Low-lying Odd-parity Nucleon Resonances in Hamiltonian Effective Field Theory

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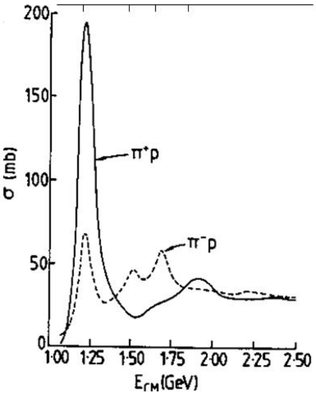
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

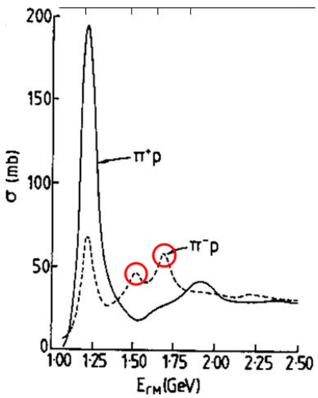
Hadron Resonances

- Manifest as "peaks" in the scattering cross-section
- Central position of the peak given as the mass, width inversely proportion to lifetime
- Identity of resonances ongoing debate
  $\Delta(1232)$: three-quark state with πN dressings
  $N^*(1440)$: generated by strong πN, $\pi\pi N$ rescattering
  $\Lambda^*(1405)$: $\overline{K}N$ bound state with $\pi\Sigma$ contributions



Odd-Parity Nucleon Resonances

- S_{11} resonances: $N^*(1535)\ \frac{1}{2}^-$ and $N^*(1650)\ \frac{1}{2}^-$
- Best studied simultaneously due to close proximity of resonance peaks
- Able to describe pole positions as dynamically generated resonances
- Able to describe $N^*(1535)$ as a quark-model-like state
- Magnetic moments from lattice QCD indicated both resonances should be three-quark states dressed by meson-baryon interactions



Hamiltonian Effective Field Theory

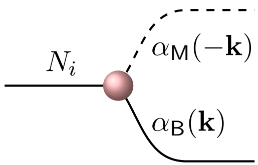
- Use experimental scattering data to constrain a Hamiltonian, and connect them to finite-volume lattice QCD
- Consider pion-Nucleon scattering in S-wave
- Construct two bare basis states: $|N_1\rangle$ with mass m_{N_1}, $|N_2\rangle$ with mass m_{N_2}
- Three scattering states:
  - $|\pi N(k)\rangle$ with energy $\omega_{\pi N}(k) = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_N^2}$
  - $|\eta N(k)\rangle$ with energy $\omega_{\eta N}(k) = \sqrt{k^2 + m_\eta^2} + \sqrt{k^2 + m_N^2}$
  - $|K\Lambda(k)\rangle$ with energy $\omega_{K\Lambda}(k) = \sqrt{k^2 + m_K^2} + \sqrt{k^2 + m_\Lambda^2}$
- Regularise with dipole form factor, with regulator parameter Λ:

$$u(k) = \frac{1}{\left(1 + (k/\Lambda)^2\right)^2}$$

Hamiltonian Effective Field Theory

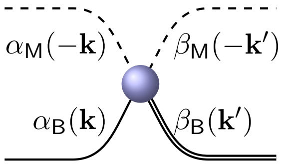
- Parametrise interaction between bare states $|N_i\rangle$ and scattering states $|\alpha(k)\rangle$ with potential $G_\alpha^{N_i}(k)$

$$G_\alpha^{N_i}(k) = \frac{\sqrt{3}\, g_\alpha^{N_i}}{2\pi f_\pi} \sqrt{\omega_{\alpha_{\mathrm{M}}}(k)}\, u(k)$$



- Parametrise interaction between scattering states $|\alpha(k)\rangle$ and $|\beta(k')\rangle$ with potential $V_{\alpha\beta}(k,k')$

$$V_{\alpha\beta}(k,k') = \frac{3\, v_{\alpha\beta}}{4\pi^2 f_\pi^2}\, \tilde{u}(k)\, \tilde{u}(k')$$



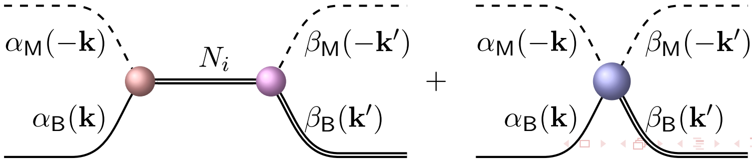
Infinite-Volume Scattering

- Formulate the coupled-channel Bethe-Salpeter equations to solve for scattering observables

$$T_{\alpha\beta}(k,k';E) = \tilde{V}_{\alpha\beta}(k,k';E) + \sum_{\gamma} \int dq\, q^2 \, \frac{\tilde{V}_{\alpha\gamma}(k,q;E)\, T_{\gamma\beta}(q,k';E)}{E - \omega_\gamma(q) + i\varepsilon} \tag{1}$$

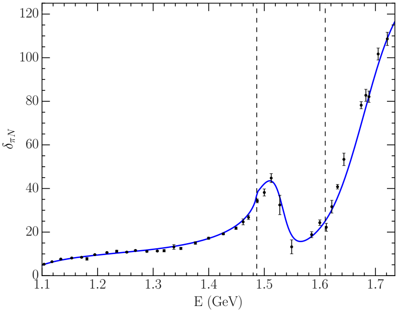
- α, β and γ run over scattering states: $|\pi N\rangle$, $|\eta N\rangle$, $|K\Lambda\rangle$

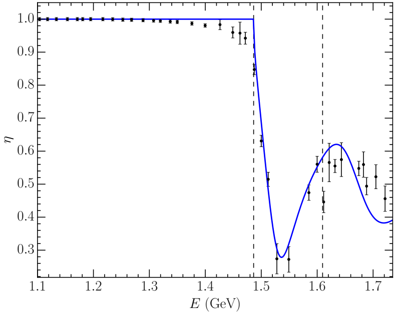
$$\tilde{V}_{\alpha\beta}(k,k';E) = \sum_{i=1}^{2} \frac{G_\alpha^{N_i}(k)\, G_\beta^{N_i}(k')}{E - m_{N_i}} + V_{\alpha\beta}(k,k') \tag{2}$$



Fitting to Scattering Data

- From T-matrix, calculate phase shifts and inelasticities and compare with experimental data.
- Vary bare masses, coupling strengths to describe this data





T-Matrix Poles

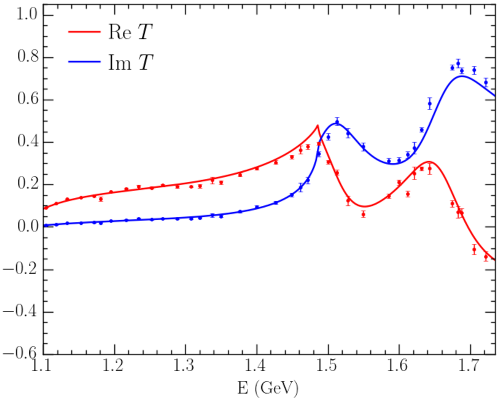
$$E_{\text{pole}} = M - \frac{\Gamma}{2}i$$

- PDG pole positions:

$$E_{N^*(1535)} = 1510 \pm 10 - (65 \pm 10)i\,,$$
$$E_{N^*(1650)} = 1655 \pm 15 - (67 \pm 18)i\,.$$

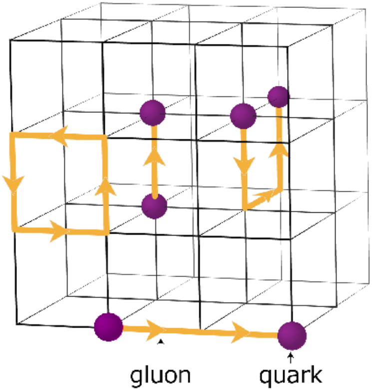
- HEFT pole positions:

$$E_1 = 1500 - 50i\,,$$
$$E_2 = 1655 - 67i\,.$$



Lattice QCD

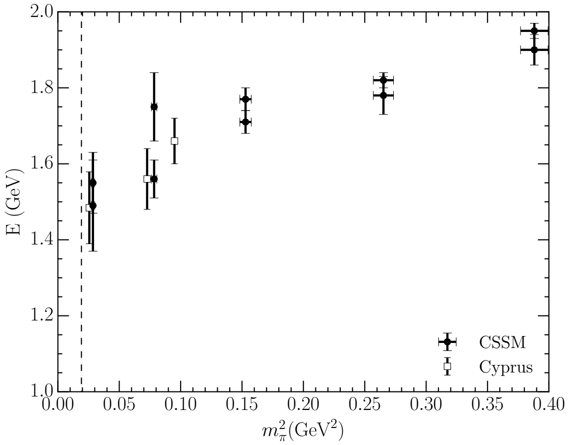
- Discretise space-time into a 4D lattice with periodic boundary conditions, of volume $L^3 \times L_{\text{time}}$
- Finite spacing a functions as a regulator
- Working at larger-than-physical pion mass reduces computational requirements



[1]

[1] 10.1051/epjconf/202024509008

Lattice QCD data at 3 fm



Finite-Volume HEFT

- Formulate HEFT in a finite-volume L^3 to compare with lattice QCD data
- Momentum is discretised

$$k_n = \frac{2\pi n}{L}\,, \quad n = \sqrt{n_x^2 + n_y^2 + n_z^2}\,, \quad n_i \in \mathbb{Z}$$

- Finite-range regulator $u(k) = \left(1 + (k/\Lambda)^2\right)^{-2}$ removes ultraviolet contributions
- Maximum momentum in the Hamiltonian matrix, k_{max}, set for $u^2(k_{\text{max}}) \sim 10^{-4}$
- Increase hadron masses with m_π^2, fit bare mass slopes to lattice QCD data

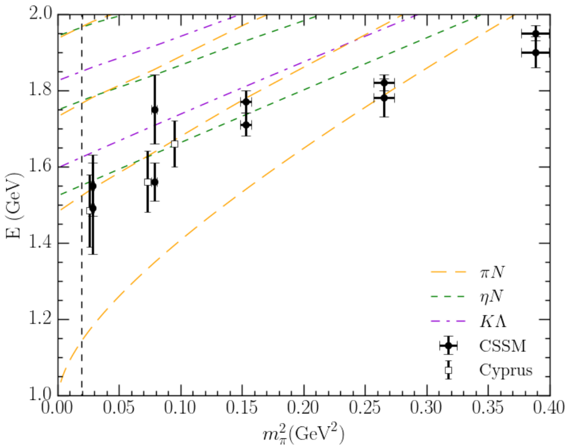
$$m_{N_i}(m_\pi^2) = m_{N_i}|_{\text{phys}} + \alpha_i \left(m_\pi^2 - m_\pi^2|_{\text{phys}}\right)$$

Finite-Volume Free Hamiltonian

- Non-interacting basis states form the free Hamiltonian

$$H_0 = \begin{pmatrix} m_{N_1} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & m_{N_2} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \omega_{\pi N}(0) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \omega_{\eta N}(0) & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \omega_{K\Lambda}(0) & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_{\pi N}(k_1) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \omega_{K\Lambda}(k_n) \end{pmatrix}$$

$L = 3$ fm: Non-interacting energy levels

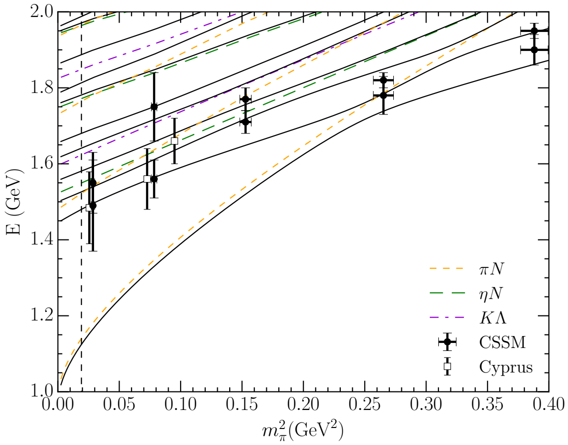


Finite-Volume Interaction Hamiltonian

$$
\begin{array}{c} \\ H_I = \end{array}
\begin{array}{c}
\begin{matrix} |N_1\rangle & |N_2\rangle & |\pi N(0)\rangle & |\eta N(0)\rangle & |K\Lambda(0)\rangle & |\pi N(k_1)\rangle & \end{matrix} \\
\begin{pmatrix}
0 & 0 & \bar{G}^{N_1}_{\pi N}(0) & \bar{G}^{N_1}_{\eta N}(0) & \bar{G}^{N_1}_{K\Lambda}(0) & \bar{G}^{N_1}_{\pi N}(k_1) & \cdots \\
0 & 0 & \bar{G}^{N_2}_{\pi N}(0) & \bar{G}^{N_2}_{\eta N}(0) & \bar{G}^{N_2}_{K\Lambda}(0) & \bar{G}^{N_2}_{\pi N}(k_1) & \cdots \\
\bar{G}^{N_1}_{\pi N}(0) & \bar{G}^{N_2}_{\pi N}(0) & \bar{V}_{\pi N \pi N}(0,0) & \bar{V}_{\pi N \eta N}(0,0) & \bar{V}_{\pi N K\Lambda}(0,0) & \bar{V}_{\pi N \pi N}(0,k_1) & \cdots \\
\bar{G}^{N_1}_{\eta N}(0) & \bar{G}^{N_1}_{\eta N}(0) & \bar{V}_{\eta N \pi N}(0,0) & \bar{V}_{\eta N \eta N}(0,0) & \bar{V}_{\eta N K\Lambda}(0,0) & \bar{V}_{\eta N \pi N}(0,k_1) & \cdots \\
\bar{G}^{N_1}_{K\Lambda}(0) & \bar{G}^{N_1}_{K\Lambda}(0) & \bar{V}_{K\Lambda \pi N}(0,0) & \bar{V}_{K\Lambda \eta N}(0,0) & \bar{V}_{K\Lambda K\Lambda}(0,0) & \bar{V}_{K\Lambda \pi N}(0,k_1) & \cdots \\
\bar{G}^{N_1}_{\pi N}(k_1) & \bar{G}^{N_2}_{\pi N}(k_1) & \bar{V}_{\pi N \pi N}(k_1,0) & \bar{V}_{\eta N \pi N}(k_1,0) & \bar{V}_{K\Lambda \pi N}(k_1,0) & \bar{V}_{\pi N \pi N}(k_1,k_1) & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

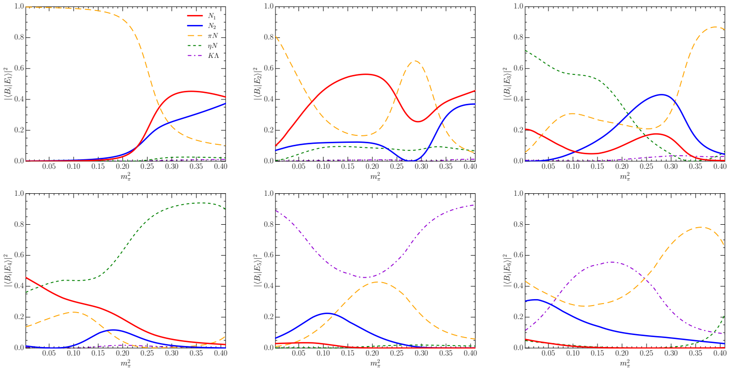
- Solve the eigenvalue equation for $H = H_0 + H_I$

$L = 3$ fm: Interacting Energy Levels

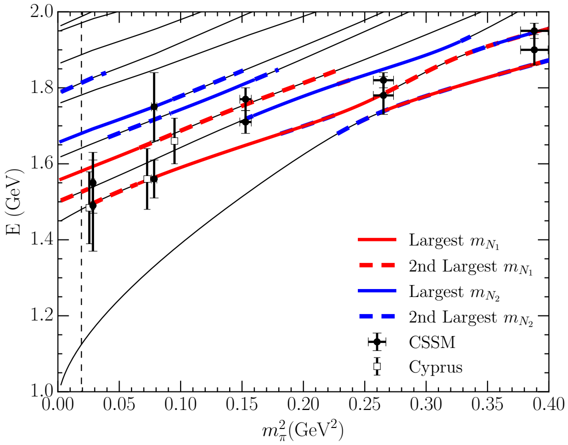


$L = 3$ fm: Eigenvectors

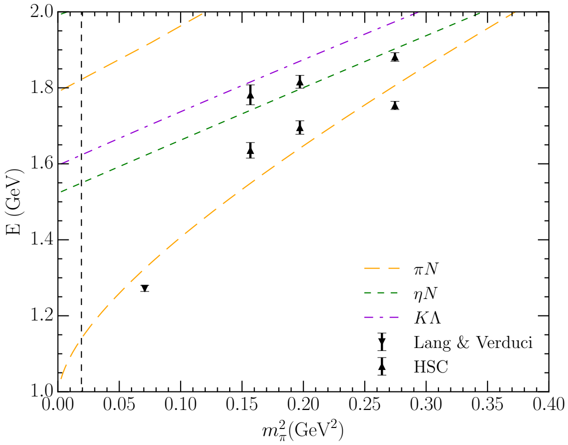
- Eigenvectors $\langle B_j | E_i \rangle$ describe contribution of basis state $|B_j\rangle$ to eigenvalue $|E_i\rangle$



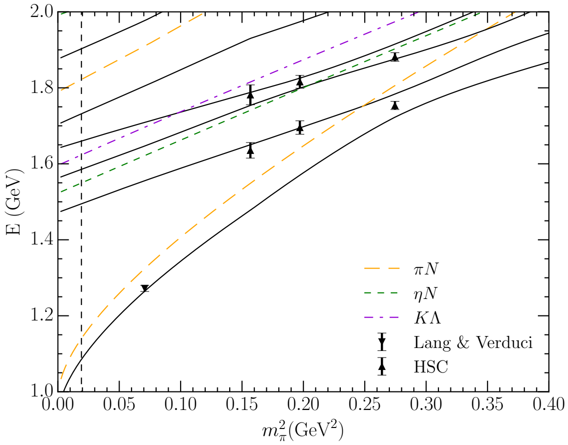
$L = 3$ fm: Bare State-Dominated Energy Levels



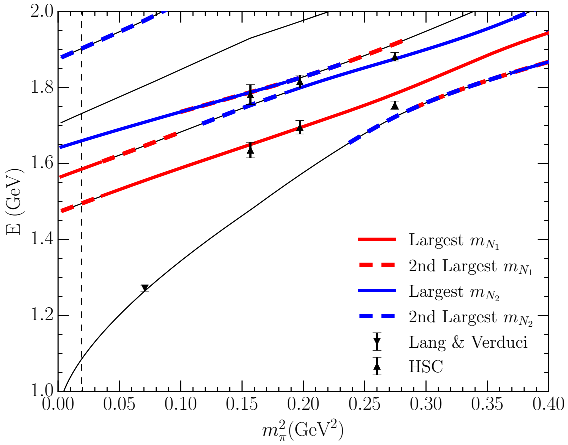
$L = 2$ fm: Non-interacting Energy Levels



$L = 2$ fm: Interacting Energy Levels



$L = 2$ fm: Bare State-Dominated Energy Levels



Conclusion

- Identity of resonances resolved by bridging experiment and lattice QCD
- Able to fit to experimental results using two bare-basis states in HEFT, describing scattering data and PDG poles
- Constructing a finite-volume Hamiltonian, able to describe 3 fm lattice QCD data by fitting the bare mass slopes
- Using these fits, able to correctly predict the 2 fm data
- Interpretation of odd-parity nucleons as three-quark states with meson-baryon dressings is consistent with both experiment and lattice QCD