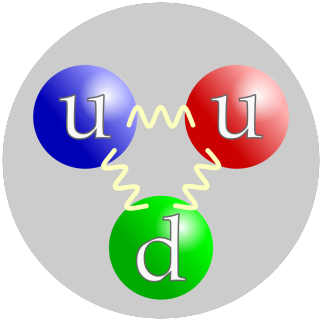
Impact of dynamical fermions on the centre-vortex structure of QCD ground-state fields and confinement

Derek Leinweber

In collaboration with:

James Biddle & Waseem Kamleh

Images of the proton found in a Google search



- Wikipedia. Grey disk not described.

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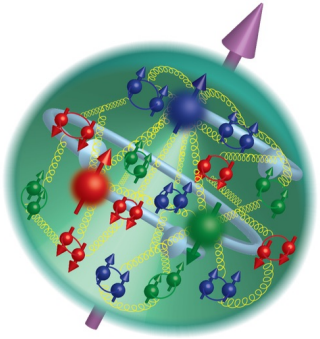

- Artistic rendering of proton structure revealing its intricate and dynamic system of quarks and gluons. (Image by Argonne National Laboratory.)

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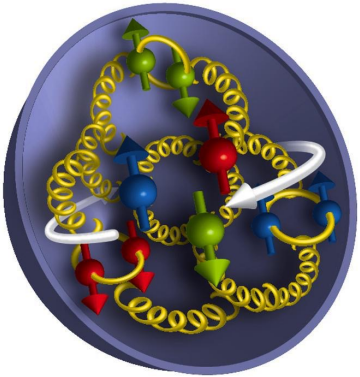

- The internal structure of a proton, with quarks, gluons, and quark spin shown. The nuclear force acts like a spring, . . . Brookhaven National Laboratory.

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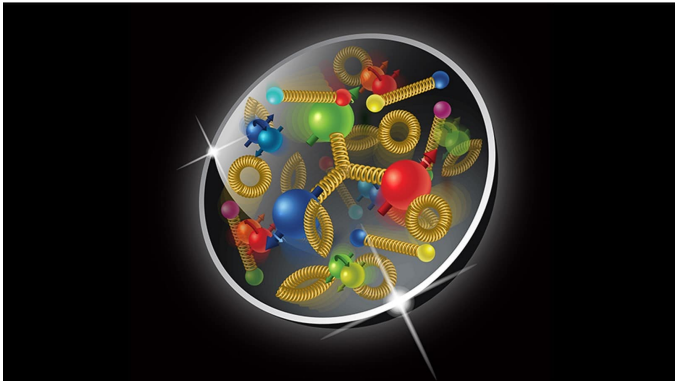
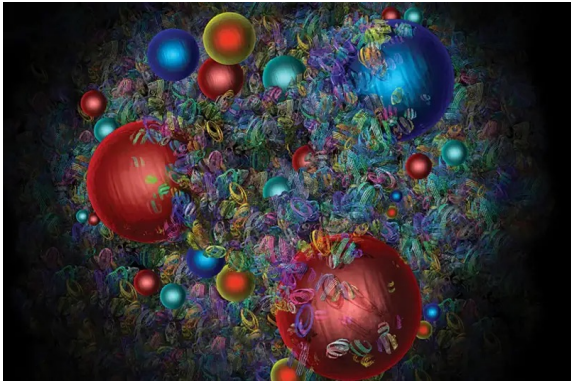


- Quark and gluon sea: in this illustration of the proton the large spheres represent the three valence quarks, the small spheres other quarks and the springs the gluons holding them together. (Courtesy: Brookhaven National Laboratory.)

Images of the proton found in a Google search

- This one's different . . .

Images of the proton found in a Google search



- **Glorious complexity** An artist's impression of the mayhem of quarks and gluons inside the proton. Credit: Daniel Dominguez.

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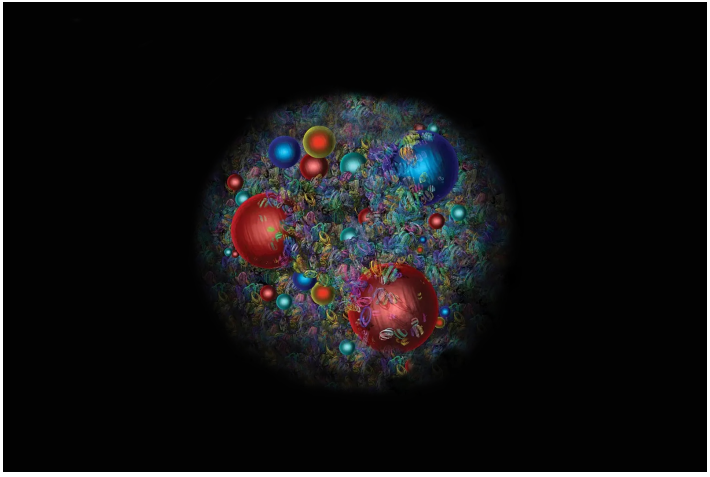
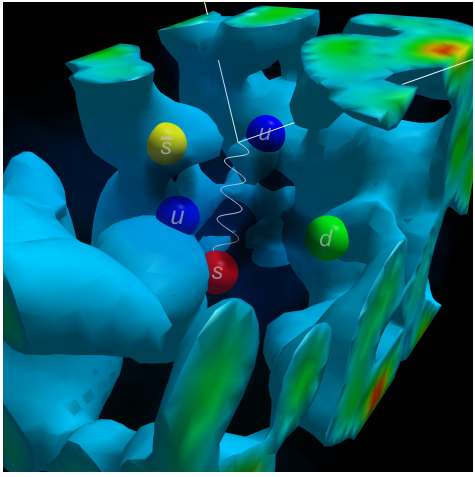


Images of the proton not found in a Google search (from 2006)



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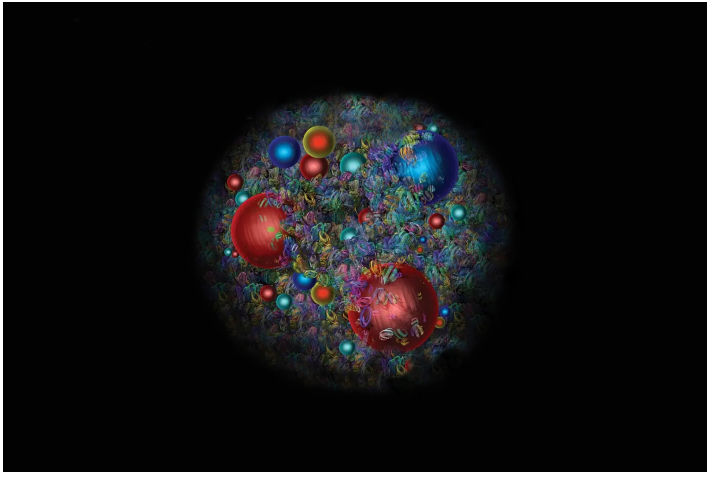

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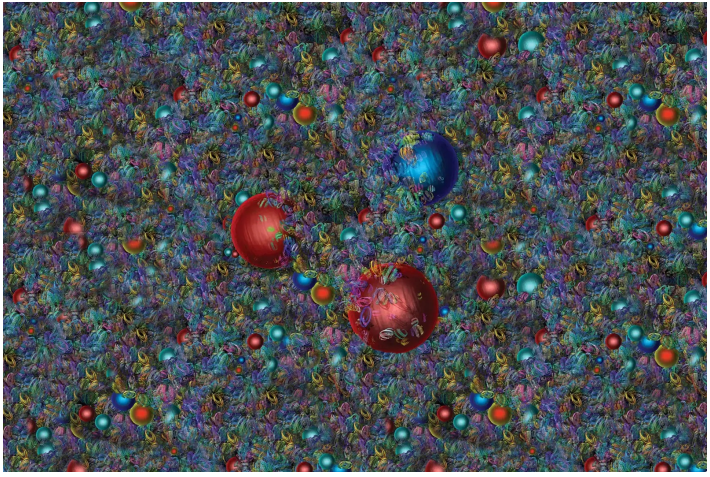

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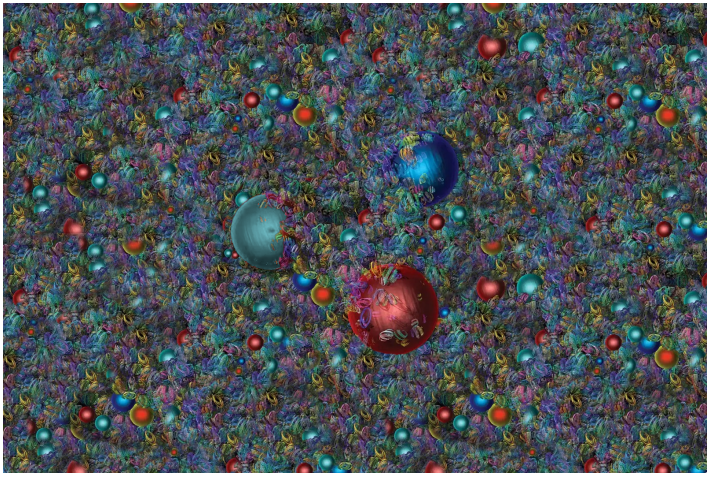

Introduction

- What aspect of the nontrivial ground-state field structure confines quarks?
- Is there an essential aspect that captures the salient features of QCD?
  - Confinement.
  - Dynamical generation of mass via Chiral Symmetry breaking.

Centre-Vortices in the Ground-State QCD-Vacuum Fields

- What are Centre Vortices and how do we locate them?

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- What do Centre Vortices look like?

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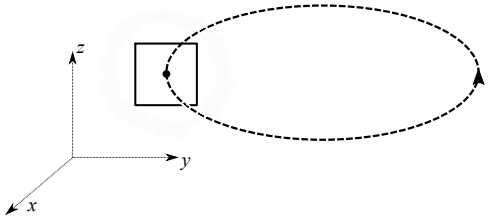
Centre-Vortices in the Ground-State QCD-Vacuum Fields

- What are Centre Vortices and how do we locate them?
- What do Centre Vortices look like?
- What is the impact of dynamical fermions on centre-vortex structure?
- How does this manifest in the emergent properties of QCD?
  - Static Quark Confinement
  - Positivity Violation in the Gluon Propagator

What Are Centre Vortices?

- Centre vortices in 3D are tube-like topological defects present in the QCD vacuum.
- We locate thin vortex lines on the lattice.
- The vortex line can be thought of as the 'axis of rotation' of the vortex.



A centre vortex (dashed line) intersecting a lattice plaquette (solid square).

Vortex Structure in the Colour Fields of the QCD Vacuum

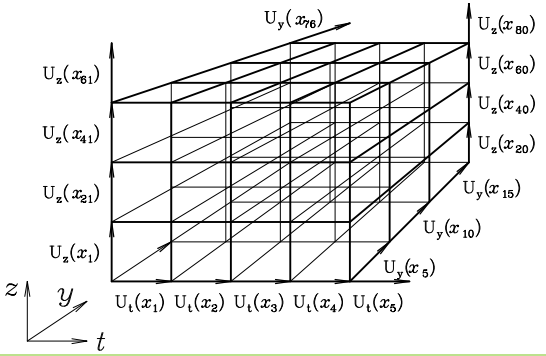
How do you find centre vortices?

Lattice Links

- On the lattice, the gluon-field is encoded in terms of the link variable

$$U_\mu^{ab}(x) \simeq \exp\left(i\, a\, g\, A_\mu^{ab}(x)\right) ,$$

  a 3×3 complex special-unitary matrix.

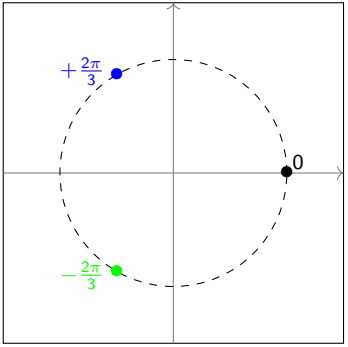


Centre Group of SU(3)

- Centre elements commute with every group element,

$$z = \exp\left(\frac{2\pi i}{3} m\right) I, \quad m \in \{-1, 0, 1\} \simeq \mathbb{Z}_3 \,.$$

- Each of the three centre phases corresponds to a centre element of SU(3).



1. Maximal Centre Gauge

- Gauge transformations bring the links close to an element of the group centre

$$z = \exp\left(\frac{2\pi i}{3}\, m\right) I\,, \;\; m \in \{-1,\, 0,\, +1\}\,.$$

- This is done by maximising the functional

$$R = \sum_x \sum_\mu |\, \mathrm{tr}[U_\mu(x)]\,|^2$$

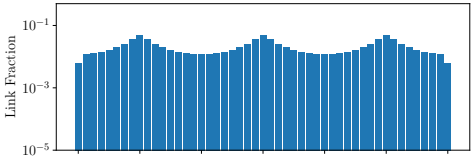
- This is called **Maximal Centre Gauge**

1. Maximal Centre Gauge

- Distribution of link phases.

$$\operatorname{tr} U_\mu^{\mathrm{MCG}}(x) \;=\; \underbrace{r_\mu(x)}_{\text{real}} \exp\underbrace{\left(\frac{2\pi i}{3}\,\phi_\mu(x)\right)}_{-\pi \,<\, \text{phase} \,\leq\, \pi}, \qquad -\frac{3}{2} < \phi_\mu(x) \leq \frac{3}{2}\,.$$

- $\phi_\mu(x)$ before gauge fixing.



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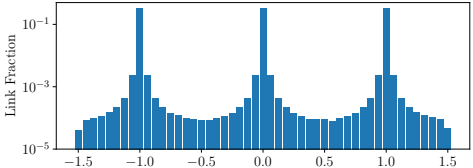

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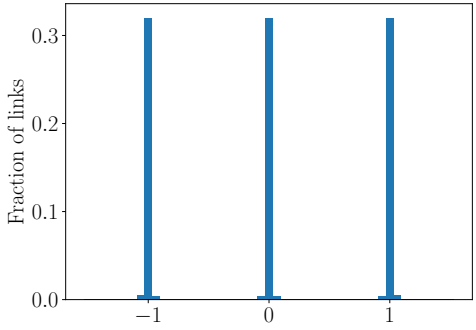
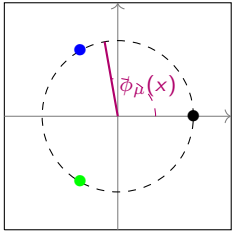


2. Centre Projection

- Project onto $Z(3)$

$$U_\mu^{\mathrm{MCG}}(x) \to Z_\mu(x) = \exp\left(\frac{2\pi i}{3}\, m_\mu(x)\right) I\,,\ \ m_\mu(x) \in \{-1,\, 0,\, +1\}\,.$$



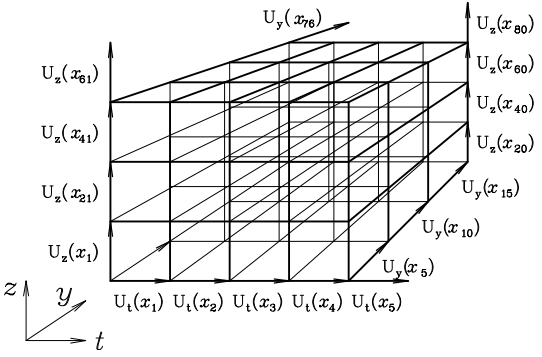
- Eight degrees of freedom are replaced by one of the three cube-roots of 1.

2. Centre Projection

- This projection allows us to define 3 sets of configurations:
  - Untouched - $U_\mu(x)$
  - Vortex Only - $Z_\mu(x)$
  - Vortex Removed - $R_\mu(x) = Z_\mu^\dagger(x)\, U_\mu(x)$

3. Identifying Vortices

- Examine the product of $Z_\mu(x)$ around each elementary square (plaquette).
- Each plaquette takes a value from $Z(3)$.



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- Non-trivial plaquettes with values

$$\exp\left(\frac{2\pi i}{3}\, m\right) \neq 1\,, \quad i.e.\ m \in \{-1, +1\}\,,$$

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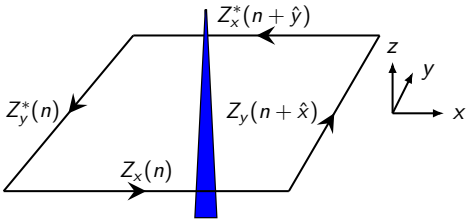
- 3 ensembles
  - $32^3 \times 64$ pure gauge (PG), spacing $a = 0.100$ fm
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.1022$ fm, $m_\pi = 701$ MeV
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.0933$ fm, $m_\pi = 156$ MeV
  - S. Aoki, *et al.* (PACS-CS), Phys. Rev. D **79**, 034503.

What do centre vortices look like?

Rendering Projected Vortices

- Vortex sheets are sliced to vortex lines in a 3D slice of the 4D lattice.
- Flow of centre charge $+1$ is indicated using a right-handed coordinate system.
- For example,
  - An $m = +1$ vortex in the x-y plane is plotted in the $+\hat{z}$ direction.



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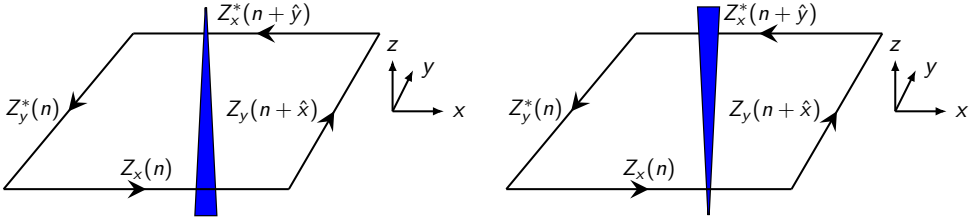
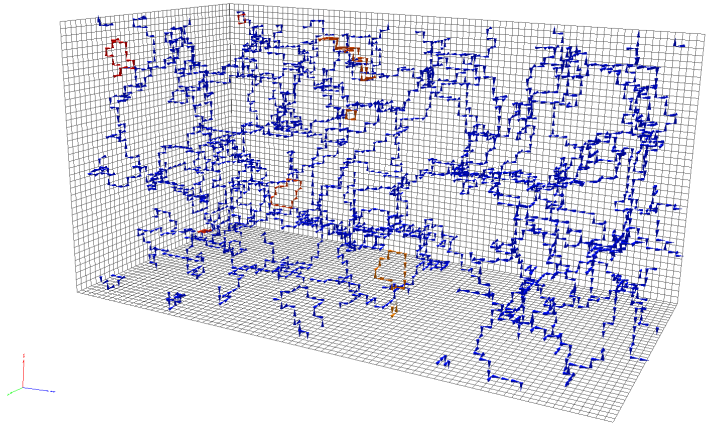
  - An $m = -1$ vortex in the x-y plane is plotted in the $-\hat{z}$ direction.

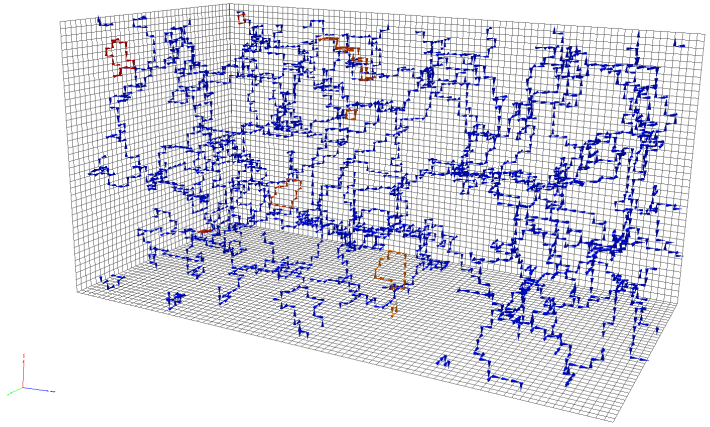


Vortices on a Pure-Gauge $32^3 \times 64$ Lattice

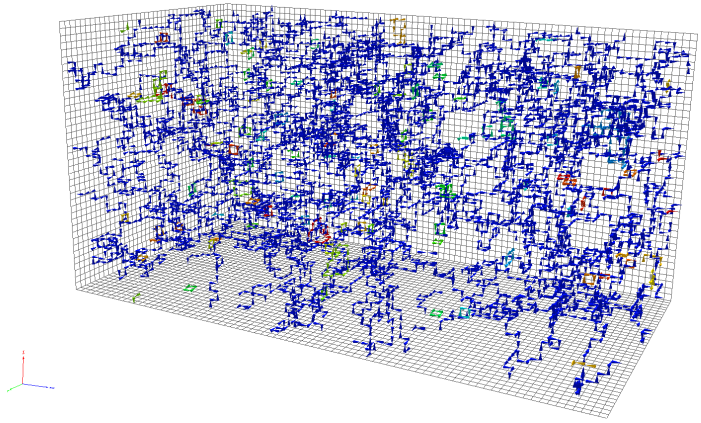


Impact of Dynamical Fermions on Centre Vortex Structure

Vortices on a Pure-Gauge $32^3 \times 64$ Lattice

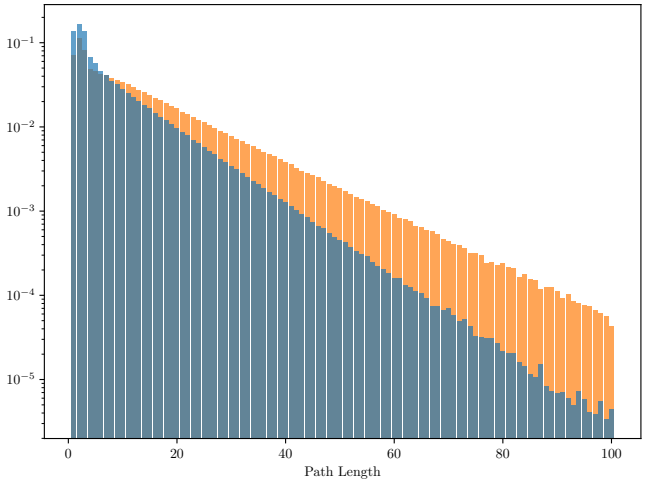


$2+1$ Flavour $32^3 \times 64$ Dynamical-Fermion Lattice $m_\pi = 156$ MeV



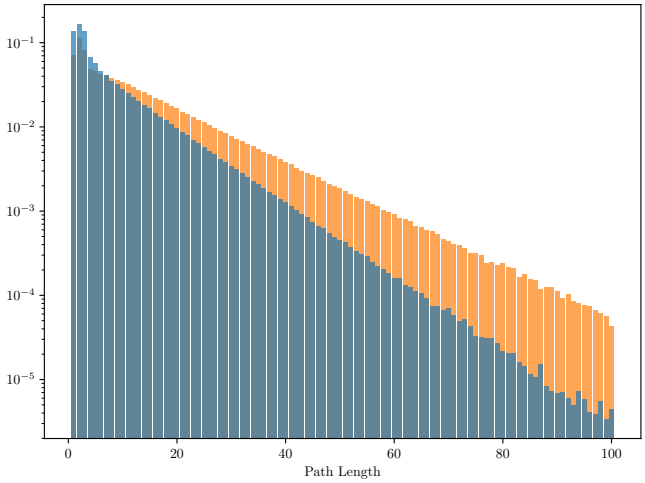
Impact of Dynamical Fermions: Vortex Path Lengths

- Histogram of vortex path lengths in the percolating cluster.
- Pure Gauge and Dynamical Fermion ensembles are illustrated.
- Path length is the number of jets from one branching point to the next.



Impact of Dynamical Fermions: Vortex Path Lengths

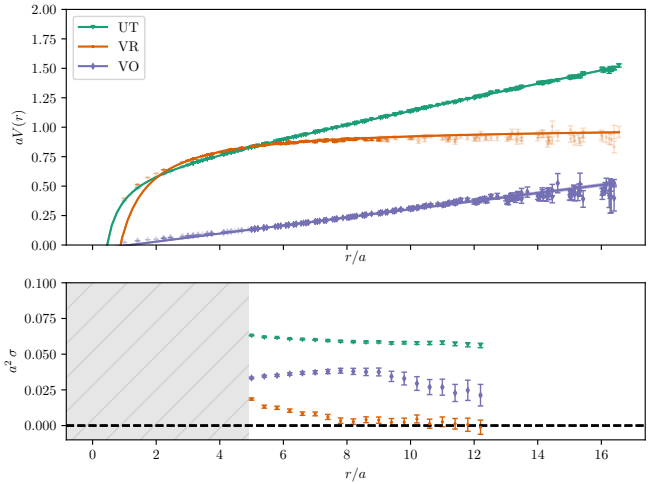
- Histogram of vortex path lengths in the percolating cluster.
- Pure Gauge and Dynamical Fermion ensembles are illustrated.
- Path length is the number of jets from one branching point to the next.
- Moderate size loops are exponentially distributed.
  - Fixed probability of branching .
  - Branching is independent of length.
  - Branching probability:
    - PG: $\sim 2/3$ fm^{-1}. DF: ~ 1 fm^{-1}.



Impact of Dynamical Fermions on the Centre-Vortex Structure of the Static Quark Potential

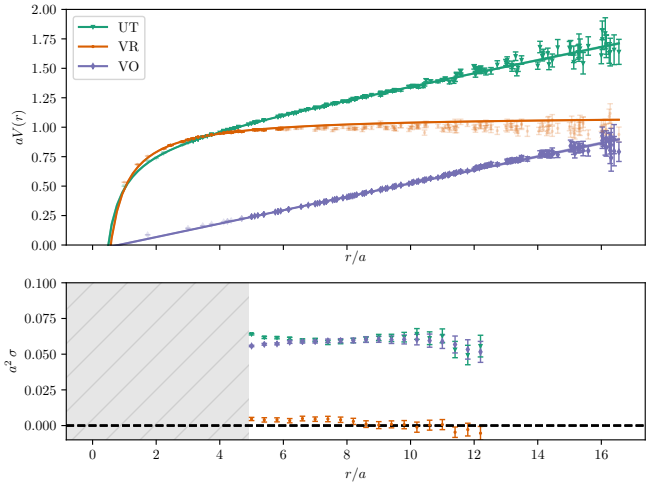
Static Quark Potential – Pure Gauge Sector

- Lower plot reports the local slope from fits to $V(r)$ over a forward-looking window from r to $r + 4a$.
- Vortex removal (VR) leaves no residual confining potential.
- Vortex-only (VO) reproduces only 62% of the original (UT) static quark potential.



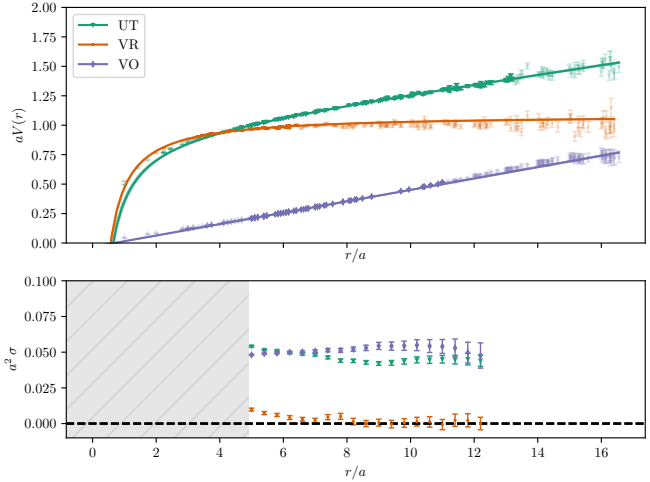
Introducing Dynamical Fermions ($m_\pi = 701$MeV)

- In the presence of dynamical fermions, vortices capture the full string tension.
- Vortex removal leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



Lighter dynamical fermions ($m_\pi = 156$MeV)

- Lighter quark masses screen the confining potential.
- Vortices continue to capture the full string tension.
- Vortex removal leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



Impact of Dynamical Fermions on the Centre-Vortex Structure of the Gluon Propagator

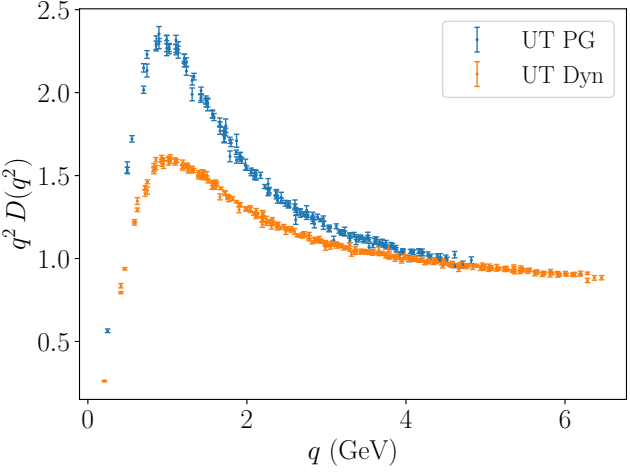
Centre Vortices and the Landau-Gauge Gluon Propagator

- The nonperturbative scalar gluon propagator in momentum space is

$$D(q^2) \equiv \frac{Z(q^2)}{q^2} \to \frac{1}{q^2} \text{ at tree level.}$$

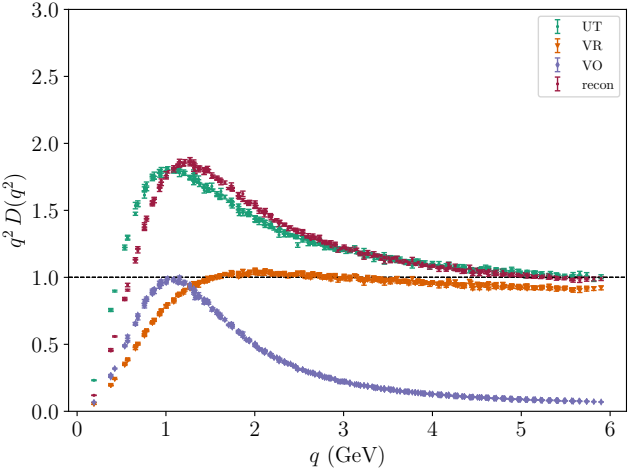
- Consider the renormalisation function

$$Z(q^2) = q^2\, D(q^2)\,.$$



Gluon Propagator – Dynamical Fermions

- Dynamical fermions (UT) suppress the overall infrared strength.
- Vortex Removal (VR) almost eliminates infrared enhancement.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstructed propagator.



The Gluon Propagator and Confinement

- The 1-dimensional Fourier transform of the gluon propagator at zero spatial momentum defines the **wall-to-wall correlator**

$$C(t) = \int_0^\infty dm\, \rho(m^2)\, e^{-mt}\,.$$

  - C(t) is negative in QCD.

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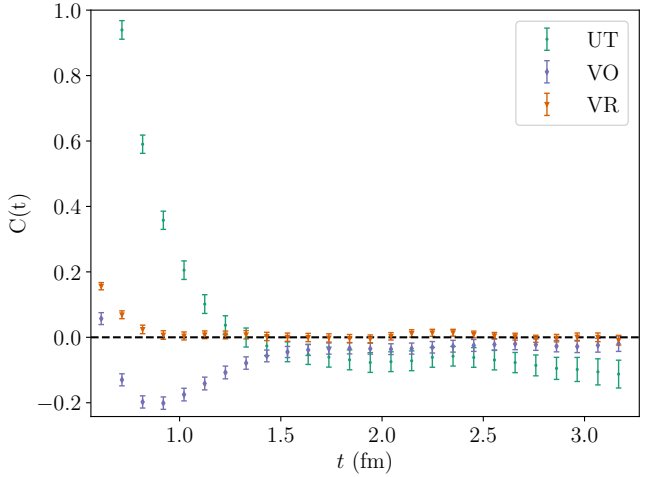
- The states are confined from the physical world.
  - J. E. Mandula and M. Ogilvie, Phys. Lett. **B185**, 127 (1987).
  - C. A. Aubin and M. C. Ogilvie, Phys. Lett. **B570**, 59 (2003), hep-lat/0306012.

Positivity Violation in QCD



Selected References

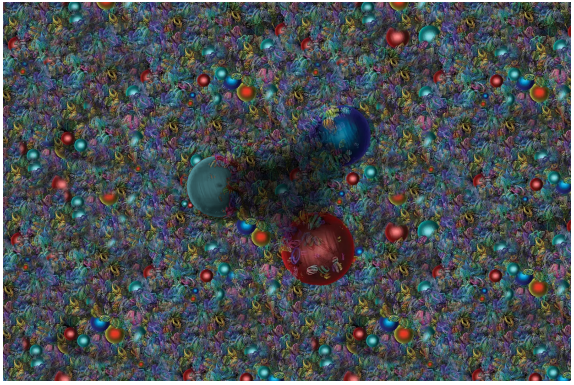
- "Impact of dynamical fermions on the center vortex gluon propagator,"
  J. Biddle, W. Kamleh and D. Leinweber, Phys. Rev. D **106** (2022) 014506 [arXiv:2206.02320 [hep-lat]].

- "Static quark potential from centre vortices in the presence of dynamical fermions,"
  J. Biddle, W. Kamleh and D. Leinweber, Phys. Rev. D **106** (2022) 054505 [arXiv:2206.00844 [hep-lat]].

- "Dynamical fermions, centre vortices, and emergent phenomena"
  D. Leinweber, J. Biddle, W. Kamleh and A. Virgili, [arXiv:2211.13421 [hep-lat]].
  Plenary presentation summary to appear in the proceedings of the XVth Quark Confinement and the Hadron Spectrum conference, 1st-6th August 2022, Stavanger, Norway.

- "Centre vortex structure in the presence of dynamical fermions,"
  J. Biddle, W. Kamleh and D. Leinweber, In preparation.

Ironic incongruity



- Valence quarks act to suppress vacuum fields as they induce flux-tube tunnels through the ground-state vacuum fields, underpinned by centre vortices.

Conclusions

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  - Restores positivity in the wall-to-wall correlator.
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  - Confinement, and the
  - Dynamical generation of mass. (Waseem Kamleh's presentation Tuesday morning.)
- Centre vortices in the foundation of matter capture the essence of nonperturbative QCD.

Perspective

- Needless to say, numerical evidence is not taken for a proof by mathematically inclined folks, and an analytic proof [for confinement] is still missing. A million dollar prize for such a proof still waits to be awarded.

  Edward Shuryak

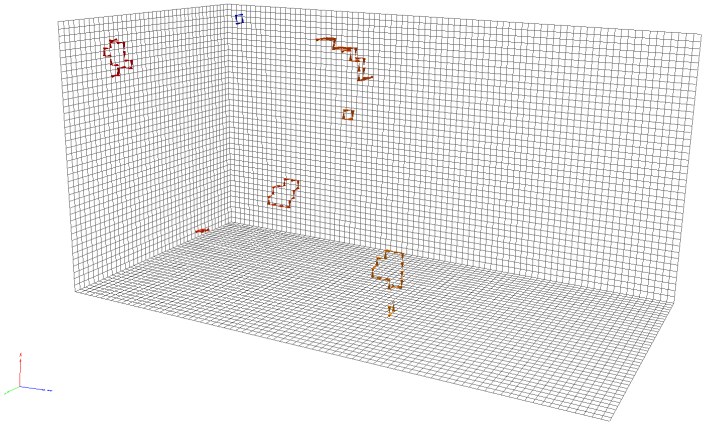
- To appear in the forthcoming book

  "50 Years of Quantum Chromodynamics"

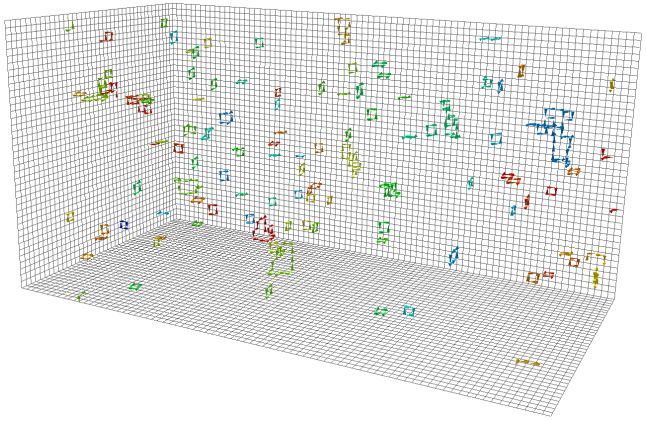
  edited by Franz Gross and Eberhard Klempt.

Additional Information

Secondary Loops on a Pure-Gauge $32^3 \times 64$ Lattice



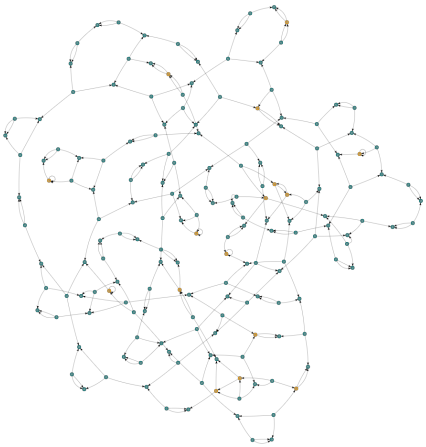
$2+1$ Flavour $32^3 \times 64$ Dynamical-Fermion Lattice $m_\pi = 156$ MeV



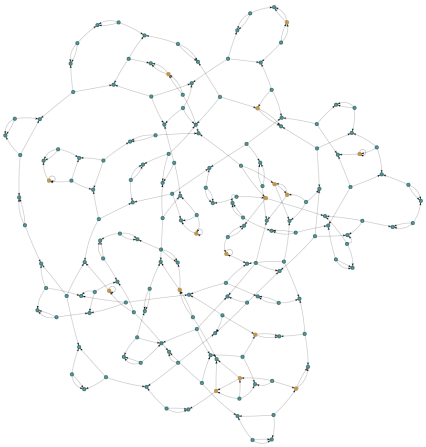
Directed Graphs

- Abstract the vortex clusters as a directed graph, independent of 3D coordinates.
- Points of vortex branching/intersection are nodes.
- Edges are weighted by the vortex path length connecting them.
- Visualisations are generated with the Pyvis package.

Directed graphs: Pure Gauge versus Dynamical Fermions

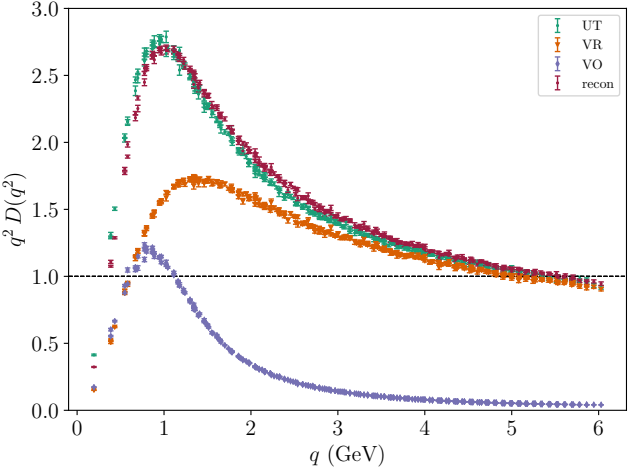


Directed graphs: Pure Gauge versus Dynamical Fermions



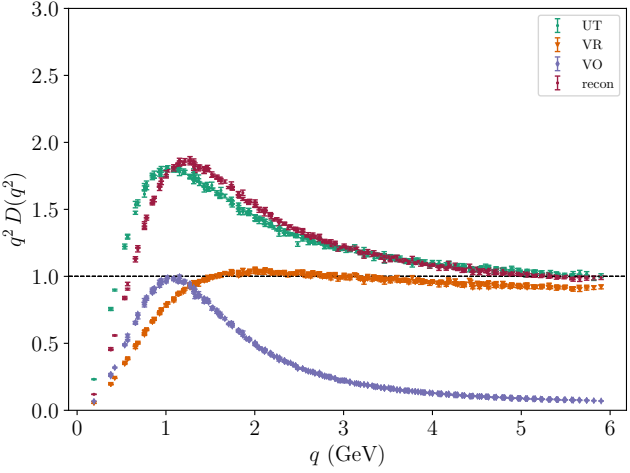
Gluon Propagator – Pure Gauge Sector

- Vortex Removal (VR) suppresses infrared enhancement whilst preserving UV perturbative behaviour.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstruction of the propagator as a linear combination of the vortex-modified parts recovers full propagator.
- Renormalisation maintains $Z(q^2) = 1$ at $q = 5.5$ GeV for UT and Recon.



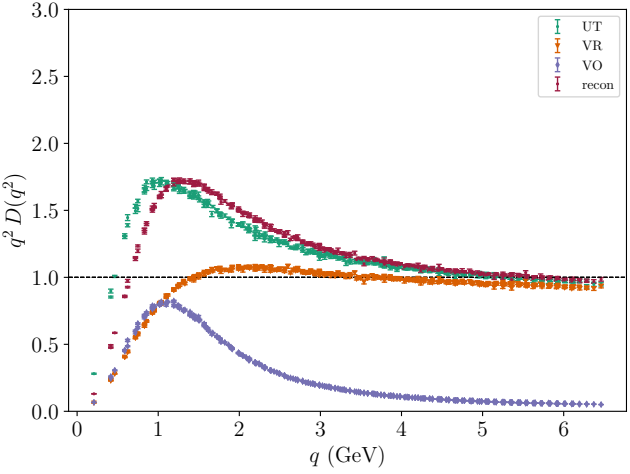
Gluon Propagator – Dynamical Fermions $m_\pi = 701$ MeV

- Dynamical fermions (UT) suppress the overall infrared strength.
- Vortex Removal (VR) almost eliminates infrared enhancement.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstruction is less perfect.

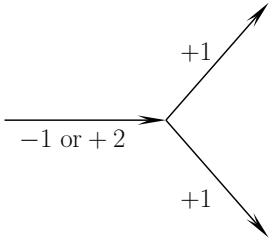


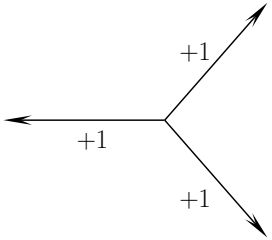
Gluon Propagator – Dynamical Fermions $m_\pi = 156$ MeV

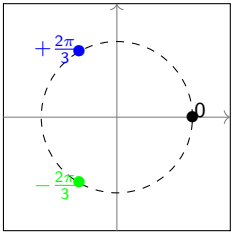
- Lighter dynamical u and d quarks further suppress the infrared enhancement.
- Centre Vortex degrees of freedom are able to capture the screening effects of dynamical fermions in QCD.



Branching Points versus Monopoles







- Our convention illustrates the directed flow of charge $m = +1$.
- Arrows indicate the direction of flow for the labelled charge.
- However, a vortex monopole with charge $+1$ flowing out of the vertex (centre) is equivalent to
  a vortex branching point with centre charge $+2$ flowing into a vertex (left).

Restoration of Chiral Symmetry

- If vortices are responsible for $D\chi SB$, then their removal should restore chiral symmetry

$$\mathrm{SU}\,2_\mathrm{L} \times \mathrm{SU}\,2_\mathrm{R} \times \mathrm{U}(1)_\mathrm{A}$$

- Expect hadrons related by chiral transformations to become degenerate

$$\begin{array}{ccc} \pi & \overset{\mathrm{U}(1)_\mathrm{A}}{\longleftrightarrow} & a_0 \\ \rho & \overset{\mathrm{SU}\,2_\mathrm{L}\times\mathrm{SU}\,2_\mathrm{R}}{\longleftrightarrow} & a_1 \\ N & \overset{\mathrm{SU}\,2_\mathrm{L}\times\mathrm{SU}\,2_\mathrm{R}}{\longleftrightarrow} & \Delta \end{array}$$

- At light quark masses, all symmetries are observed to be restored.
- A. Trewartha, W. Kamleh and DBL, J. Phys. G **44** (2017) 125002 [arXiv:1708.06789 [hep-lat]].

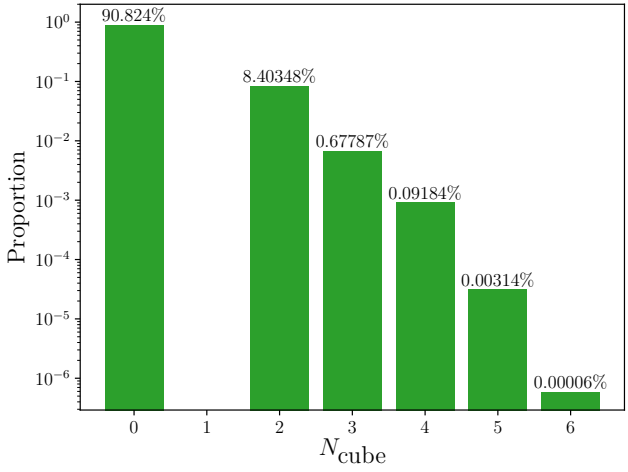
Visualising Centre Vortices

- Consider the number of vortices entering a 3D cube on the dual lattice.

$N_{\text{cube}}(\tilde{x})$	Interpretation
0	No vortices present.
1	Terminating vortex, forbidden by Bianchi*.
2	Vortex line flowing through the cube.
3	Simple three-way vortex monopole.
4	Vortex intersection.
5	Complex five-way monopole path.
6	Vortex intersections or double monopoles.

*Bianchi identity implies a continuous flow of centre vortex flux through a spatial cube.

Visualising Centre Vortices



Space-Time Oriented Vortices

Rendering Space-Time Oriented Projected Vortices

- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.

Rendering Space-Time Oriented Projected Vortices

- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.
- The three jets associated with the spatial x-y, y-z and z-x plaquettes, are complemented by
  - Jets in the three forward time x-t, y-t and z-t plaquettes, and
  - Jets in the three backward time x-t, y-t and z-t plaquettes.

Rendering Space-Time Oriented Projected Vortices

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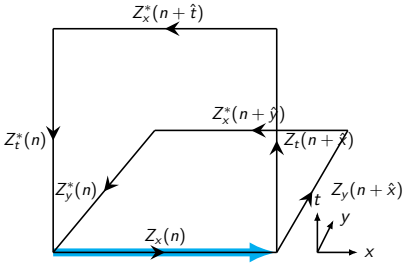
  - Jets in the three backward time x-t, y-t and z-t plaquettes.
- See "Visualization of center vortex structure," to link vortices to topological charge.
  J. C. Biddle, W. Kamleh and DBL, Phys. Rev. D **102** (2020) 034504 [arXiv:1912.09531 [hep-lat]].

Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex.



Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
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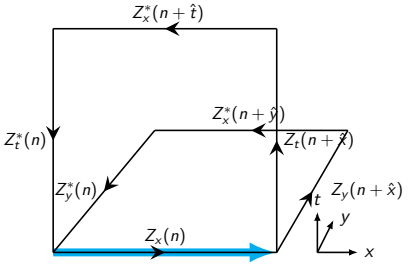
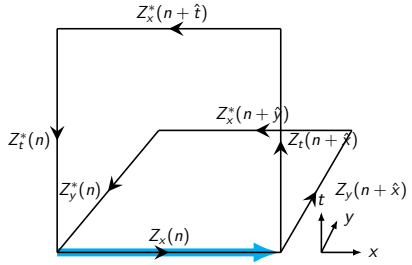
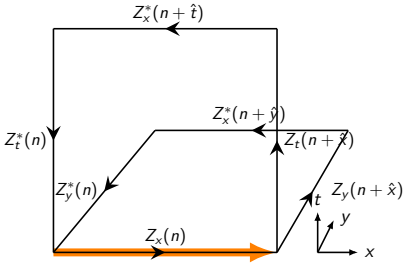
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.



Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex, and in orange for $m = -1$.
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.





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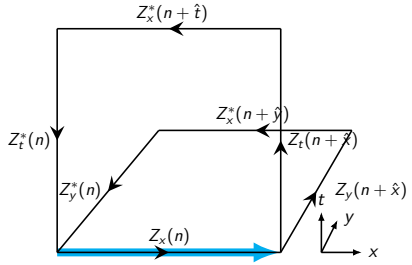
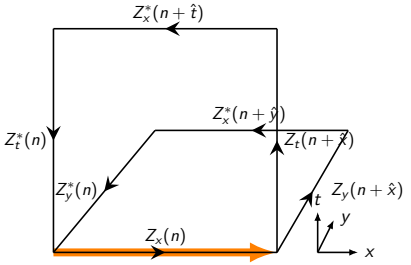
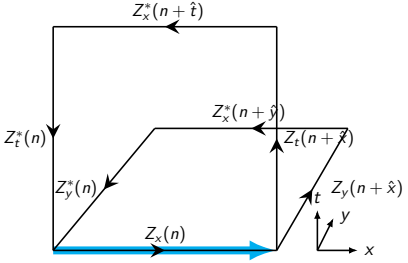
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.

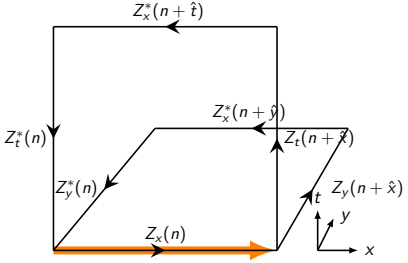




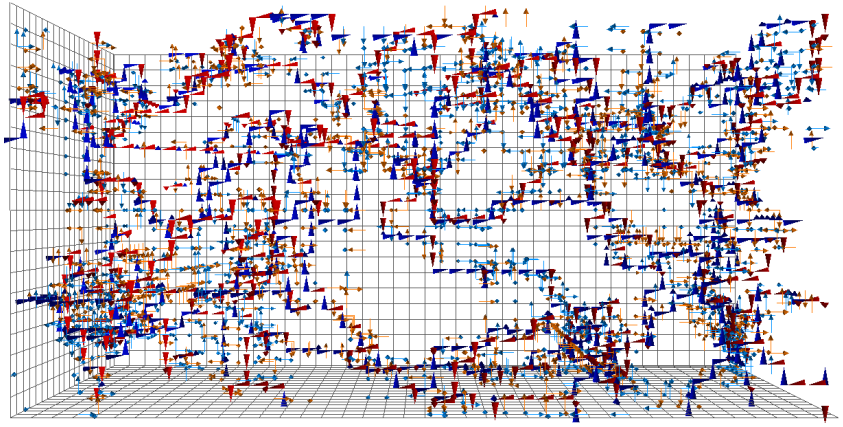
Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.
- As one steps forwards in time, positively-directed links become negatively-directed.

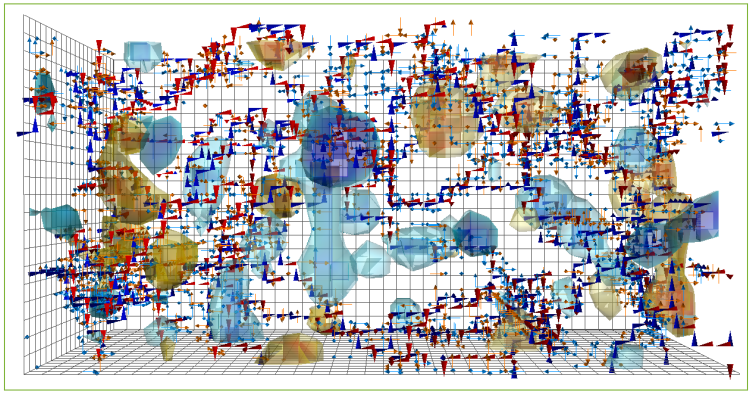




Time slice $t = 1$



Animation of Centre Vortex Structure Google: YouTube CSSM Visualisations



Interactive 3D Visualisation Techniques

- Rendered in AVS Express Visualisation Edition.
  `http://www.avs.com/solutions/express/`
- Exported in VRML.
- Converted to U3D format via pdf3d ReportGen.
  `https://www.pdf3d.com/products/pdf3d-reportgen/`
- Imported into LaTeX via media9 package.
- Viewed in Adobe acroread (Linux, use 9.4.1 when 3D support was maintained).
  `ftp://ftp.adobe.com/pub/adobe/reader/unix/9.x/9.4.1/`