Testing the Quark Model on the Δ Baryon Spectrum

Liam Hockley, Curtis Abell, Waseem Kamleh, Derek Leinweber & Anthony Thomas

Australian Institute of Physics Congress

13 Dec 2022

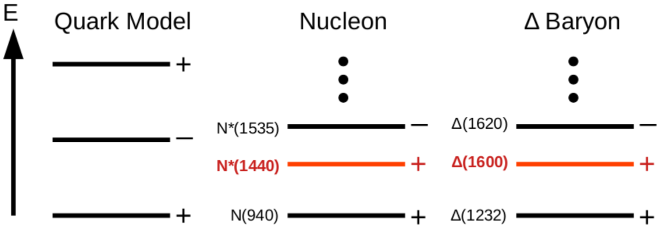
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

SUB CRUCE LUMEN

THE UNIVERSITY of ADELAIDE

The Quark Model

- Baryons modelled as 3-quark systems; good for understanding the "particle zoo"
- BUT this simple model doesn't produce the entire spectrum of baryons in nature (i.e. the Roper puzzle)



***Q*:** Is the $\Delta(1600)$ resonance *really* a 3-quark state?

Overview of our methods

Our two methods are Lattice QCD and Hamiltonian Effective Field Theory (HEFT):

- Lattice QCD allows us to compute baryon masses from first principles **on a finite volume**
- We can then construct an **infinite volume** Hamiltonian and compute various scattering observables to describe experiment
- The final step is to convert this Hamiltonian to **finite volume** so we can compare experiment with the lattice results

HEFT allows us to simultaneously compare with results on the finite volume of the lattice and with the infinite volume of the real world.

Lattice QCD

Lattice QCD is a well established method for calculating hadronic observables in the non-perturbative regime of QCD

We use standard techniques to calculate baryon masses:

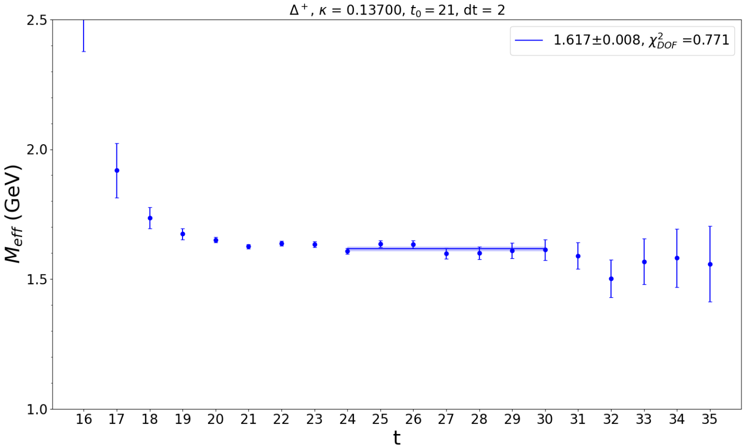
- **3-quark interpolating fields with quantum numbers of the Δ**
- Positive parity and spin-3/2 projection operators
- 4 by 4 correlation matrix of smearings extends the operator basis
- Compute two-point correlation functions and solve a GEVP to get effective masses

Gauge field ensembles used are from the PACS-CS collaboration

S. Aoki et al. [PACS-CS], Phys. Rev. D 79 (2009) 034503 [arXiv:0807.1661 [hep-lat]]

Effective Mass Fits (preliminary)

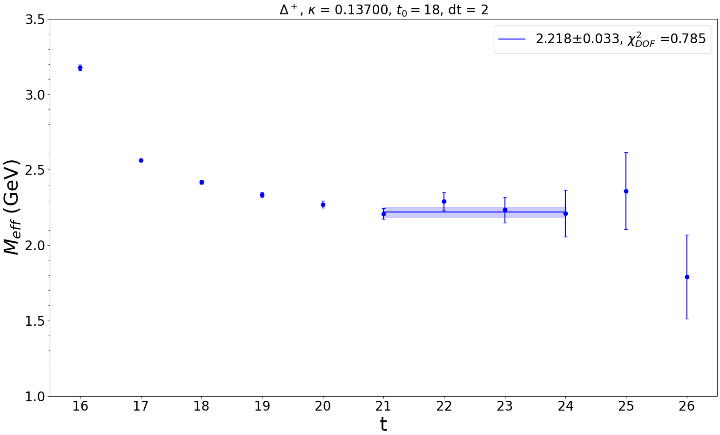
- Extract a single projected "effective" mass
- Fit a plateau (using a full covariance matrix) across multiple time slices
- Repeat across all 5 gauge field ensembles for both ground and first excited states



Ground state

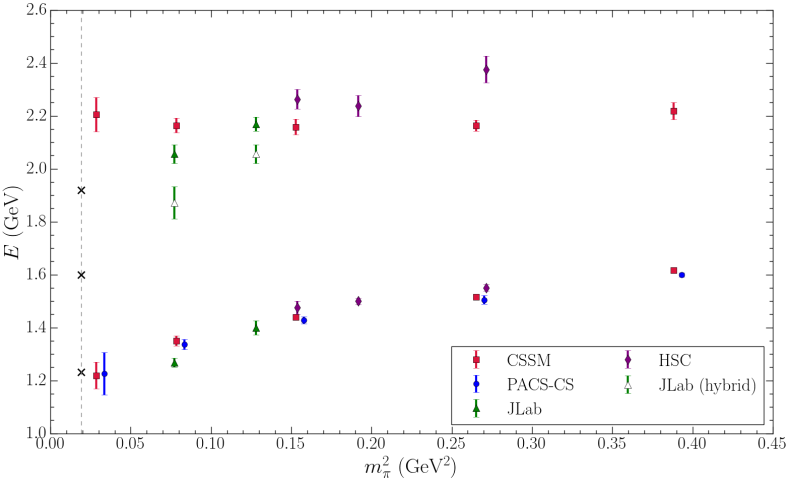
Effective Mass Fits (preliminary)

- Extract a single projected "effective" mass
- Fit a plateau (using a full covariance matrix) across multiple time slices
- Repeat across all 5 gauge field ensembles for both ground and first excited states



1st excited state

Lattice Results (preliminary)

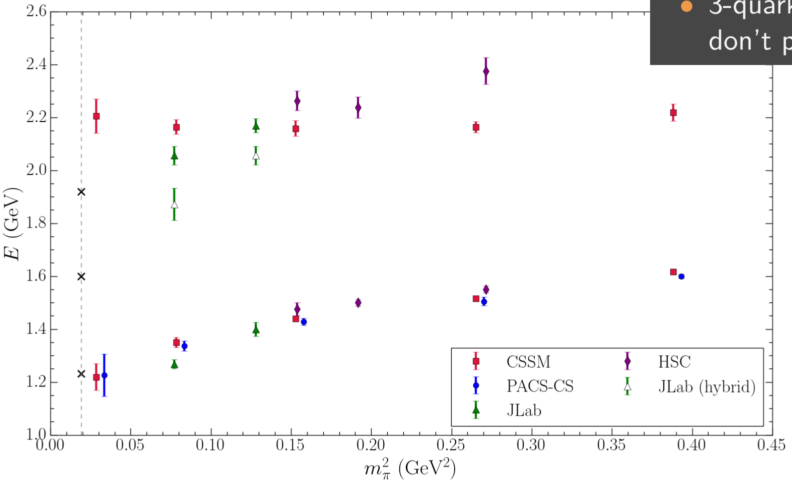


PACS-CS:
S. Aoki et al. [PACS-CS], Phys. Rev. D 79 (2009) 034503 [arXiv:0807.1661 [hep-lat]]

JLab:
T. Khan, D. Richards and F. Winter, Phys. Rev. D 104 (2021) 034503 [arXiv:2010.03052 [hep-lat]]

HSC:
J. Bulava, et al., Phys. Rev. D 82 (2010) 014507 [arXiv:1004.5072 [hep-lat]]

Lattice Results (preliminary)



- 3-quark interpolating operators don't produce $\Delta(1600)$

PACS-CS:
S. Aoki et al. [PACS-CS], Phys. Rev. D 79 (2009) 034503 [arXiv:0807.1661 [hep-lat]]

JLab:
T. Khan, D. Richards and F. Winter, Phys. Rev. D 104 (2021) 034503 [arXiv:2010.03052 [hep-lat]]

HSC:
J. Bulava, et al., Phys. Rev. D 82 (2010) 014507 [arXiv:1004.5072 [hep-lat]]

Infinite Volume Hamiltonian

Construct an infinite volume Hamiltonian $H = H_0 + H_I$ with basis states composed of:

- quark model-like, 3-quark states at rest
- non-interacting two-particle meson-baryon states, with some back-to-back momentum

Non-interacting Hamiltonian:

$$H_0 = \sum_{B_0} |B_0\rangle m_{B_0} \langle B_0| + \sum_{\alpha} \int d^3k \; |\alpha(\vec{k})\rangle \omega_\alpha(\vec{k}) \; \langle \alpha(\vec{k})|$$

$|B_0\rangle$: quark model state $\qquad$ $|\alpha(\vec{k})\rangle$: 2-particle state

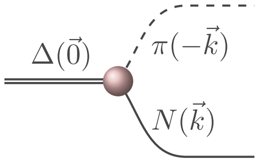
$$\omega_\alpha(\vec{k}) \equiv \sqrt{m_{\alpha_B}^2 + \vec{k}^2} + \sqrt{m_{\alpha_M}^2 + \vec{k}^2}$$

Infinite Volume Hamiltonian

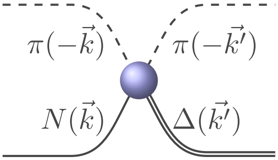
Interacting Hamiltonian:

$$H_I = g + v$$

$$g = \sum_{\alpha,\, B_0} \int d^3k \, \left\{ \, |\alpha(\vec{k})\rangle \; G^{\dagger}_{\alpha,B_0}(k) \; \langle B_0| + h.c. \right\}$$



$$v = \sum_{\alpha,\beta} \int d^3k \; d^3k' \, |\alpha(\vec{k})\rangle \, V_{\alpha,\beta}(k,k') \, \langle \beta(\vec{k'})|$$



Functional Forms

Basis states:

- Quark model-like (motivated by two states on the lattice): $|\Delta^{(1)}\rangle$, $|\Delta^{(2)}\rangle$
- Meson-baryon (motivated by PDG branching ratios): $|\pi N(\vec{k})\rangle$, $|\pi\Delta(\vec{k})\rangle$ in p-wave $\equiv |\pi\Delta_p(\vec{k})\rangle$, $|\pi\Delta(\vec{k})\rangle$ in f-wave $\equiv |\pi\Delta_f(\vec{k})\rangle$

The couplings and potentials are derived from chiral EFT i.e.

$$G^{\Delta}_{\pi N}(k) = \frac{g^{\Delta}_{\pi N}}{m_\pi} \frac{k}{\sqrt{\omega_\pi(k)}} u(k, \Lambda_{\pi N})$$

Dipole regulator: $u(k,\Lambda) = \left(1 + k^2/\Lambda^2\right)^{-2}$

$$V_{\pi N,\pi N}(k,k') = \frac{v_{\pi N,\pi N}}{(m_\pi)^2} \frac{k}{\omega_\pi(k)} \frac{k'}{\omega_\pi(k')} u(k, \Lambda^{v}_{\pi N}) u(k', \Lambda^{v}_{\pi N})$$

- Need to constrain the free parameters $m^{(i)}_{\Delta}$, $g^{(i)}_{\alpha}$, $v_{\alpha,\beta}$, Λ_α, Λ^v_α.

T-matrix

Solve for the T-matrix using the coupled channel integral equations:

$$T_{\alpha\beta}(k,k';E) = \tilde{V}_{\alpha\beta}(k,k';E) + \sum_{\gamma} \int dq\, q^2 \frac{\tilde{V}_{\alpha\gamma}(k,q,E) T_{\gamma\beta}(q,k';E)}{E - \omega_\gamma(q) + i\epsilon}$$

where we've defined the coupled channel potential

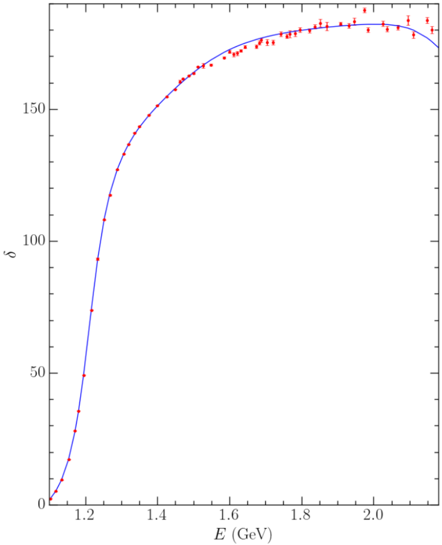
$$\tilde{V}_{\alpha\beta}(k,k',E) = \frac{G_\alpha^{B_0\dagger}(k) G_\beta^{B_0}(k')}{E - m_{B_0}} + V_{\alpha\beta}(k,k')$$

Extract scattering phase shifts and inelasticities via the S-matrix:

$$S_{\pi N,\pi N}(E) = 1 - 2i\pi \frac{\omega_\pi(k_{\rm cm})\, \omega_N(k_{\rm cm})}{E}\, k_{\rm cm}\; T_{\pi N,\pi N}(k_{\rm cm}, k_{\rm cm}; E) = \eta(E)\, e^{2i\,\delta(E)}$$

where k_{cm} satisfies $\omega_\alpha(k_{\text{cm}}) = \omega_{\alpha_M}(k_{\text{cm}}) + \omega_{\alpha_B}(k_{\text{cm}}) = E$

Fit to scattering data (preliminary)



- Fit phase shifts and inelasticities to $\pi N \to \pi N$ scattering data available from GWU SAID partial wave analysis
- Extract resonance poles to check against the PDG

Resonance	PDG Pole	This Work
$\Delta(1232)$	$1.210(1) - 0.050(1)i$	$1.211 - 0.049i$
$\Delta(1600)$	$1.510(50) - 0.135(35)i$	$1.444 - 0.219i$
$\Delta(1920)$	$1.900(50) - 0.150(100)i$	$2.262 - 0.185i$

Finite Volume Hamiltonian

- Cube with side length L with periodic b.c. $\implies$ discrete momenta

$$k_n = \frac{2\pi}{L}\sqrt{n}, \qquad n = n_x^2 + n_y^2 + n_z^2, \qquad C_3(n) \text{ counts degeneracy in } n$$

$$\int k^2\, dk = \frac{1}{4\pi}\int d^3k \to \frac{1}{4\pi}\sum_{\vec{n}\in\mathbb{Z}^3}\left(\frac{2\pi}{L}\right)^3 \to \frac{1}{4\pi}\sum_n C_3(n)\left(\frac{2\pi}{L}\right)^3$$

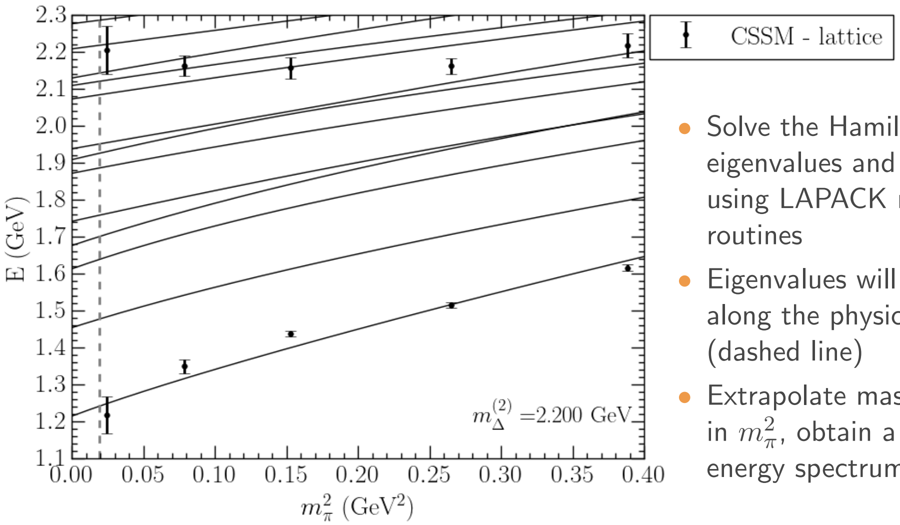
- Finite volume corrections

$$\bar{G}_\alpha^{B_0}(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{3/2} G_\alpha^{B_0}(k_n)$$

$$\bar{V}_{\alpha\beta}(k_n, k_m) = \sqrt{\frac{C_3(n)}{4\pi}}\sqrt{\frac{C_3(m)}{4\pi}}\left(\frac{2\pi}{L}\right)^3 V_{\alpha\beta}(k_n, k_m)$$

- Can solve the now finite volume Hamiltonian matrix for its eigenvalues and eigenvectors

Finite Volume Energy Spectrum (preliminary)



- Solve the Hamiltonian for eigenvalues and eigenvectors using LAPACK numerical routines
- Eigenvalues will sit at points along the physical pion mass (dashed line)
- Extrapolate masses of hadrons in m_π^2, obtain a finite volume energy spectrum in m_π^2

Eigenvector Analysis

Eigenvectors are powerful since they tell us about the composition of energy eigenstates in terms of our basis states

Consider the states which have largest overlaps with:

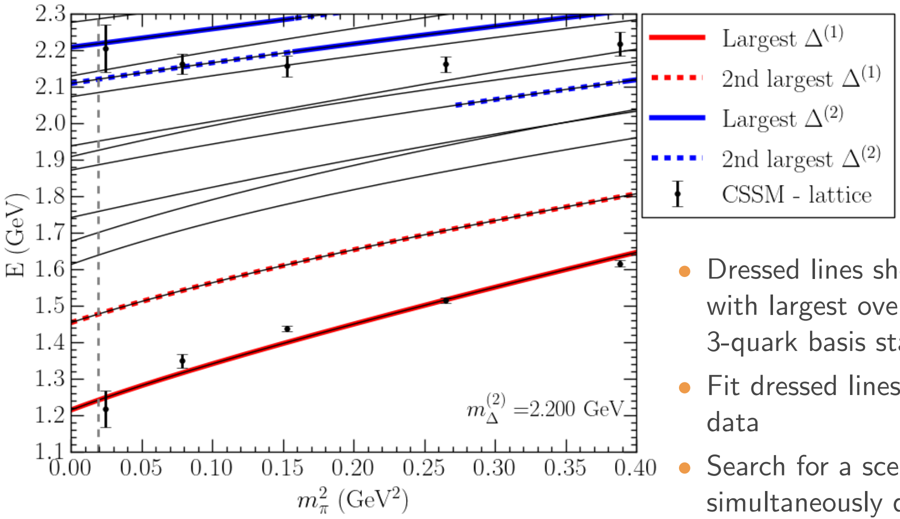
- 3-quark basis states $|\Delta^{(1)}\rangle\,,\ |\Delta^{(2)}\rangle$
- 2-particle basis states $|\pi N(k_n)\rangle\,,\ |\pi \Delta_p(k_n)\rangle\,,\ |\pi \Delta_f(k_n)\rangle$

In particular, the overlaps $\langle \Delta^{(1)} | E_i \rangle$ and $\langle \Delta^{(2)} | E_i \rangle$ tell us how dominant the quark model-like basis states are in the finite volume energy eigenstate $|E_i\rangle$

Hamiltonian eigenstates with the largest overlaps with the 3-quark states should match the lattice QCD results

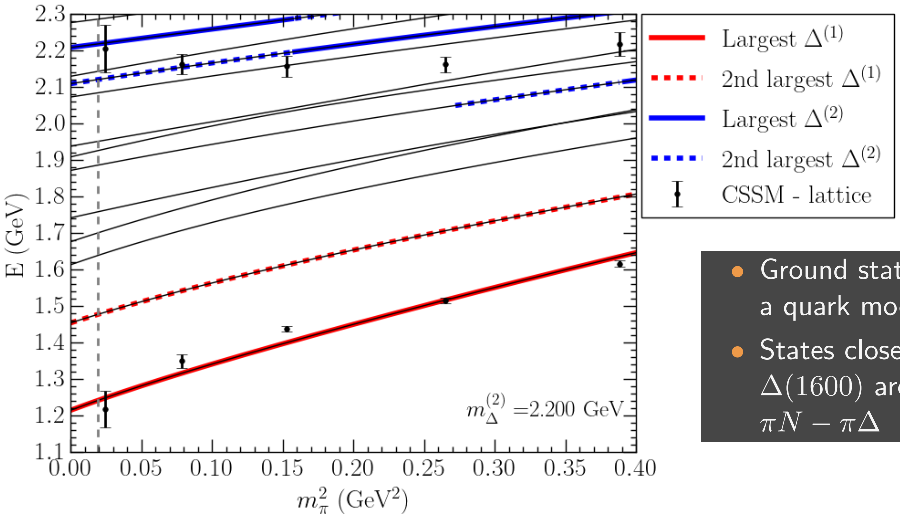
- Lattice QCD provides a second type of constraint for the HEFT results

Comparison with Lattice Data (preliminary)



- Dressed lines show the states with largest overlaps with the 3-quark basis states
- Fit dressed lines to the lattice data
- Search for a scenario which simultaneously describes experimental and lattice data

Comparison with Lattice Data (preliminary)



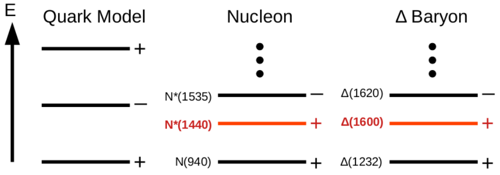
- Ground state dominated by a quark model-like state
- States close to the $\Delta(1600)$ are predominantly $\pi N - \pi\Delta$

Conclusions

HEFT provides a bridge between finite volume lattice results and infinite volume scattering data

LQCD + HEFT results suggest the following:

- $\Delta(1232)$ predominantly composed of a quark model-like state
- $\Delta(1600)$ is predominantly a dynamical resonance generated through πN and $\pi\Delta$ scattering

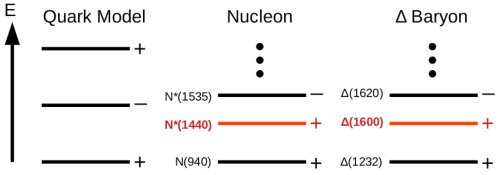


Conclusions

HEFT provides a bridge between finite volume lattice results and infinite volume scattering data

LQCD + HEFT results suggest the following:

- $\Delta(1232)$ predominantly composed of a quark model-like state
- $\Delta(1600)$ is predominantly a dynamical resonance generated through πN and $\pi\Delta$ scattering



***A*: The $\Delta(1600)$ doesn't appear to be a simple radial excitation of the $\Delta(1232)$, it appears to be dynamically generated**

Backup Slides

Lattice QCD - Gauge field configurations

- Well established method for first principles calculations of hadronic observables in the non-perturbative regime of QCD
- Note: we discretise space-time onto a 4D grid $\implies$ **finite volume**

Gauge field configurations provided by the PACS-CS collaboration

- $2+1$ flavour, dynamical-fermion configurations
- $\mathcal{O}(\alpha)$ improved Wilson fermion action $+$ Iwasaki-gauge action
- $32^3 \times 64$ lattice, periodic boundary conditions, $\beta = 1.90$
- 5 ensembles: $m_\pi = 702, 572, 413, 293, 156$ MeV

S. Aoki et al. [PACS-CS], Phys. Rev. D 79 (2009) 034503 [arXiv:0807.1661 [hep-lat]]

Lattice Ensembles

κ	m_π (MeV)	a (fm)	Number configs
0.13700	702	0.1023(15)	399
0.13727	572	0.1009(15)	397
0.13754	413	0.0961(13)	449
0.13770	293	0.0951(13)	400
0.13781	156	0.0933(13)	198*

*Lightest ensemble needs additional statistics

Hadron Spectroscopy on the Lattice

$$G_{ij}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle\Omega|T\{\chi_i(x)\bar{\chi}_j(0)\}|\Omega\rangle = \sum_{B^\pm} \lambda_i^\pm \bar{\lambda}_j^\pm \, e^{-m_{B^\pm} t}$$

- χ_i are 3-quark interpolating fields coupling to Δ baryons
- Apply positive parity and spin-3/2 projection operators
  Target $J^P = \frac{3}{2}^+$ Δ baryons: $\Delta(1232)$, $\Delta(1600)$, $\Delta(1920)$
- Use a **variational method** to project excited states

$$\left[(G(t_0))^{-1}G(t_0+\Delta t)\right]u^\alpha = e^{-m_\alpha \Delta t}u^\alpha\,, \qquad v^\alpha\left[G(t_0+\Delta t)(G(t_0))^{-1}\right] = e^{-m_\alpha \Delta t}v^\alpha$$

$$\implies M^\alpha_{\text{eff}}(t) = \ln\left(\frac{v^\alpha G(t,\vec{p}=0)u^\alpha}{v^\alpha G(t+1,\vec{p}=0)u^\alpha}\right), \qquad \alpha = \text{energy level}$$

Spin Projection Operators

$$P^{3/2}_{mn}(\vec{p}) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}(\gamma \cdot p\gamma_\mu p_\nu + p_\mu\gamma_\nu\gamma \cdot p)$$

At rest $\vec{p} = 0$:

$$P^{3/2}_{mn}(0) = \delta_{mn}\mathbb{I} - \frac{1}{3}\gamma^S_m\gamma^S_n$$

where S denotes the Sakurai representation:

$$\gamma^4_S = \gamma^0_{\text{Dirac}},\ \gamma^i_S = -i\gamma^i_{\text{Dirac}}$$

Spin projected correlator:

$$G^{3/2}_{\mu\nu}(\vec{p} = 0) = \sum_{\sigma,\rho=1}^{4} G_{\mu\sigma}g^{\sigma\rho}P^{3/2}_{\rho\nu}(0)$$

Chiral Extrapolation

Expand the bare Δ masses about the physical pion mass:

$$m_{\Delta}^{(i)} = m_{\Delta}^{(i)}\big|_{\mathsf{phys}} + \alpha_i \left(m_\pi^2 - m_\pi^2\big|_{\mathsf{phys}} \right) \qquad i = 1, 2$$

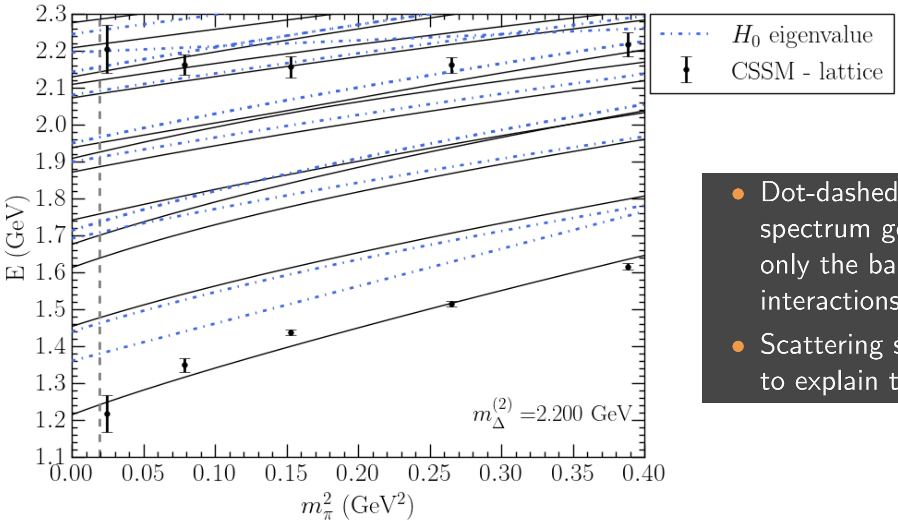
Likewise

$$m_B = m_B\big|_{\mathsf{phys}} + \alpha_B \left(m_\pi^2 - m_\pi^2\big|_{\mathsf{phys}} \right) \qquad B = N, \Delta$$

Fix the lattice size at $L = 3$ fm and solve the matrix eigenvalue problem for unphysical values of m_π^2

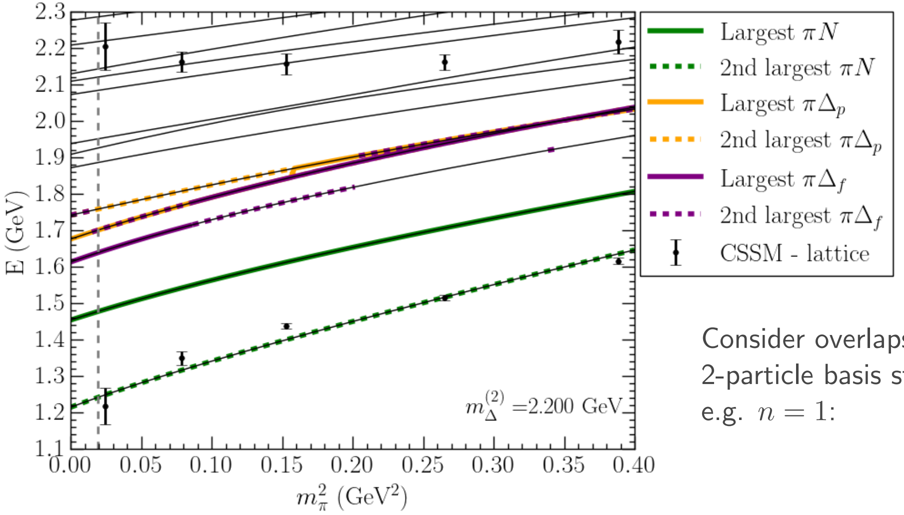
- α_N and α_Δ have known values from previous lattice studies
- Perform fits to our lattice data to obtain α_1 and α_2

Non-interacting state spectrum



- Dot-dashed lines show the spectrum generated from only the bare states, i.e. no interactions
- Scattering states necessary to explain the lattice data

2-particle dominated states



Consider overlaps with 2-particle basis states with e.g. $n = 1$:

Current Best Fit Parameters

$g^{(1)}_{\pi N}$	0.1345
$g^{(1)}_{\pi \Delta_p}$	0.0965
$g^{(1)}_{\pi \Delta_f}$	0.0234
$g^{(2)}_{\pi N}$	0.1209
$g^{(2)}_{\pi \Delta_p}$	0.1701
$g^{(2)}_{\pi \Delta_f}$	0.0755
$v_{\pi N, \pi N}$	-0.0087
$v_{\pi N, \pi \Delta_p}$	-0.0739
$v_{\pi N, \pi \Delta_f}$	-0.0938
$v_{\pi \Delta_p, \pi \Delta_p}$	-0.0199
$v_{\pi \Delta_f, \pi \Delta_p}$	0.0074
$v_{\pi \Delta_f, \pi \Delta_f}$	-0.0768

$m^{(1)}_{\Delta}$	1.3806
$m^{(2)}_{\Delta}$	2.2000
$\Lambda_{\pi N}$	0.8350
$\Lambda_{\pi \Delta_p}$	0.8645
$\Lambda_{\pi \Delta_f}$	0.7932
$\Lambda^{v}_{\pi N}$	0.6224
$\Lambda^{v}_{\pi \Delta_p}$	0.7826
$\Lambda^{v}_{\pi \Delta_f}$	0.8302