Estimation of quantum state and parameters given past and future information

Q. Yu[a,b], D. Dong[b]

[a]*Centre for Quantum Computation and Communication Technology (Australian Research Council), Centre for Quantum Dynamics, Griffith University, Yuggera Country, Brisbane, Queensland 4111, Australia.*

[b]*School of Engineering and Information Technology, University of New South Wales, Canberra, ACT 2600, Australia.*

In this work, we consider the dual quantum state-parameter smoothing problem where both the system state and unknown parameters are estimated given past and future measurement information. The quantum estimation of using past and future information has been considered in several works [1, 2, 3], among which a quantum state smoothing theory has been proposed and a smoothed quantum density operator has been successfully given [2]. For general quantum state estimation problem, we usually assume that the system is well modeled. However, environmental fluctuations or insufficient knowledge may lead to inaccuracies in system modeling. Thus, Tsang proposed a time-symmetric smoothing for the estimation of classical signals in a quantum system [4]. Further studies consider the simultaneous filtering problem of both the state and parameters by using the past measurement information, motivated by either the need to estimate the system state robustly or to estimate the parameters of interest.

Our work aims to fill the gap of dual quantum state-parameter smoothing, while the probability density distribution of the unknown parameters can be either static or dynamical. Based on the Bayes' theorem, general formulas for dual quantum filtering and smoothing are given. Moreover, solutions to systems of linear Gaussian dynamics are given. For illustration, we apply the dual quantum state-parameter smoothing to an optical parametric oscillator system which is subject to an unknown stochastic parameter.

[1] S. Gammelmark, B. Julsgaard, and K. Mølmer, "Past Quantum States of a Monitored System," *Phys. Rev. Lett.*, vol. 111, no. 16, p. 160401, 2013.

[2] I. Guevara, and H.M. Wiseman, "Quantum State Smoothing," *Phys. Rev. Lett.*, vol. 115, no. 18, p. 180407, 2015.

[3] K. Ohki, "A Smoothing Theory for Open Quantum Systems: The Least Mean Square Approach," *Proceedings of the 54th IEEE CDC,* pp. 4350–4355, 2015.

[4] M. Tsang, "Time-Symmetric Quantum Theory of Smoothing," *Phys. Rev. Lett.*, vol. 102, no. 25, p. 250403, 2009.