B Meson Flavour Tagging with Quantum Support Vector Machines

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CP violation and the Standard Model

- CP symmetry violation is necessary to explain the preponderance of matter over anti-matter, however its known sources within the standard model (SM) are insufficient to explain the magnitude of the observed asymmetry.
- While there are not many known examples of CP violation, it can be seen in certain decays of B mesons.
- Belle-II and LHCb experimentally test this CP violation, and whether the SM describes it completely, or if New Physics is required...

CP violation in the $B^0 - \overline{B}^0$ system

- CP symmetry implies equality of the decay rates Γ of B^0 and $\overline{B}^0$ to a common CP eigenstate, however the SM predicts

$$\frac{\Gamma(\overline{B}^0 \to J/\psi K_S^0) - \Gamma(B^0 \to J/\psi K_S^0)}{\Gamma(\overline{B}^0 \to J/\psi K_S^0) + \Gamma(B^0 \to J/\psi K_S^0)} \sim \sin \delta m t \sin 2\beta$$

  where β is a phase from the CKM matrix and δm is the mass difference between the two mass eigenstates.

- Need to be able to reliably distinguish B^0 from $\overline{B}^0$ in order to do this analysis.

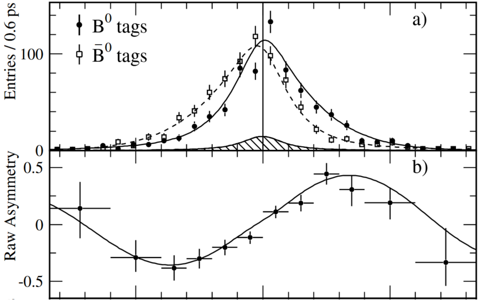


Figure taken from Ref.[1]

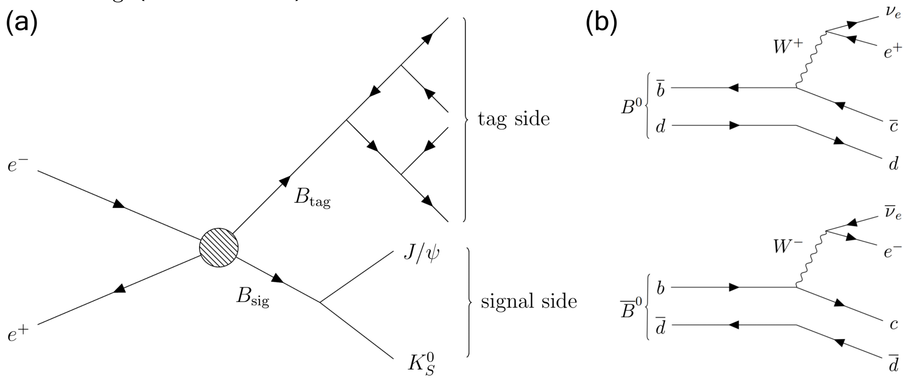
[1] B. Aubert *et al.* *Phys. Rev. Lett.* **89** 201802 (2002)

B^0 flavour tagging

- At Belle-II, $B^0 - \overline{B}^0$ pairs are created in entangled states by e^-e^+ collisions:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\left|B^0\overline{B}^0\right\rangle - \left|\overline{B}^0 B^0\right\rangle\right)$$

- *Flavour tagging* is the process of determining the quark content of the "tag-side" meson B_{tag} (i.e. $\bar{b}d$ or $b\bar{d}$)



B^0 flavour tagging at Belle-II

- Currently the state-of-the-art results at Belle-II are achieved via machine learning approaches on 130 input variables.
- The performance of the classifiers are characterised by the effective tagging efficiency Q, defined as

$$Q = \sum_{i=1}^{n_{\text{bins}}} \epsilon_i (1 - 2w_i)^2$$

  where ϵ_i is the fraction of events in the ith bin, and w_i is the fraction incorrectly tagged.
- Recent results[2] using fast boosted decision trees and deep neural networks give

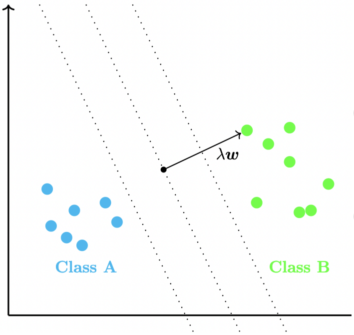
$$Q_{\text{FBDT}} = 30.0\%$$

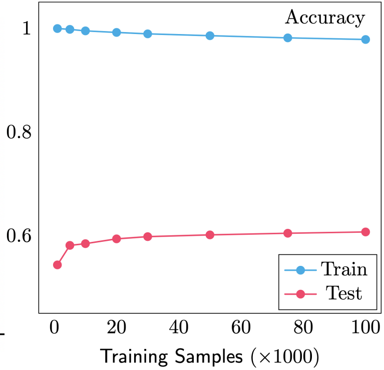
$$Q_{\text{DNN}} = 28.8\%$$

[2] Abudinen, F., et al. B-flavour tagging at Belle II. arXiv:2110.00790 (2021)

Quantum Support Vector Machines

- SVMs are linear classifiers on data which has typically been mapped to a feature space,
$$\boldsymbol{x} \mapsto \Phi(\boldsymbol{x}).$$
- In a QSVM this mapping is into a quantum Hilbert space,
$$\boldsymbol{x} \mapsto |\psi(\boldsymbol{x})\rangle = \mathcal{U}(\boldsymbol{x})\,|0\rangle$$
- QSVMs are very powerful, and therefore prone to overfitting the training data.





Continuous Variable Quantum Computers

- The fundamental unit of a conventional quantum computer is the qubit, a state of a two level quantum system:

$$|\psi\rangle_{\text{qubit}} = \alpha\,|0\rangle + \beta\,|1\rangle$$

- We will consider continuous variable (CV) quantum computers, the fundamental units of which are *qumodes*, quantum systems with continuous degrees of freedom:

$$|\psi\rangle_{\text{qumode}} = \int dx\ \psi(x)\,|x\rangle$$

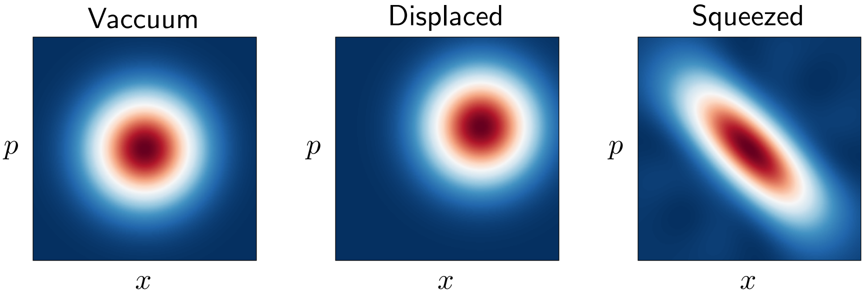
  e.g. a set of bosonic modes (harmonic oscillators).

Continuous Variable Quantum Operations

- Each qumode has a representation in terms of a pair $\hat{x}$, $\hat{p}$ of canonically conjugate operators formed from the creation and annihilation operators of the mode:

$$\hat{x} = \hat{a} + \hat{a}^\dagger, \quad \hat{p} = i\left(\hat{a}^\dagger - \hat{a}\right)$$

- We can visualise the effects of common CV operations (e.g. displacement, squeezing) in phase space via the Wigner functions of the qumodes:



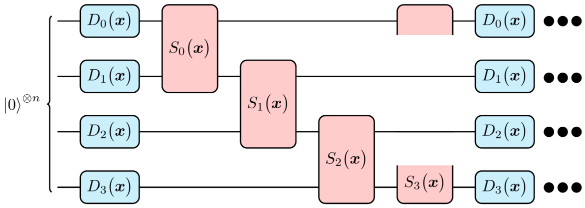
Continuous Variable Quantum Support Vector Machines

- The key component of a QSVM is the data encoding map $\boldsymbol{x} \mapsto |\psi(\boldsymbol{x})\rangle$.
- We consider various mappings readily implemented[3] on a CV quantum computer from displacement operations $D(\boldsymbol{x})$ and squeezing operations $S(\boldsymbol{x})$:

$$D(\boldsymbol{x}) = \prod_{i=0}^{n} D_i(\boldsymbol{x}) = \prod_{i=0}^{n} e^{x_i^* a_i^\dagger \,-\, x_i a_i}$$

$$S(\boldsymbol{x}) = \prod_{i=0}^{n} S_i(\boldsymbol{x}) = \prod_{i=0}^{n} e^{x_i^* a_i a_{i+1} \,+\, x_i a_i^\dagger a_{i+1}^\dagger}$$

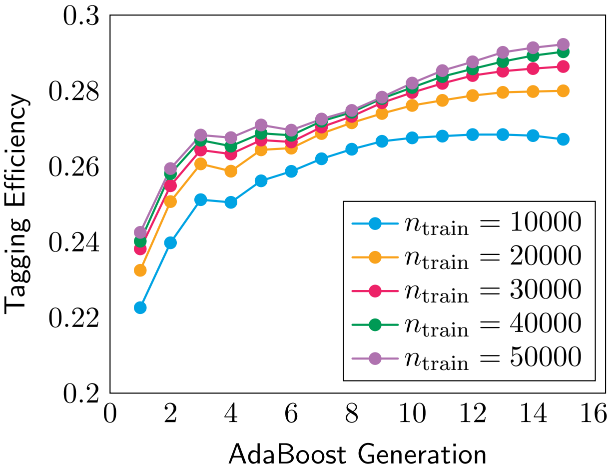
$$|\psi(\boldsymbol{x})\rangle = \underbrace{D(\boldsymbol{x})S(\boldsymbol{x})\cdots D(\boldsymbol{x})S(\boldsymbol{x})}_{l \text{ operations}} |0\rangle^{\otimes n}$$



[3] Stavenger, T. et al, Bosonic qiskit. arXiv:2209.11153 (2022)

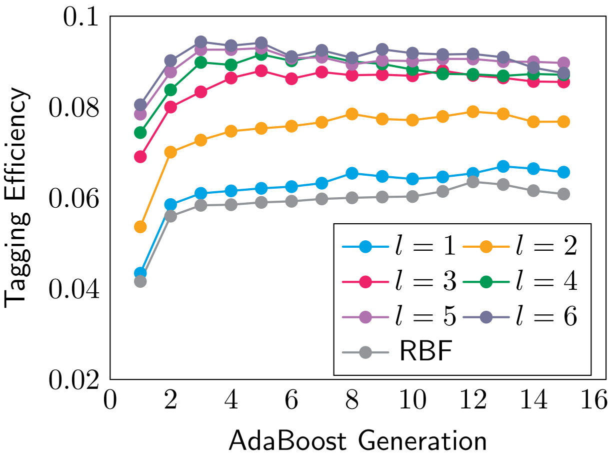
Continuous Variable Quantum Support Vector Machines: Results

- We construct boosted ensembles of 200 QSVMs, each of which “votes” on the flavour.
- Due to computational constraints, when using all 130 datapoints we are restricted to $l = 1$.
- We are able to achieve results competitive with the state of the art, but when $l = 1$ we are essentially using a classical model.



CV-QSVMs: Top 5 PCA Components

- By doing a PCA transformation we can reduce the dimensionality of the data and employ more powerful QSVMs.
- Increasing the depth l of the CV-QSVMs allows us to significantly outperform the classical RBF kernel.



Summary

- B meson flavour tagging is an important component of experiments which probe CP violation and heavy quark mixing.
- By using boosted ensembles of QSVMs we can achieve flavour tagging at level commensurate with state of the art classical algorithms.
- There is a tantalising prospect for outperforming classical methods as quantum computer hardware matures and it becomes possible to perform large-scale entangled kernels.