On the query complexity of connectivity with global queries

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We study the query complexity of determining if a graph is connected with global queries. The first model we look at is matrix-vector multiplication queries to the adjacency matrix. Here, for an n-vertex graph with adjacency matrix A, one can query a vector $x \in \{0,1\}^n$ and receive the answer Ax. We give a randomized algorithm that can output a spanning forest of a weighted graph with constant probability after $O(\log^4(n))$ matrix-vector multiplication queries to the adjacency matrix. This complements a result of Sun et al.[1] that gives a randomized algorithm that can output a spanning forest of a graph after $O(\log^4(n))$ matrix-vector multiplication queries to the signed vertex-edge incidence matrix of the graph. As an application, we show that a quantum algorithm can output a spanning forest of an unweighted graph after $O(\log^5(n))$ cut queries, improving and simplifying a result of Lee, Santha, and Zhang [2], which gave the bound $O(\log^8(n))$.

In the second part of the paper, we turn to showing lower bounds on the linear query complexity of determining if a graph is connected. If w is the weight vector of a graph (viewed as an $\binom{n}{2}$ dimensional vector), in a linear query one can query any vector $z \in^{\binom{n}{2}}$ and receive the answer $\langle z, w \rangle$. We show that a zero-error randomized algorithm must make $\Omega(n)$ linear queries in expectation to solve connectivity. As far as we are aware, this is the first lower bound of any kind on the unrestricted linear query complexity of connectivity. We show this lower bound by looking at the linear query *certificate complexity* of connectivity, and characterize this certificate complexity in a linear algebraic fashion.

[1] Xiaoming Sun, David P Woodruff, Guang Yang, and Jialin Zhang. *Querying a matrix through matrix-vector products*. In Proceedings of the 46th International Colloquium on Automata, Languages, and Programming (ICALP 2019), pages 94:1–94:16, 2019.

[2] Troy Lee, Miklos Santha, and Shengyu Zhang. *Quantum algorithms for graph problems with cut queries*. In Daniel Marx, editor, Proceedings of the 2021 ACM-SIAM Symposium on Discrete Algorithms, SODA 2021, pages 939–958. SIAM, 2021.