Progress toward uncovering the spin of a vortex

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Low-dimensional physical systems have proven to provide a fertile ground on which the interplay between gravity and quantum field theory can be studied. In particular, a $(2+1)$-dimensional gravity has been studied intensively since Witten discovered a dualism which connects it to $\mathrm{SO}(1,2)_k$ Chern–Simons theory, where the level k on one side relates to the cosmological constant on the other. This suggests that a (2+1)-dimensional gravity is a topological field theory implying that interactions are only manifest as non-local effects. Theories of this type can be simulated in superfluid condensates through *analog gravity*. In addition to gravity, it has also been shown that electromagnetic field theory as well as quantum field theory can be simulated through the generation of quantum turbulence. Hence, we may refer to a model of this kind as a *superfluid universe*.

Adopting a Maxwell–Einstein picture of a planar superfluid allows us to study the interaction between vortices, which play the role of electrons in such an analogue universe, and analogue electromagnetic as well as gravitational fields. By employing the tetradic formalism we cast the gravitational part of the theory into the form of a gauge gravity theory in which the Poincaré group plays the role as gauge group, and the spin-connection ω^{μ}_{ν} as the gauge field. The existence of such a non-abelian gauge group suggests that vortex modes, also known a *kelvons*, possess an intrinsic spin. A question highly pertinent thus is what this spin corresponds to physically. It is well known that the presence of a kelvon quasi-particle within the vortex core causes the core structure to undergo deformation, which in turn breaks the $\mathrm{SO}(2)$ symmetry of the vortex. This breaking of rotational symmetry induces multi-pole moment contributions in the vortex velocity field. It may therefore be natural to conjecture that the second term in the multi-pole expansion, the dipole, relates to the kelvon spin. Numerical results indicate that the dipole moment indeed couple to gravity and that it may acquire a gravitational non-abelian geometric phase akin to a spin-$\frac{1}{2}$ particle in a magnetic field.

In light of the aforementioned dualism connecting $(2+1)$-dimensional gravity and Chern–Simons theory, the question whether gravitational interactions can be mediated in a topological fashion arise. We show via explicit calculations that there exists an infinite set of asymptotically flat analog space-times that render the interactions topological, thus making possible the implementation of topologically protected transformations. In particular, we show that an arbitrary non-abelian phase may be acquired by simply manipulating the density landscape of the fluid in a particular way. Finally, applications within topological quantum computation is discussed and the notion of computational universality is illuminated upon.