# Progress towards uncovering the spin of a vortex

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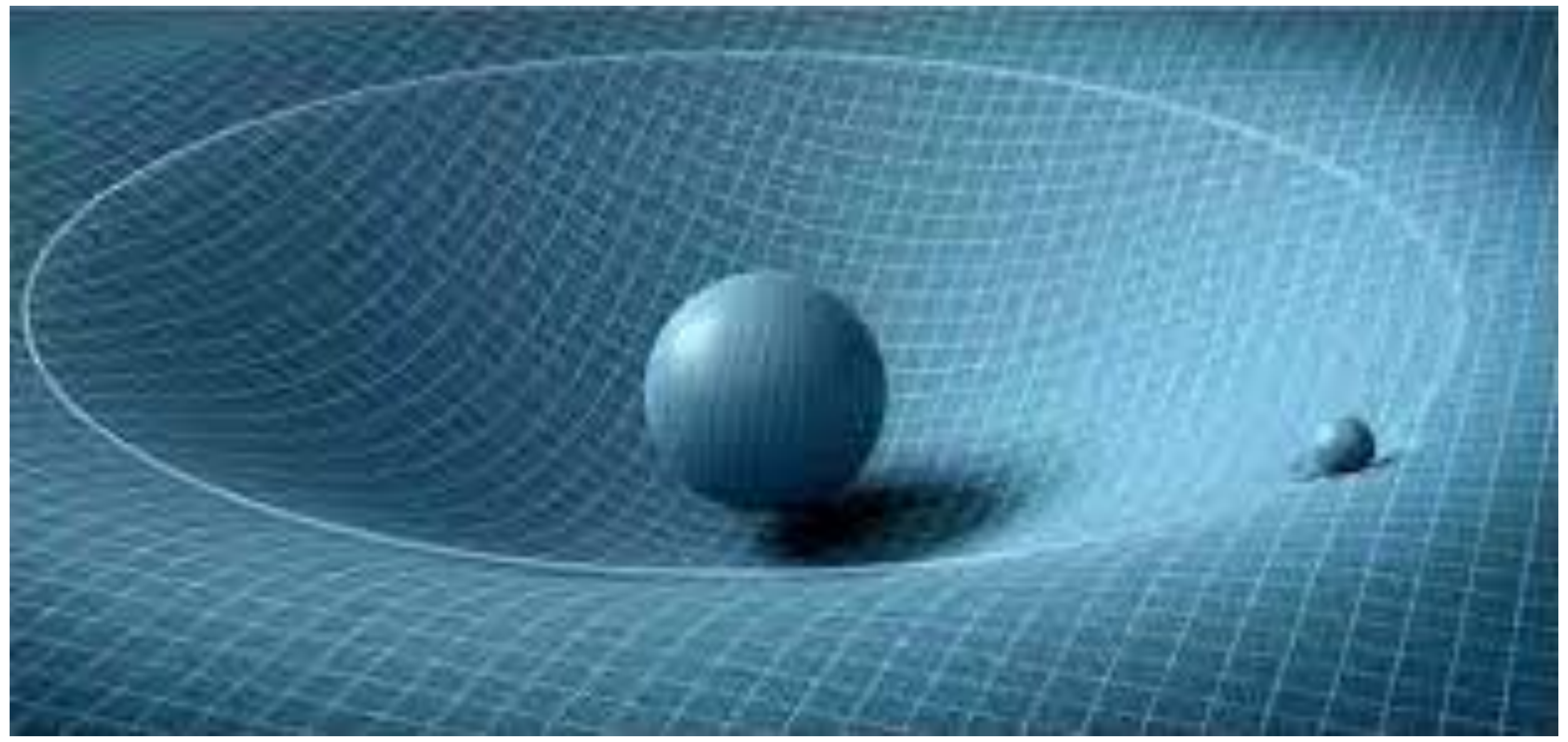

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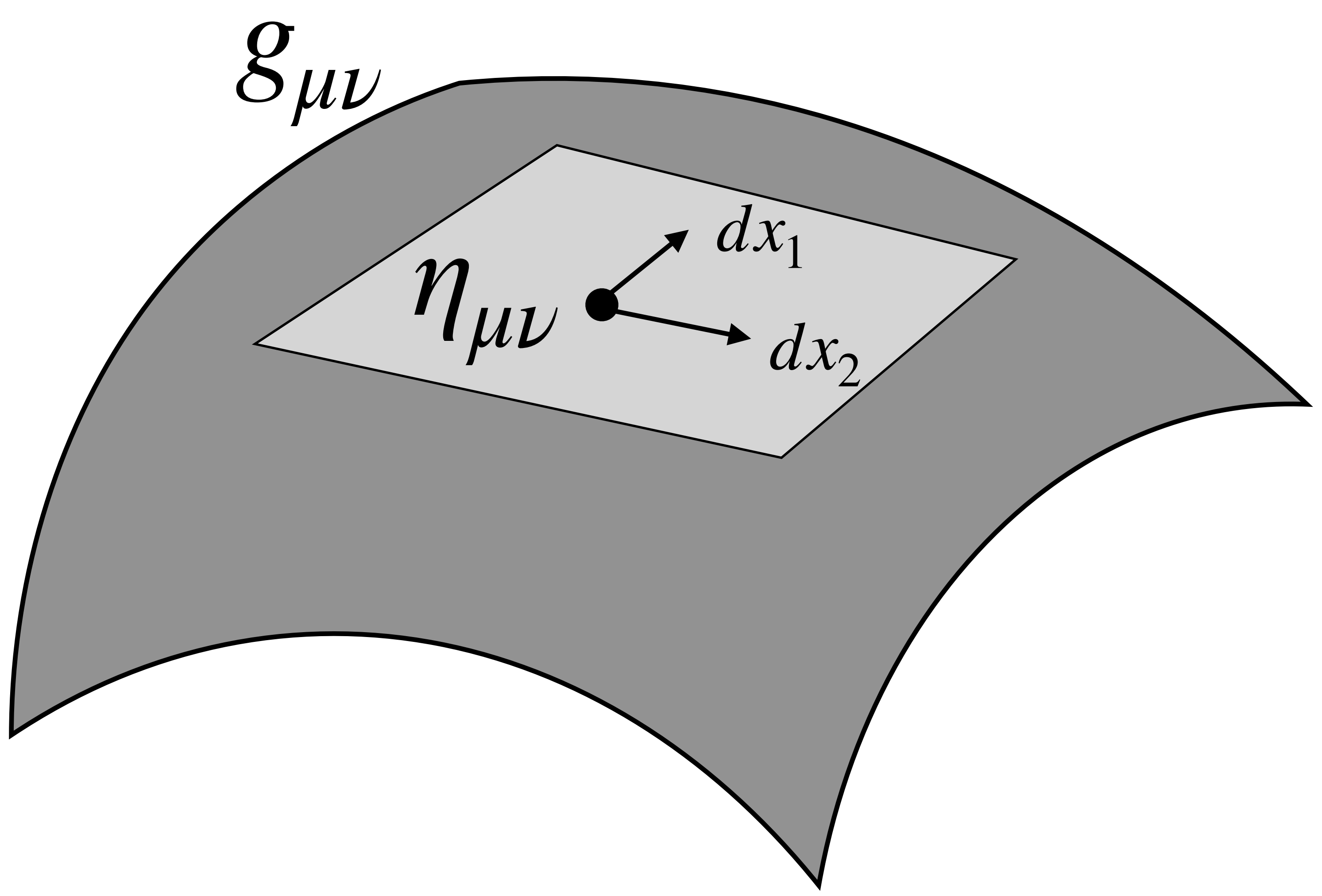
# Outline

- Spin in GR
    - Gravity as a gauge theory
- Vortex electrodynamics and analogue GR
    - Maxwell—Einstein/superfluidity correspondence
- Vortex angular momentum
    - Numerical simualtions
    - Bloch sphere mapping

# Spin in a curved space-time

- No finite dimensional spinorial representations of $\mathrm{GL(d+1)}$
  - Solution: stick the spinor in the flat tangent space
- Introduce tetrad fields ${e^a}_\mu$ such that $g_{\mu\nu} = {e^a}_\mu {e^b}_\nu \eta_{ab}$
  - $\partial_\mu \longrightarrow D_\mu = \partial_\mu + {e^a}_\mu P_a + {\omega^{ab}}_\mu M_{ab}$




- Cartan's structure equations
  - $T^a = de^a + {\omega^a}_b \wedge e^b$
  - ${R^a}_b = d{\omega^a}_b + {\omega^a}_c \wedge {\omega^c}_b$

# Vortex electrodynamics

- Start from the Gross—Pitaevskii equation

  - Madelung transformation $\psi(\vec{r},t) = |\psi(\vec{r},t)|\, e^{i\theta(\vec{r},t)}$

  - Action: $S = \int dr^3 [\underbrace{\nabla\theta^2}_{F^{\mu\nu}F_{\mu\nu}} + \underbrace{\left(\frac{\nabla|\psi|}{|\psi|}\right)^2}_{R^{\lambda}_{\mu\nu}} + int.]$

- Interpretation

  - Vortex EM: $E_{sf} \propto |\psi|^{2x+2}\nabla\theta$ and $B_{sf} \propto |\psi|^{2x}\partial_t\theta$ , $x \in \mathbf{R}$

  - Gravity: $F_G \propto \frac{\nabla|\psi|}{|\psi|}$

# Analogue gravity

- Perturb the system linearly

  - $\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi) = 0$ (Unruh 81')

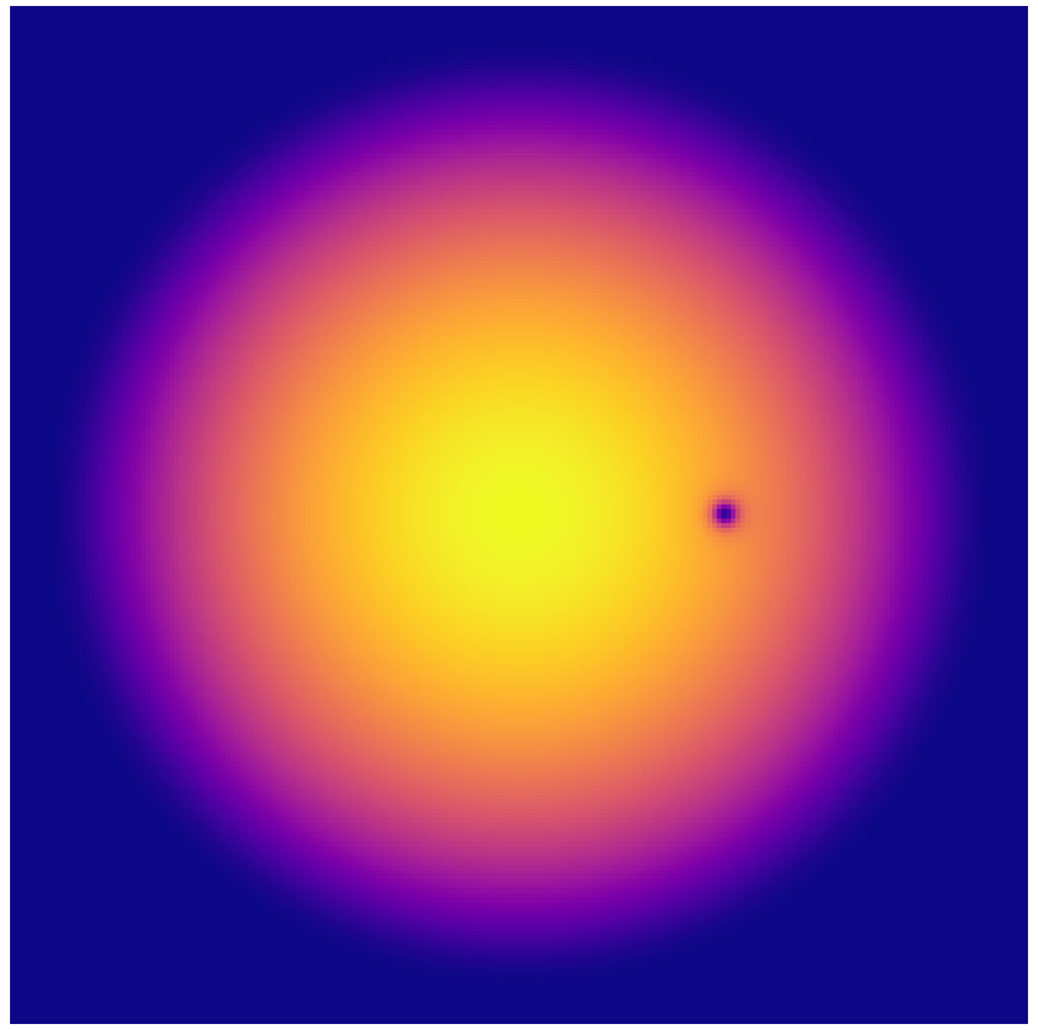
  - $ds^2 = n^2[-c_s^2 dt^2 + (d\vec{r} - \vec{v}_s dt)^2]$ (SR)

  - Promote $n$ to $n(r) = |\psi(r)|^2$ to obtain GR

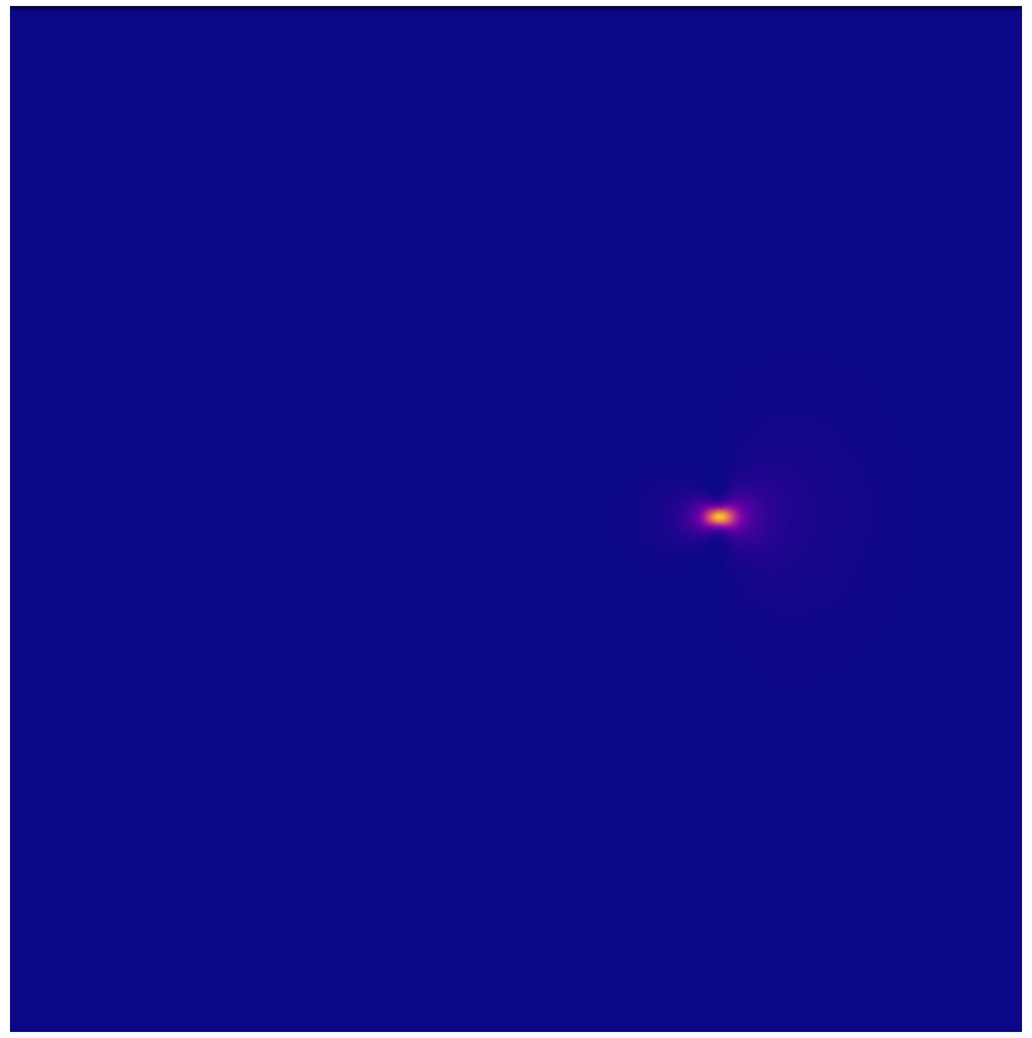
- Simulating GR

  - Holonomy, Lorentz boost etc.

Vortex $\psi(r)$



Kelvon $\hat{L}_z\psi(r)$



# A Maxwell—Einstein picture

- What is the full theory?

  - Strategy: start from $\{F_{\mu\nu}(n, \nabla\theta, \bar{\partial}), \quad g_{\mu\nu}(n), \quad \Gamma^{\lambda}_{\mu\nu}(n)\}$

  - Build a Maxwell theory coupled to gravity

- Solutions

  - $E_{sf} \propto |\psi(r)|^{2x+2} \nabla\theta$ and $B_{sf} \propto |\psi(r)|^{2x} \partial_t\theta$

  - Correspondence between planar superfluidity and Maxwell—Einstein theory in $(d+1)-D$ where $x = x(d)$

# Vortex-gravity coupling

- Apply the tetrad formalism to the fluid

  - Decompose the effective metric $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$

  - Long distance $T^a \longrightarrow 0$

  - Solve $0 = de^a + \omega^a{}_b \wedge e^b$ :

$$\begin{cases} \omega^r{}_\theta = -\,\omega^\theta_r = (r\dfrac{\partial_r n(r)}{n(r)} + 1)d\theta \\ \omega^t{}_r = -\,\omega^r_t = c_s\dfrac{\partial_r n(r)}{n(r)}dt \end{cases}$$

- Interpretation

  - Density gradient velocity $v_d \propto \dfrac{\partial_r n(r)}{n(r)}$

# Lorentz rotation

- Each $n(r) \longrightarrow$ unique gravity

  - Vortex precesses with $v_d$

- Geometric phase from holonomy:

  - $\phi = exp(i\int_{\gamma} \omega^{r}{}_{\theta} \hat{L}_z d\theta)$

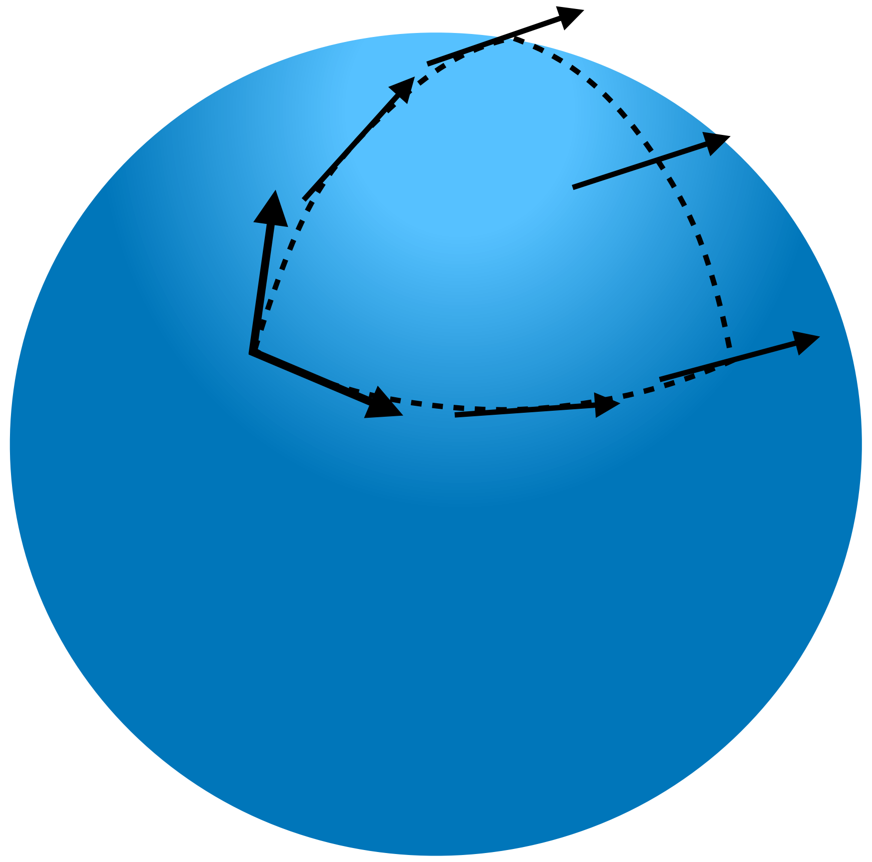
  - $\omega^{r}{}_{\theta} = (rv_d + 1)d\theta$ where $rv_d = r \times v_d \cdot \hat{e}_z \propto l_z$ (angular momentum)

# Topological space-time

- Topological phase?

  - Find $n(r)$ s.t. $\omega^{r}{}_{\theta}$ is flat

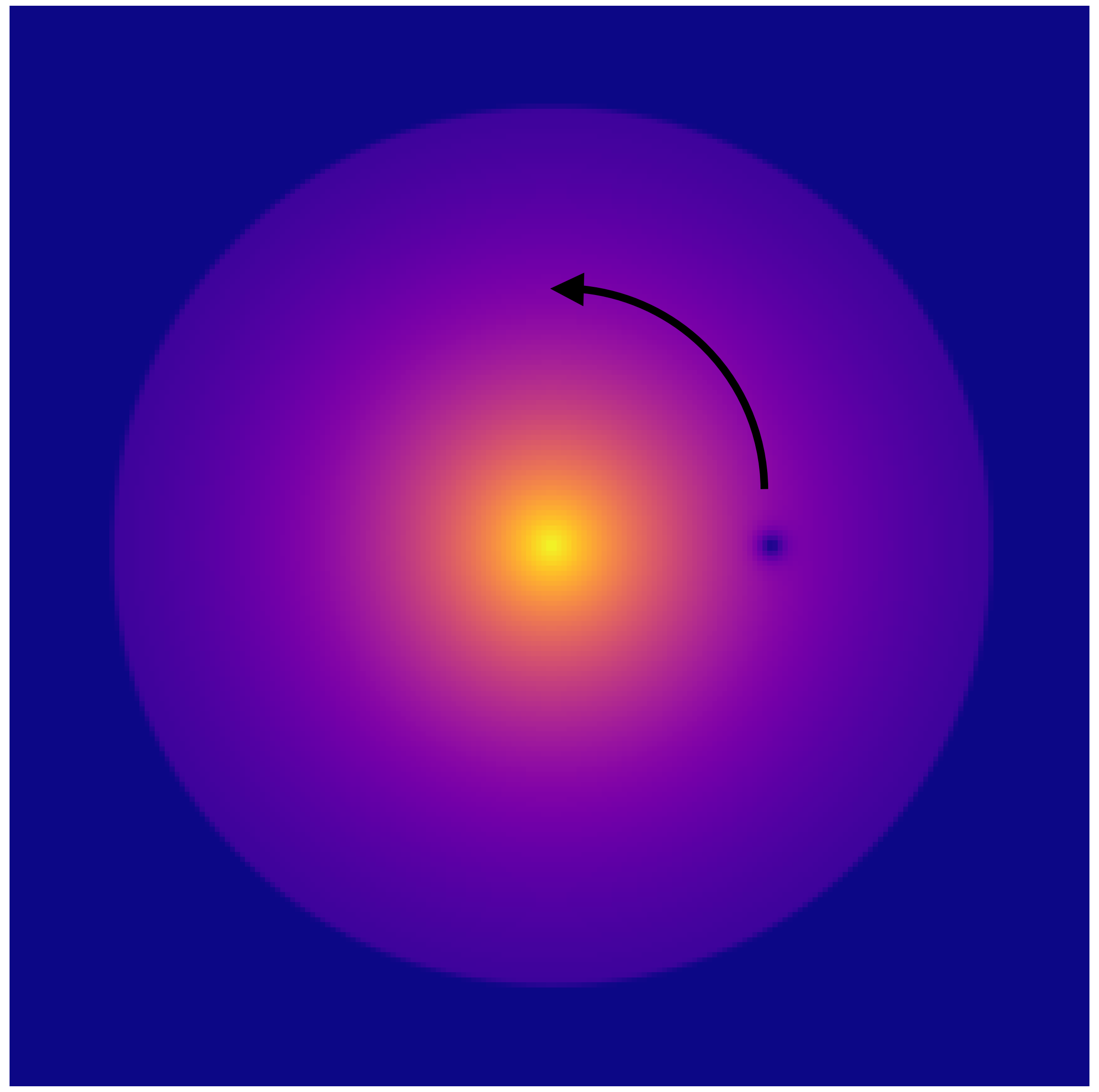
  - $n(r) = r^{c} \longrightarrow exp(i2\pi c\hat{L}_z)$ for any $c \in \mathbf{R}$



- Example: $c = -\frac{1}{4}$ and evolve in real time

  - Precession with $v_d \propto -\frac{1}{4r} \longrightarrow l_z = -\frac{1}{4}$

$n(r) = r^{-\frac{1}{4}}$



- Gravions

  - $lim_{r \longrightarrow 0} \;\; n(r) = \begin{cases} 0, & c > 0 \\ \infty, & c < 0 \end{cases}$, so that $R^{a}{}_{b} \neq 0$

# Lorentz boost

- Temporal part $\omega^{t}{}_{r}$

  - Each $n(r) \longrightarrow$ unique $exp(i\int_{\Delta t}\omega^{a}{}_{b}M^{b}{}_{a}) = exp(i\int_{\Delta t}\omega^{t}{}_{r}\hat{K}_{x}dt)$

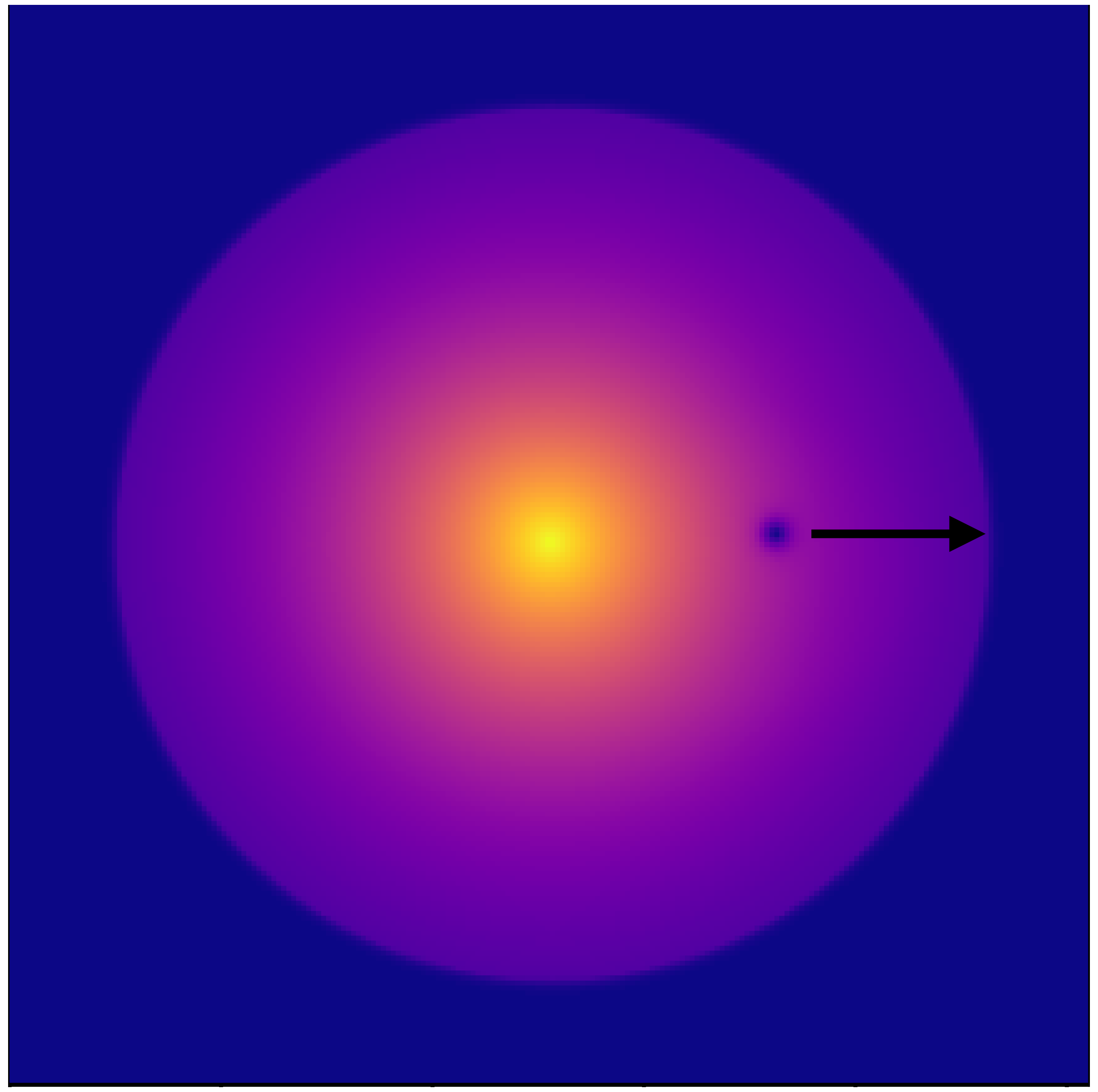
- Can we make it topological?

  - Pick $n(r) = e^{cr}$

- Example: $c = -\dfrac{1}{4}$ and evolve in imaginary time

  - Radial motion with rapidity $\alpha = ct$

$n(r) = e^{-\frac{1}{4}cr}$



# Kelvon dynamics

- Internal structure

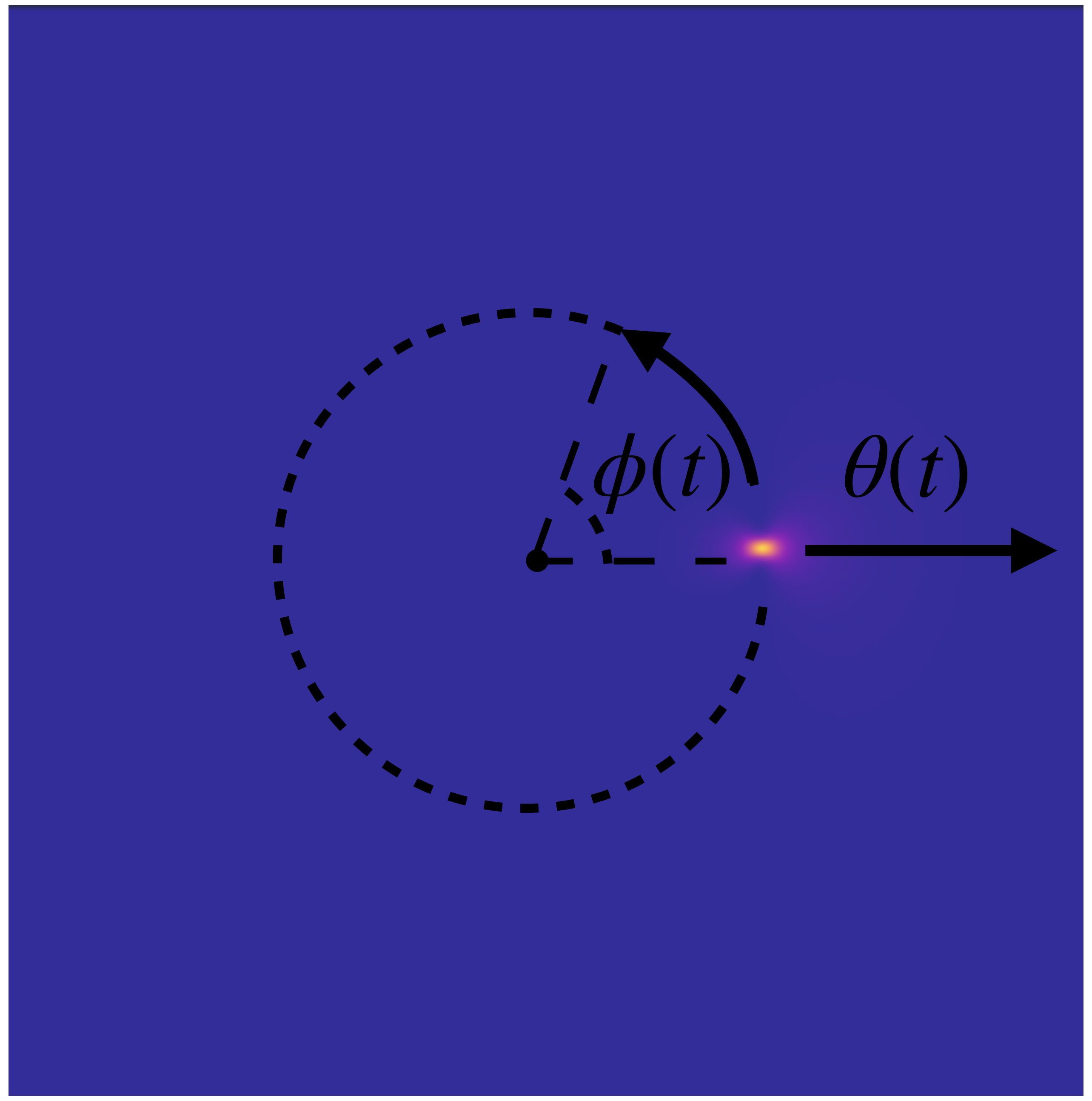
  - Kelvon $(u, v)^T$

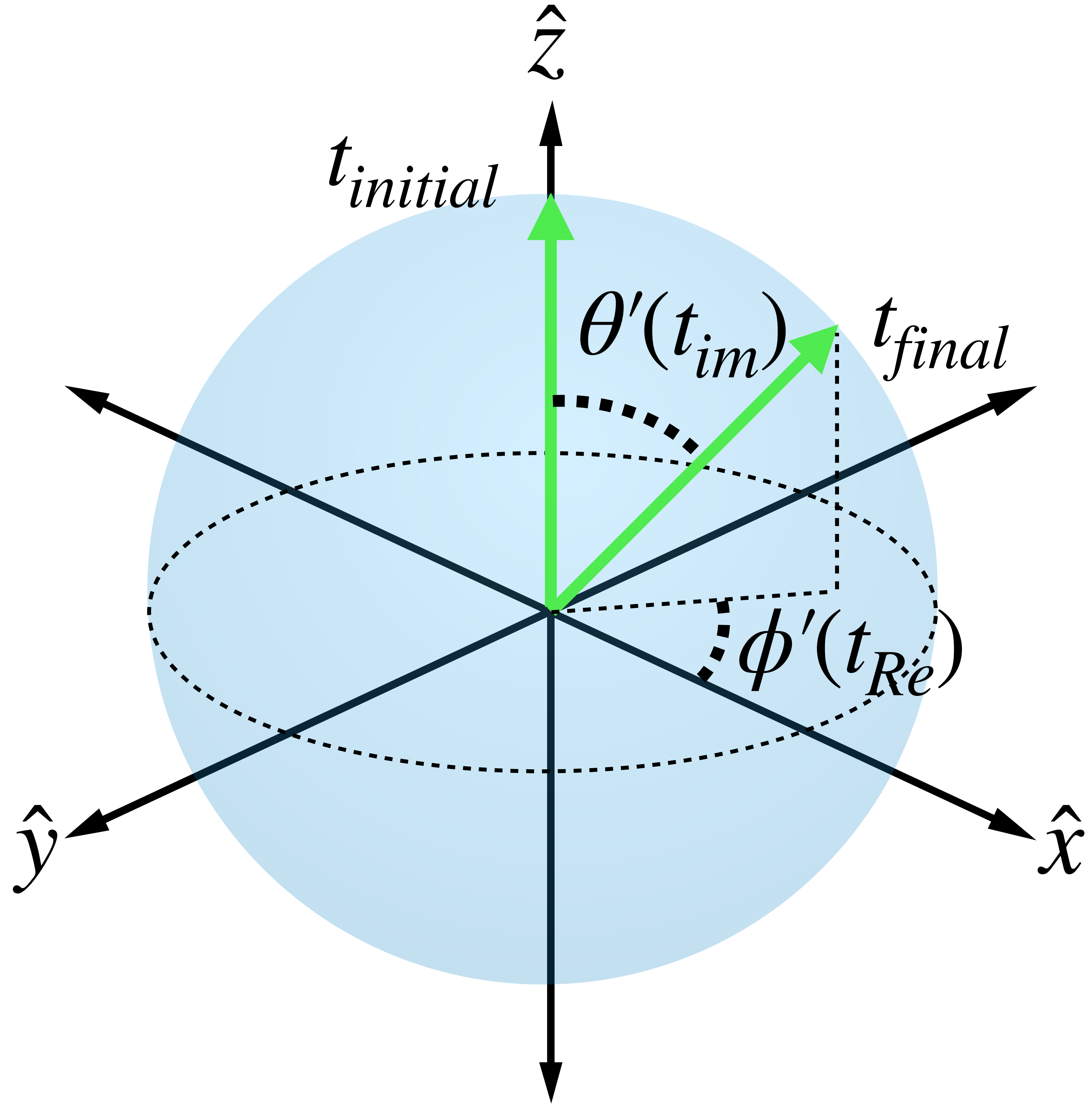
- Kelvon transformations: $\hat{L}_z \longrightarrow \sigma_z$ and $\hat{K}_x \longrightarrow i\sigma_x$

  - Rotations: $exp(i\int_\gamma \omega^r{}_\theta \sigma_z d\theta)$

  - Boosts: $exp(-\int_{\Delta t} \omega^t{}_r \sigma_x d\theta)$

  - Electron in a magnetic field



# Summary

- Vortex EM and gravity

  - Superfluidity/Maxwell—Einstein correspondence

- Gravity ⟶ vortex angular momentum

  - Topological solutions

- Kelvon quasi-particles

  - Dynamics-Bloch sphere mapping

- Future work

  - Extend to spinor BECs

  - Vortex QCD coupled to gravity