Abstract for AIPC 2022

Quantum mean states are nicer than you think: finding states maximizing average fidelity

A. Afham[a], Richard Kueng[b], and Chris Ferrie[a]

[a]*Centre for Quantum Software and Information, University of Technology Sydney, NSW 2007, Australia.*

[b]*Institute for Integrated Circuits, Johannes Kepler University Linz, 4040 Linz, Austria*

Fidelity is arguably the most popular figure of merit in quantum sciences. However, many of its properties are still unknown. In this work, we resolve the open problem of maximizing average fidelity over arbitrary finite ensembles of quantum states and derive new upper bounds. Given a finite ensemble of quantum states $\rho_1, \ldots, \rho_n$ and associated weights $w_1, \ldots, w_n$, we find:

$$\arg\max_{\sigma} \sum_{i=1}^{n} w_i \mathrm{F}(\rho_i, \sigma),$$

where σ is varied across all quantum states and $\mathrm{F}(\rho, \sigma) = \mathrm{Tr}\left(\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\right)$ is the fidelity between quantum states ρ and σ.

We first construct a semidefinite program whose optimal value is the maximum average fidelity and then derive fixed-point algorithms that converge to the optimal state. The fixed-point algorithms outperform the semidefinite program in terms of numerical runtime. We also derive expressions for near-optimal states that are easier to compute and upper and lower bounds for maximum average fidelity that are exact when all the states in the ensemble commute. Finally, we discuss how our results solve some open problems in Bayesian quantum tomography.

The work is available on arXiv at https://arxiv.org/abs/2206.08183.