The shareability of steering in two-producible states

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Quantum steering was originally introduced as the phenomenon whereby one party (Alice) can steer the quantum system of another party (Bob) into distinct ensembles of states by performing different measurements on her subsystem. Here, we investigate steering in a network scenario involving n parties, where the global quantum state shared between them is produced using only two-party entangled states $|\psi_\varepsilon\rangle = \sqrt{1-\varepsilon}|0\rangle|0\rangle + \sqrt{\varepsilon}|1\rangle|1\rangle$, and mixing with ancillary separable states $|0\rangle$. We introduce three scenarios which can be straightforwardly implemented on standard quantum optics architecture, which we call random $\frac{n}{2}$-pair entanglement, random pair entanglement and semi-random pair entanglement (SRPE). For example, the SRPE scenario is where, among n parties, a fixed party shares the entangled state $|\psi_\varepsilon\rangle$ with a random party, and other $n-2$ parties are prepared in single-qubit pure states $|0\rangle$.

We derive analytically necessary and sufficient steering criteria for the states in the three scenarios under different measurement settings. Strikingly, using the SRPE construction, one party can steer any one of the $n-1$ other parties, for arbitrarily large n, using only two measurements. Then, exploiting symmetry, we study various small network configurations for three- and four-parties in the three scenarios, under different measurements and parameter ε in the state $|\psi_\varepsilon\rangle$. Motivated by these results, we investigate whether the phenomenon of collective steering could be observed in the SRPE scenario, where two parties must cooperate in order to steer a third. This is known to have applications in quantum secret sharing schemes. Using semi-definite programming techniques, we find collective steering possible, and robust to noise.

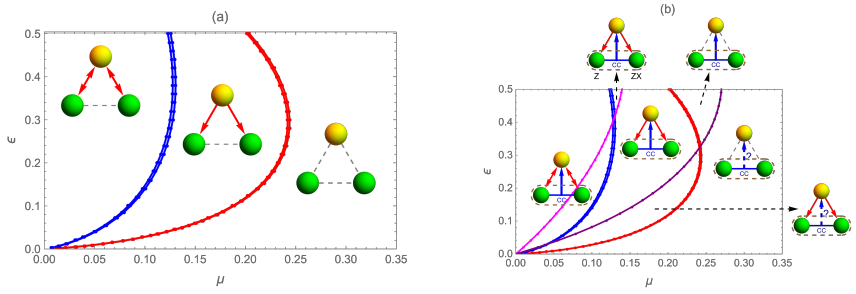

Figure 1: (**a**) Noise-tolerant steering of the tripartite SRPE state. (**b**) Collective steering of the tripartite SRPE state with noise μ. Red arrows indicate directions where steering between parties (coloured balls) is possible. The magenta (purple) curve represents collective steering bound which is obtained by Bob1 making one (two) measurement(s) and Bob2 two (three) measurements. The blue arrow denotes collective steering by Bob1 and Bob2, with classical communication (CC).