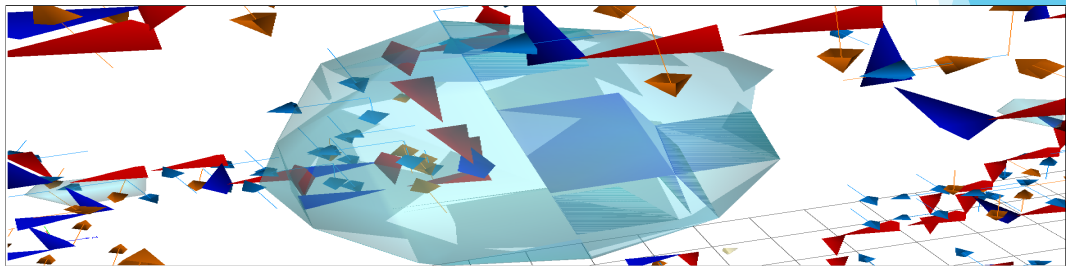# The emergent origin of mass

Waseem Kamleh

Collaborators
Derek Leinweber, Adam Virgili

Australian Institue of Physics Congress
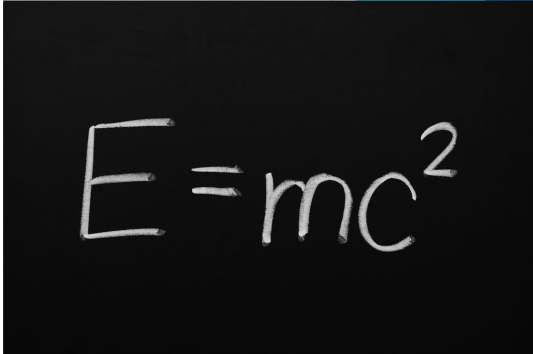Adelaide Convention Centre, 11-16 December 2022

Origin of mass

Mass as an input parameter

Proton mass

$$3 + 3 + 5 \stackrel{?}{=} 938 \text{ MeV}$$

Higgs contributes $\sim 1\%$ of your mass

Where does the rest come from?

Emergent phenomena

An emergent behaviour or emergent property can appear when a number of simple entities (agents) operate in an environment, forming more complex behaviors as a collective.

Emergent features of QCD

Non-trivial vacuum state (c.f. QED)
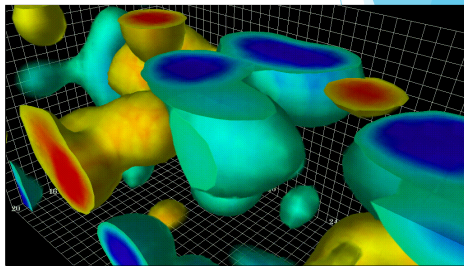
    Requires a non-perturbative description.

Two key *emergent* features

    Confinement of quarks

    Dynamical mass generation

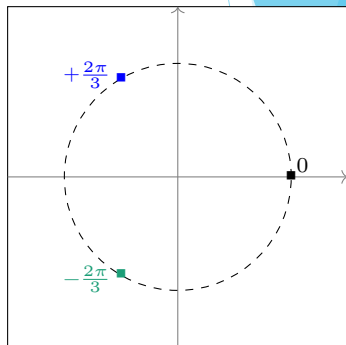Can centre vortices explain the emergent features of QCD?

Centre group of SU(3)

Centre elements commute with every group element,

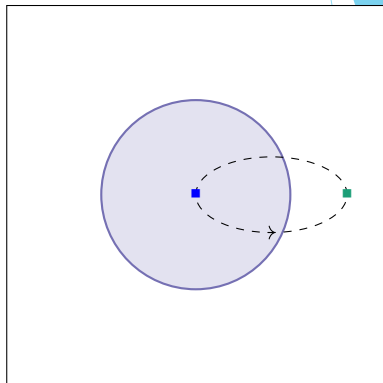$$z = \exp\left(\frac{2\pi i}{3}m\right)I, \quad m \in \{-1, 0, 1\} \simeq \mathbb{Z}_3$$

Each of the three centre phases corresponds to a centre element of SU(3)

Centre vortices

A (thin) *centre vortex* represents a flow of centre flux along a closed surface in four dimensions.

A Wilson loop $W(C)$ along a curve $C = \partial A$ is topologically linked if the vortex pierces the enclosed area A only once.
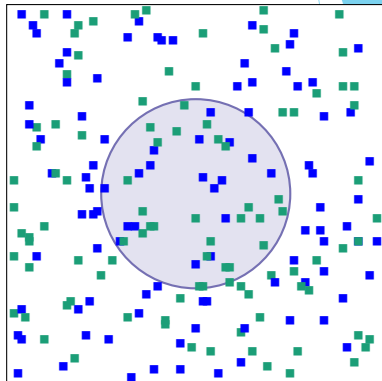
Confinement

The piercing vortex generates a non-trival centre phase z,

$$W(C) \rightarrow zW(C)$$

If centre vortices percolate through a volume with density ρ, this gives rise to an area law for the Wilson loop
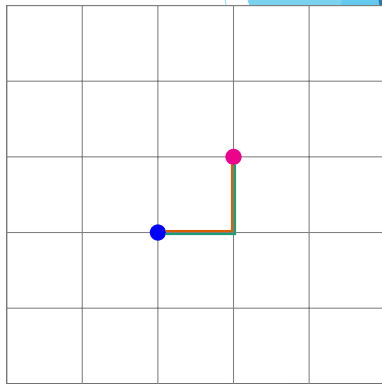
$$W(C) = e^{-2\rho A}$$

Lattice QCD

Discretise space-time onto 4D hypercube

Gauge field $U_\mu(x) \in SU(3)$ becomes unitary

$32^3 \times 64$ (periodic) lattice volume

Dynamical $m_\pi = 156$ MeV, $a = 0.0933$ fm

S. Aoki *et al* (PACS-CS), Phys. Rev. D **79**, 034503
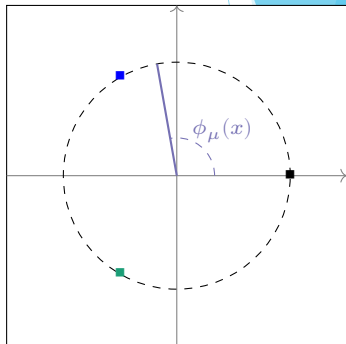
Identifying centre vortices

Transform to Maximal Centre Gauge

$$\sum_{x,\mu} \mathrm{Re\,Tr}[U_\mu^\Omega(x) Z_\mu^\dagger(x)] \to \mathrm{Max}$$

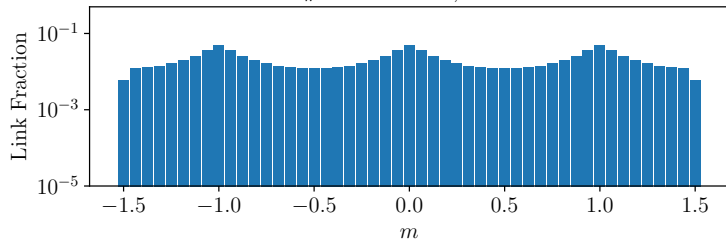$\Omega(x)$ maximises overlap with centre elements.

Project onto $\mathbb{Z}_3$ by choosing closest centre element to the phase of

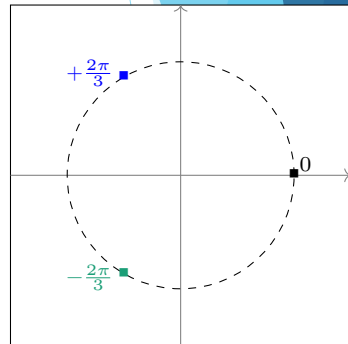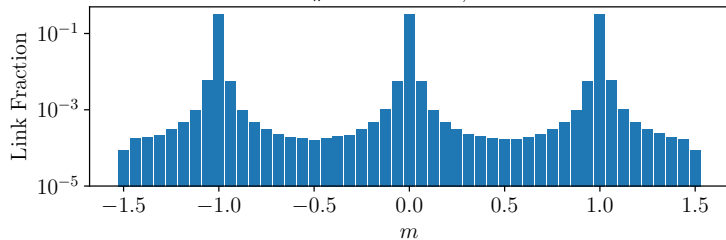$$\frac{1}{3}\mathrm{Tr}\,U_\mu^\Omega(x) = r_\mu(x)\exp(i\phi_\mu(x))$$

Centre phase distribution

$m_\pi = 156$ MeV, before

$m_\pi = 156$ MeV, after

Identifying centre vortices

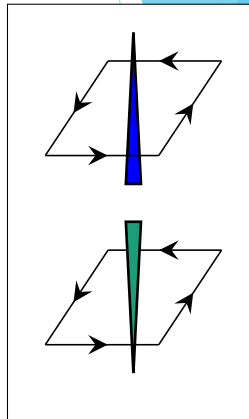The centre vortex field lives on the dual lattice,

$$\bar{x} = x + \frac{a}{2}(\hat{\mu} + \hat{\nu} - \hat{\lambda} - \hat{\kappa})$$

The centre flux $m_{\kappa\lambda}(\bar{x})$ through an elementary plaquette is

$$P_{\mu\nu}(x) = \exp\left(\frac{\pi i}{3}\,\epsilon_{\kappa\lambda\mu\nu}m_{\kappa\lambda}(\bar{x})\right)$$

Centre-projected plaquette is pierced by a (P-)vortex if

$$P_{\mu\nu}(x) = Z_\mu(x)Z_\nu(x+\mu)Z_\mu^\dagger(x+\nu)Z_\nu^\dagger(x)$$
$$= \exp\left(\frac{\pm 2\pi i}{3}\right)I$$

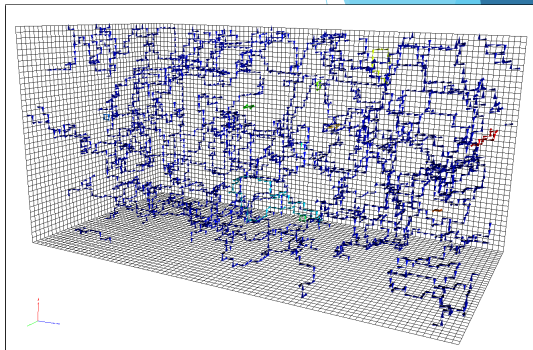# Centre vortices on the lattice

Untouched configurations

$$U_\mu(x)$$

Vortex-only configurations

$$Z_\mu(x) = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right]\mathrm{I}$$

Vortex removed configurations

$$R_\mu(x) = Z_\mu^\dagger(x) U_\mu^\Omega(x)$$

Visualization of center vortex structure
J.C. Biddle, WK, D.B. Leinweber
Phys. Rev. D 102 (2020) 3, 034504
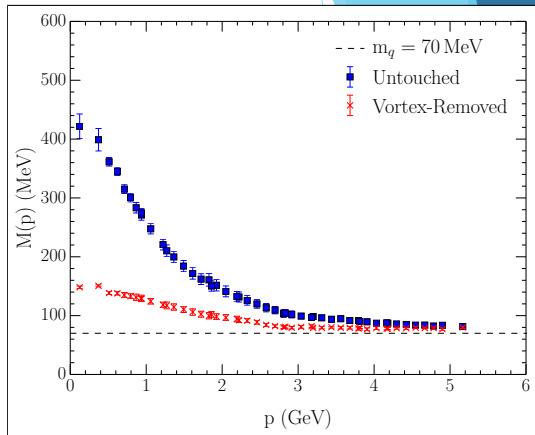
Overlap quark propagator - pure gauge

$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

$M(p)$ is the mass function

$Z(p)$ is the renormalisation function

Calculated in Landau gauge

Overlap fermion valence quarks

Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
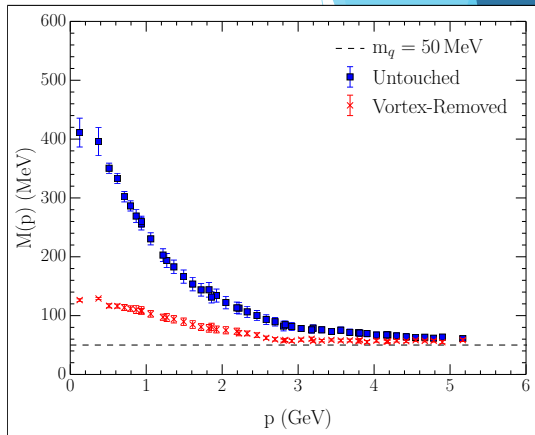A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - pure gauge

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Infrared behaviour of $M(p)$

Dynamical mass generation

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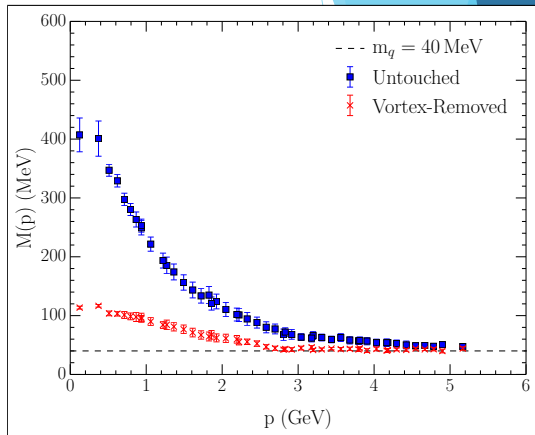# Overlap quark propagator - pure gauge

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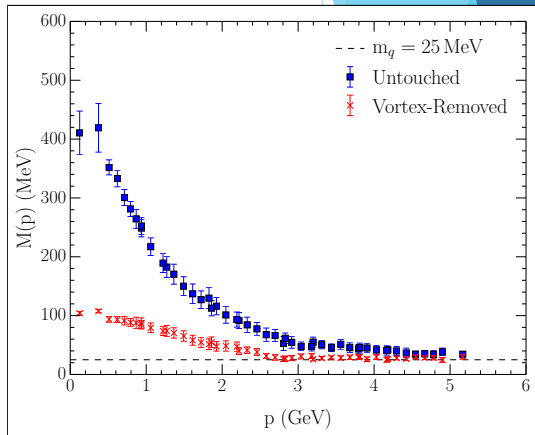A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

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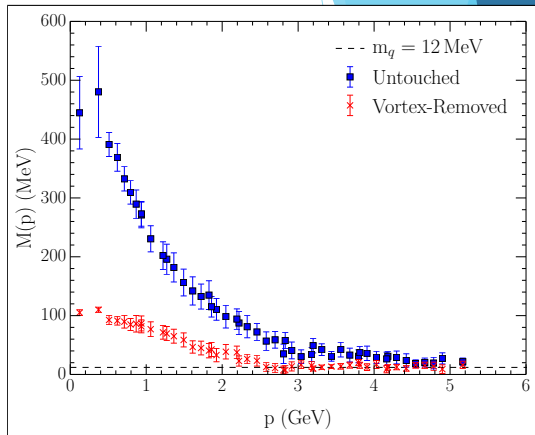A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - pure gauge

Vortex only field is very rough

Overlap has smoothness condition

$\rightarrow$ 10 sweeps of cooling

$\rightarrow$ Smooth untouched to compare

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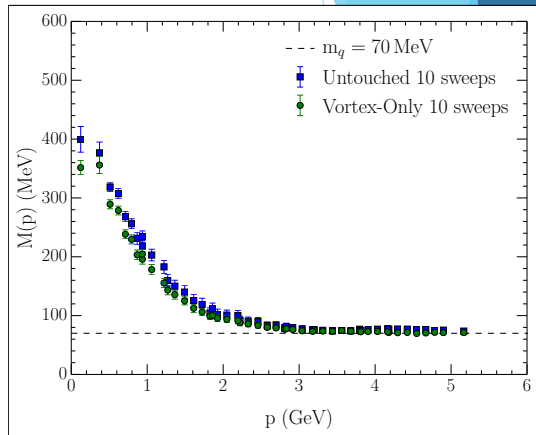A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

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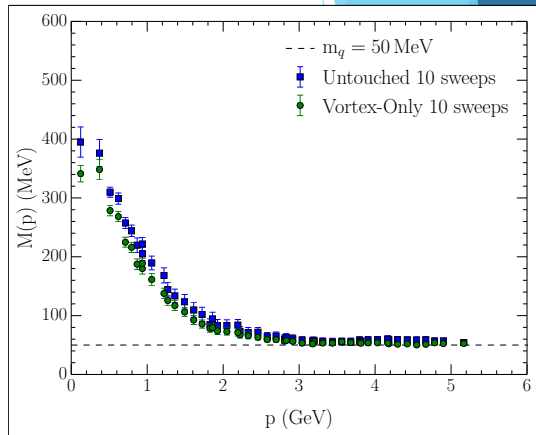A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377
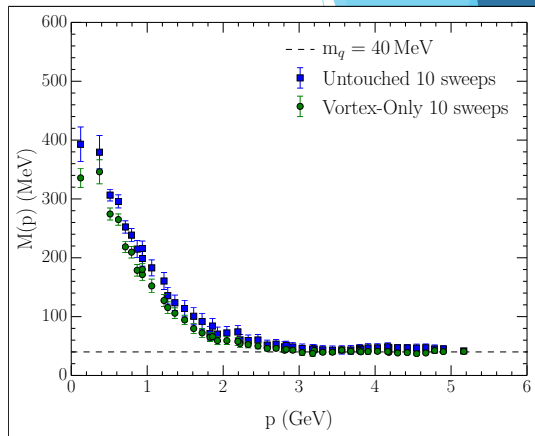
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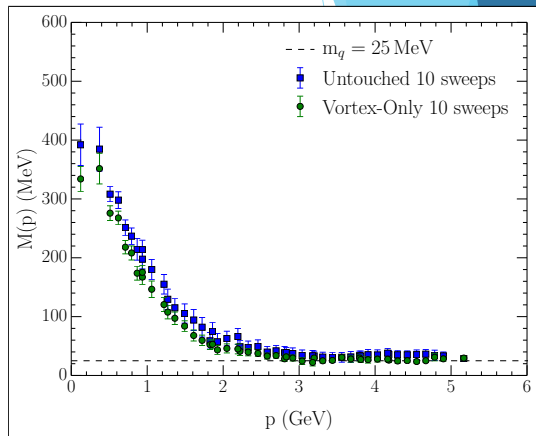
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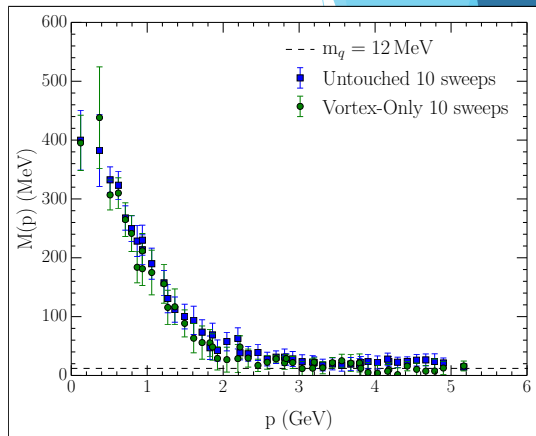
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Centrifuge preconditioned smoothing

Work with centre phase field

$$e^{i\lambda_\mu(x)}I \to [\lambda_\mu(x), \lambda_\mu(x), \lambda_\mu(x)]$$

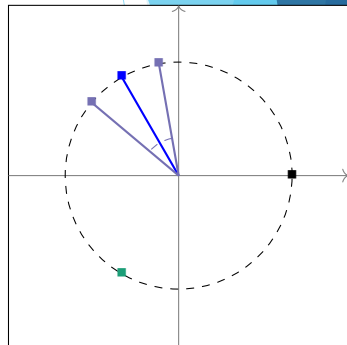Define the staple phase

$$\sigma_\mu(x) = \frac{1}{6}\sum_{\nu\neq\mu}\left(\begin{array}{c}\end{array} + \begin{array}{c}\end{array}\right)$$

Select a pair of indices randomly for each link,

$$\lambda_\mu(x) \to (1 \mp \omega)\,\lambda_\mu(x) \pm \omega\,\sigma_\mu(x)$$

Update corresponds to a phase rotation by $\mp\omega(\lambda - \sigma)$

Smoothing algorithms for projected center-vortex gauge fields
A. Virgili, WK, D.B. Leinweber, Phys.Rev.D 106 (2022) 1, 014505
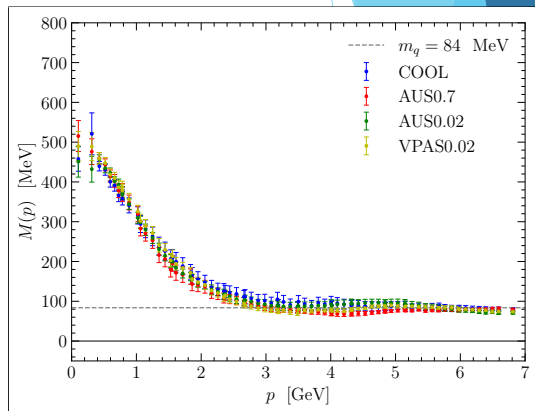
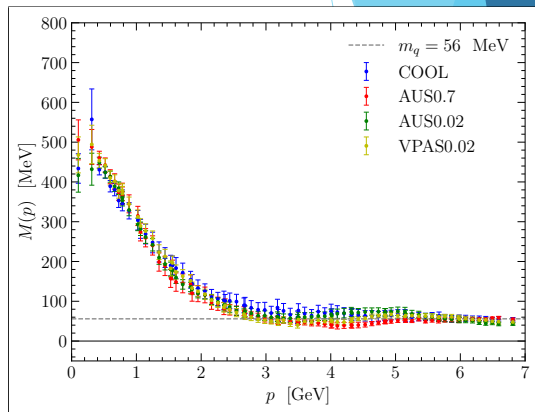# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

Centrifuge preconditioning $\omega = 0.02$
  Annealed smoothing, $\alpha = 0.7$
  Vortex preserving, $\alpha = 0.02$
10 sweeps of cooling for comparison

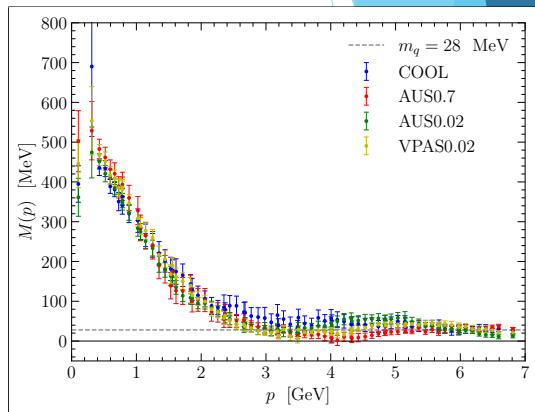# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

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Dynamical mass generation from smoothed centre vortices

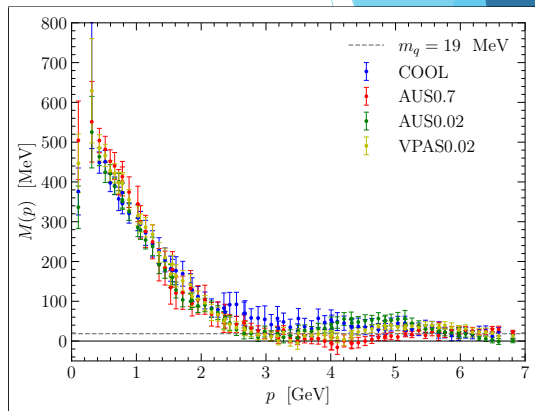# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

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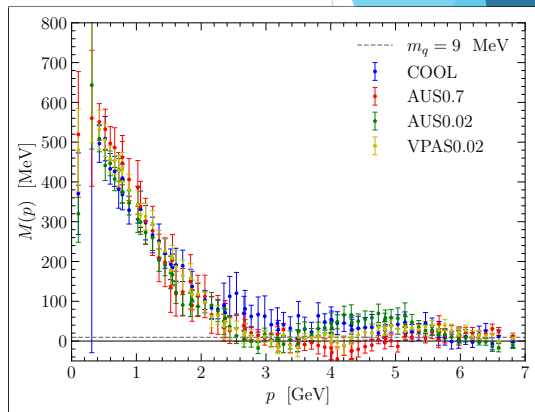# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

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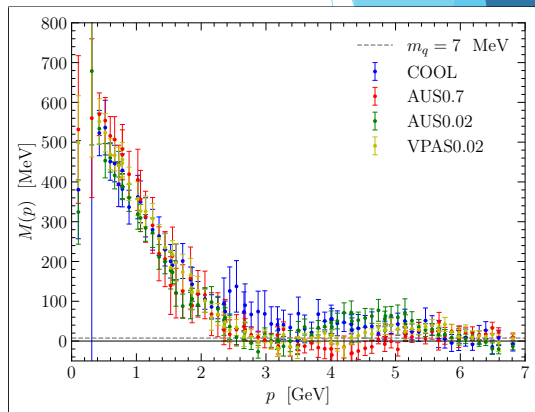# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

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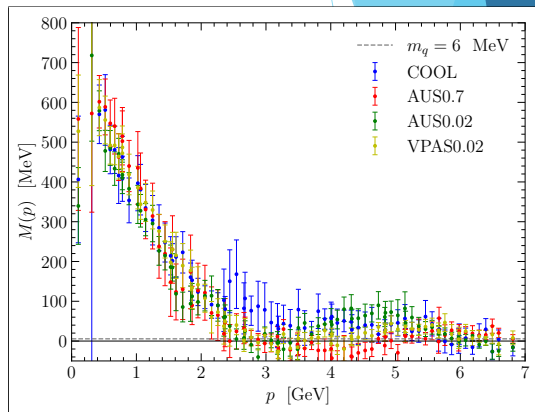# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

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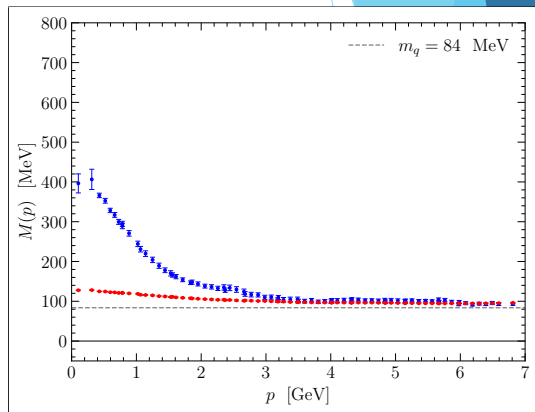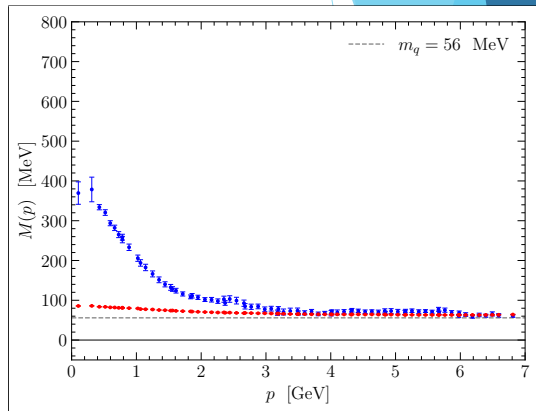Dynamical mass generation from smoothed centre vortices

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Dynamical mass generation
$\rightarrow$ Increases as m_q decreases
Vortex removed dynamical mass
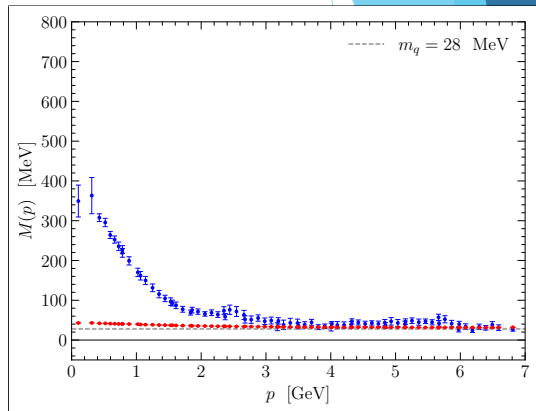$\rightarrow$ Significantly reduced at large m_q

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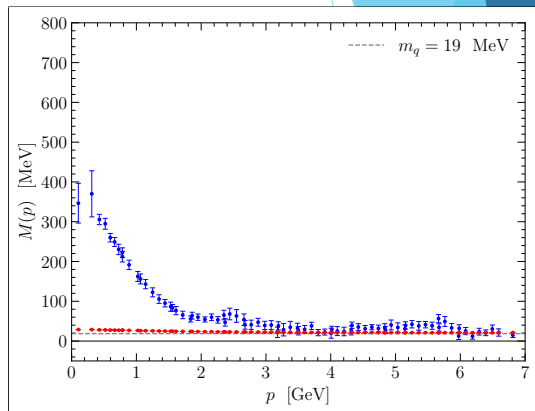$\rightarrow$ Significantly reduced at large m_q

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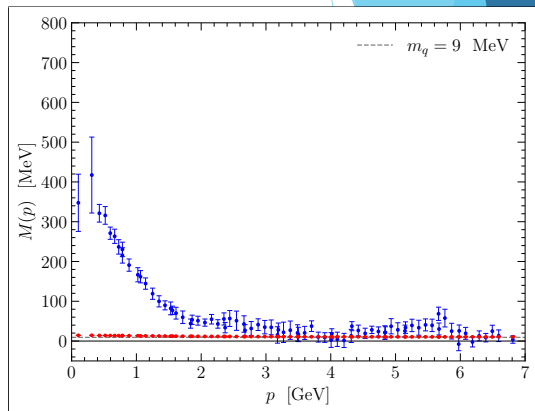$\rightarrow$ Significantly reduced at large m_q

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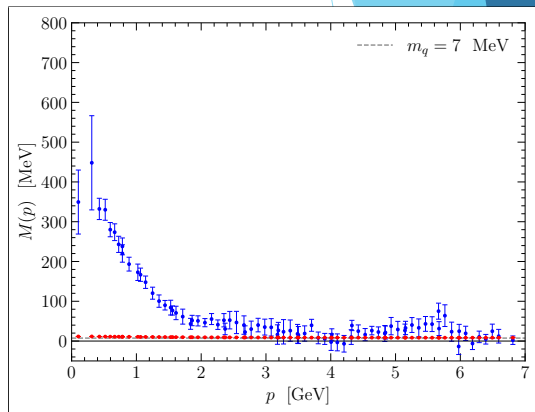$\rightarrow$ Significantly reduced at large m_q

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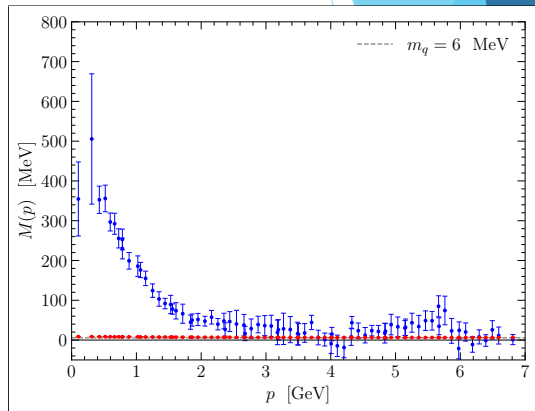# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

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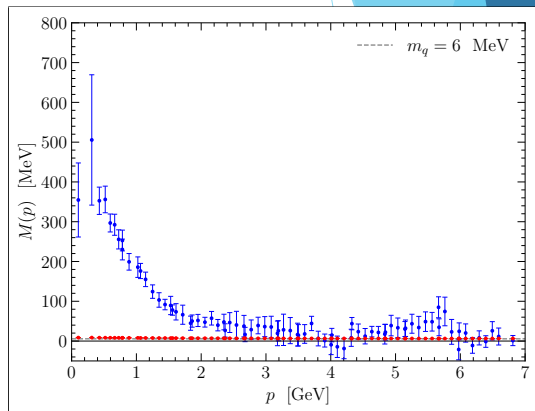# Overlap quark propagator - dynamical $m_\pi = 156$ MeV

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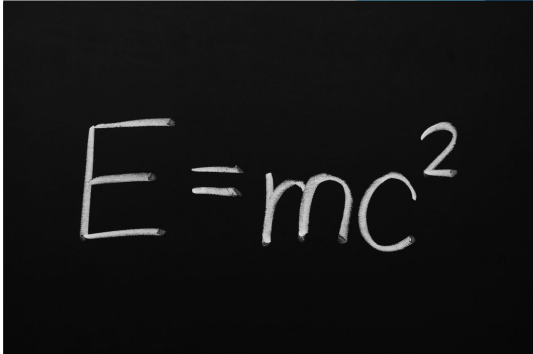VR transforms QCD to weakly interacting ($\sim$free) quark theory

Origin of mass

Proton mass

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Higgs contributes $\sim 1\%$ of your mass

Centre vortices responsible for
dynamically generating the rest!