Lattice QCD Determination of Transverse Force Distributions in the Proton

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Motivation

- The Electron Ion Collider (EIC) at Brookhaven National Laboratories begins construction in 2024, with expected operation to begin in the early 2030's.
- Deep Inelastic Scattering (DIS) allows us to probe the insides of hadrons.

Motivating Question

What forces keep quarks within hadrons?

- The static quark potential predicts a constant restoring force at large distances, but is this true?

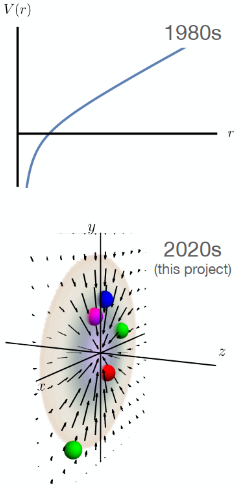

Figure: Changing ideas about QCD forces.

What is and Why Lattice QCD?

- Coupling constant for QCD is large at low energy scales.
- **Cannot use perturbation theory** to compute observables.
- Lattice QCD discretises spacetime and introduces a cut-off momenta to regulate divergent integrals.
- **Allows for numerical computation of non-perturbative observables** with systematically improvable precision.
- Calculations often performed under unphysical conditions - need to bring back to the physical point.

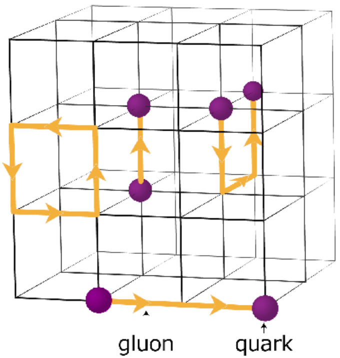

Figure: Discretisation of spacetime in Lattice QCD [1].

Transverse Forces from DIS

- Lepton scattering from transversely polarised targets allows for the extraction of higher-twist matrix elements.
- The twist-3 part of the nucleon structure function $g_2(x, Q^2)$ does not have a single particle interpretation.
- Alternative interpretation: **twist-3 matrix elements represent transverse forces** [2].

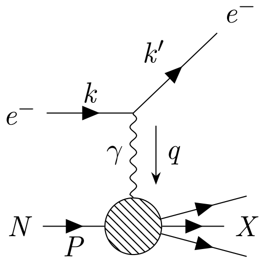

Figure: Effective Feynman diagram for DIS.

Transverse Forces from DIS

- The x^2 moment of the twist-3 part of the g_2 structure function can be related to a local light-cone matrix element.

$$3\int_{-1}^{1} dx\, x^2 \tilde{g}_2(x) = d_2 = \frac{1}{2mP^+P^+S^x} \langle P,S|\overline{\psi}(0)\gamma^+ gG^{+y}(0)\psi(0)|P,S\rangle \,. \qquad (1)$$

- Untangling the gluon field strength tensor component, we find:

$$\begin{aligned} G^{+y} &= \frac{1}{\sqrt{2}}\left(G^{0y} + G^{zy}\right) = \frac{1}{\sqrt{2}}\left(-E_c^y + B_c^x\right), \\ &= -\frac{1}{\sqrt{2}}\left[\vec{E}_c + \vec{v}\times\vec{B}_c\right]^y = -\frac{1}{\sqrt{2}} F^y! \end{aligned} \qquad (2)$$

- **Suggests a connection between d_2 and a “Colour-Lorentz” force** [3].

Developing Position-Space Densities

- Decompose our matrix element into momentum-dependent form factors, $\Phi_i(-\Delta^2)$, much like EM form factors,

$$\langle p', \lambda' | j^\mu | p, \lambda \rangle = \overline{u}(p', \lambda') \left[\gamma^\mu F_1(-\Delta^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} F_2(-\Delta^2) \right] u(p, \lambda) \quad (3)$$

- Taking the **2D Fourier Transform in the Infinite Momentum Frame yields a position-space density** [4].

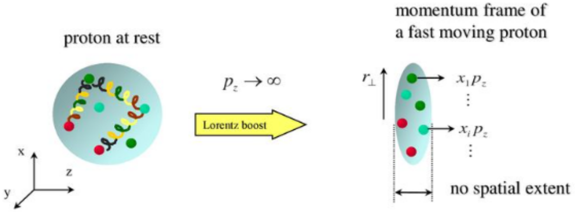

Figure: Infinite Momentum Frame kinematics [5].

Developing Position-Space Densities

$$F_1(Q^2) \xleftrightarrow{\text{2D FT}} \rho(\vec{b})$$

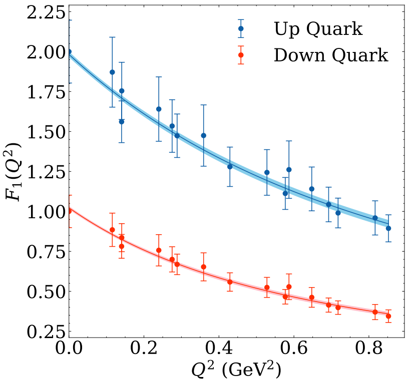

Figure: EM Dirac F_1 Form Factor.

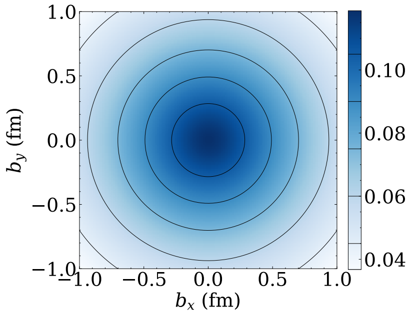

Figure: Up quark density in an unpolarised proton.

Form Factor Results

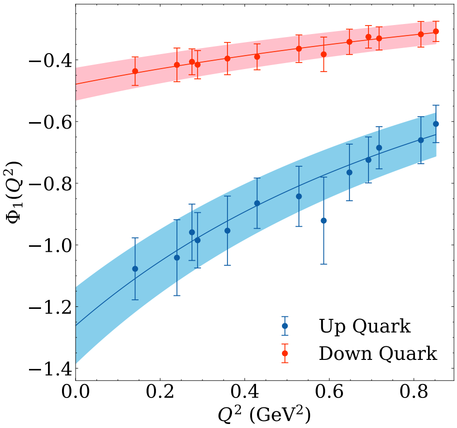

Figure: Bare Results for Φ_1 Form Factor.

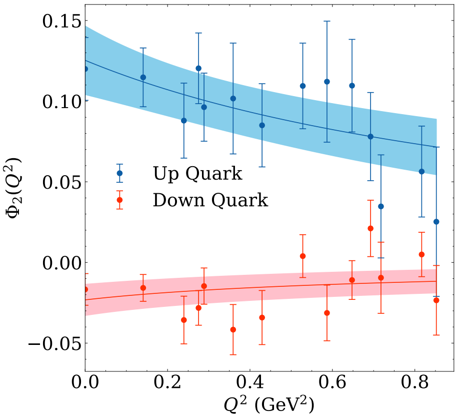

Figure: Bare Results for Φ_2 Form Factor.

Transverse Force Densities

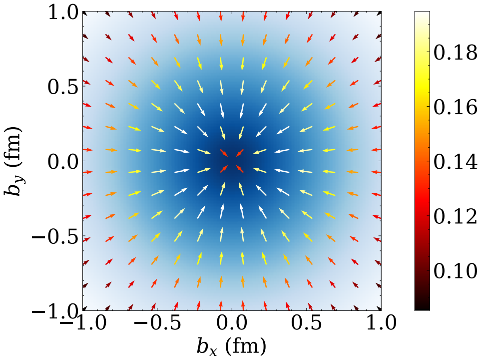

Figure: Force density for unpolarised up quark in an unpolarised proton.

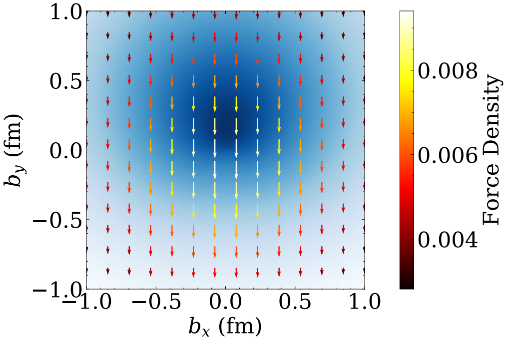

Figure: Force density for unpolarised up quark in an $\hat{x}$-polarised proton.

Discussion

- Comparing the magnitudes of the colour-Lorentz force and unpolarised quark density:

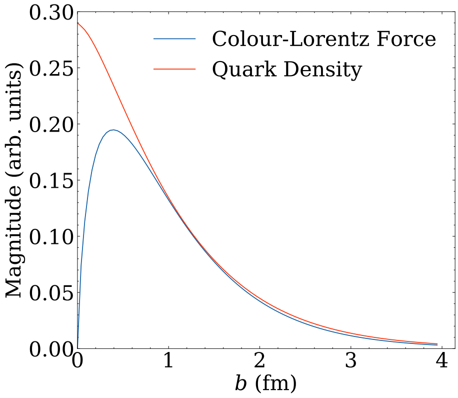

Figure: Comparison of force density and quark density profiles.

Summary and Conclusions

- Transverse force tomography is a novel perspective on forces in QCD.
- Construction of a 'Colour-Lorentz' force allows for new, interesting questions to be asked about forces in QCD.
- Lattice QCD is well-equipped to compute the required matrix elements.
- Relevant form factors have been extracted and preliminary distributions of these forces developed.
- Further control of lattice systematics and model dependence of form factors required.

References I

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Colour-Lorentz Forces

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Acknowledgements

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