Exploring higher order dispersion: Families of exact soliton solutions

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Solitons are pulses which are unchanged in shape as they propagate, due to the balance of nonlinear effects with the effect of dispersion. Traditionally, studies on solitons have focused on quadratic dispersion, where the (inverse) group velocity depends linearly on frequency. This effect often dominates higher orders of dispersion, however recent studies have shown that there in fact exist large families of soliton solutions where higher order dispersion dominates \cite{1, 2}. New experimental capabilities now allow for repeatable generation of these soliton solutions in a fibre laser with a pulse shaper \cite{3}. These experimental results have coincided with increased numerical and analytic studies of soliton solutions involving exclusively a combination of 2\textsuperscript{nd} and 4\textsuperscript{th} order dispersion \cite{2}. We have extended these results to include 6th order dispersion, providing greater understanding of existing solutions, and predicting the characteristics of solutions at higher dispersion orders. Within these higher order dispersion regimes, we find families of exact analytic soliton solutions at arbitrary dispersion order and methods to obtain the associated dispersion coefficients. In Fig. 1, we show an example, of an exact analytic solution of the form $u = A \text{sech}^p(\alpha \tau)$ at different dispersion orders, up to 100\textsuperscript{th} order. These analytic results introduce a foundation for future studies of solitons at any dispersion order, as numerical and experimental capabilities continue to improve and expand into higher dispersion domains.

![Figure 1: Power of solutions of the form $u = A \text{sech}^p(\alpha \tau)$, with normalized power $\gamma P = 1.000 \text{ mm}^{-1}$ and $\alpha = 0.5000 \text{ ps}^{-1}$ kept constant, for $p = 3$ (blue), $p = 4$ (red), $p = 15$ (yellow), $p = 50$ (purple) on (a) a linear scale, and (b) a log scale.](image)

\cite{1} R. Parker, et al, Physica D: Nonlinear Phenomena 422, 132890 (2021)
