

arxiv:2206.08183

# Quantum mean states are nicer than you think

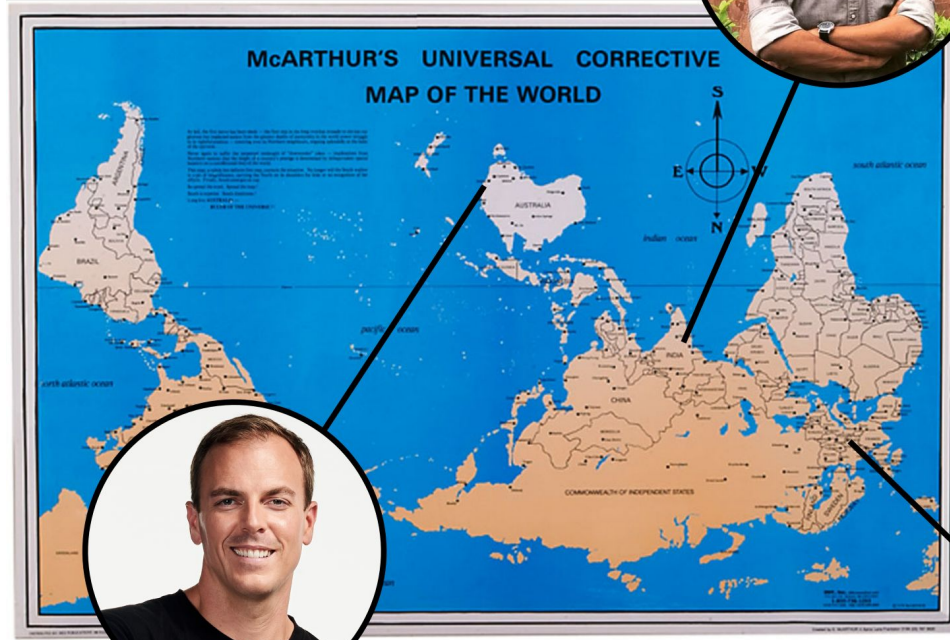
**Chris Ferrie**

(UTS Centre for Quantum Software and Information)

 @csferrie

**We show you how  
to find the state  
that maximizes  
average fidelity.**

**Afham**  
(UTS Centre for Quantum  
Software and Information)

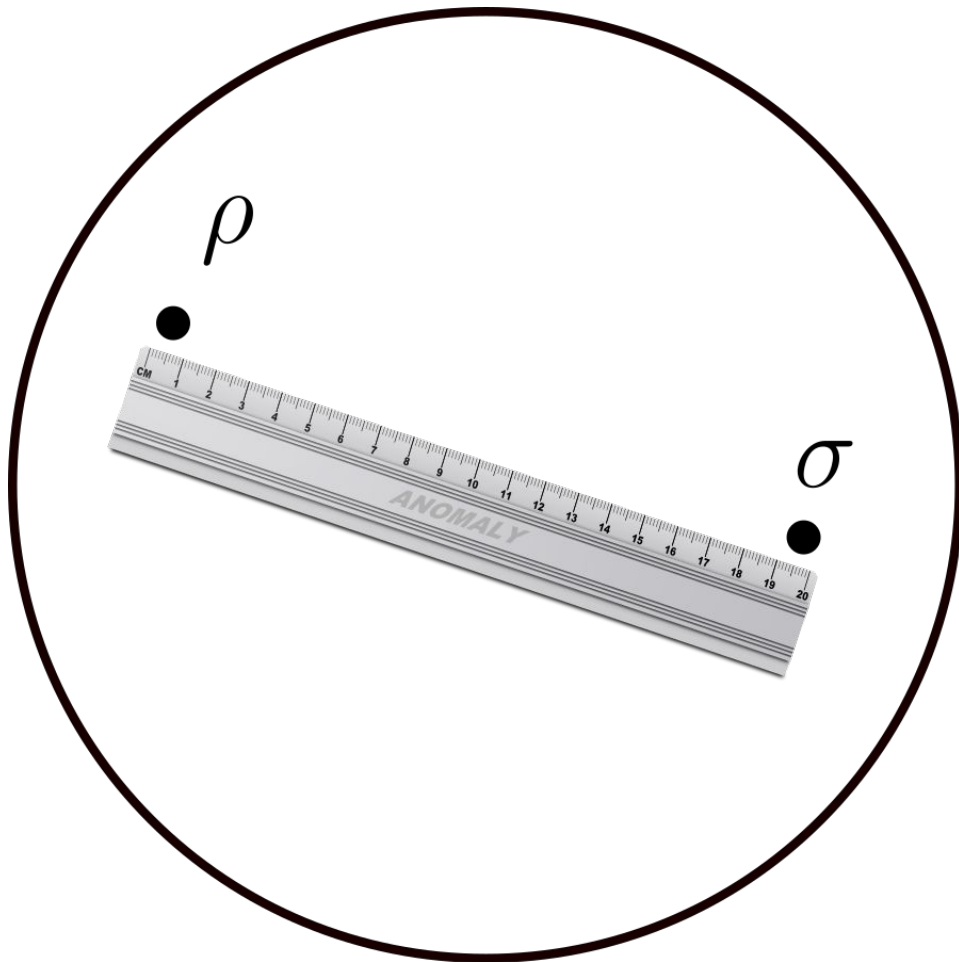


**Chris Ferrie**  
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Software and Information)

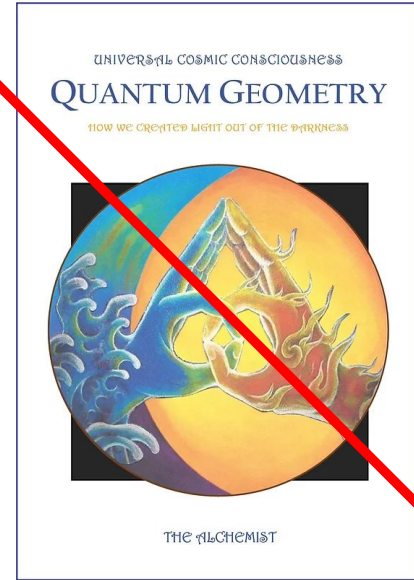
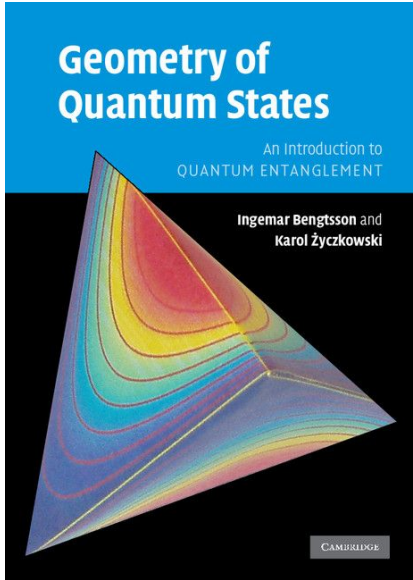


**Richard Kueng**  
(Johannes Kepler  
University Linz)

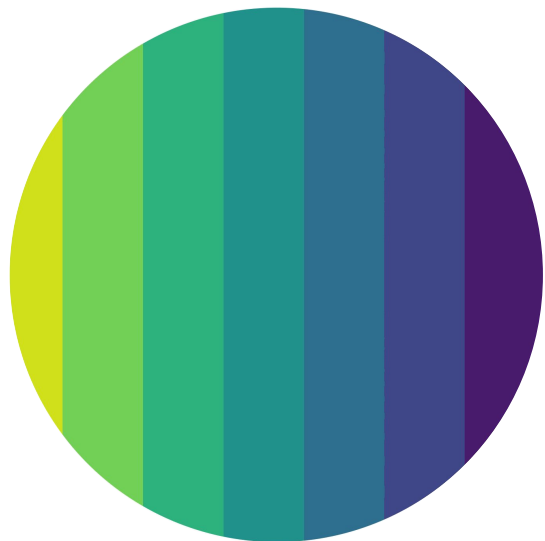




$$F(\rho, \sigma) = \text{Tr} \sqrt{\sqrt{\rho} \sigma \sqrt{\rho}}$$



Fidelity



Frobenius



Trace

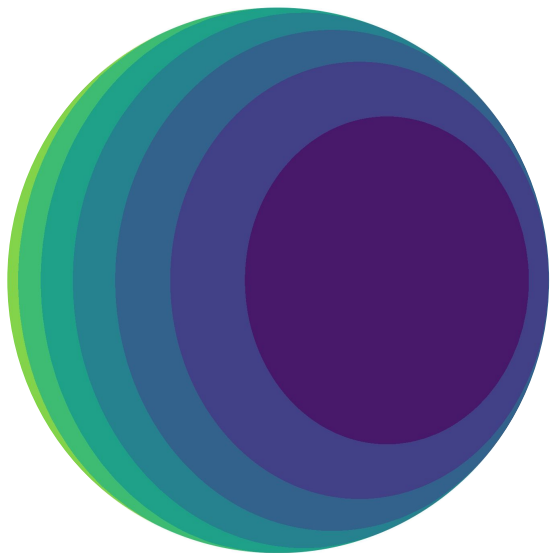


Far



Close

Fidelity



Frobenius



Trace



Far



Close



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outlook  
Ageing

# nature



## HIGH FIDELITY

Silicon qubits cross key error-correction threshold for quantum computing

### Coronavirus

Counting the global death toll of the COVID-19 pandemic

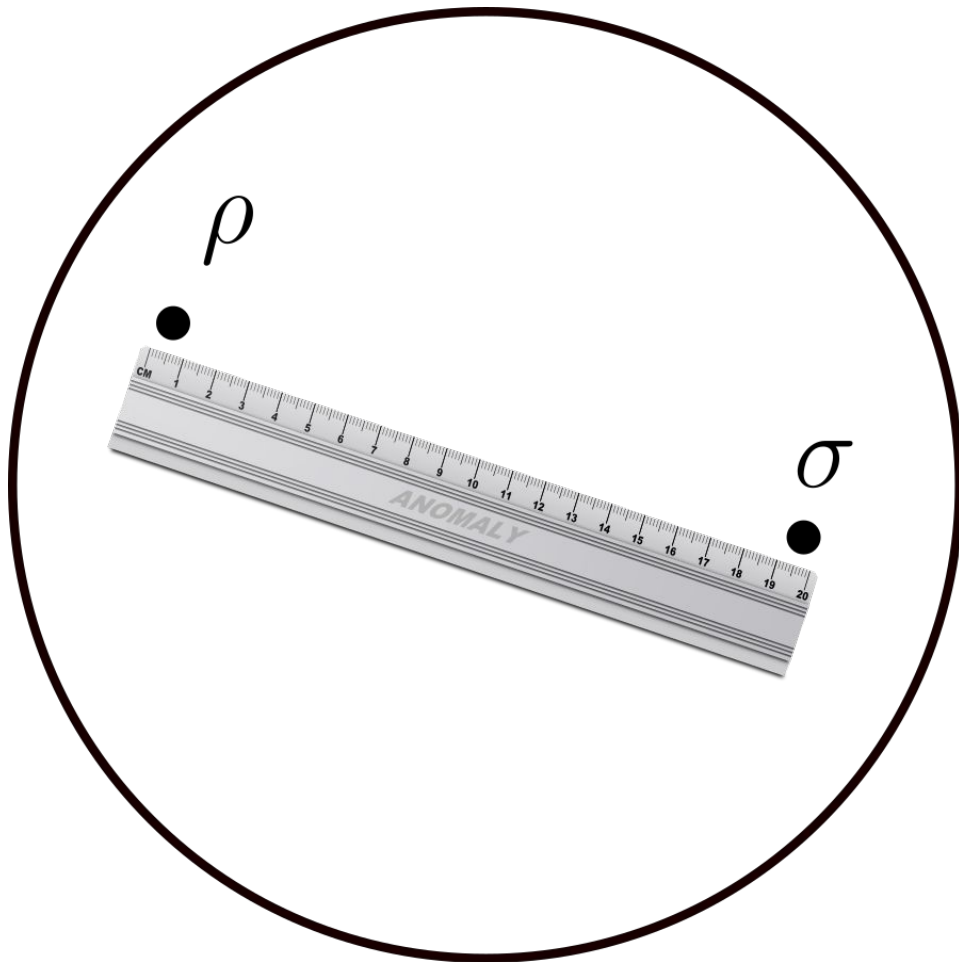
### Pregnancy health

Cell-free RNA in blood samples can predict risk of pre-eclampsia

### Precision dating

Detailed timelines for global networks in the Viking Age

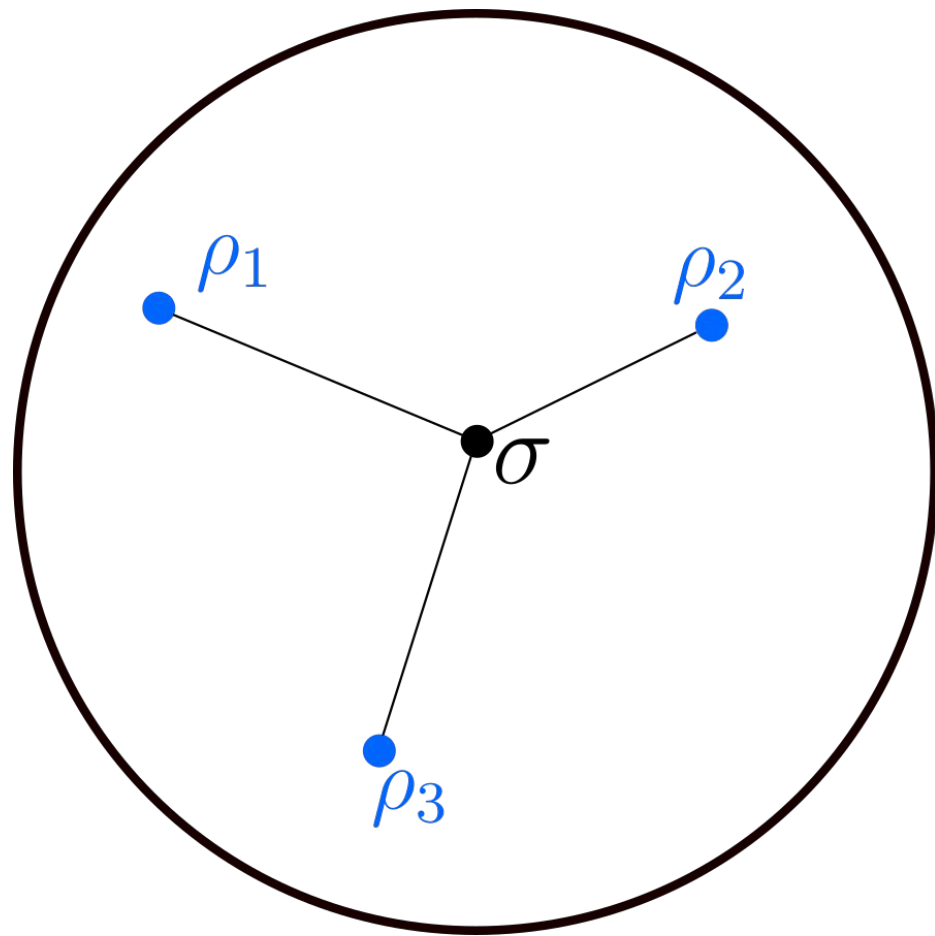
Vol 601 No 7883  
nature.com



Fixed, given

$$F(\rho, \sigma) = \text{Tr} \sqrt{\sqrt{\rho} \sigma \sqrt{\rho}}$$

Variable, to solve for



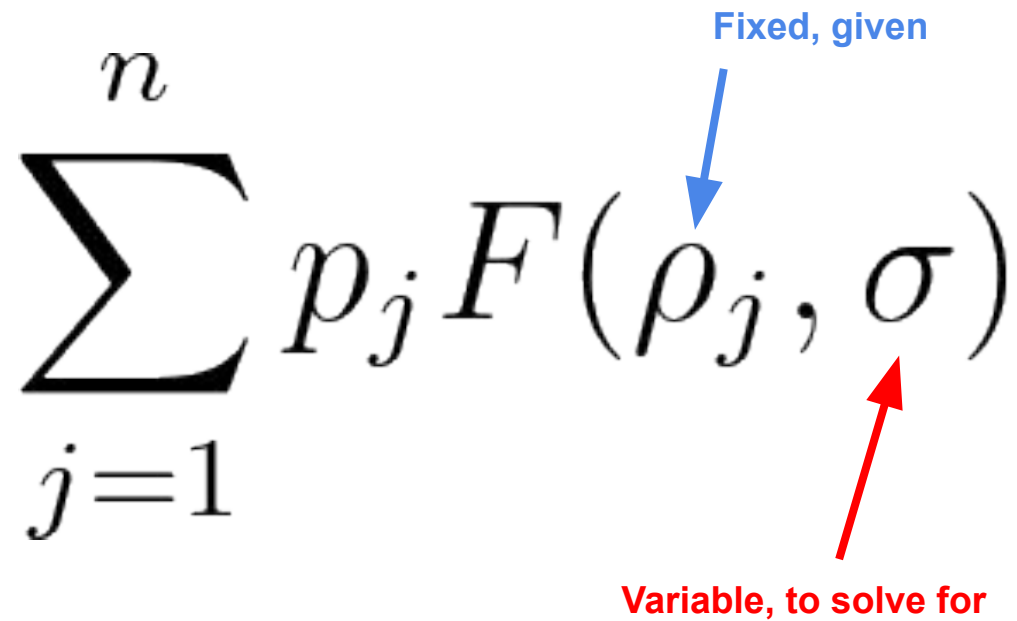
$$\frac{1}{n} \sum_{j=1}^n F(\rho_j, \sigma)$$

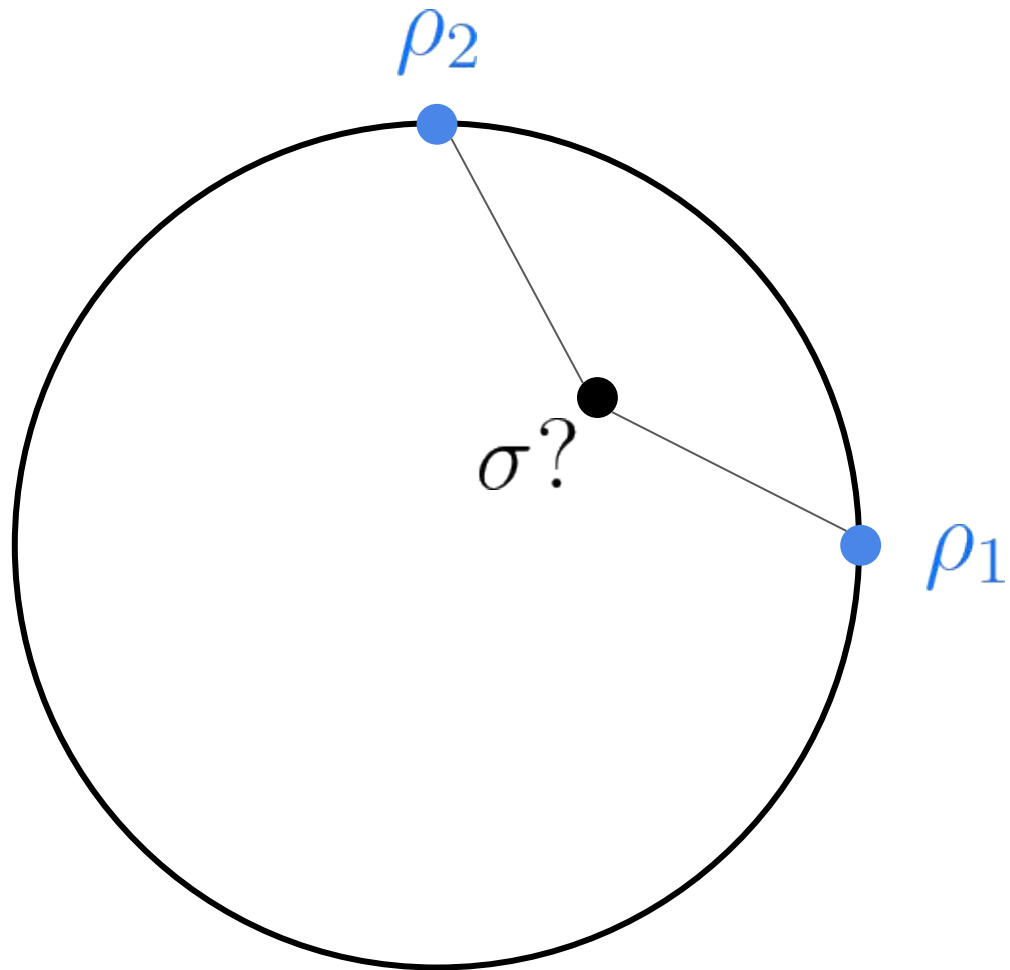
$$\sum_{j=1}^n p_j F(\rho_j, \sigma)$$

$$\sum_{j=1}^n p_j F(\rho_j, \sigma)$$

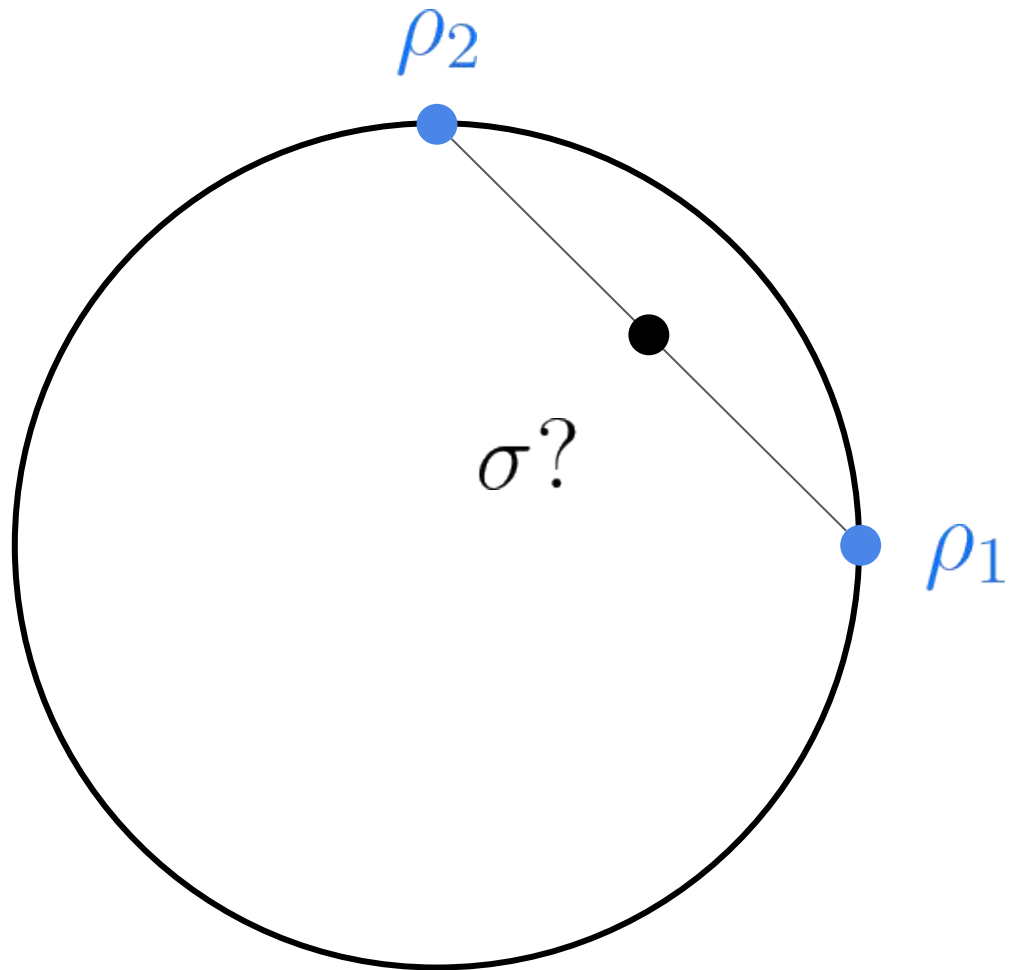
Fixed, given

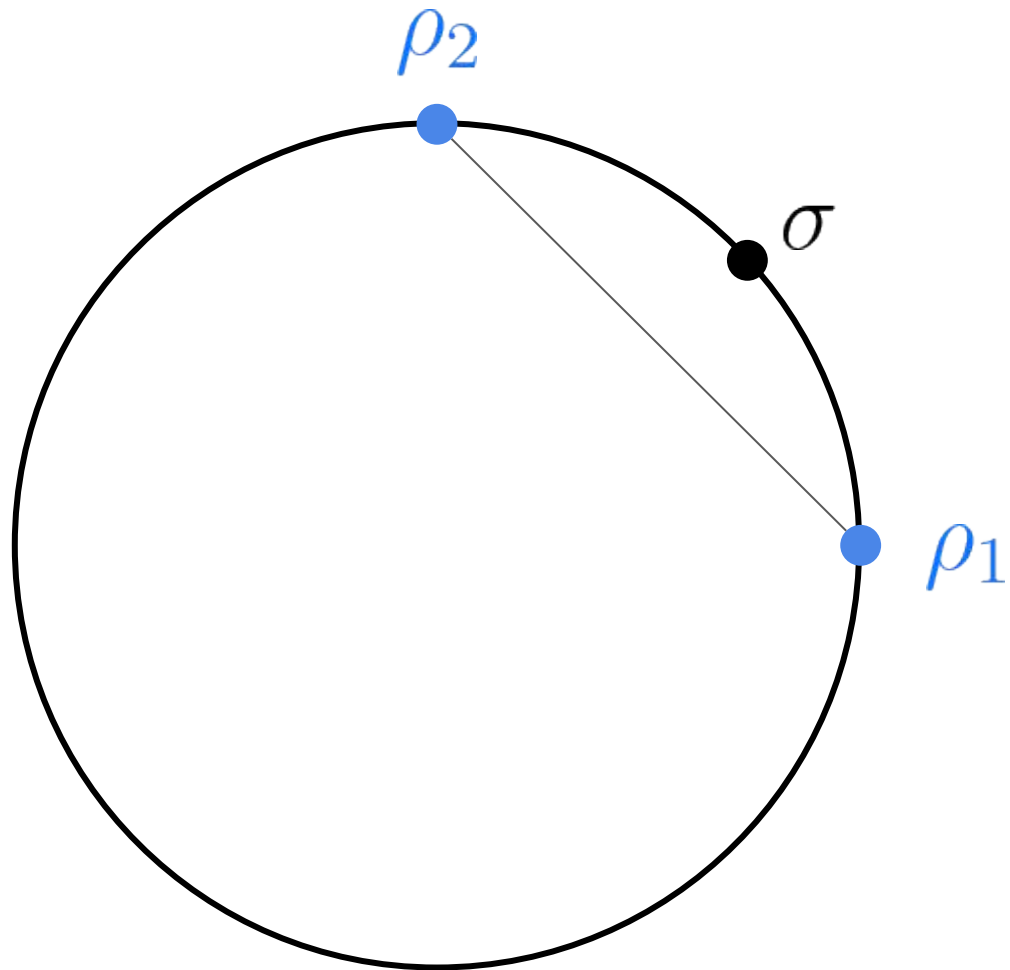
Variable, to solve for

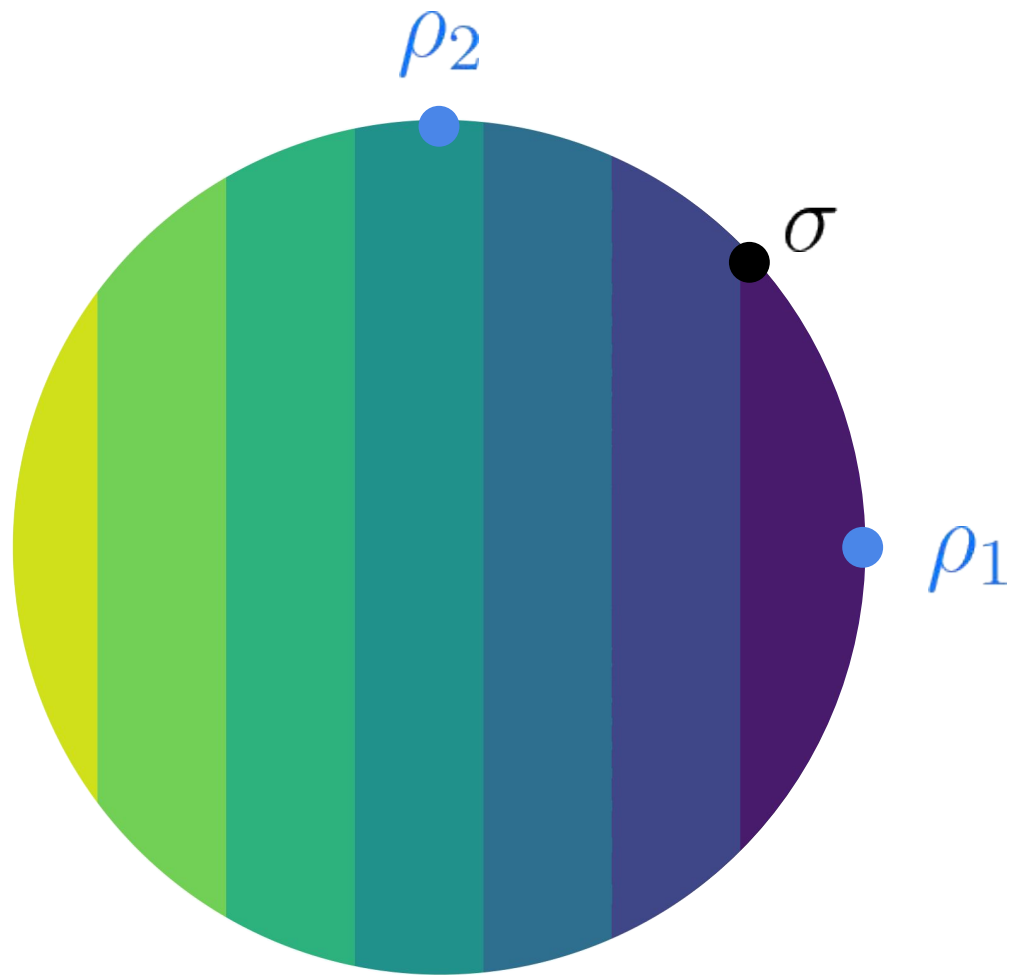




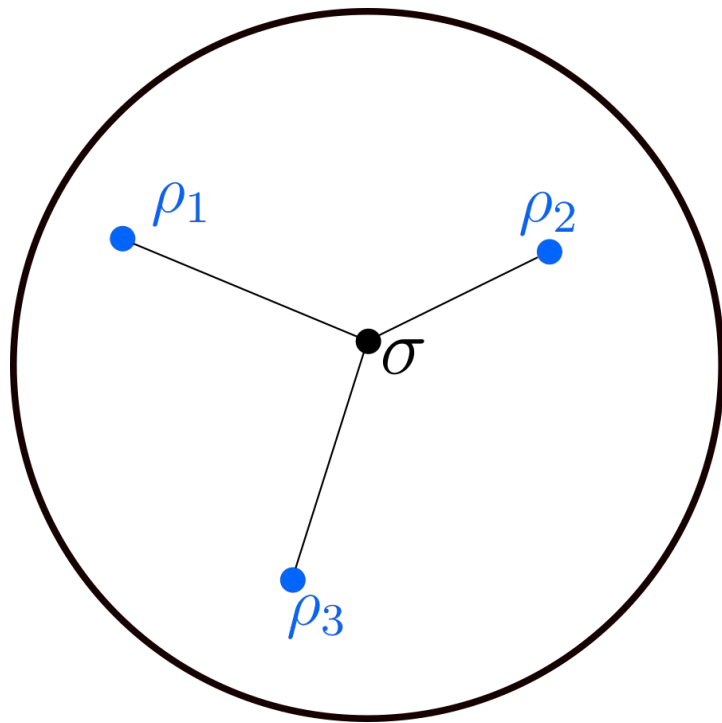


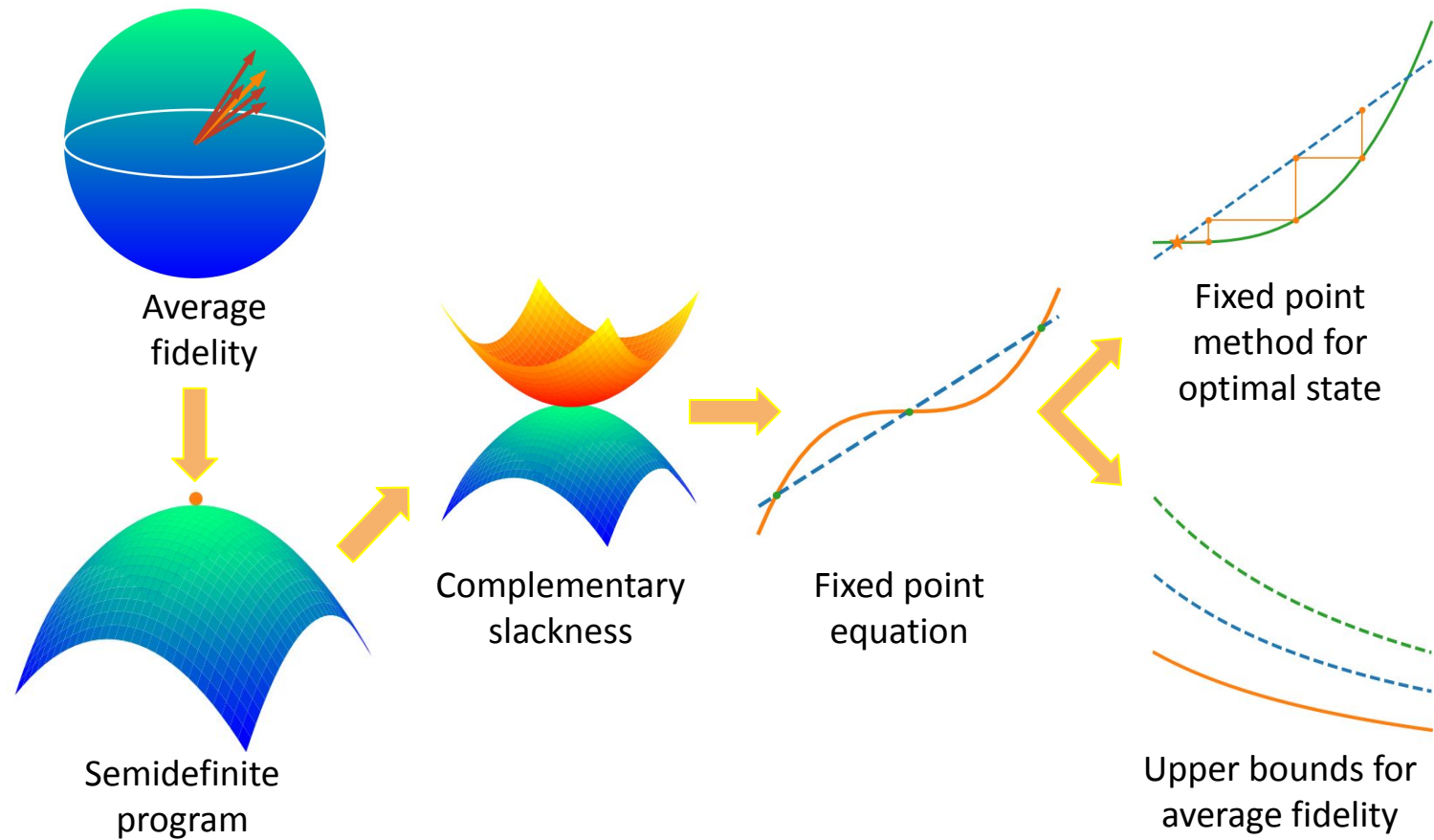






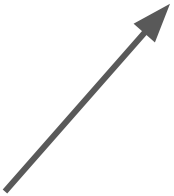
$$\sum_{j=1}^n p_j F(\rho_j, \sigma)$$






$$\sigma_{(k+1)} \propto \sigma_{(k)}^{-1/2} \left( \sum_{i=1}^n p_i \sqrt{\sigma_{(k)}^{1/2} \rho_i \sigma_{(k)}^{1/2}} \right)^2 \sigma_{(k)}^{-1/2}$$

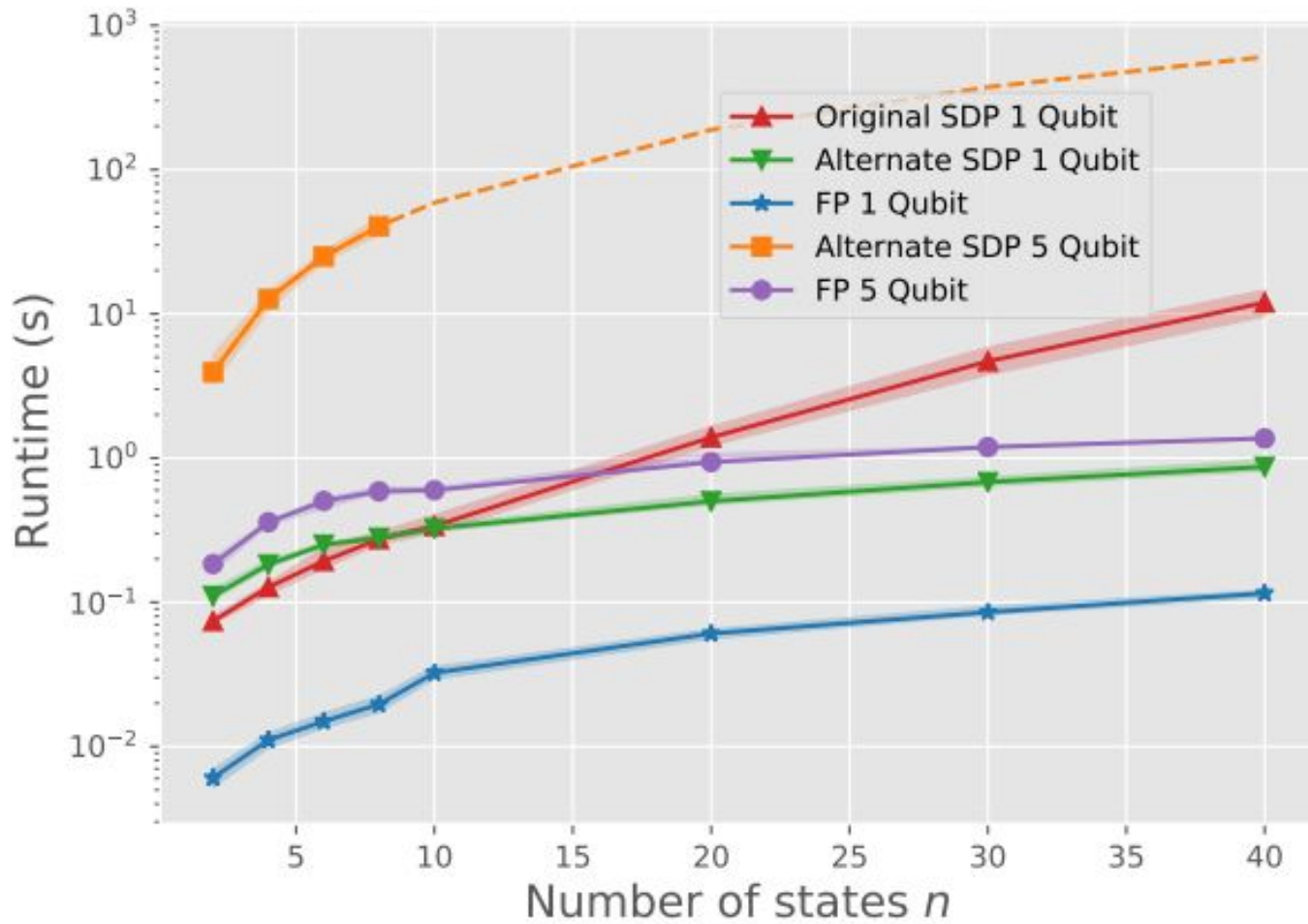
Iterative  
algorithm  
(numerical  
approximation)



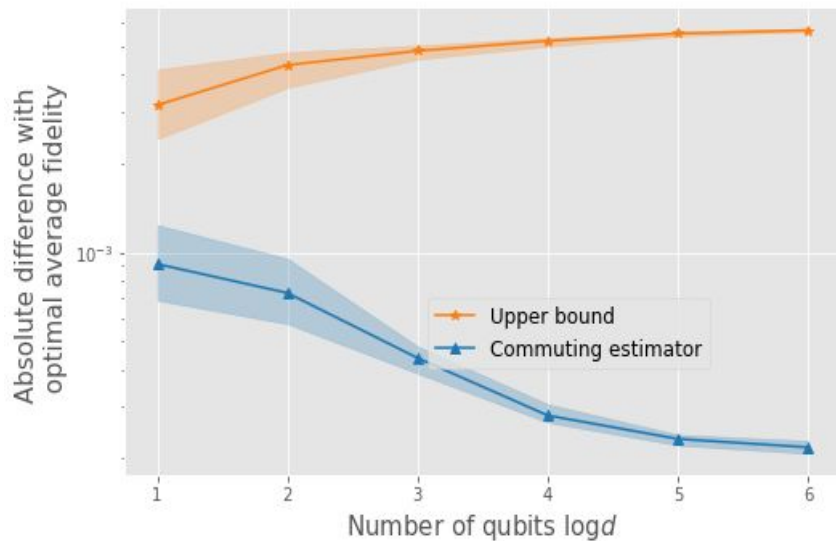
Exact  
solution  
(commuting  
approximation)



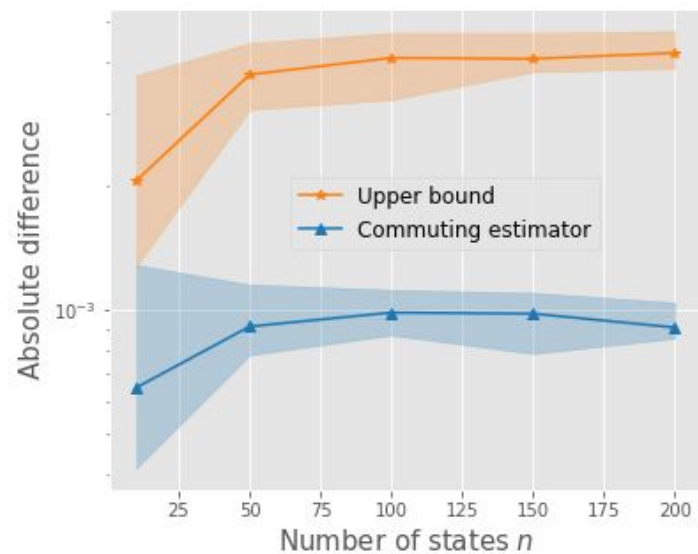
$$\sigma \propto \left( \sum_{j=1}^n p_j \sqrt{\rho_j} \right)^2$$



n = 20



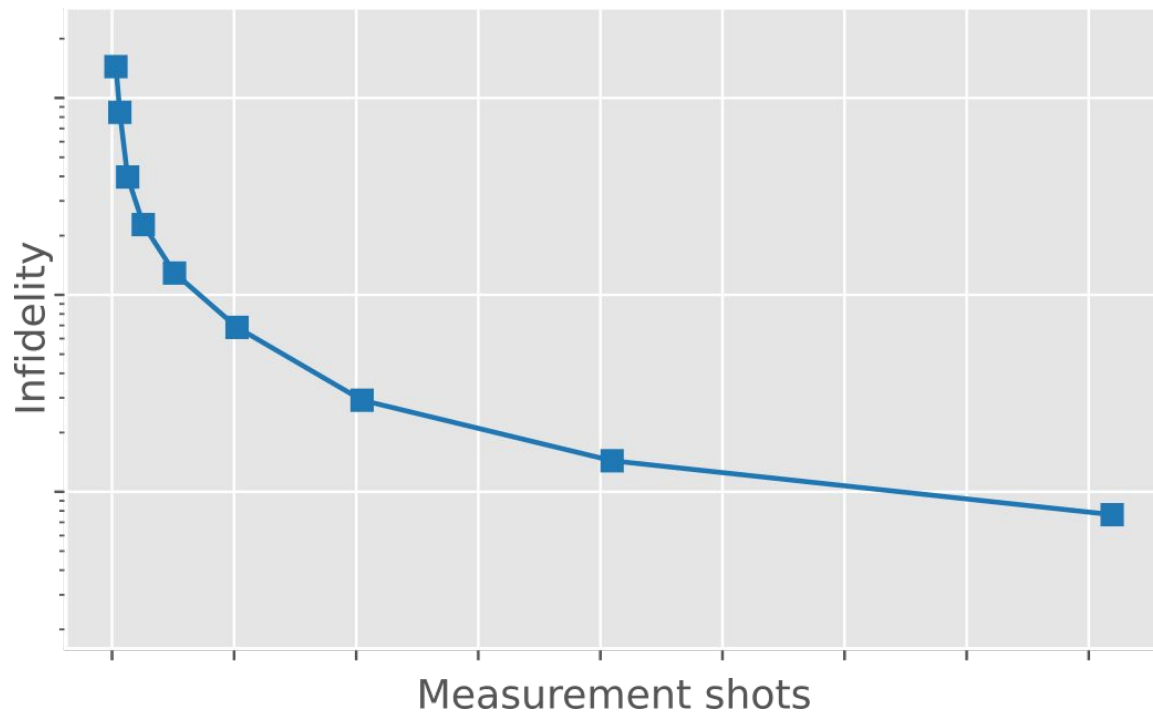
d = 2

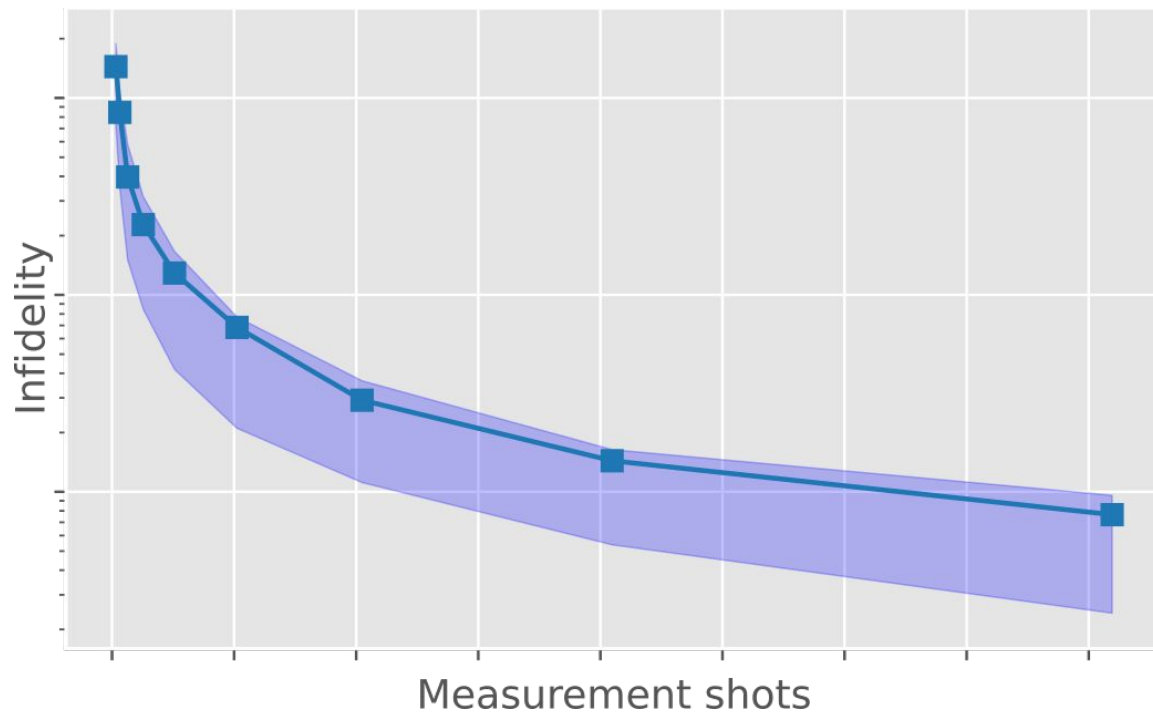


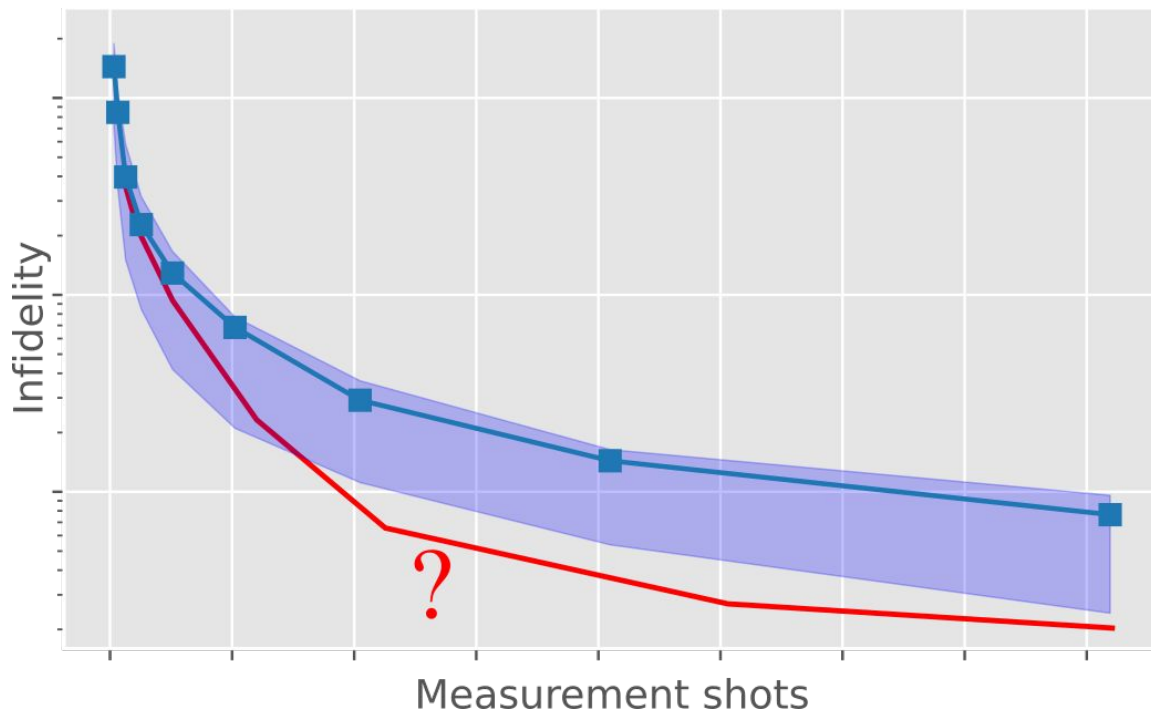
$$\sqrt{\sum_{i,j=1}^n p_i p_j F(\rho_i, \rho_j)}$$

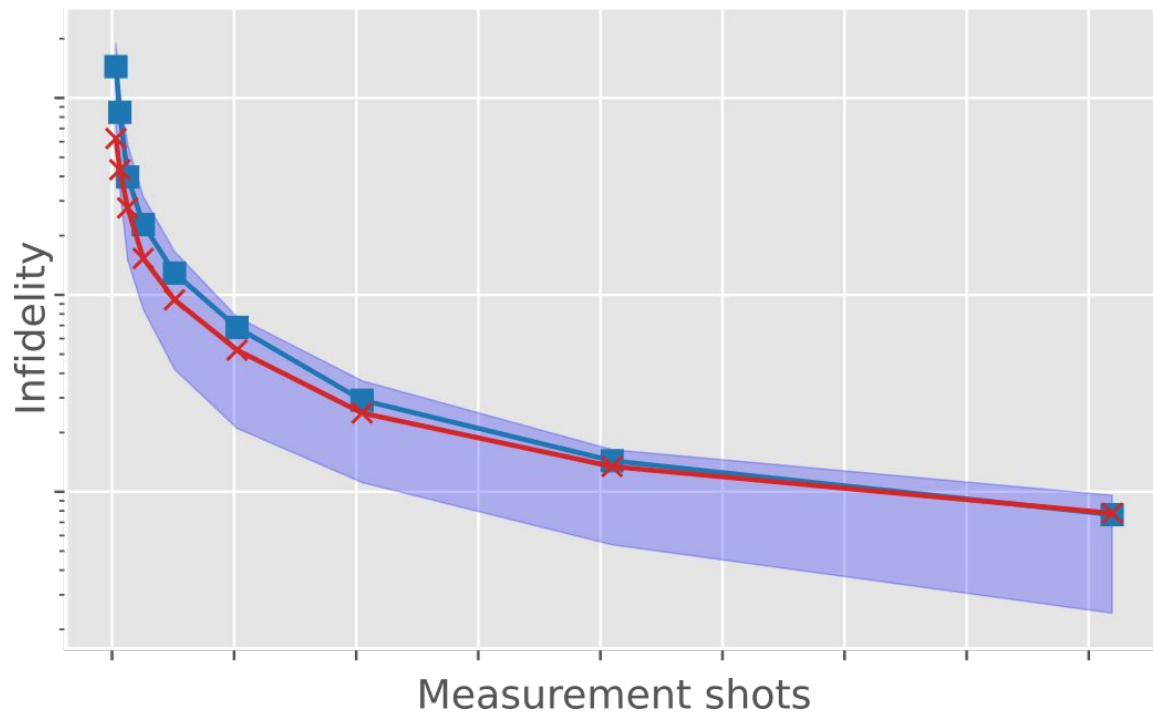
$$\sigma \propto \left( \sum_{i=1}^n p_i \sqrt{\rho_i} \right)^2$$











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