The Quark-Gluon Interactions in Low Energies

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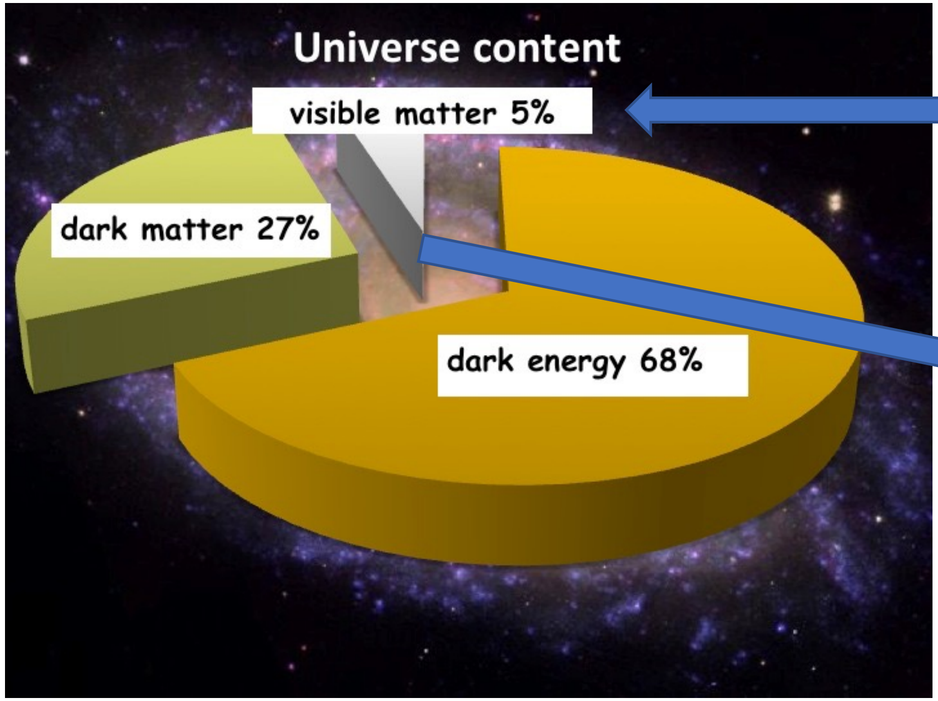
ADELAIDE

AIP- AUSTRALIAN INSTITUTE OF PHYSICS CONGRESS

MASS is a MYSTERIOUS CONCEPT!



Protons , neutrons, electrons … : it is us

0.1% of visible matter is due to the "HIGGS" mechanism

What about the remaining of the visible matter?

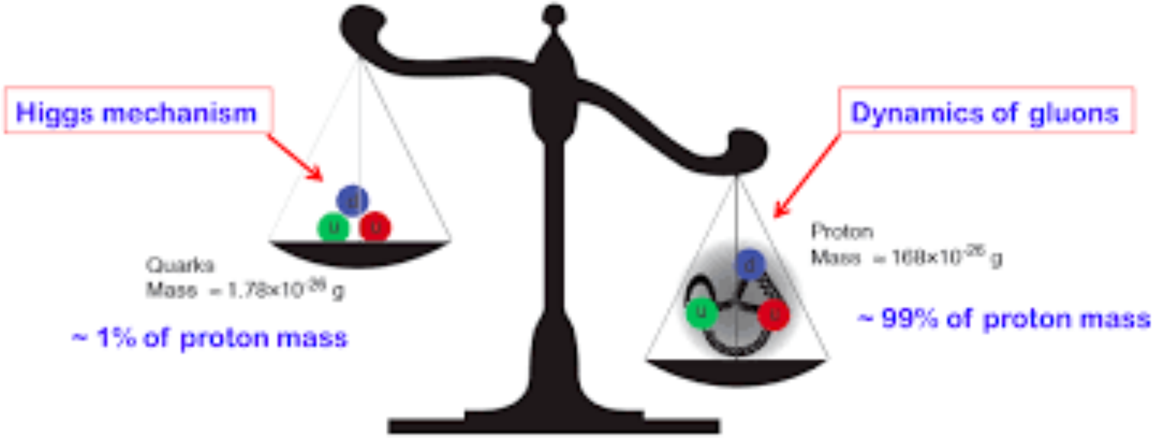
The rest emerges from the interactions to keep quarks together inside hadrons

Visible world: mainly made of light quarks

Existence of our Universe:

Proton (uud) : massive and stable

Proton mass ~ 940 MeV (~1 GeV)



QCD

Gluon Field · Quark Field · Quark Mass

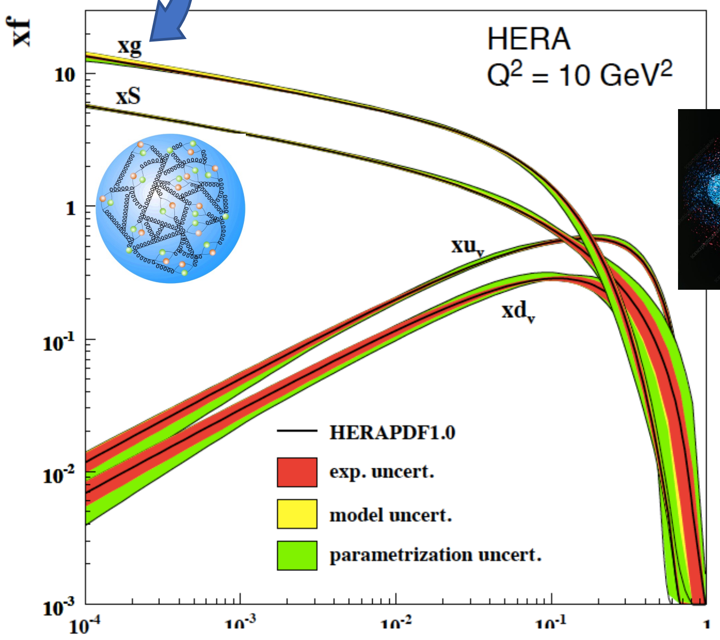
$$\mathcal{L}_{QCD} = -g\overline{\Psi}_i\gamma^\mu A^a_\mu T^a_{ij}\Psi_j + \overline{\Psi}_i(i\not{\partial} - m)\Psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \boxed{g f^{abc} A^b_\mu A^c_\nu}$$

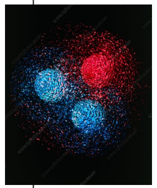
Non-Abelian

Gluons and sea quarks dominate the proton structure at x<0.1

Density of quarks or gluons inside the proton



Fraction of overall proton momentum carried by q,g



Confinement and Mass Generation are IR dynamics of QCD

NONPERTURBATIVE

SDE, FRG, Lattice QCD

Asymptotic Freedom is UV dynamics of QCD

PERTURBATIVE

Proton Size ~10^-15m~ 1fm

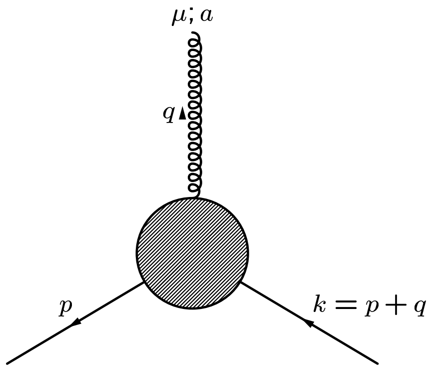
Confinement ~ 1fm

Quark and Gluon Size ~10^-17m

Asymptotic freedom ~ 1/10 fm

Schwinger-Dyson (Gap) Equation:

- Inserting gluon propagator in gap equation with a bare vertex gives insufficient DCSB
- Abelian (Ball-Chiu) vertex also gives insufficient enhancement
- Nontrivial tensor structure is crucial for hadron physics
- Non-perturbative running coupling (Effective charge) most naturally defined from quark-gluon vertex?



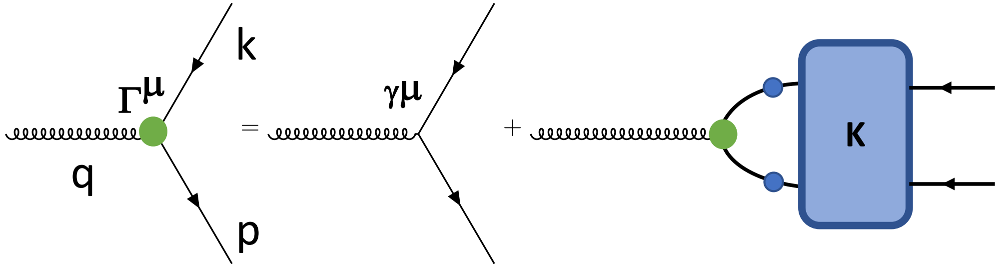
QUARK-GLUON VERTEX

We study the quark-gluon vertex in the limit of vanishing gluon momentum using lattice QCD with two flavors for several lattice spacings, volumes and quark masses

A. Kızılersü, O. Oliveira, P. J. Silva, J.-I. Skullerud, A. Sternbeck, Phys. Rev. D 103 (11) (2021) 114515

Non-Perturbative Quark-Gluon Vertex:

$$\left(\Lambda^a_\mu\right)^{ij}_{\beta\rho} = t^a_{ij}\ \left(\Lambda_\mu\right)_{\beta\rho} =$$



$$\mathbf{\Lambda^\mu_F(p,k,q)} = \sum_{\mathbf{i=1}}^{\mathbf{4}} \lambda^{\mathbf{i}}(\mathbf{p^2,k^2,q^2},\xi,\mathbf{m})\, \mathbf{L^\mu_i(p,k)} + \sum_{\mathbf{i=1}} \tau^{\mathbf{i}}(\mathbf{p^2,k^2,q^2},\xi,\mathbf{m})\, \mathbf{T^\mu_i(p,k)}$$

Non-Transverse Part

Transverse Part

$$L_{1,\mu} = \gamma_\mu$$
$$L_{2,\mu} = -\ \not{P}\, P_\mu$$
$$L_{3,\mu} = -iP_\mu$$
$$L_{4,\mu} = -i\sigma_{\mu\nu}\, P_\nu$$

$$T_{1,\mu} = -i\ell_\mu$$
$$T_{2,\mu} = -\ \not{P}\, l_\mu$$
$$T_{3,\mu} = -iP_\mu$$
$$T_{4,\mu} = -i\left[q^2\,\sigma_{\mu\nu}P_\nu + 2q_\mu\sigma_{\nu\lambda}p_\nu k_\lambda\right]$$
$$T_{5,\mu} = -i\sigma_{\mu\nu}\, q_\nu$$
$$T_{6,\mu} = (q\cdot P)\gamma_\mu - \not{q}\, P_\mu$$
$$T_{7,\mu} = -\frac{i}{2}(q\cdot P)\sigma_{\mu\nu}\, P_\nu - iP_\mu\sigma_{\nu\lambda}p_\nu k_\lambda$$
$$T_{8,\mu} = -\gamma_\mu\,\sigma_{\nu\lambda}\, p_\nu k_\lambda$$

with $P_\mu \equiv p_\mu + k_\mu,$ $\qquad \ell_\mu \equiv (p\cdot q)\ k_\mu - (k\cdot q)\ p_\mu$

$q =$ gluon momentum $\qquad k, p =$ quark momenta

Non-Perturbative Quark-Gluon Vertex:

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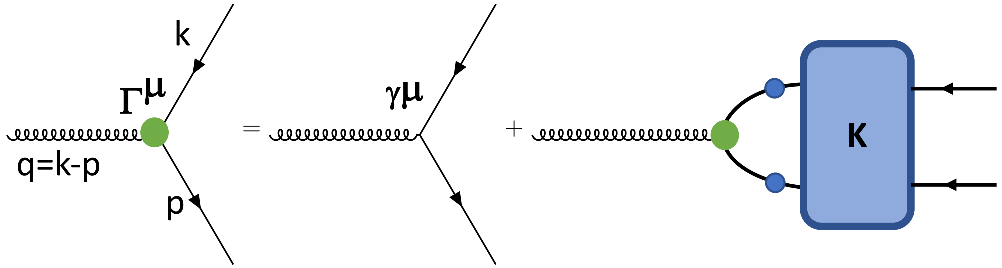


Slavnov-Taylor Identities

Normal STI

$$q_\mu \Lambda^\mu(p,q,k) = G_h(q^2)\ \left[\bar{H}(k,-p,-q)S^{-1}(k) - S^{-1}(p)H(-p,k,-q)\right]$$

Quark-Gluon Vertex

Ghost Dressing function

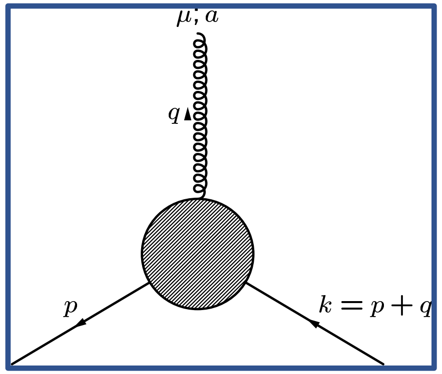
Ghost-Quark Scattering Kernel

Quark Propagator

Transverse STI

$$\begin{aligned} iq^\mu \Lambda^\nu_V(p_1,p_2) - iq^\nu \Lambda^\mu_V(p_1,p_2) &= S_F^{-1}(p_1)\sigma^{\mu\nu} + \sigma^{\mu\nu} S_F^{-1}(p_2) \\ &+ 2m\Lambda^{\mu\nu}_T(p_1,p_2) \\ &+ (p_{1\lambda} + p_{2\lambda})\,\epsilon^{\lambda\mu\nu\rho}\Lambda_{A\rho}(p_1,p_2) \\ &- \int \frac{d^4k}{(2\pi)^4} 2k_\lambda\, \epsilon^{\lambda\mu\nu\rho}\,\Lambda_{A\rho}(p_1,p_2;k) \end{aligned}$$

Non-Perturbative Quark-Gluon Vertex:



Transverse Projection

$D_{\mu\nu}^{-1}$ does not exist, so we will be looking at transverse projection

$$\tilde{\Lambda}^T_\mu(p,k,q) = P^T_{\mu\nu}(q)\Lambda_\nu = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Lambda_\nu(p,k,q)$$

Transverse Projected Form Factors :

$$\lambda_1' = \lambda_1 - q^2\tau_3 \quad ; \quad \lambda_2' = \lambda_2 - \frac{q^2}{2}\tau_2 \quad ; \quad \tau_5' = \tau_5' \quad ; \quad \tau_6' = \tau_6'$$

$$\lambda_3' = \lambda_3 - \frac{q^2}{2}\tau_1 \quad ; \quad \lambda_4' = \lambda_4 + q^2\tau_4 \quad ; \quad \tau_7' = \tau_7' \quad ; \quad \tau_8' = \tau_8'$$

J.Skullerud, A.Kizilersu, JHEP09(2002)013
J. Skullerud,P. Bowman, A.Kizilersu, D.Leinweber,A.Williams, JHEP04(2003)047

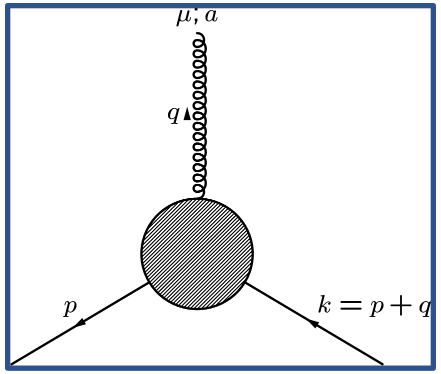
Abelian Non-Transverse Vertex :

$$\lambda_1 = \frac{1}{2}\left(A(p^2) + A(k^2)\right) \quad ; \quad \lambda_2 = \frac{A(p^2) - A(k^2)}{2\,(p^2 - k^2)}$$

$$\lambda_3 = \frac{B(p^2) - B(k^2)}{(p^2 - k^2)} \quad ; \quad \lambda_4 = 0$$

$$S_F(p) = \frac{1}{i\,\not{p}A(p^2) + B(p^2)}$$

Form Factor Extraction



Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$

$$(\overline{\Lambda}^a_\mu) = -ig_0 \left(\lambda_1 \; [\gamma_\mu] + \lambda_2 \; [-4 \; \not{p} p_\mu] + \lambda_3 \; [-2ip_\mu] \right)$$

Covariant Form factors in Continuum:

- $\lambda_1 \;=\; \frac{1}{(-ig_0)} \left\{ \frac{1}{3} \left[\mathrm{Tr}_4(\gamma_\mu \overline{\Lambda}_\mu) - \frac{p_\mu p_\nu}{p^2} \mathrm{Tr}_4(\gamma_\nu \overline{\Lambda}_\mu) \right] \right\}$
- $\lambda_2 \;=\; \frac{1}{(-ig_0)} \left\{ \frac{1}{3p^2} \left[\mathrm{Tr}_4(\gamma_\mu \overline{\Lambda}_\mu) - 4 \frac{p_\mu p_\nu}{p^2} \mathrm{Tr}_4(\gamma_\nu \overline{\Lambda}_\mu) \right] \right\}$
- $\lambda_3 \;=\; \frac{1}{(-ig_0)} \left\{ \frac{i}{2} \frac{p_\mu}{p^2} \mathrm{Tr}_4(I \, \overline{\Lambda}_\mu) \right\}$

ν=μ

P_μ=0

ν ≠ μ

Non-covariant Form factors in Continuum :

- $\lambda_1 \;=\; \frac{1}{(-ig_0)} \left\{ \left[Tr_4(\gamma_\alpha \overline{\Lambda}_\mu) \right] \Big|_{\substack{\alpha=\mu \\ p_\mu = 0}} \right\}$
- $\lambda_2 \;=\; \frac{1}{(-ig_0)} \left\{ -\frac{1}{4\,p^2} \frac{p_\alpha p_\mu}{p^2} \left[Tr_4(\gamma_\alpha \overline{\Lambda}_\mu) \Big|_{\alpha \neq \mu} \right] \right\}$
- $\lambda_3 \;=\; \frac{1}{(-ig_0)} \left\{ \frac{i}{2} \frac{p_\mu}{p^2} Tr_4(I \, \overline{\Lambda}_\mu) \right\}$

MOM Renormalisation: $\lambda_1^R(\mu^2, 0, \mu^2) = 1$ $\;\Rightarrow\;$ $\Gamma^{\mathrm{lat}}_\mu(p, k, q) = Z_1 \Gamma^R_\mu(p, k, q)$

Lattice Parameters of Gauge Ensembles in this Study (N_f=2)

Lattice action

- Wilson gauge action
- ***(Sheikholeslami-Wohlert)*** clover fermion action
- $\mathcal{O}(\alpha)$ improved rotated propagator
- Landau gauge $(\xi = 0)$

Name	β	κ	a [fm]	V	m_π [MeV]	m_q [MeV]	N_{cfg}	N_{src}
L08	5.20	0.13596	0.081	$32^3 \times 64$	280	6.2	900	4
H07	5.29	0.13620	0.071	$32^3 \times 64$	422	17.0	900	4
L07	5.29	0.13632	0.071	$32^3 \times 64$	295	8.0	908	4
L07-64	5.29	0.13632	0.071	$64^3 \times 64$	290	8.0	750	2
H06	5.40	0.13647	0.060	$32^3 \times 64$	426	18.4	900	2
Q07	6.16	0.13400	0.071	$32^3 \times 64$	1000	130	998	4

Acknowledgements

N_f = 2 gauge ensambles are provided by RQCD collaboration (Regensburg), *S. Bali et all, Phys Rev D91, 054501 (2014)*

A. Kizilersu, O. Oliveira, P.J. Silva, J. Skullerud and A. Sternbeck, Phys.Rev.D103 (2021)114515

Non-Perturbative Quark-Gluon Vertex:

Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$

Non-Perturbative Quark-Gluon Vertex in Continuum :

$$(\overline{\Lambda}^a_\mu) = -ig_0 \left(\lambda_1 [\gamma_\mu] + \lambda_2 [-4 \not{p} p_\mu] + \lambda_3 [-2ip_\mu]\right)$$

Lattice momenta : $p_\mu \to K_\mu(p) = \frac{1}{a}\sin(p_\mu a)$ where p_μ = Fourier mode.

Non-Perturbative tree-level Quark-Gluon Vertex on Lattice :

$$\overline{\Lambda}^{(0)}_{R,\mu}(p,0,p) = (-ig_0) \quad \frac{(1+b_q am)}{(1+am/2)^2} \frac{1}{\left(1+c_q^2 a^2 K^2(p)\right)^4}$$

$$\times \quad \Big\{ \gamma_\mu \left[\left(1+c_q^2 a^2 K^2(p)\right)^2 C_\mu(p)\right]$$

$$-4a^2 K_\mu \not{K}(p) \left[2c_q^2 C_\mu(p) - c_q\left(1 - c_q^2 a^2 K^2(p)\right)\right]$$

$$-2iaK_\mu \left[-2c_q^2 a^2 K^2(p) + \frac{1}{2}\left(1 - c_q^2 a^2 K^2(p)\right) - 2c_q\left(1 - c_q^2 a^2 K^2(p)\right) C_\mu(p)\right] \Big\}$$

Lattice momenta: $K_\mu(p) = \frac{1}{a}\sin(p_\mu a)$, $C_\mu(p) = \cos(p_\mu a)$

- Two different lattice tensors for each of $L_{2\mu}$ and $L_{3\mu}$

Lattice form factors and Tree-Level Corrections

Continuum form factors

Tree-level corrected, lattice equivalents of the form factors

Lattice momentum variables: $\mathbf{p}_\mu \rightarrow \mathbf{K}_\mu(\mathbf{p}) \equiv \frac{1}{\mathbf{a}} \sin(\mathbf{p}_\mu \mathbf{a}), \;\; \mathbf{C}_\mu(\mathbf{p}) = \cos(\mathbf{p}_\mu \mathbf{a})$

$$\lambda_1 = \frac{1}{(-ig_0)} \left\{ \left[Tr_4(\gamma_\alpha \overline{\Lambda}_\mu) \right] \Big|_{\substack{\alpha=\mu \\ p_\mu=0}} \right\}$$

$$\lambda_2 = \frac{1}{(-ig_0)} \left\{ -\frac{1}{4\,p^2} \frac{p_\alpha p_\mu}{p^2} \left[Tr_4(\gamma_\alpha \overline{\Lambda}_\mu) \Big|_{\alpha\neq\mu} \right] \right\}$$

$$\lambda_3 = \frac{1}{(-ig_0)} \left\{ \frac{i}{2} \frac{p_\mu}{p^2} Tr_4(I\,\overline{\Lambda}_\mu) \right\}$$

$$\lambda_1(p^2,0,p^2) = \frac{\mathrm{Im}}{g_0} \left\{ \left[\mathrm{Tr}_4(\gamma_\alpha \overline{\Lambda}_\mu) \right] \Big|_{\substack{\alpha=\mu \\ p_\mu=0}} \right\} \Big/ \lambda_1^{(0)}$$

$$\lambda_2(p^2,0,p^2) = \frac{\mathrm{Im}}{g_0} \left\{ -\frac{1}{4\,K(p)^2} \frac{K_\alpha(p) K_\mu(p)}{K(p)^2} \left[\mathrm{Tr}_4(\gamma_\alpha \overline{\Lambda}_\mu) \Big|_{\alpha\neq\mu} \right] \right\} - \left(\lambda_2^{(0)} + \overline{\lambda}_{2(\mu)}^{(0)} \right)$$

$$\lambda_3(p^2,0,p^2) = \frac{\mathrm{Re}}{(-g_0)} \left\{ \frac{1}{2} \frac{K_\mu(p)}{K^2(p)} \mathrm{Tr}_4(I\,\overline{\Lambda}_\mu) \right\} - \left(\lambda_3^{(0)} + \overline{\lambda}_{3(\mu)}^{(0)} \right)$$

- Divide λ_1 or subtract $\lambda_{2,3}$ tree level expression

$$\lambda_1^{(0)} = F(p)\left(1 + c_q^2 a^2 K^2(p)\right)^2$$

$$\lambda_2^{(0)} + \overline{\lambda}_{2(\mu)}^{(0)} = a^2 F(p) \left[-c_q \left(1 - c_q^2 a^2 K^2(p)\right) + 2 c_q^2 a\, C_\mu(p) \right]$$

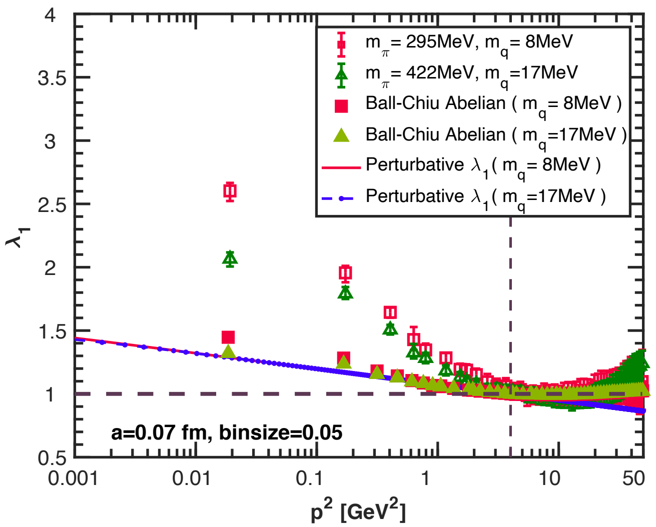
$$\lambda_3^{(0)} + \overline{\lambda}_{3,(\mu)}^{(0)} = \frac{a}{2} F(p) \left[\left(1 - c_q^2 a^2 K^2(p)\right)^2 - 4 c_q^2 a^2 K^2(p) - 4 c_q \left(1 - c_q^2 a^2 K^2(p)\right) C_\mu(p) \right]$$

- Two different lattice tensors for each of $L_{2\mu}$ and $L_{3\mu}$

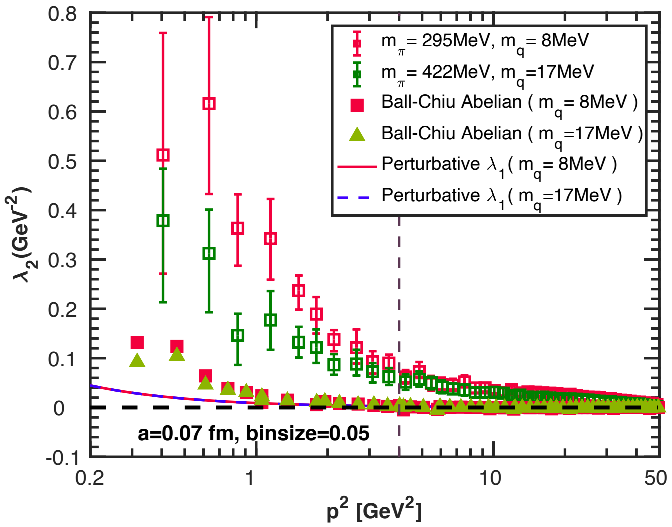
Quark Mass Dependence

$$S_F(p) = \frac{F(p^2)}{\not p - M(p^2)} = \frac{1}{A(p^2)\ \not p - B(p^2)}$$

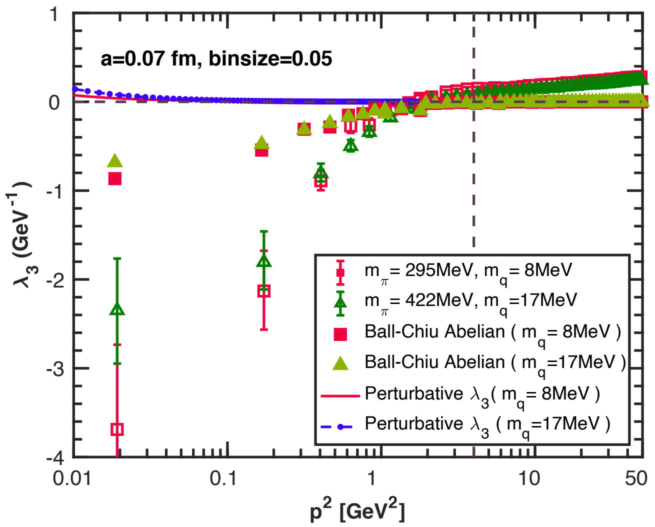
Moderate quark mass effect



$$\lambda_1^{BC} = A(p^2)$$



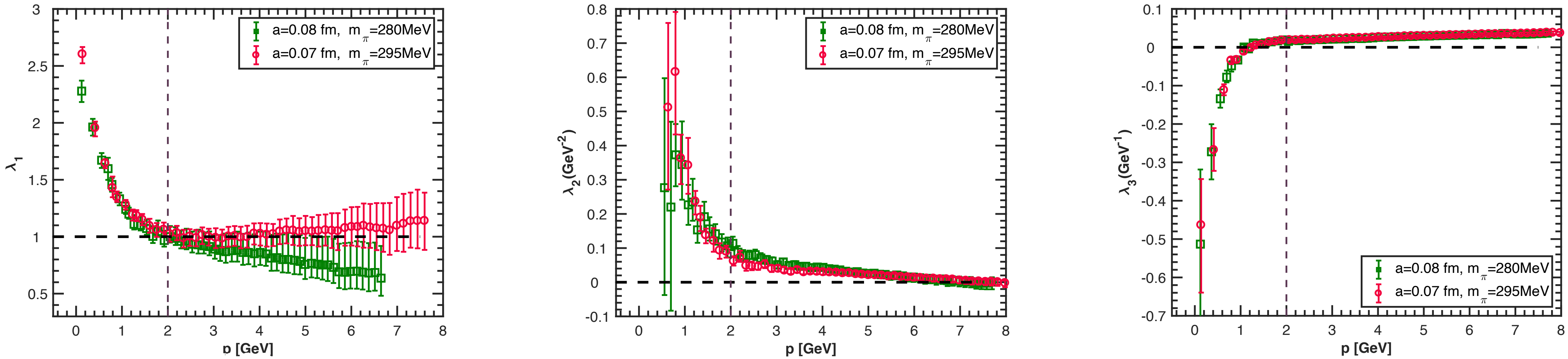
$$\lambda_2^{BC} = -\frac{1}{2}\frac{dA(p^2)}{dp^2}$$



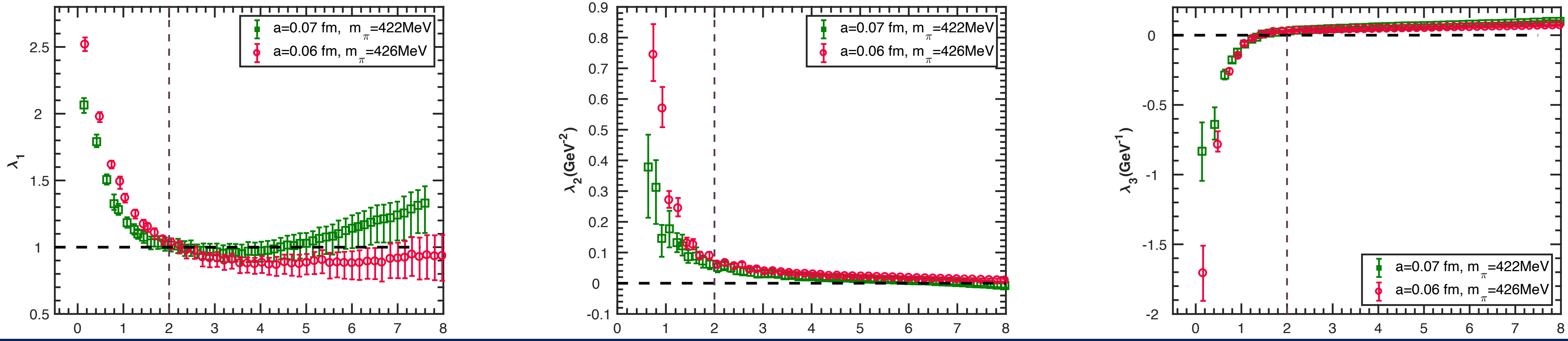
$$\lambda_3^{BC} = \frac{dB(p^2)}{dp^2}$$

Lattice Spacing

Top 290MeV; Bottom 420 MeV

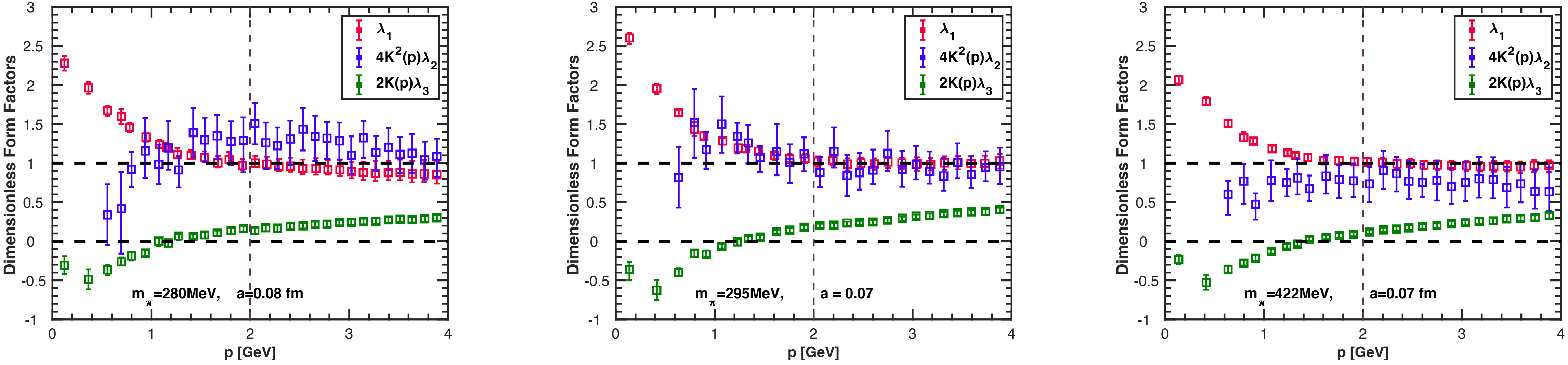


Stronger IR enhancement with reduced lattice spacing



Dimensionless Form Factors

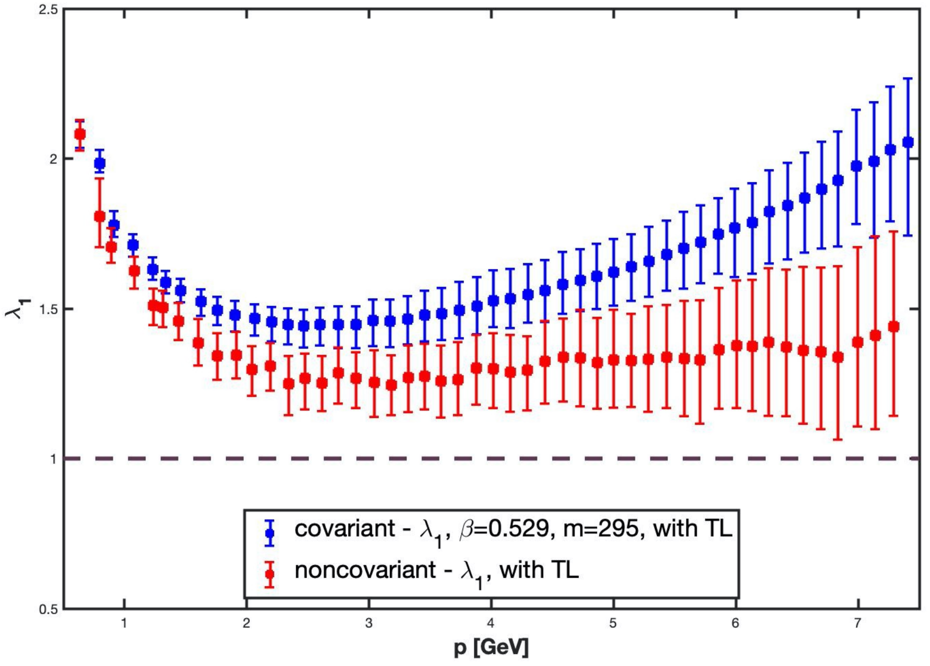
$$K_\mu(p) = \frac{1}{a} sin(p_\mu a)$$

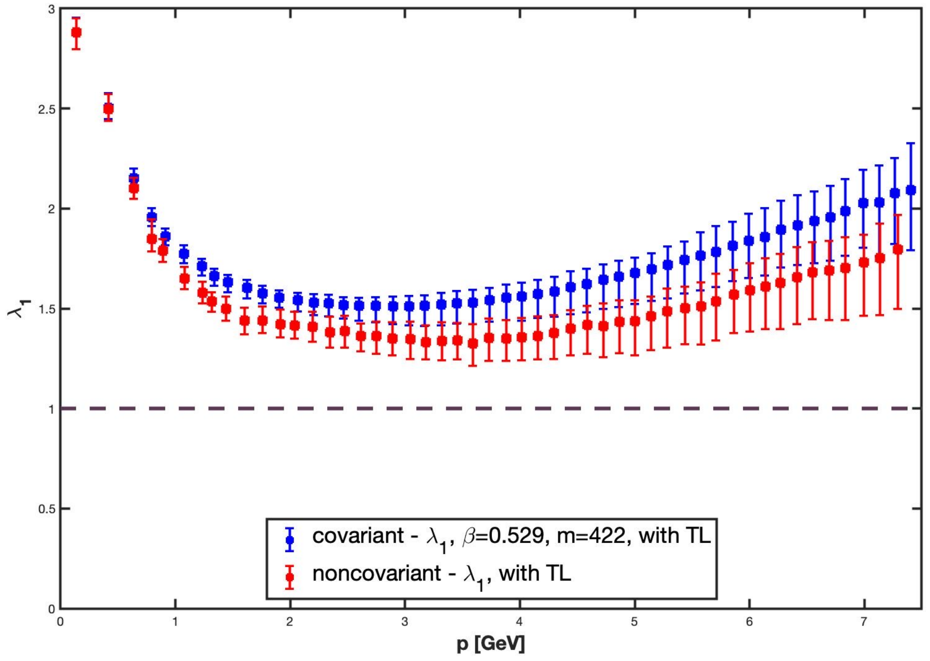


- Greater infrared enhancement with lighter quarks
- Moderate lattice spacing effects
- Significant contribution from DCSB form factor λ_3

Covariant Form Factor

PRELIMINARY





SUMMARY and OUTLOOK

First ever study of Quark-Gluon Vertex in Soft Gluon Kinematics for Landau Gauge with N_f=2 dynamical fermions

Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$

- Enhancement in infrared -- driver of chiral symmetry breaking
- Stronger enhancement
  - with dynamical quarks
  - for smaller quark masses and smaller lattice spacing
  - towards continuum limit
- Significant contribution from DCSB form factor λ_3
- Covariant and non-covariant extraction is in good agreement for
- Other kinematics, form factors in progress λ_1

Orthogonal Kinematics : $q \cdot P = 0 \qquad k^2 = p^2$ $\boldsymbol{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_6, \tau_5, \tau_7}$