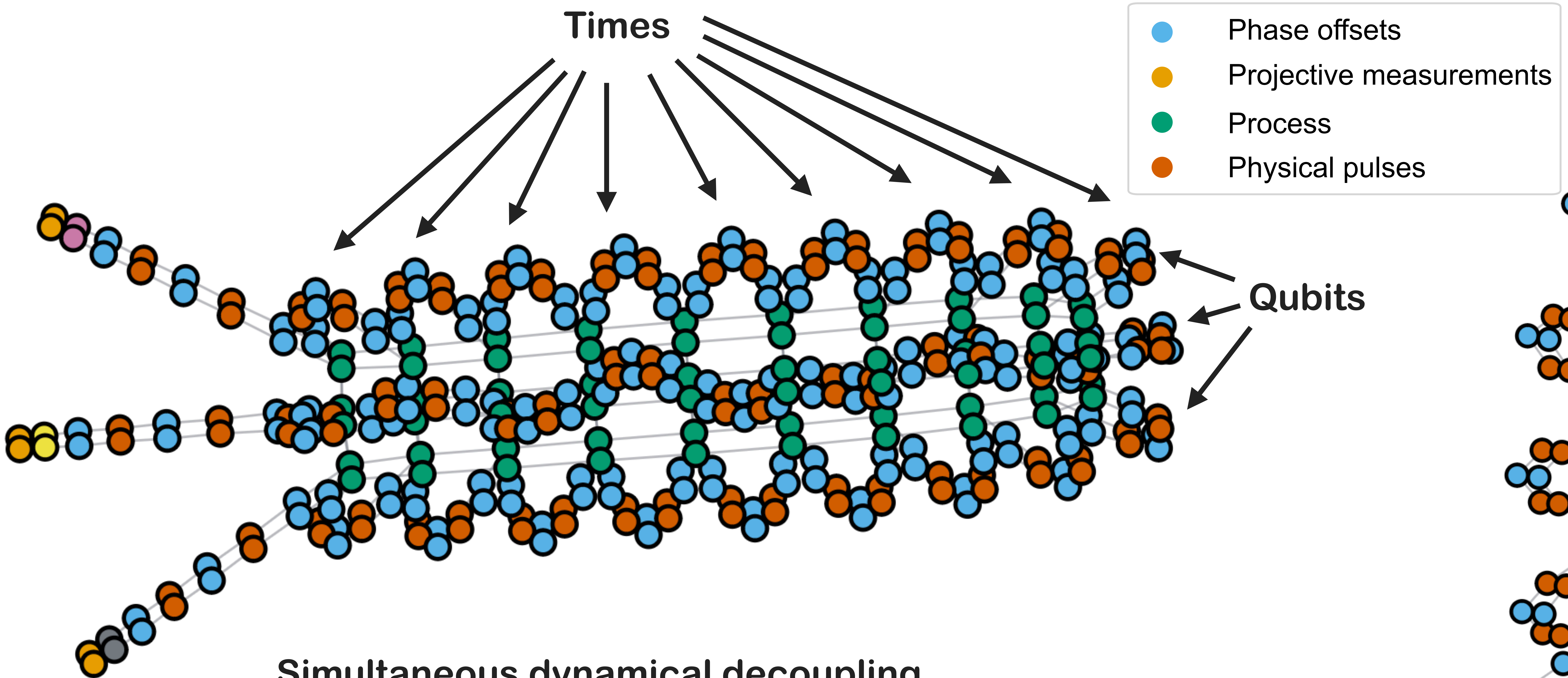

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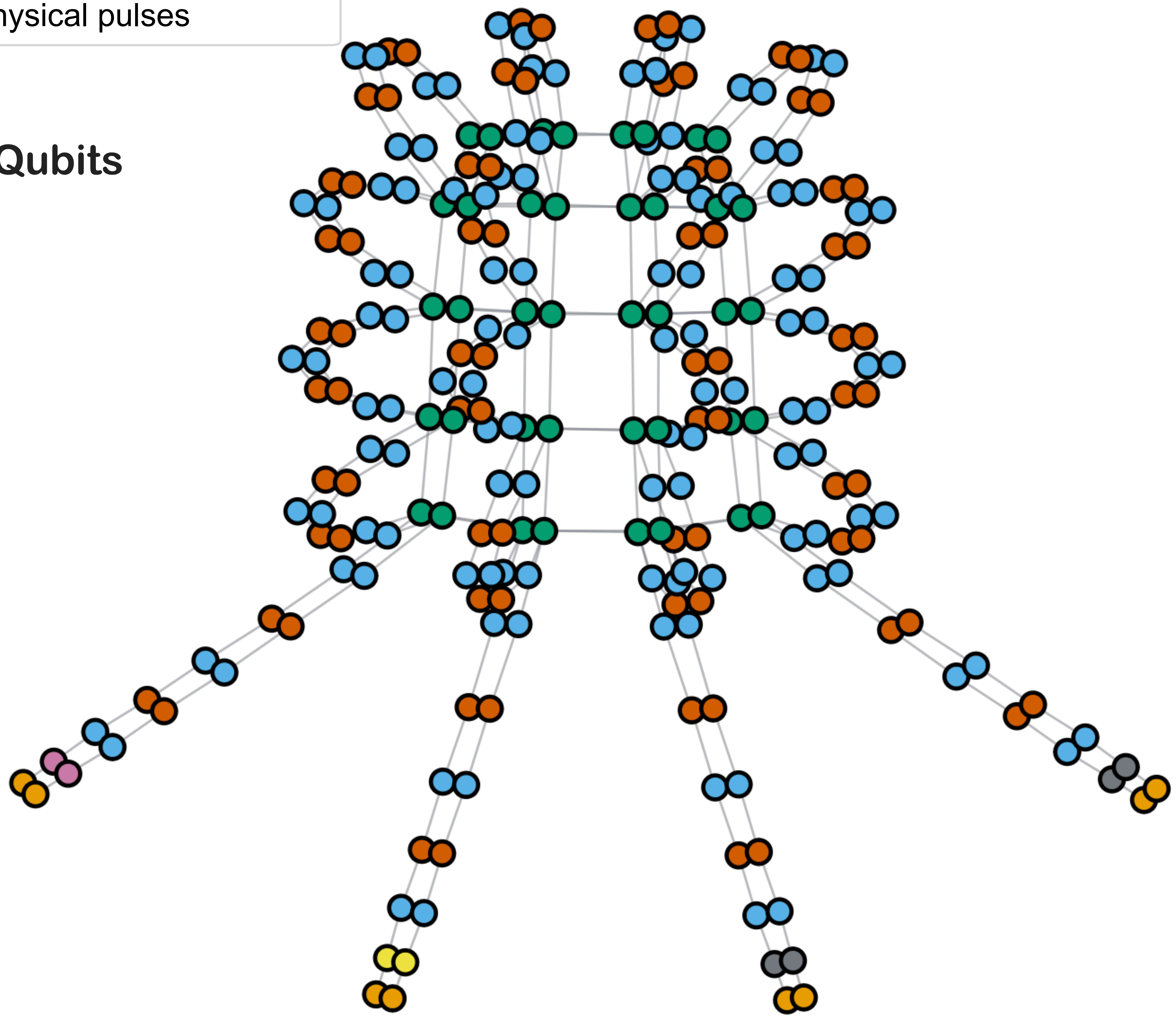

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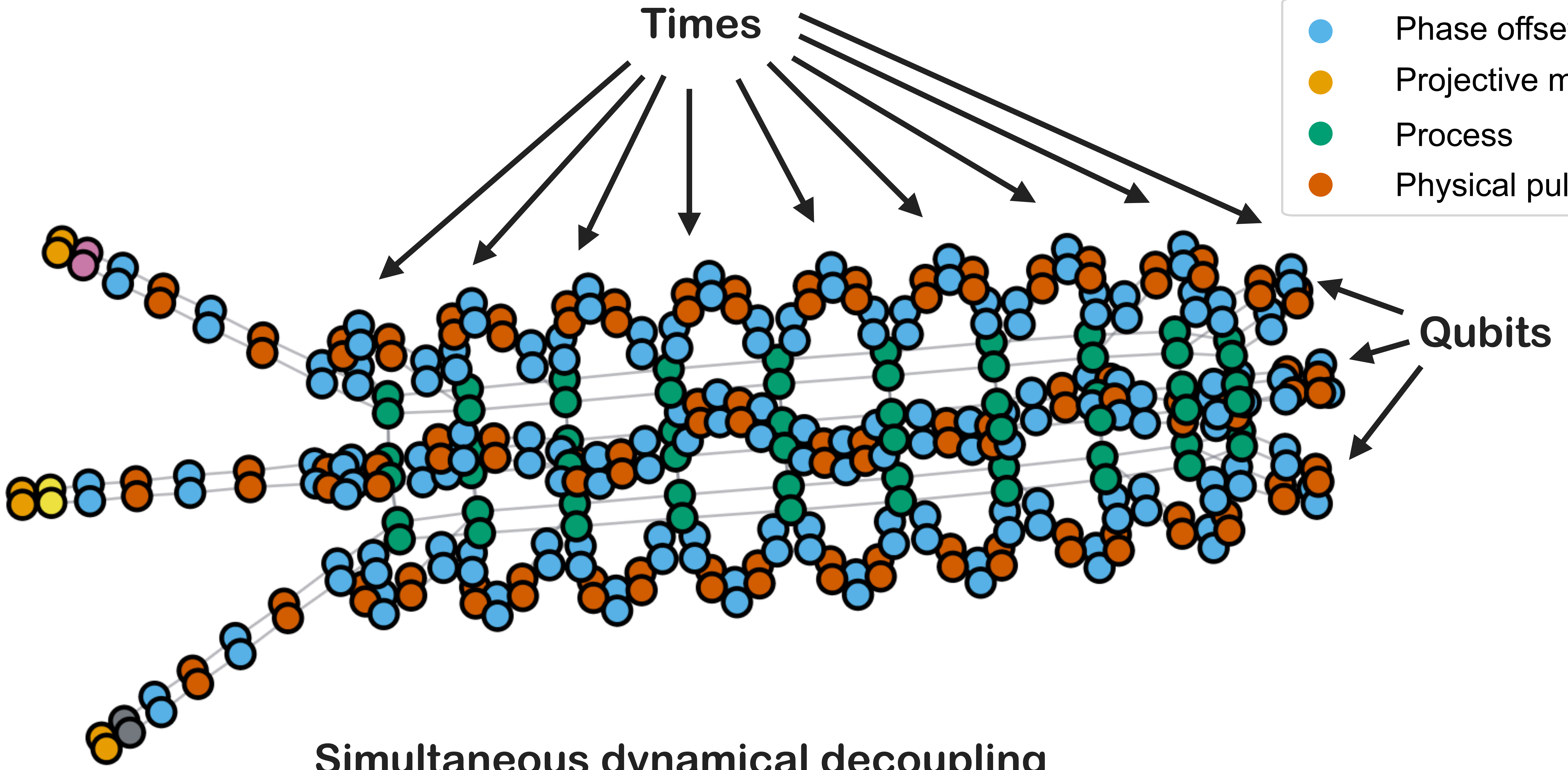

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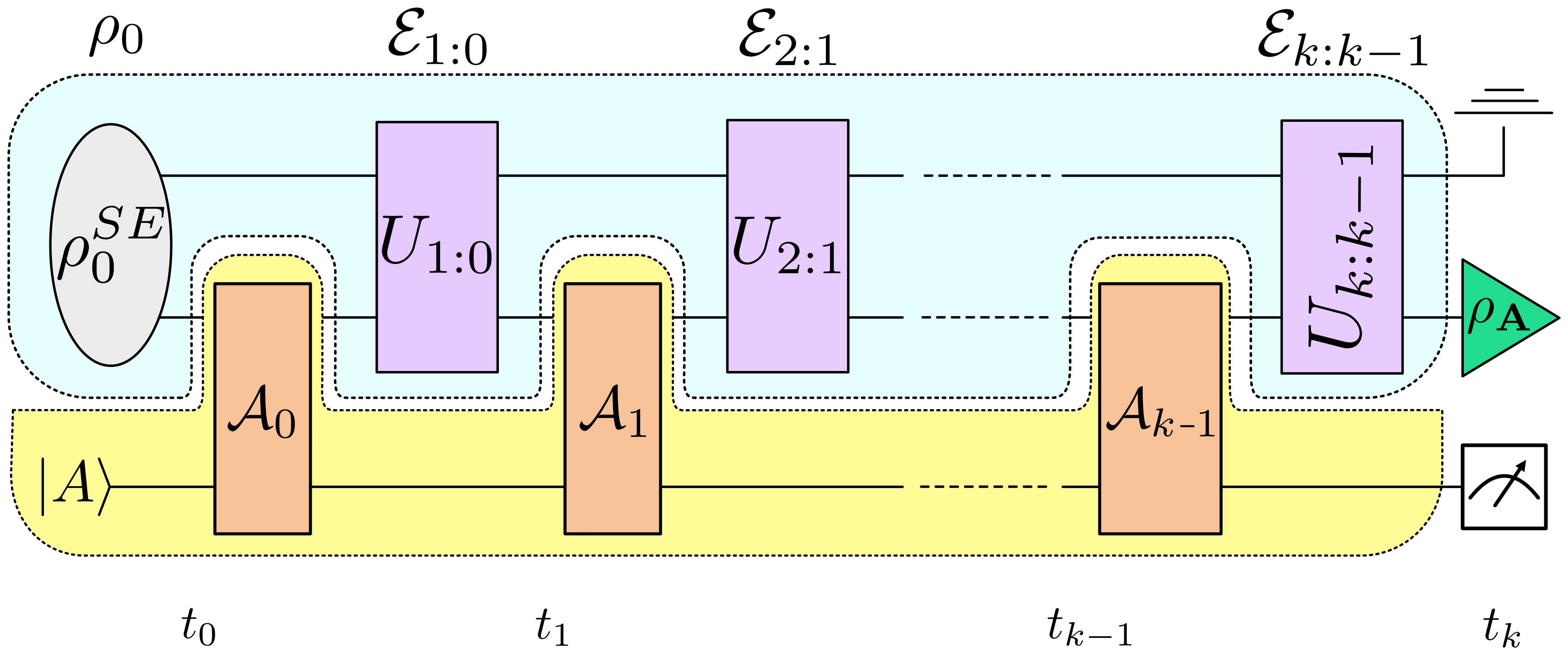

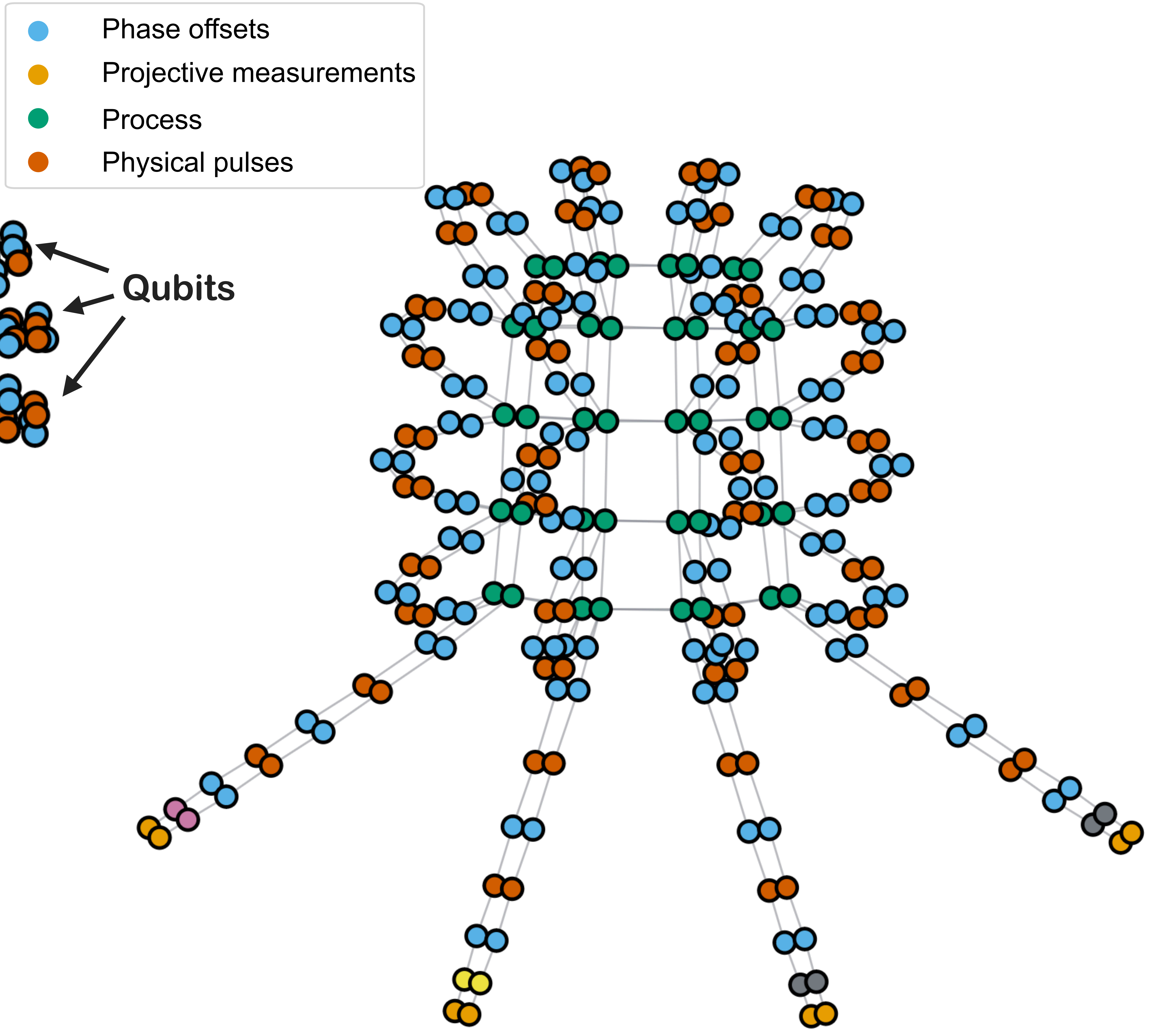

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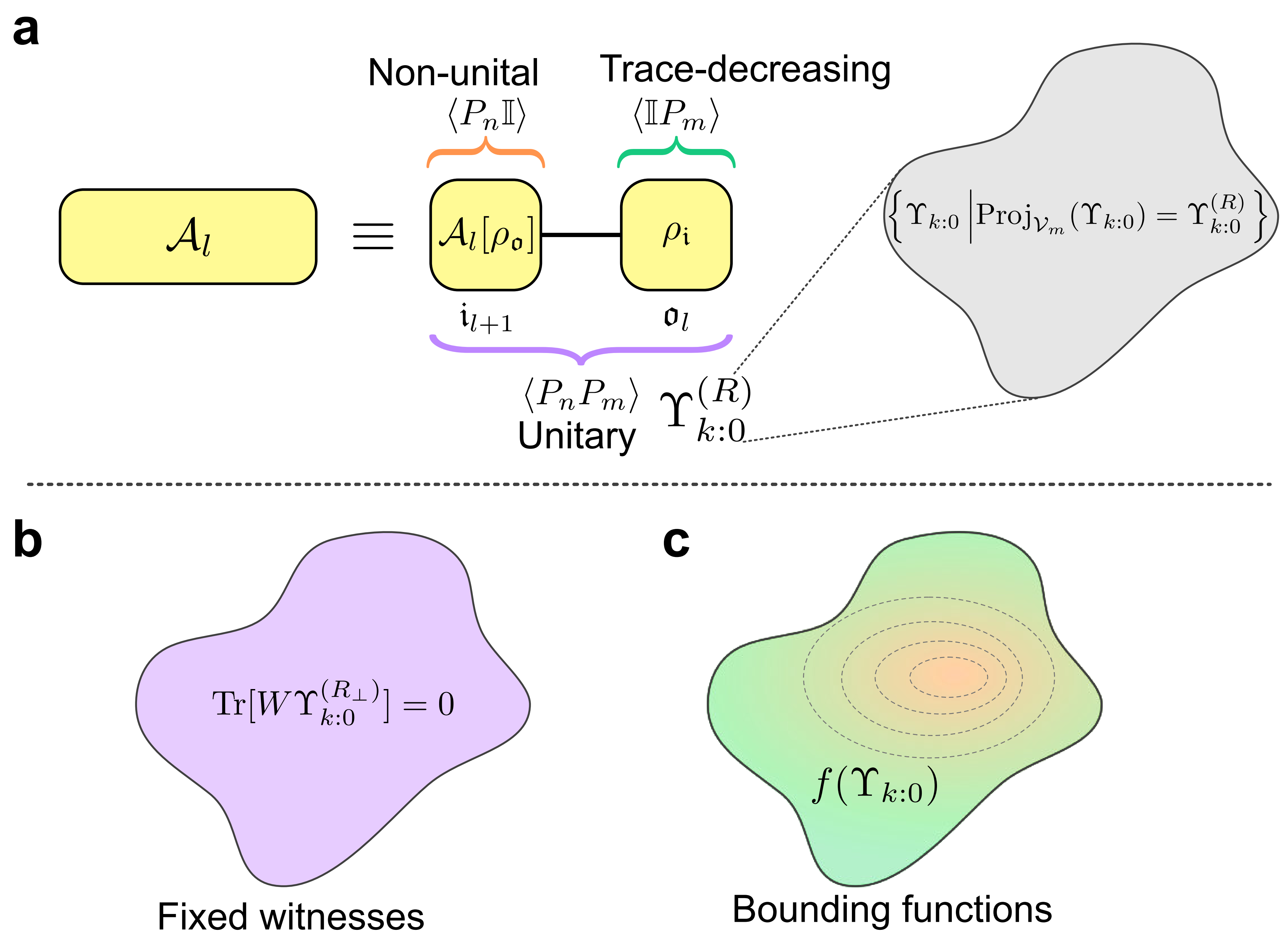
Unitary-Restricted Observables



$$\mathcal{O} = \sum_{i,\vec{\mu}} \alpha_{i,\vec{\mu}} P_i \otimes \bigotimes_{j=0}^{k} P_{\mu_j}, \qquad \text{where}$$

$$\alpha_{i,\vec{\mu}} \in \mathbb{R},$$
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$$\min_{W \in \mathcal{W}} \mathrm{Tr}[W \Upsilon_{k:0}],$$
$$\mathcal{W} = \{W | \, \forall\, M : \exists\, P_M, Q_M \succcurlyeq 0 : W = P_M + Q_M^{\Gamma_M},$$
$$\mathrm{Tr}[W] = 1, W \in \mathfrak{U}\}.$$

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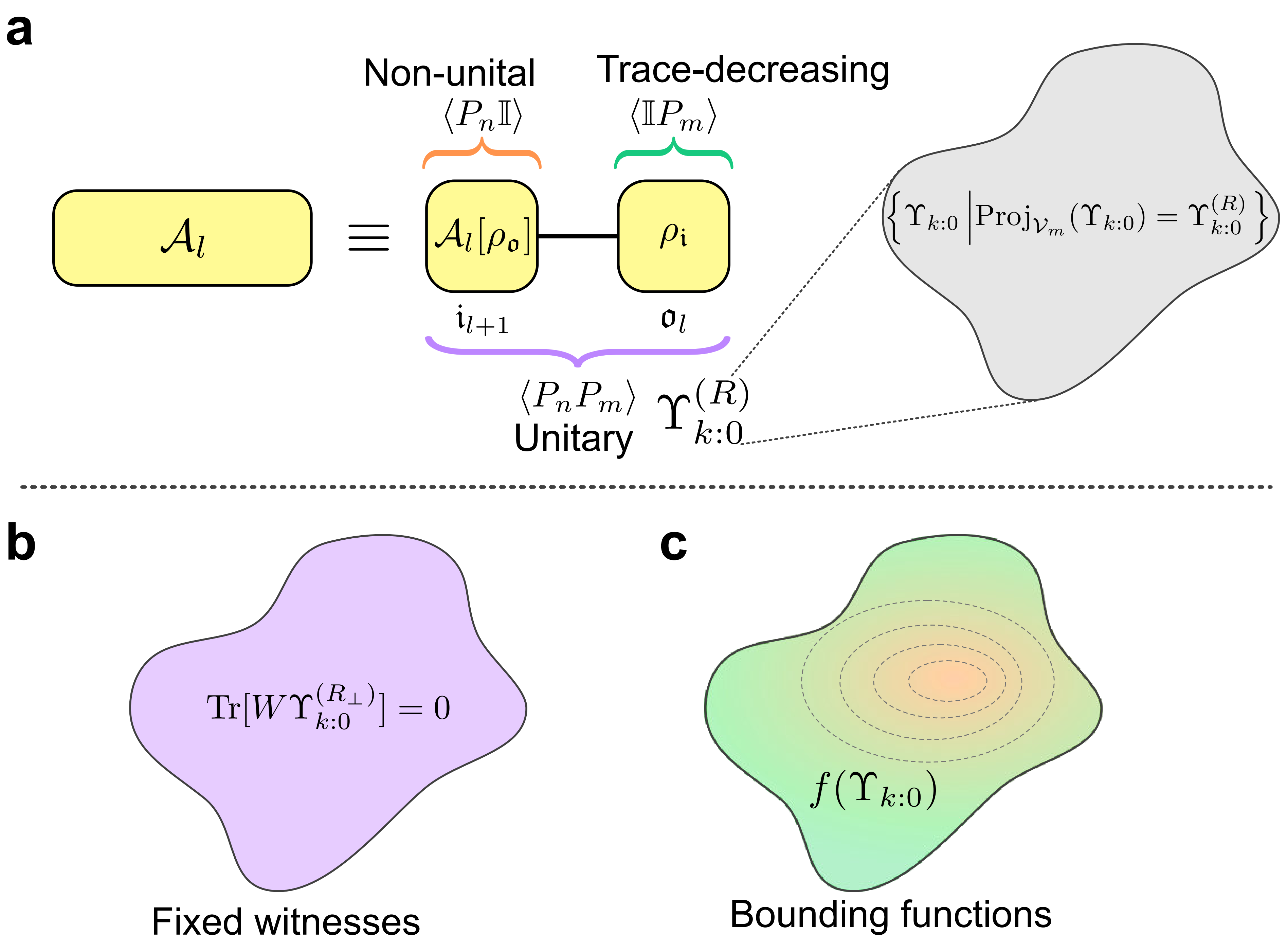

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$$\Upsilon = \Upsilon^{(R)} + \Upsilon^{(R_\perp)}$$

Fixed **Non-unique**

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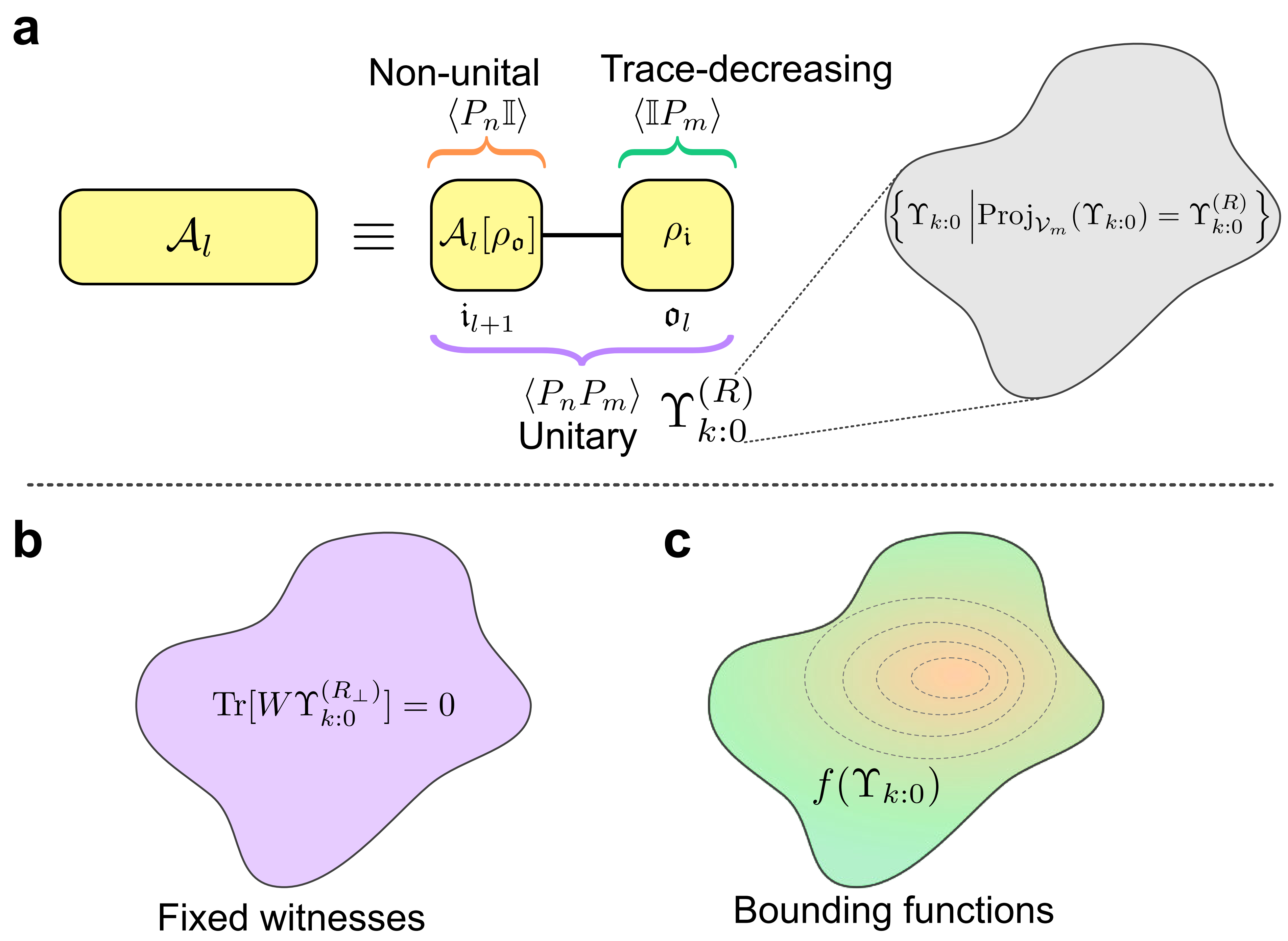

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Fixed **Non-unique**

- Also look at 3-step/7-body processes with different backgrounds

Setup	QMI (min, max)	Negativity	Purity	Fidelity
#1. System Alone	(0.298, 0.304)	(0.0179, 0.0181)	(0.8900, 0.8904)	(0.9423, 0.9427)
#2. $\vert+\rangle$ Nearest Neighbours (NN)	(0.363, 0.369)	(0.0255, 0.0259)	(0.7476, 0.7485)	(0.7239, 0.7244)
#3. Periodic CNOTs on NNs in $\vert+\rangle$	(0.348, 0.358)	(0.0240, 0.0246)	(0.7796, 0.7811)	(0.7264, 0.7272)
#4. $\vert 0\rangle$ NNs with QDD	(0.358, 0.373)	(0.0205, 0.0210)	(0.8592, 0.8612)	(0.8034, 0.8045)
#5. $\vert+\rangle$ Long-range Neighbours	(0.329, 0.339)	(0.0209, 0.0214)	(0.8594, 0.8608)	(0.9252, 0.9260)
#6. $\vert+\rangle$ NN, delay, $\vert+\rangle$ next-to-NN	(0.322, 0.329)	(0.0209, 0.0213)	(0.8534, 0.8549)	(0.9077, 0.9083)

ibmq_casablanca