THE UNIVERSITY of ADELAIDE

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

The Compton Amplitude and Structure Functions of the nucleon

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Partially based on:

Can et al., PRD102, 114505 (2020), arXiv:2007.01523 [hep-lat]

Batelaan et al., arXiv:2209.04141 [hep-lat]

in collaboration with QCDSF/UKQCD:

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Y. Nakamura (RIKEN, Kobe)

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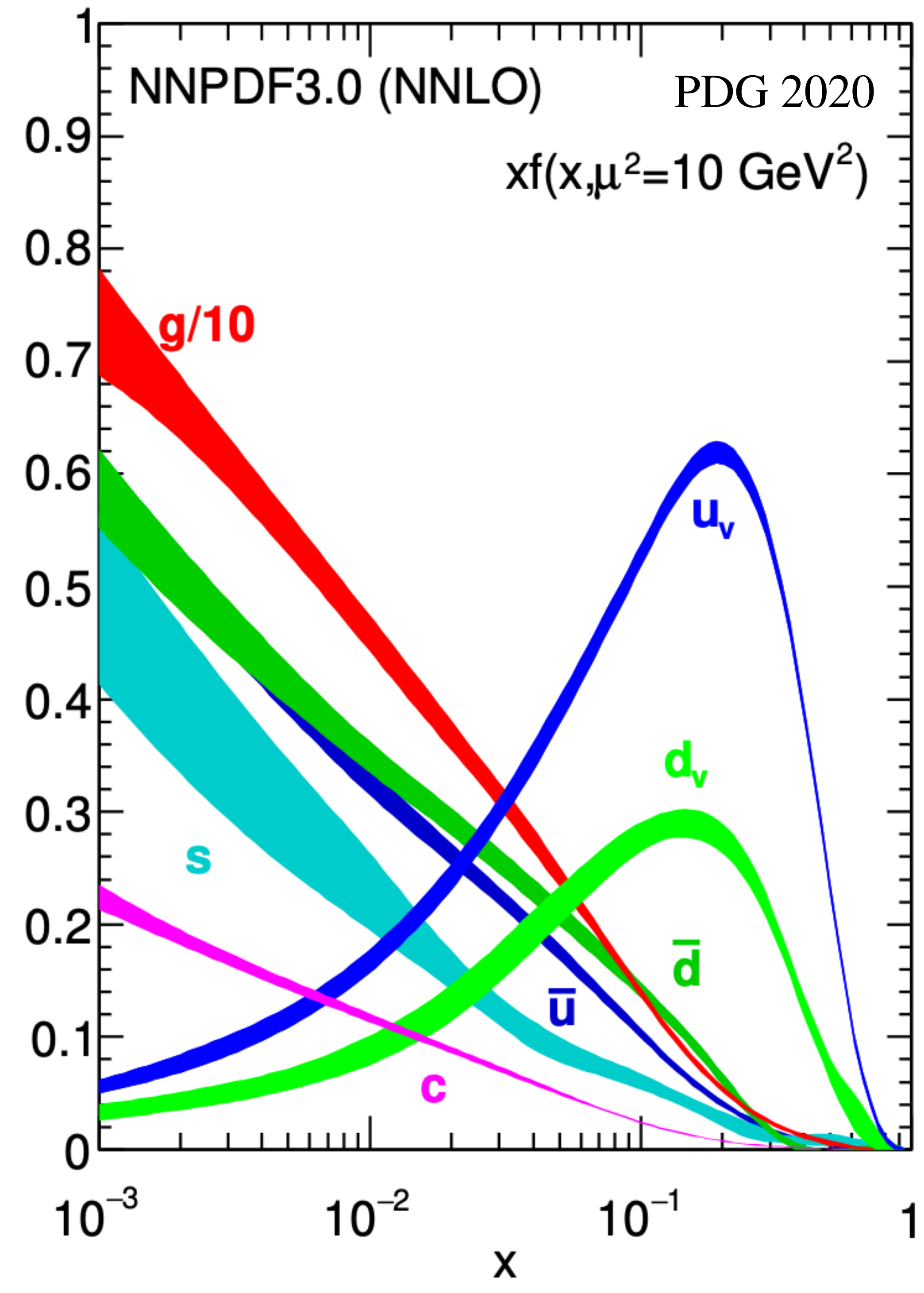
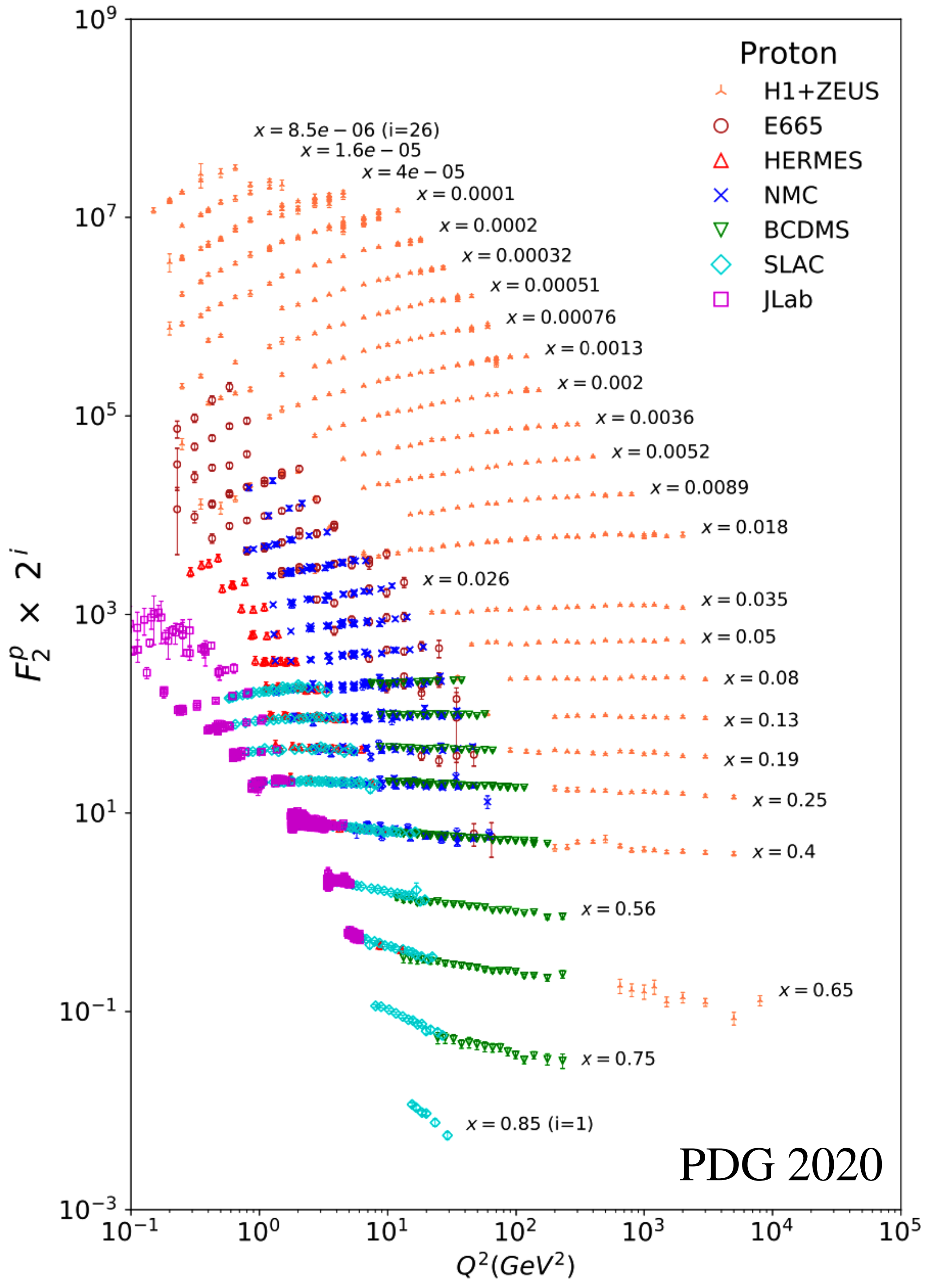
The numerical configuration generation (using the BQCD lattice QCD program)) and data analysis (using the Chroma software library) was carried on the DiRAC Blue Gene Q and Extreme Scaling (EPCC, Edinburgh, UK) and Data Intensive (Cambridge, UK) services, the GCS supercomputers JUQUEEN and JUWELS (NIC, Jülich, Germany) and resources provided by HLRN (The North-German Supercomputer Alliance), the NCI National Facility in Canberra, Australia (supported by the Australian Commonwealth Government) and the Phoenix HPC service (University of Adelaide). RH is supported by STFC through grant ST/P000630/1. PELR is supported in part by the STFC under contract ST/G00062X/1. KUC, RDY and JMZ are supported by the Australian Research Council grants DP190100297 and DP220103098.

Motivation I

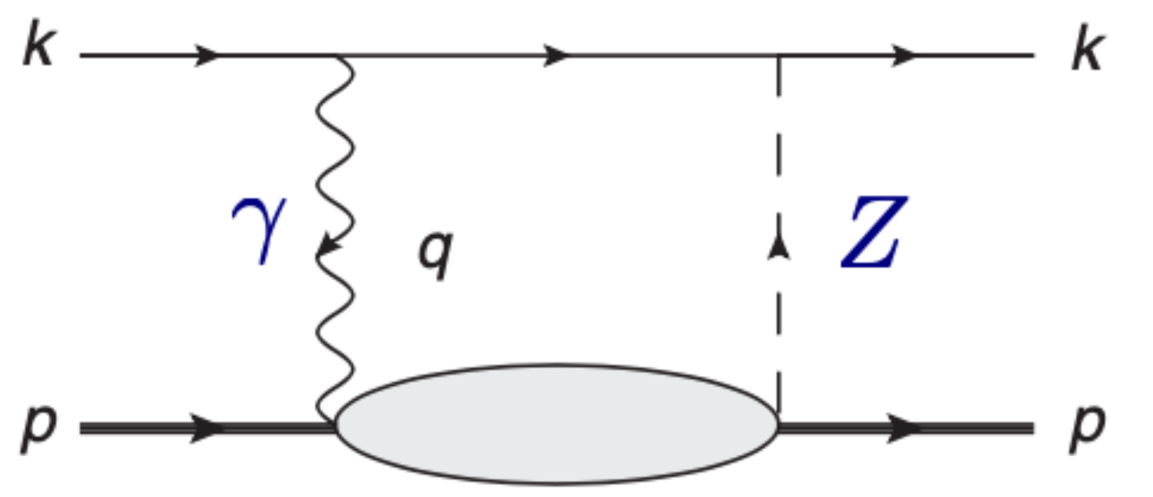
- <u>Nucleon structure</u> (leading twist)
  - Structure functions from first principles
  - Understanding the behaviour in the high- and low-x regions

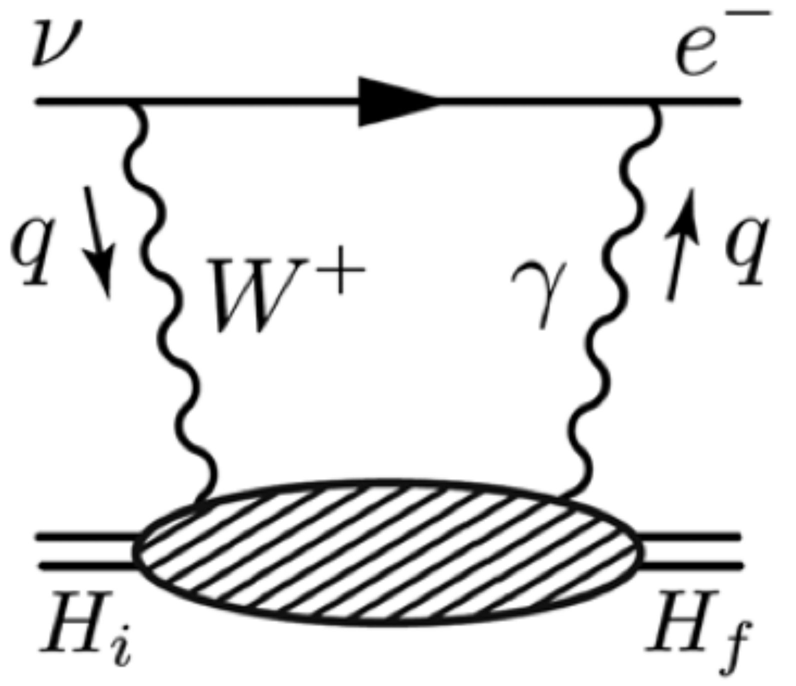


- <u>Scaling and Power corrections/ Higher twist effects</u>
  - Q^2 cuts of global QCD analyses
  - Twist-4 contributions
  - Kinematic effects



- <u>New physics searches</u>
  - Weak charge of the proton
  - $\gamma - W/Z$ interference





Motivation II

- <u>Technical issues</u>
  - Operator mixing/renormalisation issues in OPE approach in LQCD

$$\mu(Q^2) = c_2(a^2Q^2)\,v_2(a) + \frac{c_4(a^2Q^2)}{Q^2}\,v_4(a) + \cdots$$

twist-2

twist-4

$1/a^2$ divergence

mixing

  - 4-point functions are costly; harder to tackle
  - Feynman-Hellmann (FH) approach needs 2-point functions only

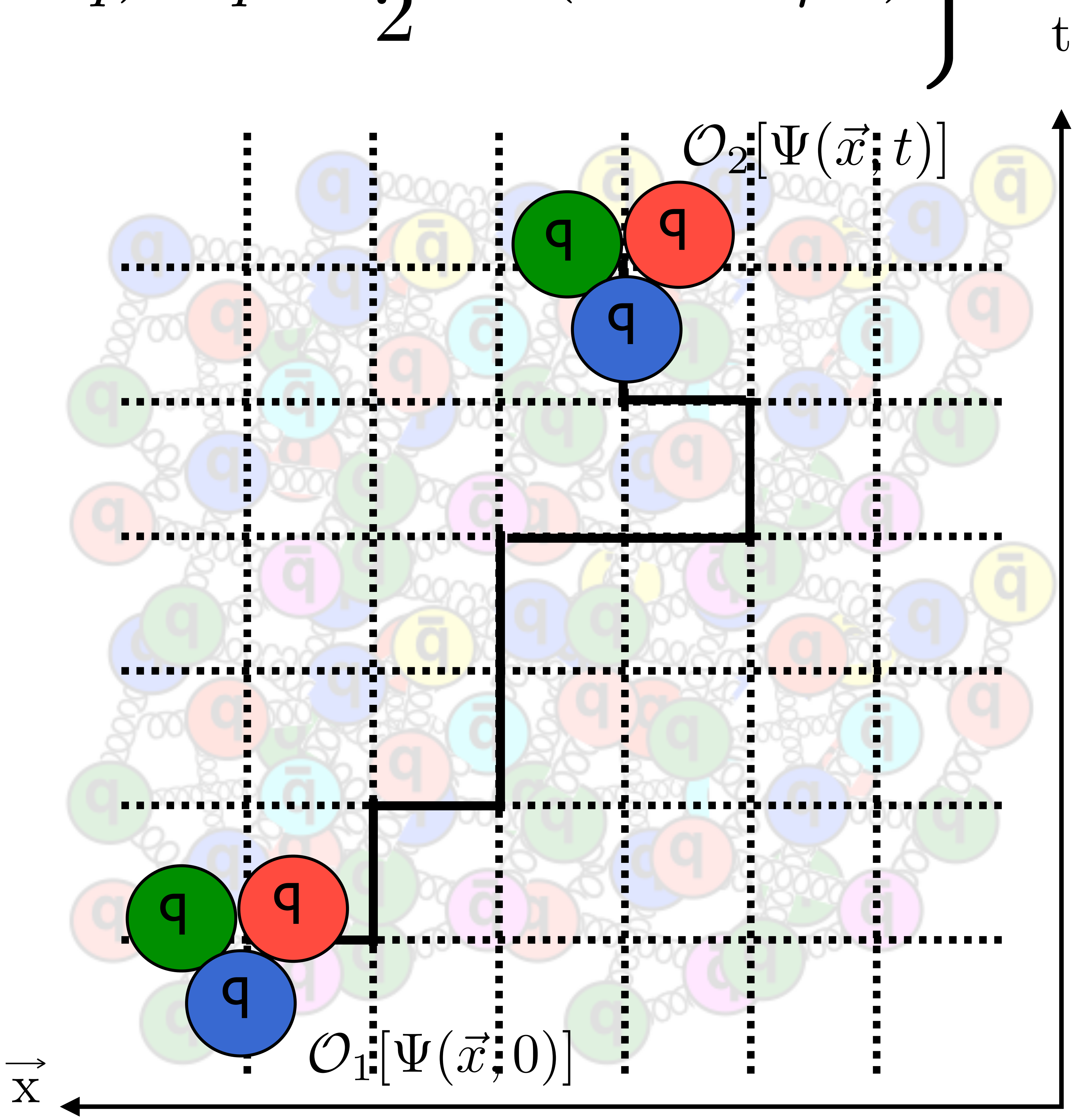
Lattice QCD

$$\mathcal{S}_{QCD}[\psi,\bar{\psi},A] = \int d^4x \left\{ \sum_q \bar{\psi}_q \left(i\gamma^\mu D_\mu - m_q \right) \psi_q - \frac{1}{2} \mathrm{Tr}\left(G^{\mu\nu} G_{\mu\nu} \right) \right\}$$

$$\langle \hat{O}_2(t)\hat{O}_1(0)\rangle = \frac{\int D[\Psi] e^{-S_E[\Psi]} O_2[\Psi(x,t)] O_1[\Psi(x,0)]}{\int D[\Psi] e^{-S_E[\Psi]}}$$

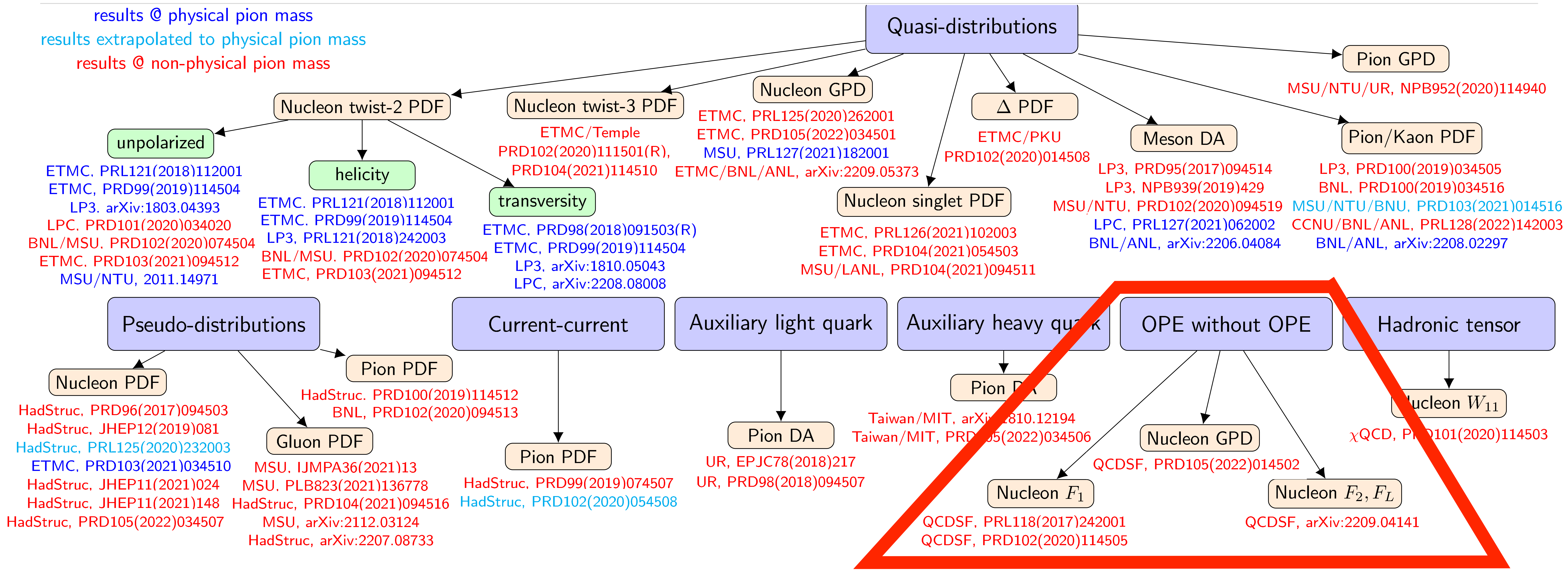
- Discretise the space-time continuum: regularises the theory
- "Measure" the observables via supercomputer simulations
  - i.e. approximate the infinite dimensional path integral

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} \mathcal{O}[\Psi]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}} = \lim_{N\to\infty} \sum_{n=1}^{N} \mathcal{O}[U_n]$$



LQCD landscape

© K. Cichy, INT-22-83W



- **QCDSF/UKQCD Collaboration**
- **F_1, F_2 and F_L**
- **Study of higher-twist**

Forward Compton Amplitude

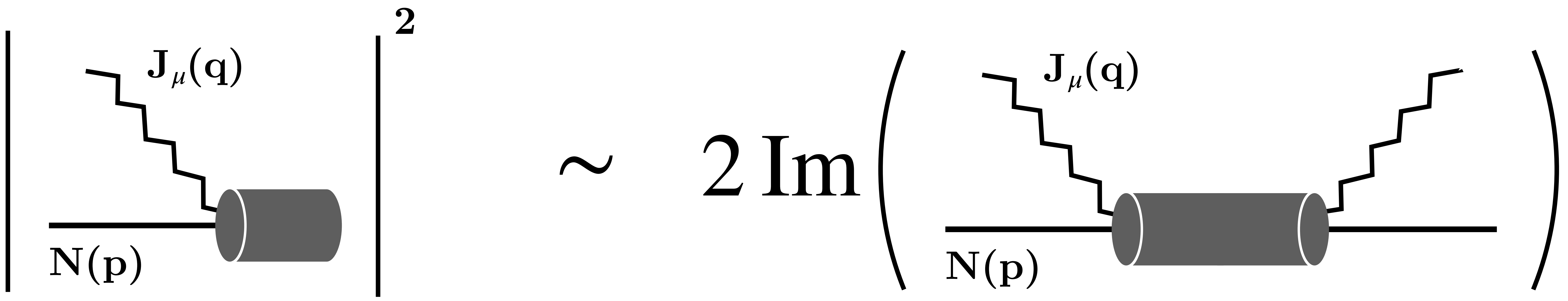
$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathscr{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$, spin avg. $\rho_{ss'} = \frac{1}{2}\delta_{ss'}$

$\omega = \frac{2p\cdot q}{Q^2}$

Same Lorentz decomposition as the Hadronic Tensor

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)



DIS Cross Section ~ Hadronic Tensor

Forward Compton Amplitude ~ Compton Tensor

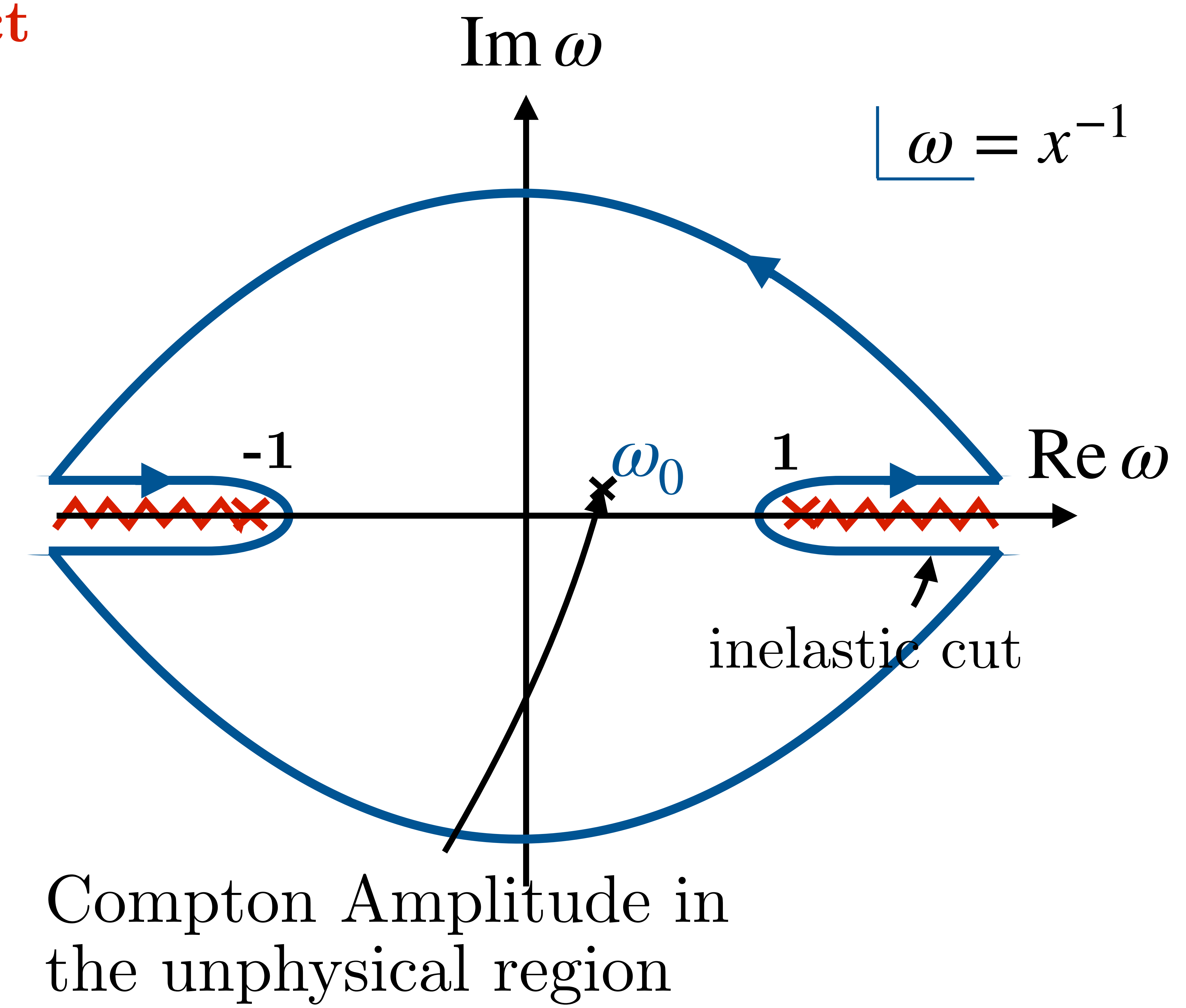
Nucleon Structure Functions

- **we can write down dispersion relations and connect Compton SFs to DIS SFs:**

$$\underbrace{\mathcal{F}_1(\omega,Q^2)-\mathcal{F}_1(0,Q^2)}_{\equiv\overline{\mathcal{F}}_1(\omega,Q^2)} = 2\omega^2\int_0^1 dx\frac{2x\,F_1(x,Q^2)}{1-x^2\omega^2-i\epsilon}$$

$$\mathcal{F}_2(\omega,Q^2) = 4\omega^2\int_0^1 dx\frac{F_2(x,Q^2)}{1-x^2\omega^2-i\epsilon}$$

$$\underbrace{\mathcal{F}_L(\omega,Q^2)+\mathcal{F}_1(0,Q^2)}_{\equiv\overline{\mathcal{F}}_L(\omega,Q^2)} = \frac{8M_N^2}{Q^2}\int_0^1 dxF_2(x,Q^2) + 2\omega^2\int_0^1 dx\frac{F_L(x,Q^2)}{1-x^2\omega^2-i\epsilon}$$



Nucleon Structure Functions

- Mellin moments

$$\omega = \frac{2p\cdot q}{Q^2} \equiv x^{-1}$$

$$\overline{\mathcal{F}}_{1,L}(\omega,Q^2) = \sum_{n=0}^{\infty} 2\omega^{2n} M_{2n}^{(1,L)}(Q^2)\ , \text{ where } M_{2n}^{(1)}(Q^2) = 2\int_0^1 dx\, x^{2n-1} F_1(x,Q^2)\ , \text{ and } \quad M_0^{(1)}(Q^2) = 0$$

$$\mathcal{F}_2(\omega,Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1} M_{2n}^{(2)}(Q^2), \text{ where } M_{2n}^{(2,L)}(Q^2) = \int_0^1 dx\, x^{2n-2} F_{2,L}(x,Q^2)\ , \text{ and } \quad M_0^{(L)}(Q^2) = \frac{4M_N^2}{Q^2} M_2^{(2)}(Q^2)$$

- $\mu = \nu = 3$ and $p_3 = q_3 = 0 \implies \mathcal{F}_1(\omega,Q^2) = T_{33}(p,q)$
- $\mu = \nu = 0$ and $p_3 = q_3 = q_0 = 0 \implies \mathcal{F}_2(\omega,Q^2) = \left[T_{00}(p,q) + T_{33}(p,q)\right] \dfrac{Q^2\omega}{2E_N^2}$

Once we have the Compton amplitude, $T_{\mu\nu}(p,q)$, we can extract the Mellin moments!

FH Theorem at 1st order

in Quantum Mechanics:

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

H_λ: perturbed Hamiltonian of the system

E_λ: energy eigenvalue of the perturbed system

ϕ_λ: eigenfunction of the perturbed system

- **expectation value of the perturbed system is related to the shift in the energy eigenvalue**

in Lattice QCD: energy shifts in the presence of a weak external field

e.g. local bilinear operator

$$S \to S(\lambda) = S + \lambda \int d^4x\, \mathcal{O}(x) \longrightarrow \bar{q}(x)\Gamma_\mu q(x) \quad , \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \ldots\}$$

real parameter

@ 1st order

$$\frac{\partial E_\lambda}{\partial \lambda} = \frac{1}{2E_\lambda} \langle 0 | \mathcal{O} | 0 \rangle$$

$\boxed{E_\lambda \to \text{spectroscopy, 2-pt function}}$

$\boxed{\langle 0|\mathcal{O}|0\rangle \to \text{determine 3-pt}}$

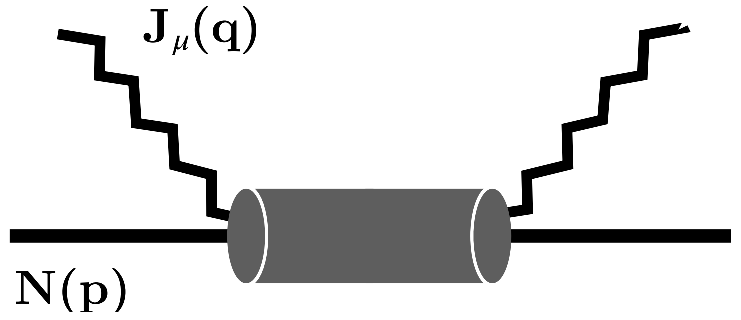
Applications:

- σ **- terms**
- **Form factors**

Compton Amplitude from FHT at 2nd order

- **unpolarised Compton Amplitude**

$$T_{\mu\mu}(p,q) = \int d^4z e^{i\mathbf{q}\cdot\mathbf{z}} \langle N(p) | \mathcal{T}\{J_\mu(z) J_\mu(0)\} | N(p)\rangle$$



- **Action modification**

local EM current

$J_\mu(z) = \sum_q e_q \bar{q}(z)\gamma_\mu q(z)$

$$S \to S(\lambda) = S + \lambda \int d^4z \left(e^{i\mathbf{q}\cdot\mathbf{z}} + e^{-i\mathbf{q}\cdot\mathbf{z}}\right) J_\mu(z)$$

- **2nd order derivatives of the 2-pt correlator, $G_\lambda^{(2)}(\mathbf{p};t)$, in the presence of the external field**

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \left(\frac{\partial^2 A_\lambda(\mathbf{p})}{\partial\lambda^2} - tA(\mathbf{p})\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right) e^{-E_N(\mathbf{p})t}$$

from spectral decomposition

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \frac{A(\mathbf{p})}{2E_N(\mathbf{p})} t e^{-E_N(\mathbf{p})t} \int d^4z (e^{iq\cdot z} + e^{-iq\cdot z}) \langle N(\mathbf{p})|\mathcal{T}\{\mathcal{J}(z)\mathcal{J}(0)\}|N(\mathbf{p})\rangle$$

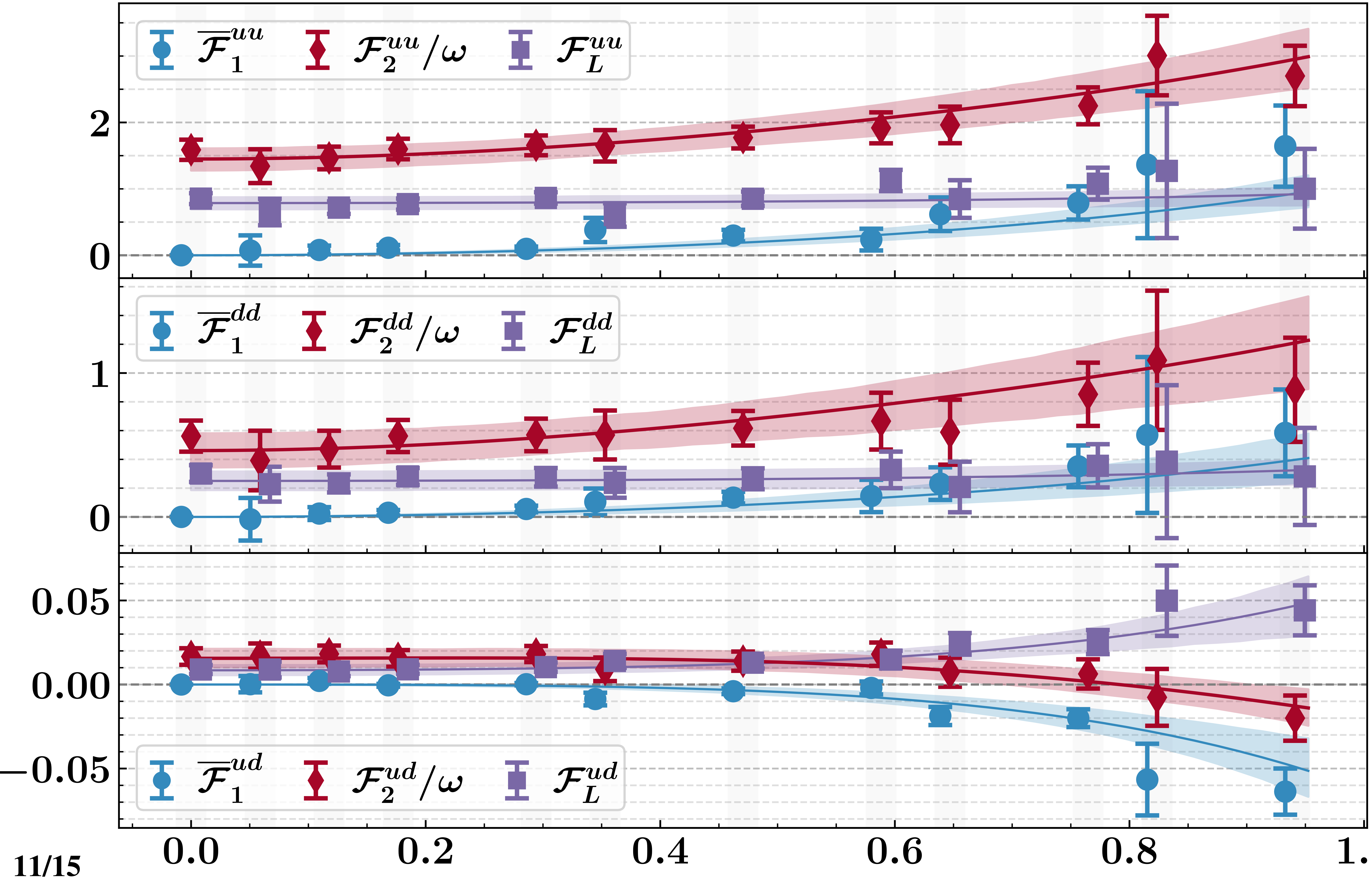
from path integral

- **equate the time-enhanced terms:**

$$\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right|_{\lambda=0} = -\frac{1}{2E_N(\mathbf{p})} \overbrace{\int d^4z (e^{iq\cdot z} + e^{-iq\cdot z}) \langle N(\mathbf{p})|\mathcal{J}(z)\mathcal{J}(0)|N(\mathbf{p})\rangle}^{T_{\mu\mu}(p,q)} + (q \to -q)$$

<u>**Compton amplitude is related to the second-order energy shift**</u>

Compton Structure Functions

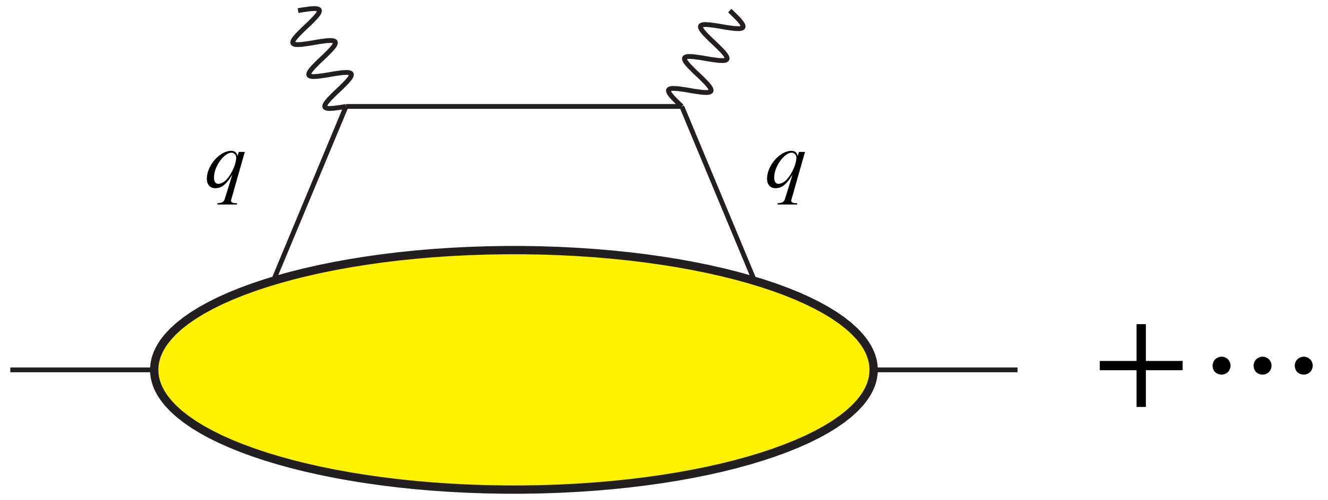


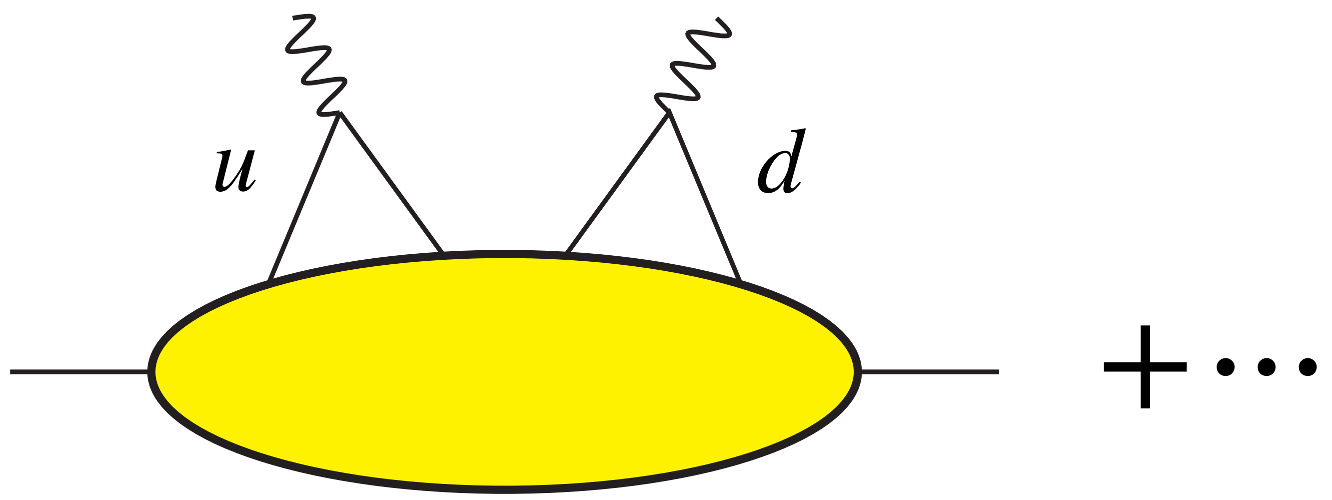
48^3x96, 2+1 flavour

$a = 0.068\,\mathrm{fm}$

$m_\pi \sim 420\,\mathrm{MeV}$

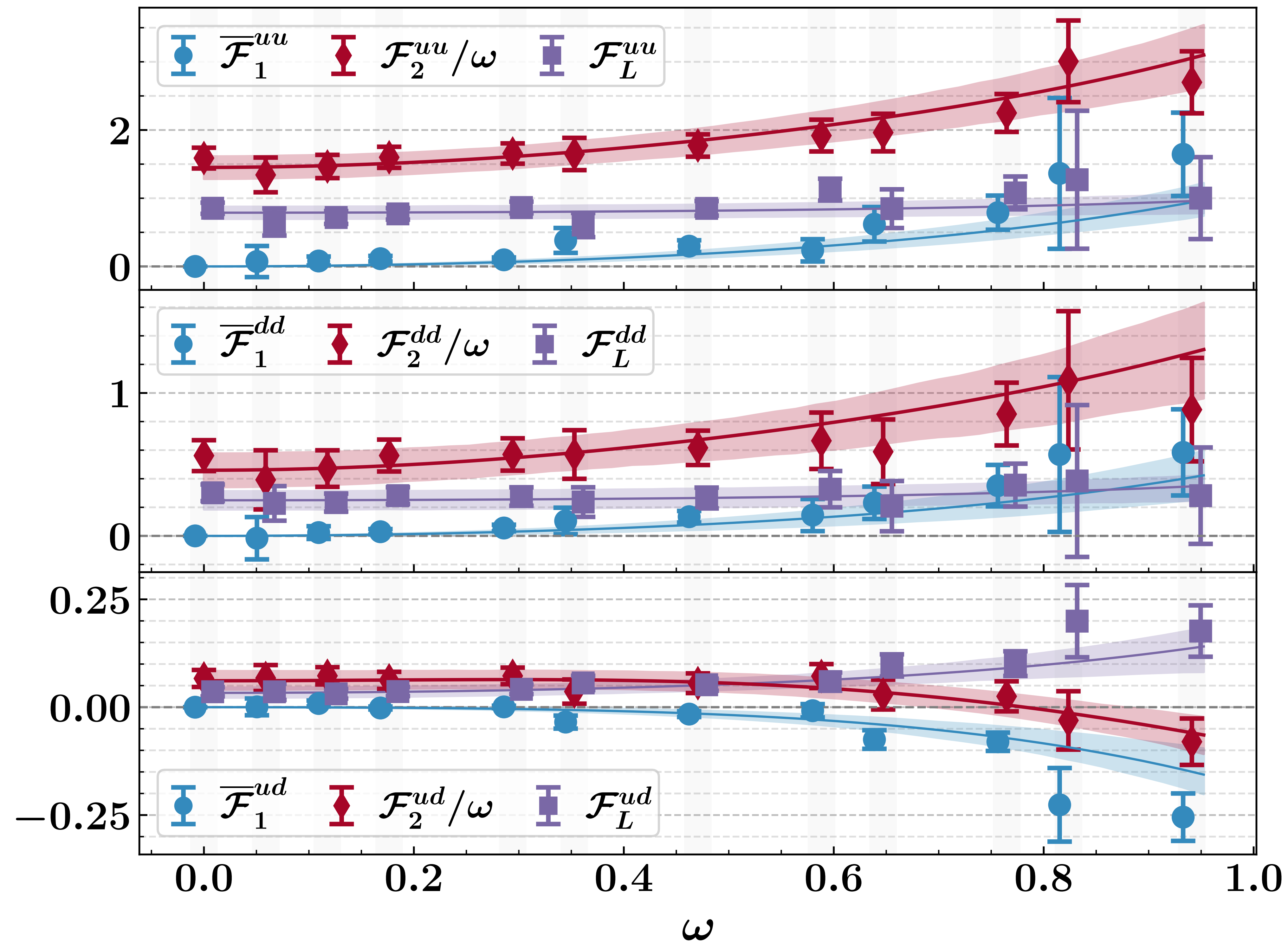
$Q^2 = 4.9\,GeV^2$





$\omega = x^{-1} = 2p \cdot q/Q^2$

Moments | Fit details



$$\overline{\mathscr{F}}_1^{qq}(\omega, Q^2) = \sum_{n=0}^{\infty} 2\omega^{2n} M_{2n}^{(1)}(Q^2)$$

$$\frac{\mathscr{F}_2^{qq}(\omega, Q^2)}{\omega} = \frac{\tau}{1+\tau\omega^2} \sum_{n=0}^{\infty} 4\omega^{2n} \left[M_{2n}^{(1)} + M_{2n}^{(L)}\right](Q^2), \text{ where } \tau = \frac{Q^2}{4M_N^2}$$

- **Enforce monotonic decreasing of moments for u and d only, not necessarily true for $u-d$**

$$M_2^{(1)}(Q^2) \geq M_4^{(1)}(Q^2) \geq \cdots \geq M_{2n}^{(1)}(Q^2) \geq \cdots \geq 0$$

We truncate at $n = 6$
No dependence to truncation order for $3 \leq n \leq 10$

- **Bayesian approach by MCMC method**

Sample the moments from Uniform priors *individually for u- and d-quark*

$$M_2(Q^2) \sim \mathscr{U}(0, 1)$$

$$M_{2n}(Q^2) \sim \mathscr{U}\left(0, M_{2n-2}(Q^2)\right)$$

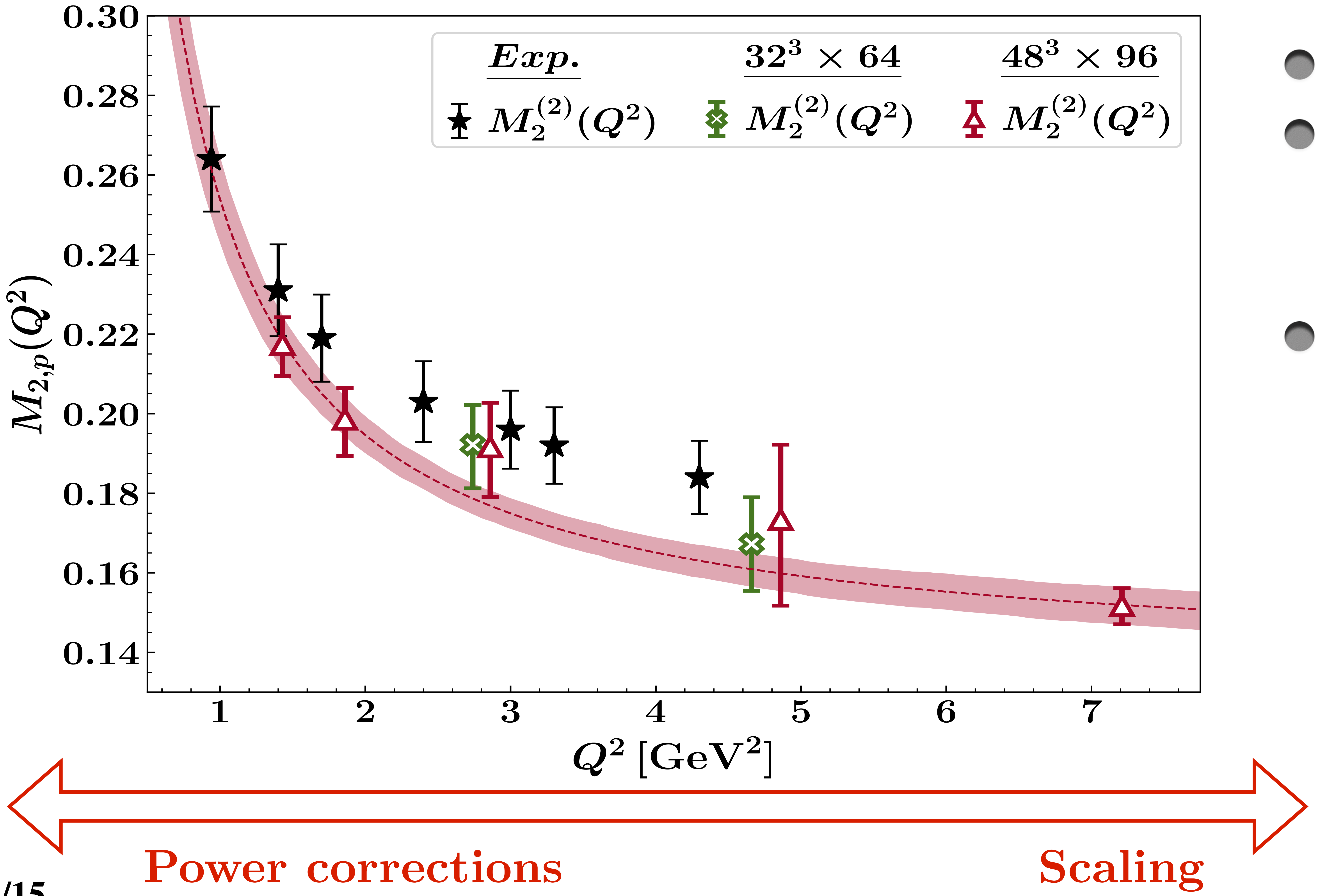
Normal Likelihood function, $exp(-\chi^2/2)$

$$\chi^2 = \sum_i \frac{\left(\overline{\mathscr{F}}_i - \overline{\mathscr{F}}^{obs}(\omega_i)\right)^2}{\sigma_i^2}$$

errors via bootstrap analysis

Moments | proton F_2

- **Unique ability to study the Q^2 dependence of the moments!**



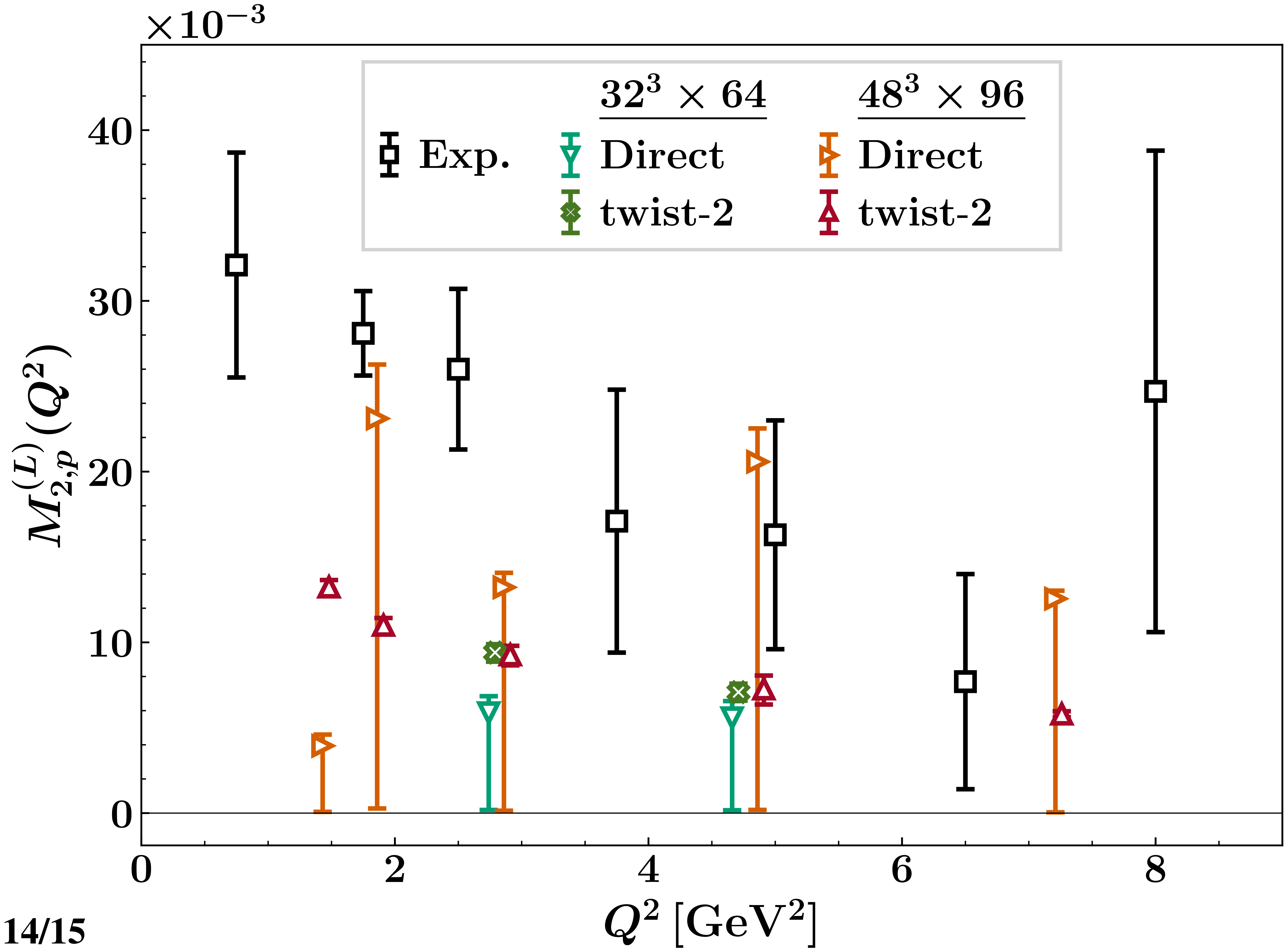
- Global PDF-fit cuts $\sim 10\,GeV^2$
- Need $Q^2 > 10\,GeV^2$ data to reliably extract partonic moments

- Power corrections below $\sim 3\,GeV^2$?
  - Modelling via
  - $M_2^{(2)}(Q^2) = M_2^{(2)} + C_2^{(2)}/Q^2$

Exp $M_2^{(2)}$: C. S. Armstrong, R. Ent, C. E. Keppel, S. Liuti, G. Niculescu, and I. Niculescu, Phys. Rev. D 63, 094008 (2001), arXiv:hep-ph/0104055.

Moments | proton F_L

- **Unique ability to study the moments of F_L!**



Possible for the first time in a lattice QCD simulation!

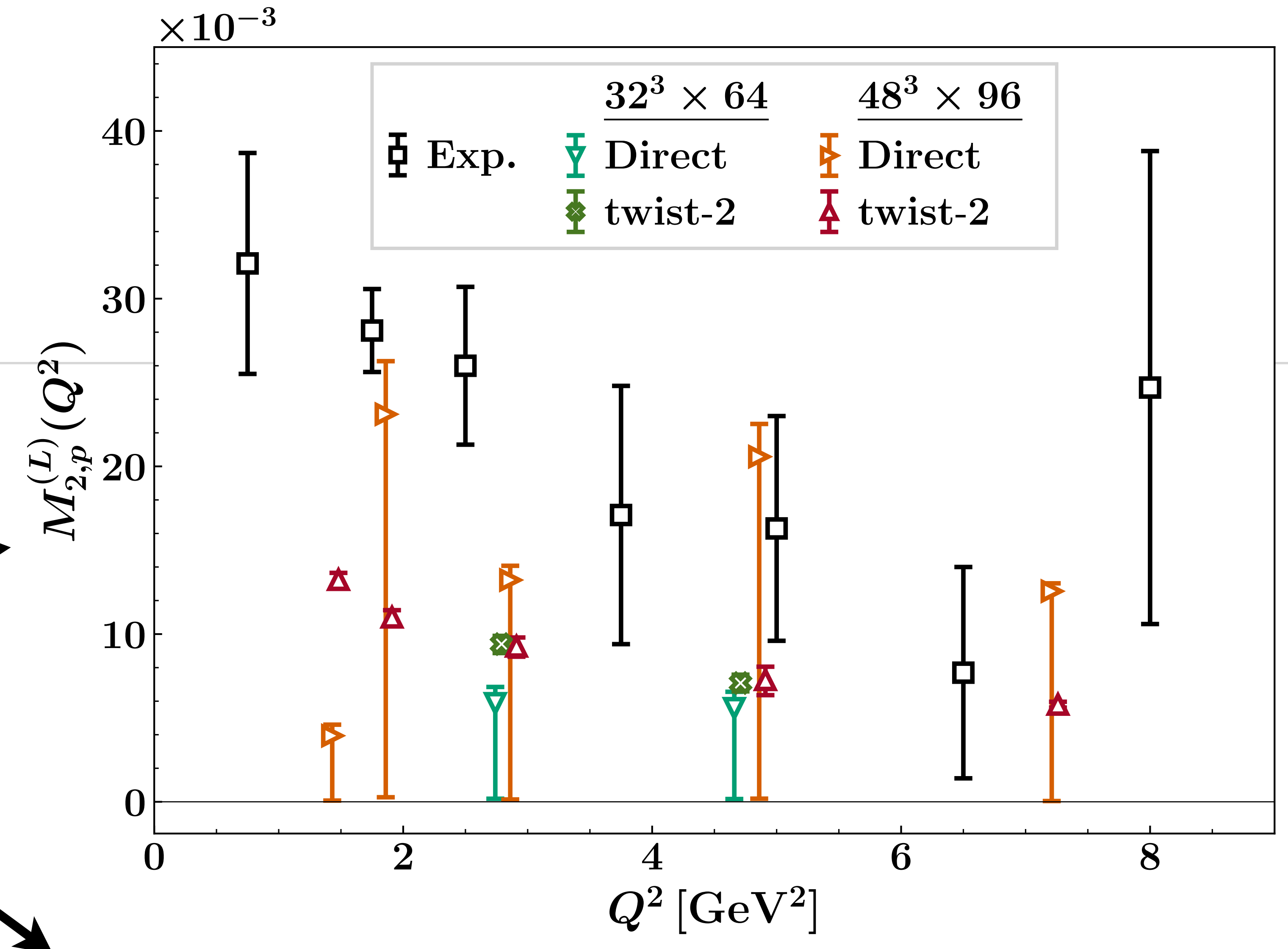
- **Direct:** Fit to data points
  - Determines upper bounds
- **Twist-2:** Use the moments of F_2:
  - $M_2^{(L),QCD}(Q^2) = \dfrac{4}{9\pi}\alpha_s(Q^2)\, M_2^{(2)}(Q^2)$
  - Better precision, good agreement with exp. behaviour

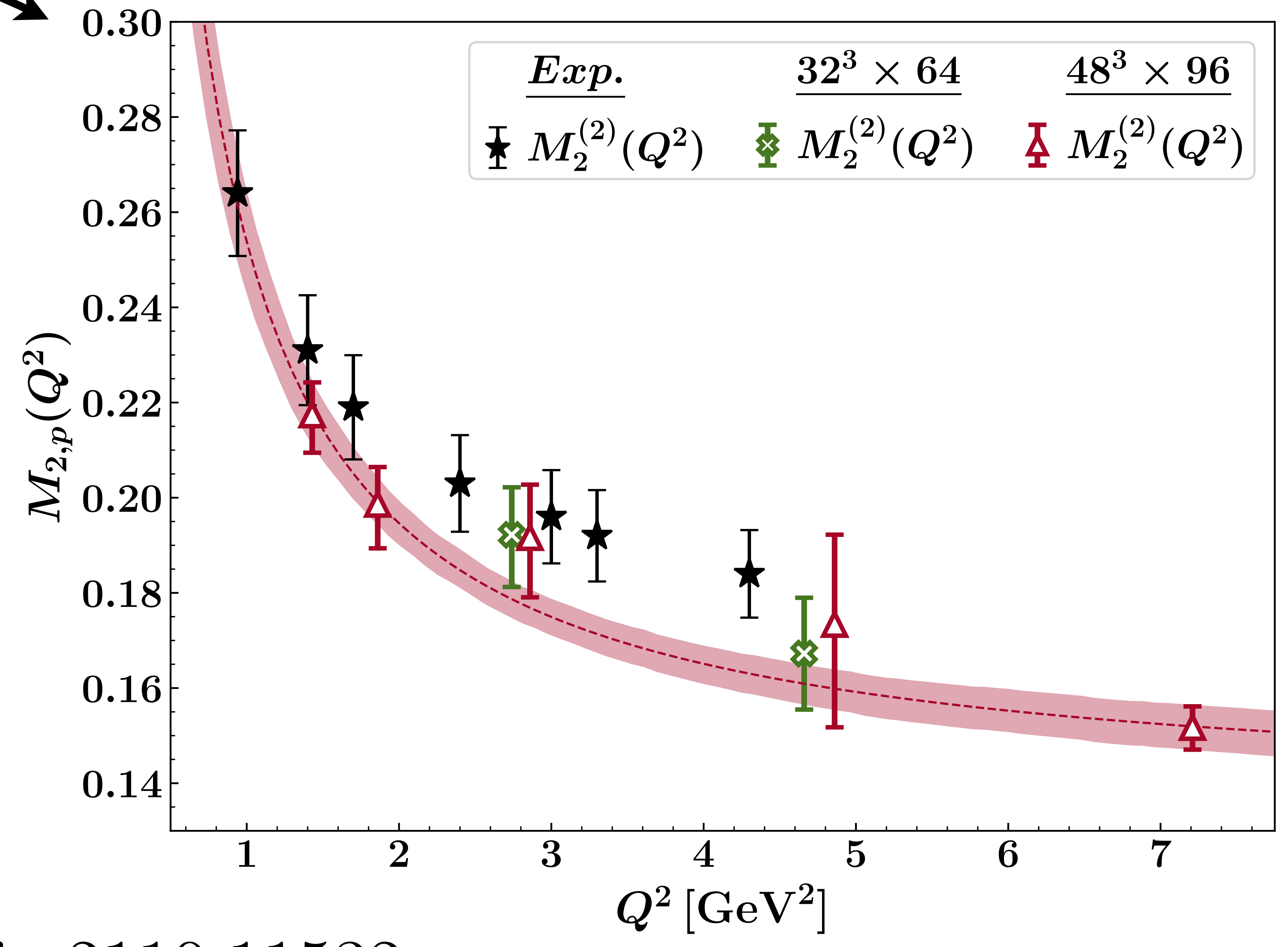
Exp Nachtmann $M_2^{(L)}$: P. Monaghan, A. Accardi, M. E. Christy, C. E. Keppel, W. Melnitchouk, and L. Zhu, Phys. Rev. Lett. 110, 152002 (2013), arXiv:1209.4542 [nucl-ex].

Summary

- A versatile approach! F_1, F_2, and F_L
- Systematic investigation of power corrections, higher-twist effects and scaling is within reach
- Overcomes the operator mixing/renormalisation issues
- Can be extended to:
  - mixed currents, interference terms
  - spin-dependent structure functions (on going)
  - GPDs: A. Hannaford-Gunn et al. Phys. Rev. D **105**, 014502, arXiv:2110.11532

    see Alec's talk on 15 Dec, 16:45 @ NUPP8





Backup

Simulation Details

QCDSF/UKQCD configurations

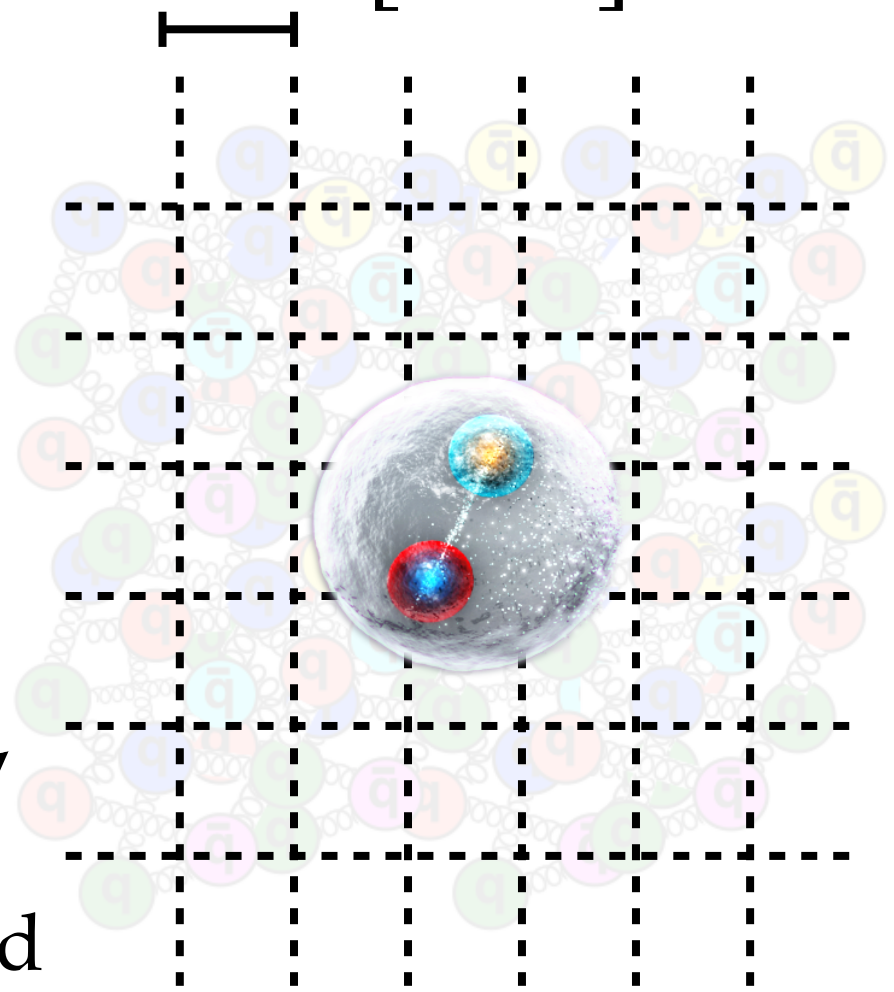
$\begin{pmatrix} 32^3 \times 64 \\ 48^3 \times 96 \end{pmatrix}$, 2+1 flavor (u/d+s)

$\beta = \begin{pmatrix} 5.50 \\ 5.65 \end{pmatrix}$, NP-improved Clover action

Phys. Rev. D 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$m_\pi \sim \begin{bmatrix} 470 \\ 420 \end{bmatrix}$ MeV, ~SU(3) sym.

$m_\pi L \sim \begin{bmatrix} 5.6 \\ 6.9 \end{bmatrix}$ $\quad a = \begin{bmatrix} 0.074 \\ 0.068 \end{bmatrix}$ fm



Unmodified QCD background

- FH implementation at the valence quark level
  - Valence u/d quark props with modified action, $S(\lambda)$
  - Local EM current insertion, $J_\mu(x) = Z_V \bar{q}(x)\gamma_\mu q(x)$
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Several current momenta in the range, $1.5 \lesssim Q^2 \lesssim 7\,GeV^2$
- Up to $\mathcal{O}(10^4)$ measurements for each pair of Q^2 and λ
- Access to a range of $\omega = 2p \cdot q/Q^2$ values for several (p, q) pairs
  - An inversion for each q and λ, varying p is relatively cheap
- Connected 2-pt correlators calculated only, <u>no disconnected</u>