Scattering amplitudes of massive spin-2 Kaluza-Klein particles in Extra Dimensions

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With

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THE UNIVERSITY of ADELAIDE

ARC CENTRE OF EXCELLENCE FOR DARK MATTER PARTICLE PHYSICS

The validity of a Quantum Field Theory

- Perturbative Quantum Field Theories describe a physical system until some high energy scale
- For renormalizable theories in d space-time dimensions, the scale is determined by the fundamental couplings of the theory

EFT action in d dimensions

$$S = \int d^{\mathsf{d}}x \; \mathscr{L}(x)$$

$$[\mathscr{L}(x)] = \mathsf{d}$$

Operator expansion with coefficients

$$\mathscr{L}(x) = \sum_i c_i \, O_i(x)$$

$$S = \int d^{\mathsf{d}}x \; \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi \qquad S = \int d^{\mathsf{d}}x \; \bar{\psi} \; i\partial\!\!\!/ \; \psi$$

$$[\phi] = \frac{1}{2}(\mathsf{d} - 2) \qquad [\psi] = \frac{1}{2}(\mathsf{d} - 1)$$

EFT Expansion

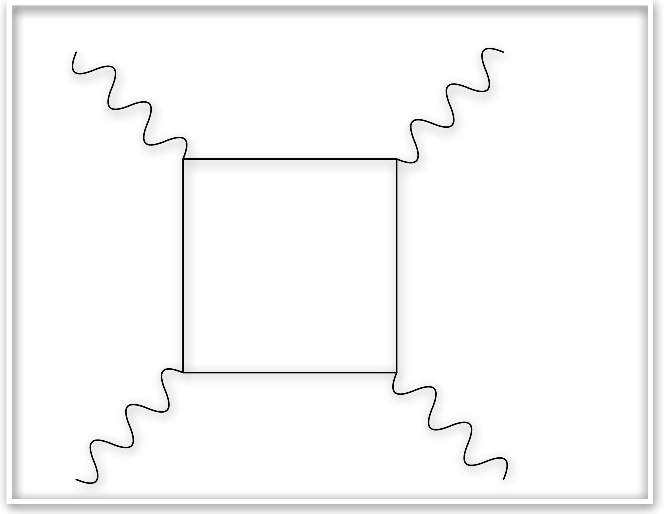
$$\mathscr{L}_{\mathrm{EFT}} = \sum_{\mathscr{D} \geq 0, i} \frac{c_i^{(\mathscr{D})} O_i^{(\mathscr{D})}}{\Lambda^{\mathscr{D}-d}} = \sum_{\mathscr{D} \geq 0} \frac{\mathscr{L}_{\mathscr{D}}}{\Lambda^{\mathscr{D}-d}}$$

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D} \leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \ldots$$

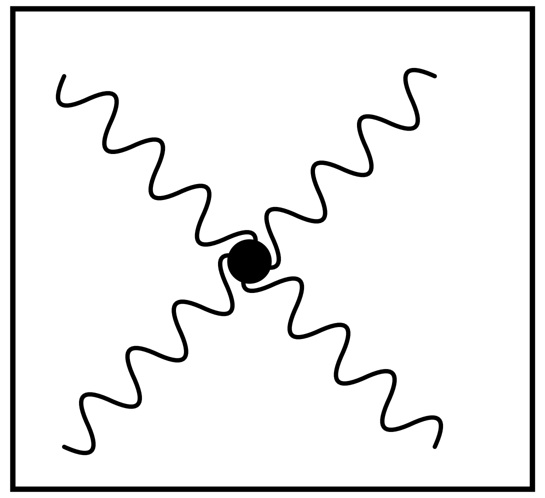
The validity of a Quantum Field Theory

How do we understand the validity of an EFT ?

Through Scattering Amplitudes



$\xrightarrow{\quad F^4_{\mu\nu} \quad}$



$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{m_e^4}\left[c_1\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + c_2\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right] + \ldots .$$

$$\mathscr{A} \sim \left(\frac{p}{\Lambda}\right)^{\mathscr{D}-\mathrm{d}}$$

$$\mathscr{A} \sim \frac{\alpha^2\omega^4}{m_e^4}$$

The mass of the electron determines the validity of this EFT

$$\sigma \sim \left(\frac{\alpha^2\omega^4}{m_e^4}\right)^2 \frac{1}{\omega^2}\frac{1}{16\pi} \sim \frac{\alpha^4\omega^6}{16\pi m_e^8}$$

What if I did not know about gauge invariance in nature ?

$$\mathscr{L} = c\,\alpha^2(A_\mu A^\mu)^2$$

$$\sigma \sim \alpha^4/(16\pi\omega^2)$$

The ratio 10^{48}!

Gravity as a valid Effective Field Theory

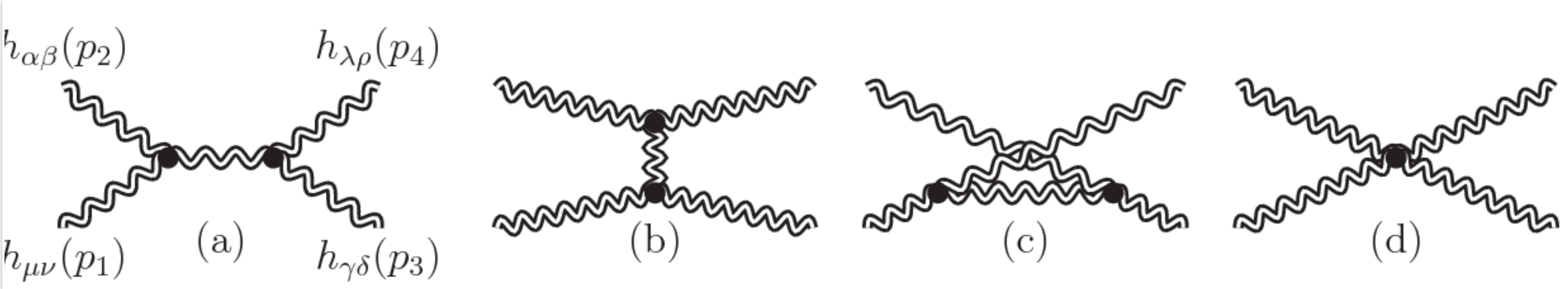
$$S_G = \frac{2}{\kappa^2}\int d^4x\sqrt{-g}R$$

$\kappa = \sqrt{32\pi G_N} = \frac{2}{M_{pl}} = \frac{1}{2.5\times 10^{15}\text{TeV}}$

$g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}$

Graviton

A spin-2 particle

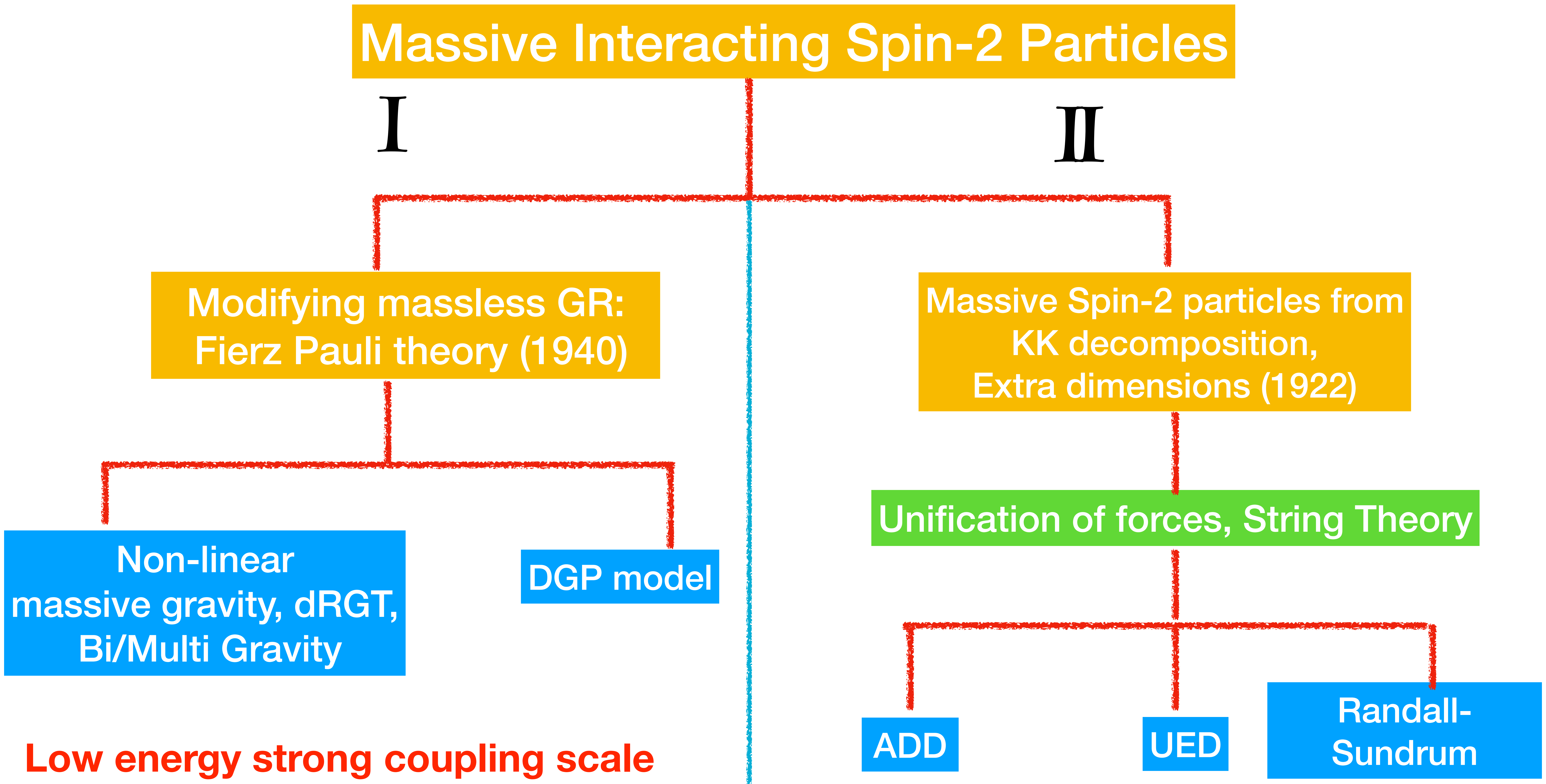


Scattering Amplitude grows as s/M_{Pl}^2 Classical/Semi-Classical gravity is a valid EFT until the Planck Scale

What about theories with a massive graviton ?

Mass is constrained by gravitational wave experiments to be less than 1.22 X 10^{-22} eV/c^2

Invitation : Massive Spin-2 particles and where to find them



Low energy strong coupling scale

arXiv : 1105.3735 (Hinterbichler)
arXiv: 1401.4173 (De-Rham)

- **EFT scale determined by underlying 5D theory**
- **Compactified on a flat torus : 5D Planck scale**
- **AdS : Curvature determines emergent scale**

Massive Spin-2 particles, Fierz-Pauli Theory and beyond

Massless GR :

$$S_G = \int d^4x \sqrt{g} R$$

Diffeomorphism/Gauge Invariance/Co-ordinate Invariance :

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

D.O.F counting in d dimensions for the massless graviton

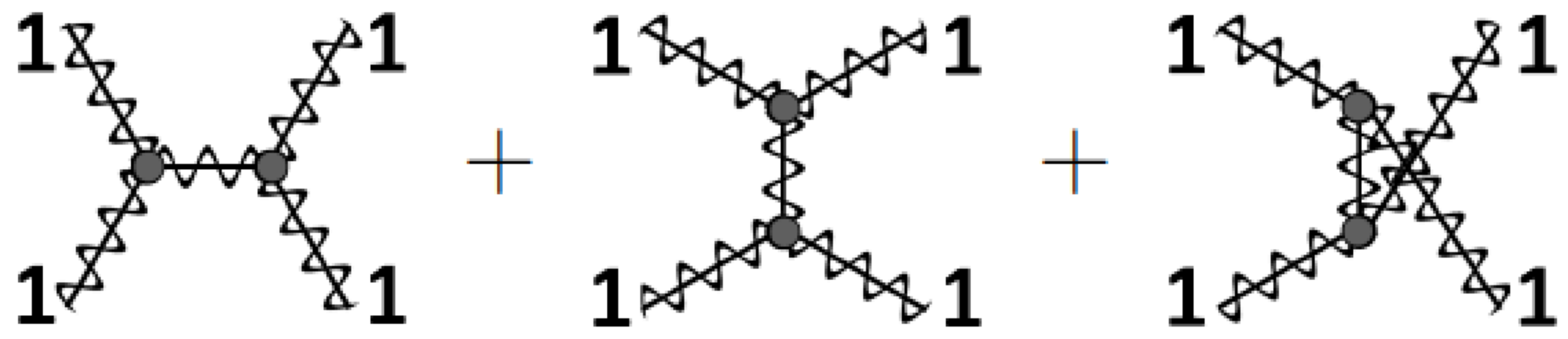
$$d(d+1)/2 - 2d = d(d-3)/2$$

D.O.F for d=4 : 2
D.O.F for d=4 : 5

Fierz-Pauli theory : 1940

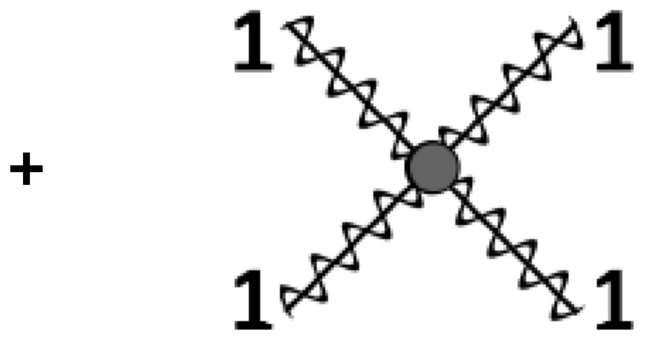
$$S_G = \int d^4x \sqrt{g} R + m^2((h_{\mu\nu})^2 - h^2)$$

Low Energy Scale in Massive Gravity



$$[\mathcal{O}(s^7) \to \mathcal{O}(s^5)]$$

$$S_G = \int d^4x\sqrt{g}R + m^2((h_{\mu\nu})^2 - h^2)$$



$$\to \frac{s^5}{m^8 M_{pl}^2}$$

$$\epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$

Unitarity is violated at a scale $\Lambda_5 = (M_{pl}m^4)^{1/5} \ll M_{pl}$

Van Damn-Veltman-Zakharov (VDVZ) discontinuity : 1963

Fierz-Pauli theory and extensions

1. **Resembles a Brans-Dicke theory.**
2. **Does not reduce to GR in the massless limit (van-Dam Veltman Zakahrov (vDVZ) discontinuity).**
3. **Vainshtein -> vDVZ discontinuity artifact of the linear theory, (Vainshtein screening).**
4. **Boulware-Deser -> Generic non-linear extensions of FP theory introduces ghosts.**
5. **dRGT theory (2010) -> A ghost free construction of massive gravity by tuning generic coefficients**

De-Rham, Gabadadze, Tolley (2010)
Cheung and Remen (2018)
Bonifacio, Rosen, Hinterbichler(2019)
Georgi, Arkani-Hamed, Schwartz(2001)
Schwartz (2003)

Extra Dimensions

- Soon after Einstein published, Theodore Kaluza suggested a 4^{th} space dimension (5^{th} spacetime dimension).

WHY ?

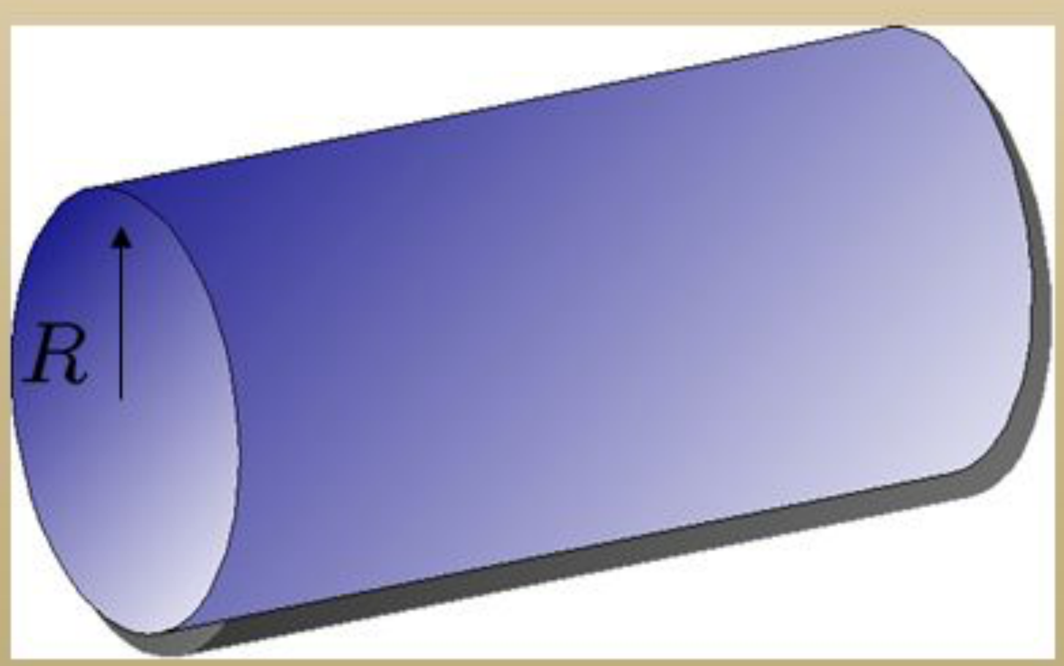
He recognised that gravity in the extra dimension would look like electromagnetism in regular space!

But wouldn't we notice another dimension?

COMPACTIFICATION

Why should I care about compactified theories

- New motivation for Extra Dimensions came from string theory (1980s)
- 6 extra dimensions are predicted in consistent string models
- They were considered to be tiny small $l_P \sim 10^{-34}$ mts

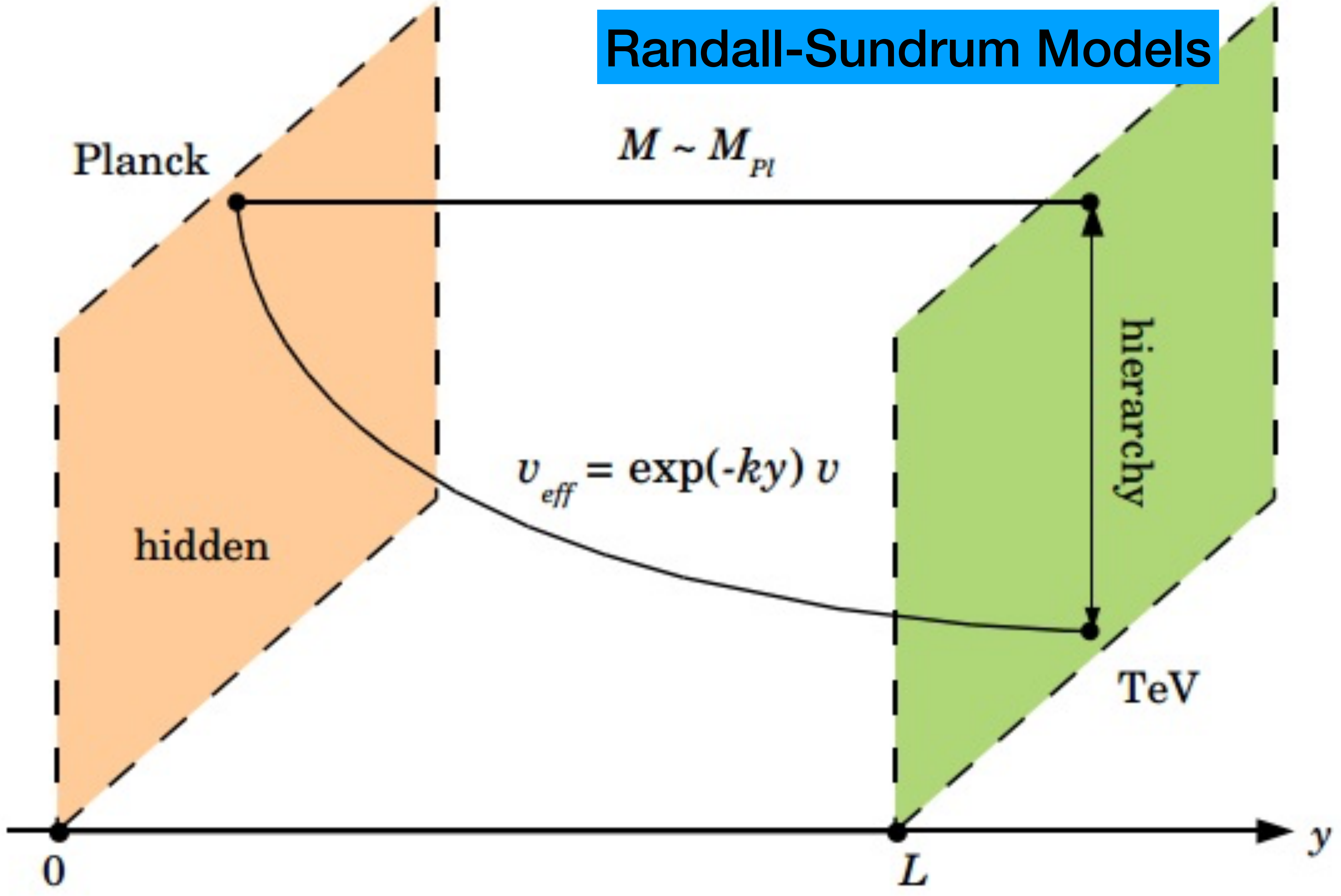


- Higher dimensional fields decompose in massless modes plus modes with masses

$$m_{KK} \sim n/R \sim nM_P$$

- ED effects irrelevant at low energies

Randall-Sundrum Models



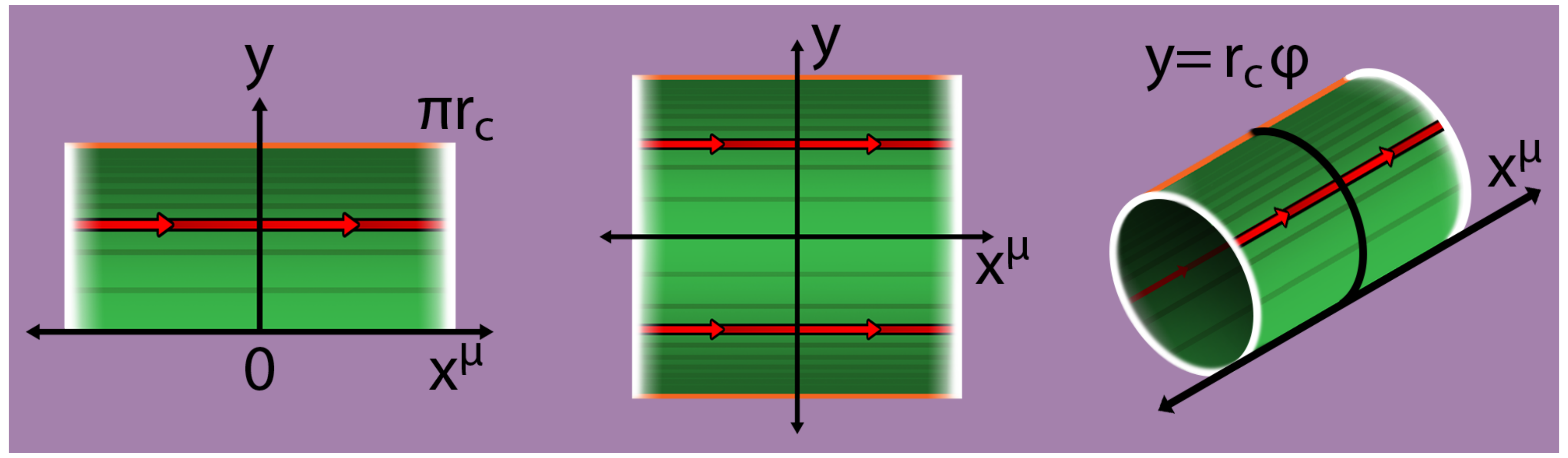
$$\mathcal{S}^{EH}_{5D} \propto \frac{1}{M_5^3} \int d^5x \sqrt{-g} R_{5D}$$

5D diffeomorphism with a 5D Planck mass

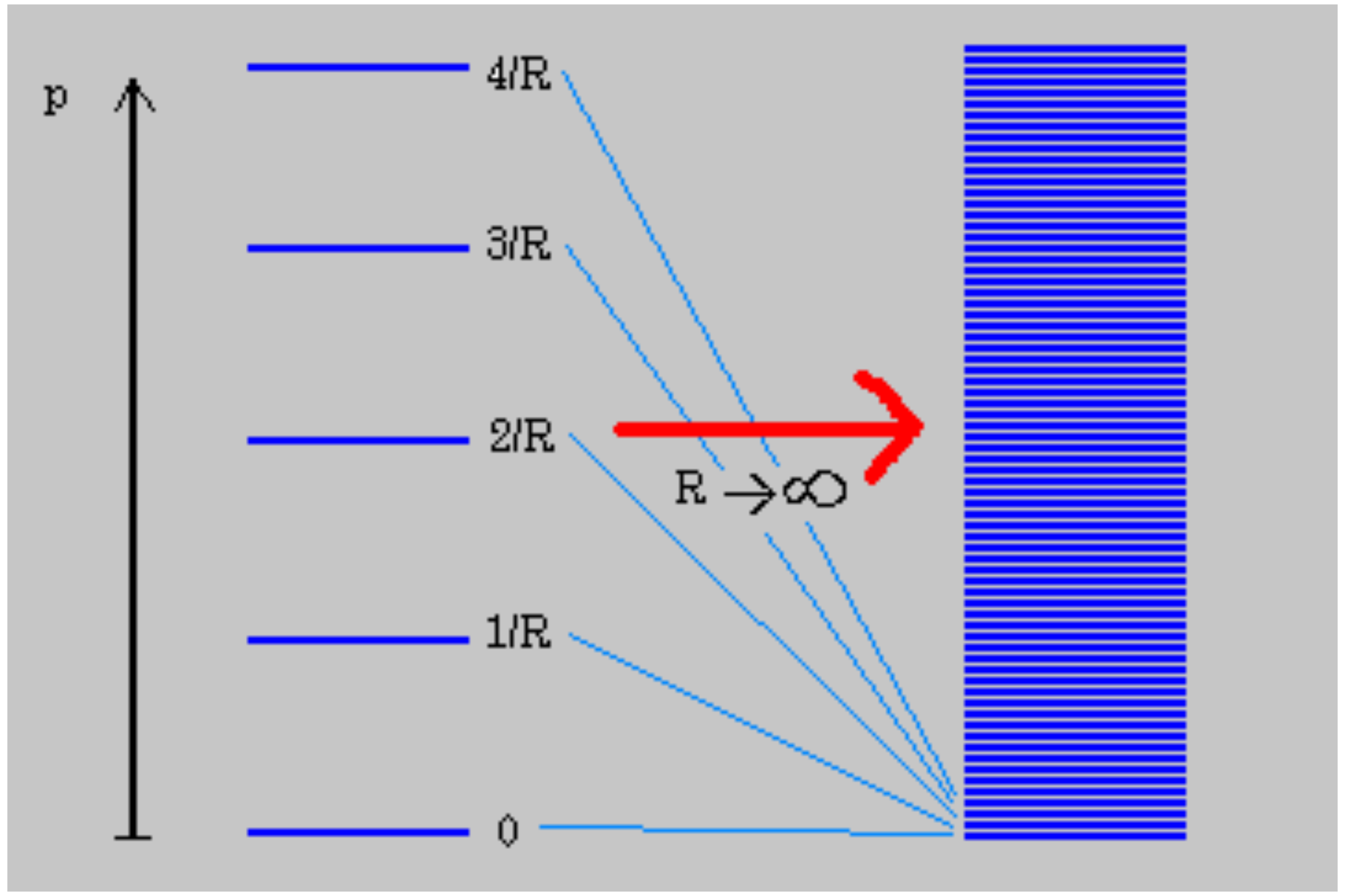
Why should I care about compactified theories

Compactify an Extra Dimension : A tower of massive Spin-2 KK states (+ massless graviton)

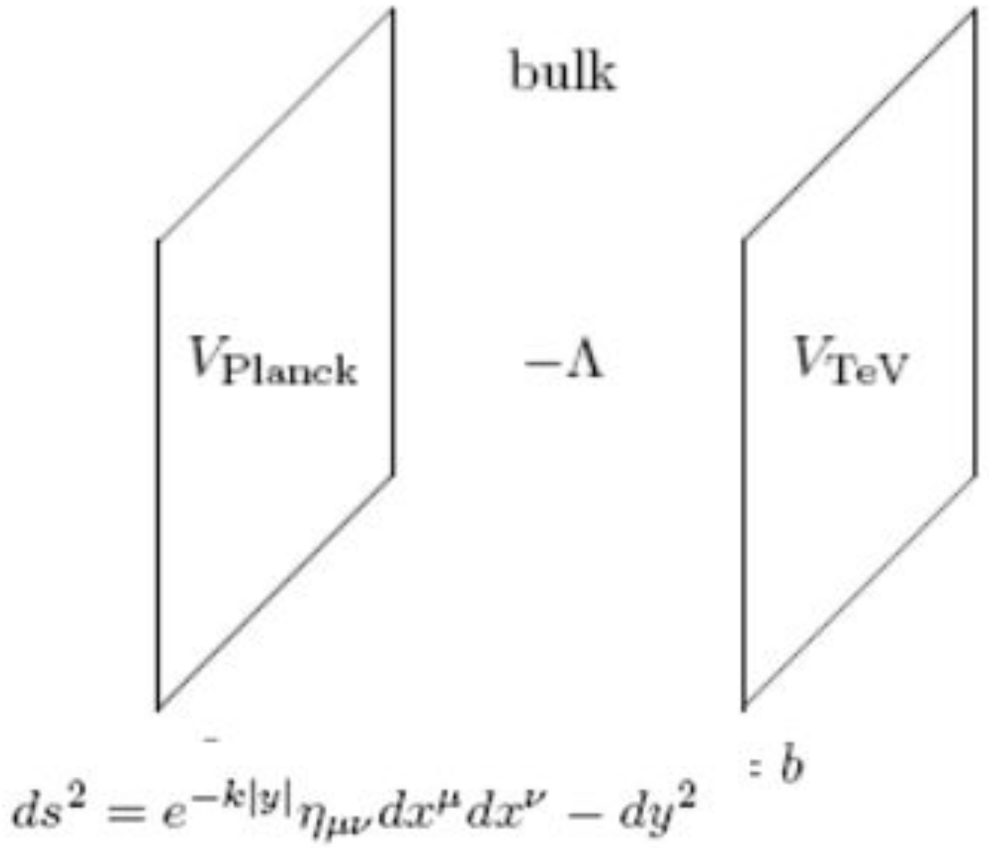
Flat extra dimensions

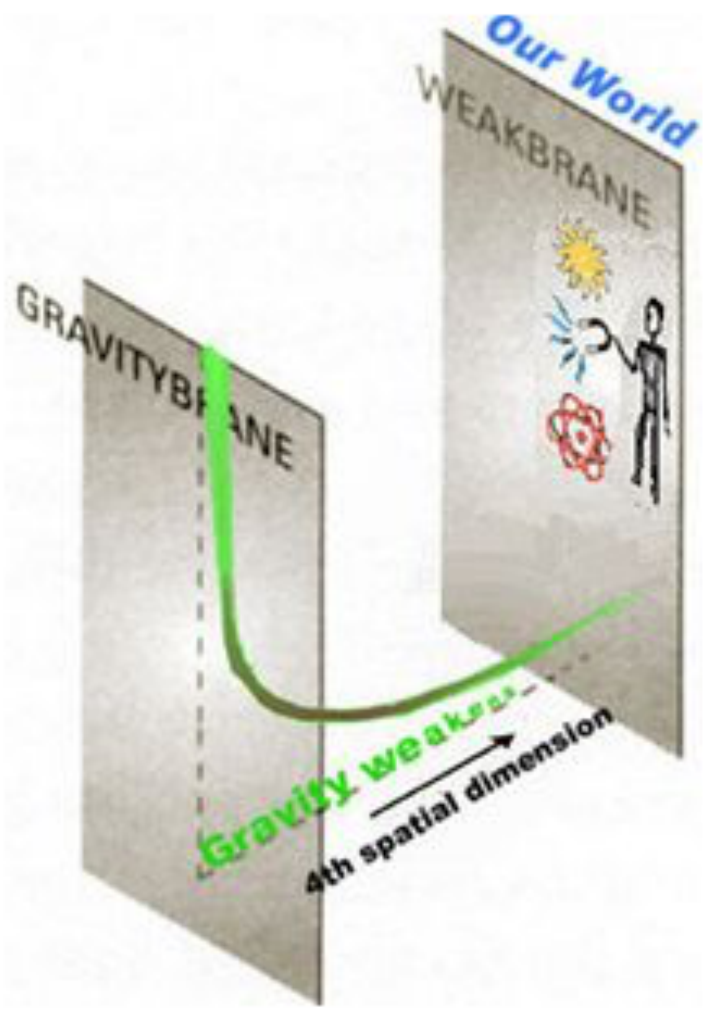


Choose even solutions on $[-\pi r_c, \pi r_c] \implies$ massless particles



Warped extra dimensions





Compactified 5D theory

$$S_{5D}^{EH} \propto \frac{1}{M_5^3}\int d^5x\sqrt{-g}R_{5D}$$

5D diffeomorphism with a 5D Planck mass

Compactification (IR phenomenon) should not change the high energy (UV) behavior,

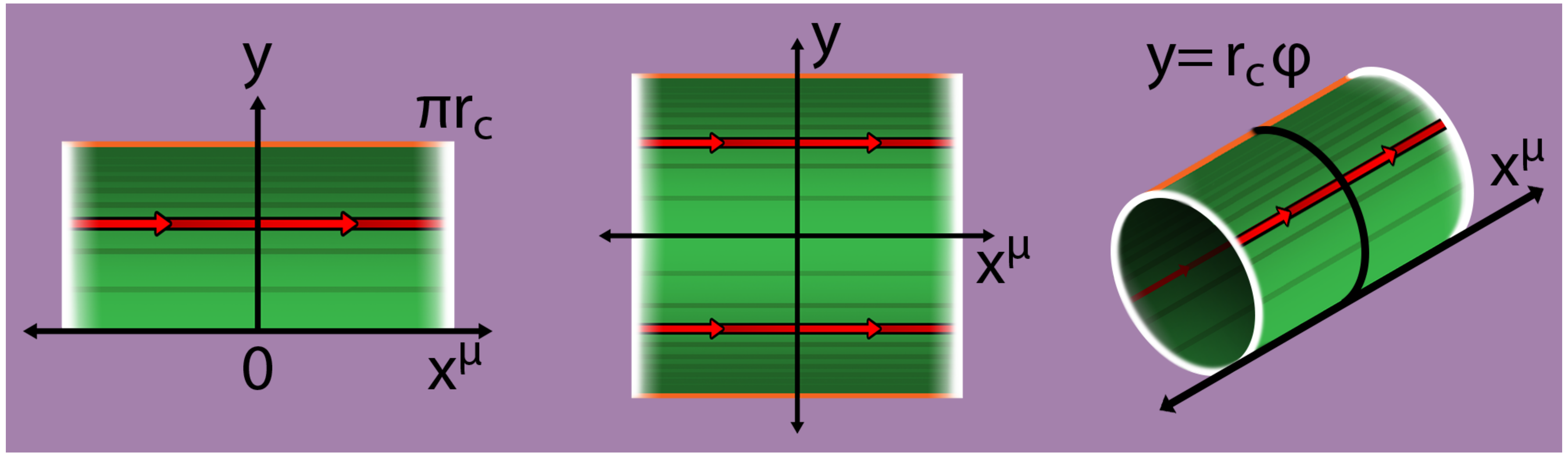
High energy growth s/M_{Pl}^2

Coupled channel analysis $s^{3/2}/M_5^3$

Examine Strong coupling scale

A. **Flat Extra dimension compactified on a torus**
B. **The Randall Sundrum Model (ADS)**

Compactified theories : Orbifolded Torus



5D orbifolded torus

Choose even solutions on $[-\pi r_c, \pi r_c] \implies$ massless particles

$$G^{(5\mathrm{DOT})}_{MN} = \begin{pmatrix} e^{\frac{-\kappa \hat{r}}{\sqrt{6}}}(\eta_{\mu\nu} + \kappa \hat{h}_{\mu\nu}) & 0 \\ 0 & -\left(1 + \frac{\kappa \hat{r}}{\sqrt{6}}\right)^2 \end{pmatrix}$$

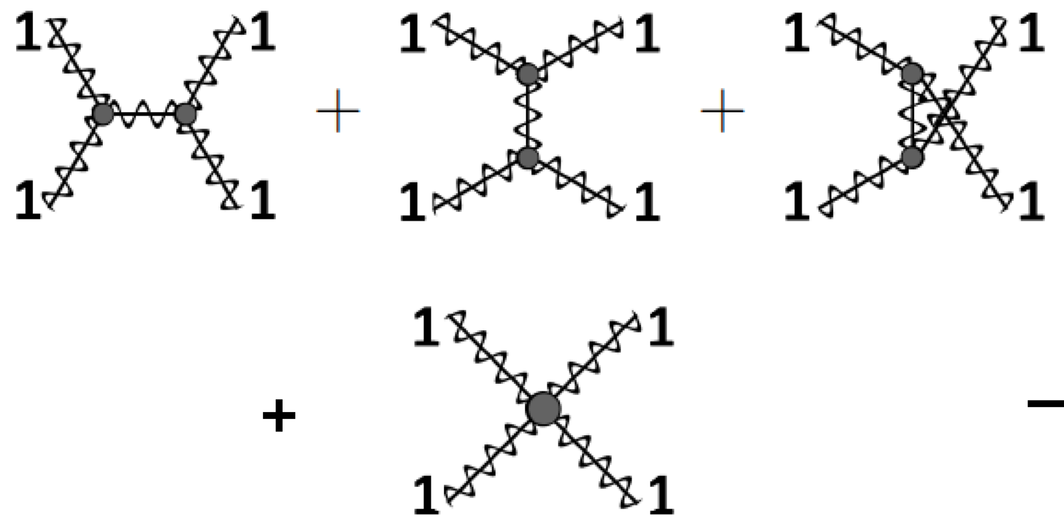
Parametrized for Canonical kinetic and mass terms

$$S = M_5^3 \int d^4x dy \sqrt{G} R^{(5)}$$

Integrate over extra dimension: EFT with a cut off

Discrete momentum conservation in extra dimensions ⟶ **KK number conservation**

Compactified theories : Orbifolded Torus



$[\mathcal{O}(s^7) \to \mathcal{O}(s^5)]$

$$\to \frac{s^5}{m^8 M_{pl}^2} \qquad \epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$

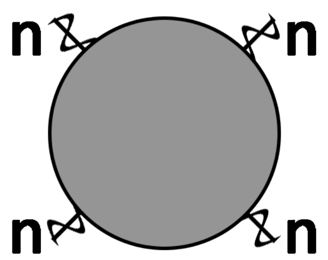
Recall: we're interested in how the high-energy limits of…

Massless 5D Gravity: $\mathcal{M}_{2\to2} \to \mathcal{O}(\sqrt{N_0}s)$

Massive 4D Gravity: $\mathcal{M}_{2\to2} \to \mathcal{O}(s^5)$

are consistent *despite* massless 5D gravity's KK expansion involving infinitely many massive spin-2 modes.

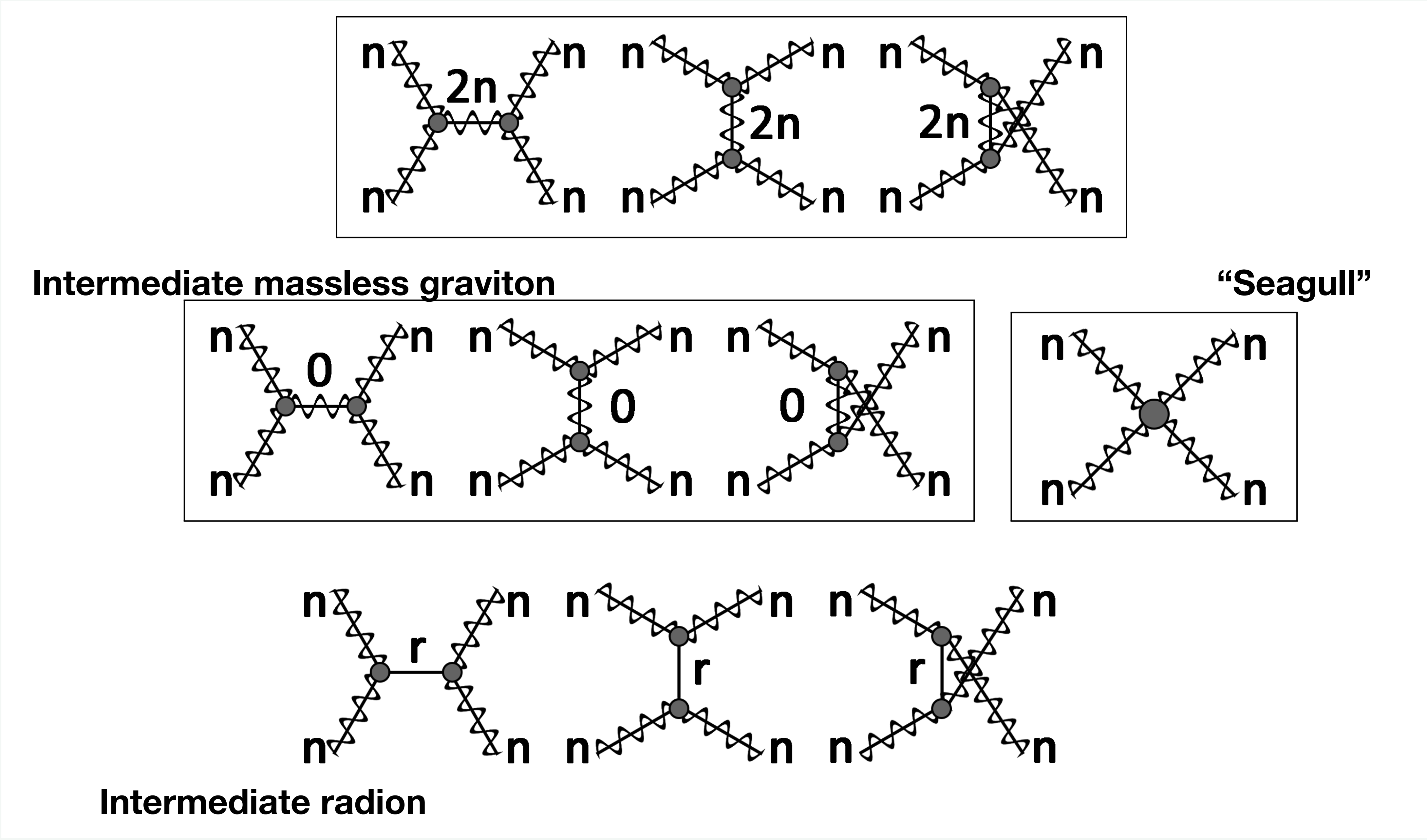
To see how this is resolved, we focus on the $h^{(n)}h^{(n)} \to h^{(n)}h^{(n)}$ **pure longitudinal helicity amplitude $\mathcal{M}_{nn\to nn}$ at $\mathcal{O}(\kappa^2)$.**

$$\mathcal{M}_{nn\to nn} =$$  $$= \sum_k \mathcal{M}^{(k)}_{nn\to nn}\, s^k$$

$$\mathcal{M} = \mathcal{M}_{\text{contact}} + \mathcal{M}_{\text{radion}} + \sum^{+\infty} \mathcal{M}_j$$

$$\mathcal{M}_{\text{contact}} \text{ and } \mathcal{M}_0 \sim \mathcal{O}(s^5)$$

$$\mathcal{M}_{\text{radion}} \sim \mathcal{O}(s^3)$$



Compactified theories: Orbifolded torus

	s^5	s^4	s^3	s^2
$\mathcal{M}_{contact}$	$-\frac{\kappa^2 r_c^7[7+c_{2\theta}]s_\theta^2}{3072n^8\pi}$	$\frac{\kappa^2 r_c^5[63-196\,c_{2\theta}+5\,c_{4\theta}]}{9216n^6\pi}$	$\frac{\kappa^2 r_c^3[-185+692\,c_{2\theta}+5\,c_{4\theta}]}{4608n^4\pi}$	$-\frac{\kappa^2 r_c[5+47\,c_{2\theta}]}{72n^2\pi}$
$\mathcal{M}_{2n}$	$\frac{\kappa^2 r_c^7[7+c_{2\theta}]s_\theta^2}{9216n^8\pi}$	$\frac{\kappa^2 r_c^5[-13+c_{2\theta}]s_\theta^2}{1152n^6\pi}$	$\frac{\kappa^2 r_c^3[97+3\,c_{2\theta}]s_\theta^2}{1152n^4\pi}$	$\frac{\kappa^2 r_c[-179+116\,c_{2\theta}-c_{4\theta}]}{1152n^2\pi}$
$\mathcal{M}_0$	$\frac{\kappa^2 r_c^7[7+c_{2\theta}]s_\theta^2}{4608n^8\pi}$	$\frac{\kappa^2 r_c^5[-9+140\,c_{2\theta}-3\,c_{4\theta}]}{9216n^6\pi}$	$\frac{\kappa^2 r_c^3[15-270\,c_{2\theta}-c_{4\theta}]}{2304n^4\pi}$	$\frac{\kappa^2 r_c[175+624\,c_{2\theta}+c_{4\theta}]}{1152n^2\pi}$
$\mathcal{M}_{radion}$	0	0	$-\frac{\kappa^2 r_c^3 s_\theta^2}{64n^4\pi}$	$\frac{\kappa^2 r_c[7+c_{2\theta}]}{96n^2\pi}$
Sum	0	0	0	0

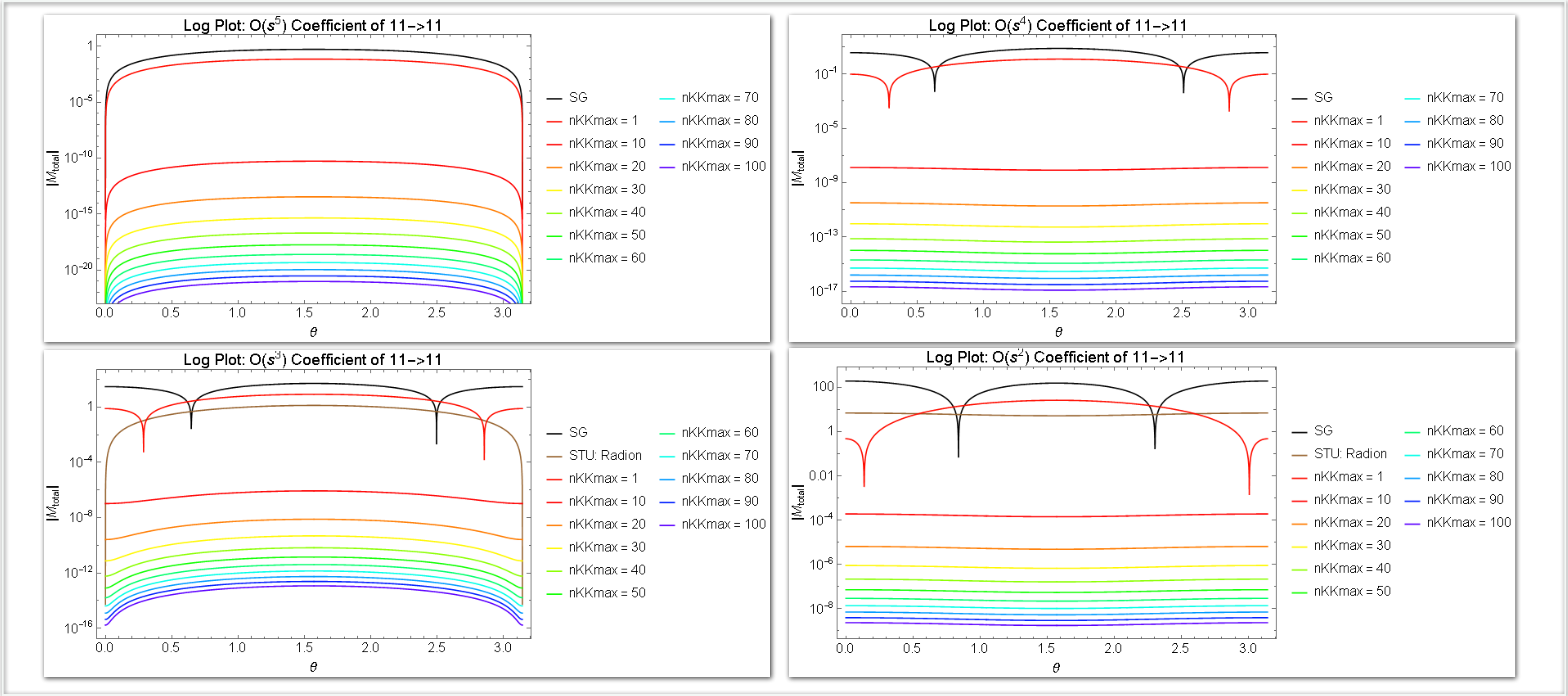
$$\overline{\mathcal{M}}^{(5)} = \overline{\mathcal{M}}^{(4)} = \overline{\mathcal{M}}^{(3)} = \overline{\mathcal{M}}^{(2)} = 0$$

$$\overline{\mathcal{M}}^{(1)} = \frac{x_{klmn}\kappa^2}{256\pi r_c}\,[7+\cos(2\theta)]\csc^2\theta$$

Amplitude grows as s/M_{Pl}^2

Can be proved analytically from symmetry arguments and Sturm-Liouville Theory of compact dimensions

Compactified theories: Randall Sundrum model



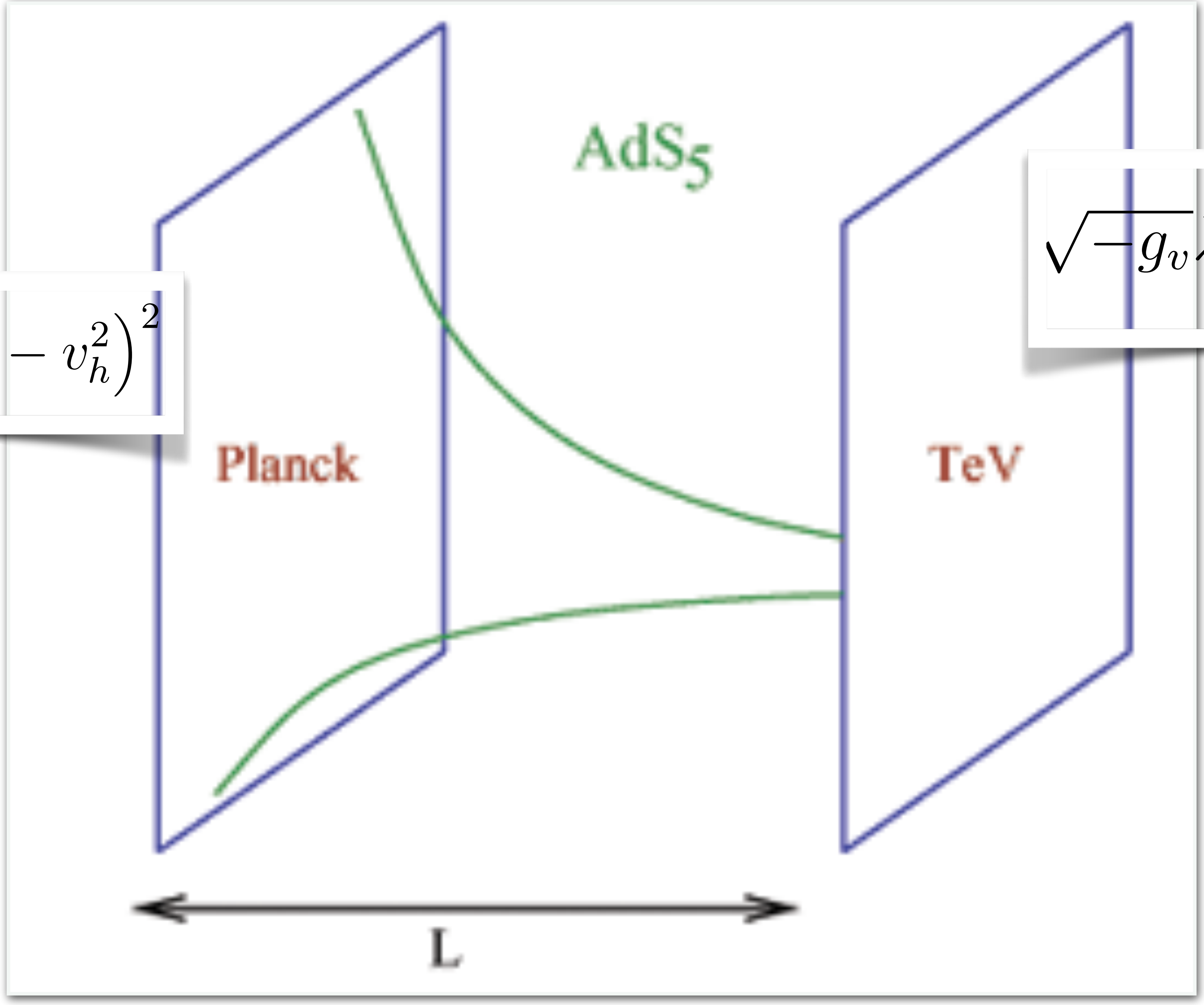
Cancellations a function of intermediate KK states, ideally sum to infinity, limited by machine precision

Residual growth $\quad \overline{\mathcal{M}}^{(k)}_{N_{max}} \propto \mathcal{O}\left(\frac{1}{N^{2k+1}_{max}}\right) \quad k \in \{2,3,4,5\} \quad$ vanish in the $N_{max} \to \infty$ limit

Residual s growth : Angular structure same as torus
Divide torus contribution by RS for fixed m_1, M_{Pl} ⟶ strong at an energy scale $\sqrt{s} \simeq \Lambda_\pi$

Why Stabilize ?

Goldberger, Wise 2000
De-Wolfe, Freedman, Karch, Gubser 2001
Tanaka, Garriga 2001
Peloso, Koffman 2001



$$\sqrt{-g_h}\lambda_h\left(\Phi^2 - v_h^2\right)^2$$

$$\sqrt{-g_v}\lambda_v\left(\Phi^2 - v_v^2\right)^2$$

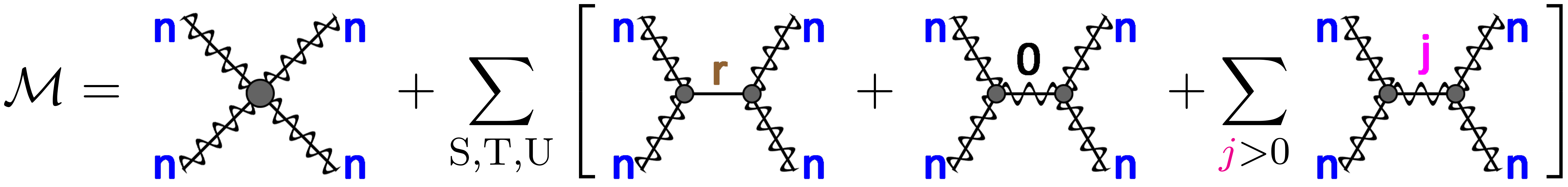
- No dynamical mechanism for the radius
- Matter on the brane + massless scalar metric fluctuation
- = Brans-Dicke theory

Mix the radial model of the metric (G_{55}) with the bulk scalar -> Generate an effective potential

$V_\Phi(r_c)$ **Minimize** $kr_c = \left(\frac{4}{\pi}\right)\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right]$ **Stabilization from Dynamics**

Radion Mass $\frac{k^2 v_v^2}{3M^3}\epsilon^2 e^{-2kr_c\pi}$ Changes Background Metric : Changes the description of the 4D EFT

Contributions to the matrix Element

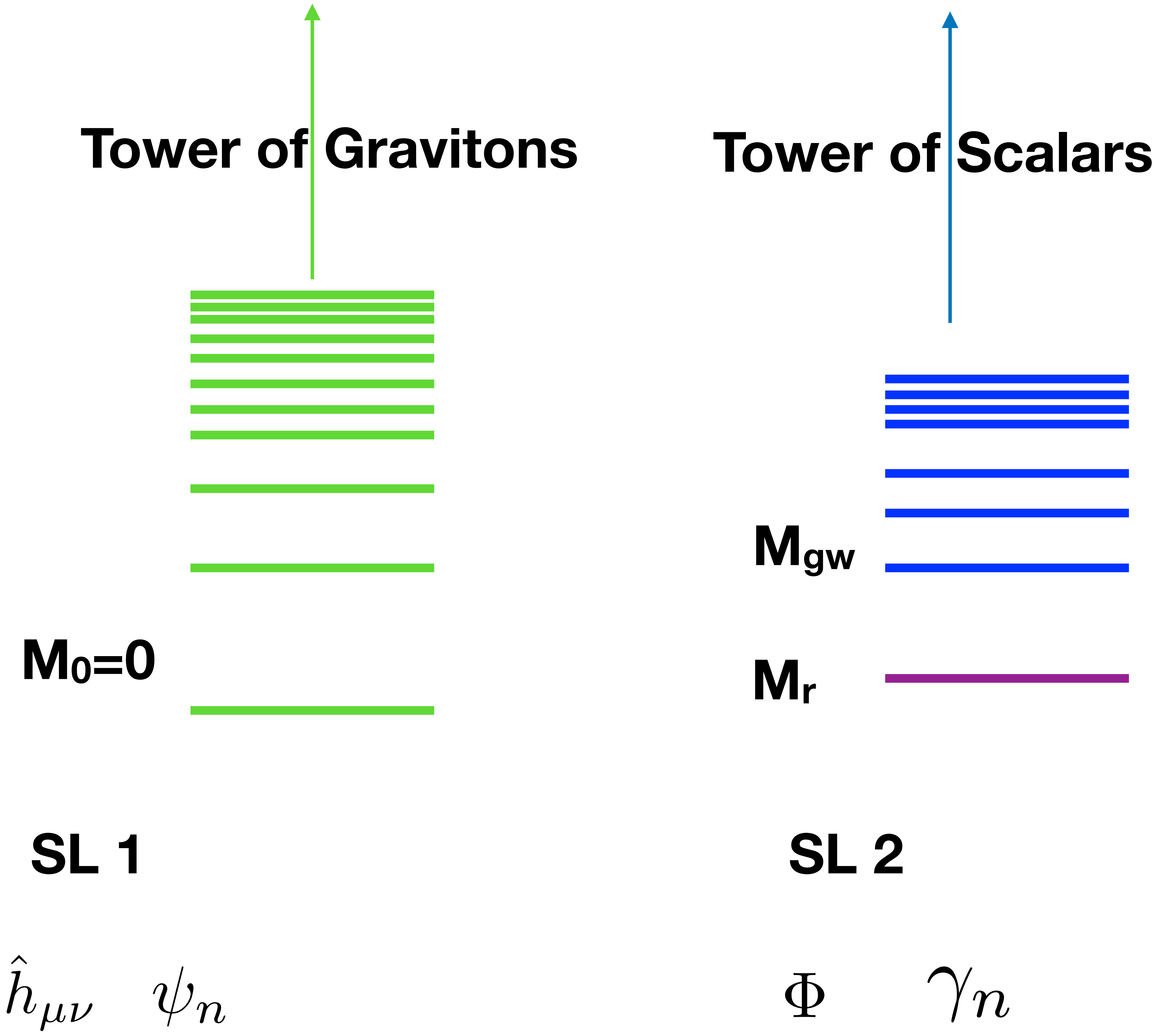


Add a naive mass term for the radion ⟶ **massive radion propagator**

$$\mathcal{M} \propto \frac{s^2 m_r^2}{\Lambda_\pi^2 m_n^4}$$ ⟶ **Re-introduction of a low energy cut-off**

Conjecture : Contribution must cancel from from the dynamics of the larger problem

SL problem of the Stabilized model



- Two Connected SL problems
- Radion piece must cancelled by a combination GW scalars and gravitons

Summary -again

How do bad high energy growth get tamed in Extra Dimensional Models

MASSIVE SPIN-2 PARTICLES AND ENERGY GROWTHS $\rightarrow \dfrac{s^5}{m^8 M_{pl}^2}$

MATRIX ELEMENTS AND CANCELLATIONS IN TOROIDAL AND AND ADS COMPACTIFICATIONS → s/M_{Pl}^2 → ✔ Boundary value problem, Sum rules

STABILIZING THE EXTRA DIMENSION VIA GOLDBERGER-WISE MECHANISM → $\dfrac{s^2 m_r^2}{\Lambda_\pi^2 m_n^4}$ → Naive re-introduction of bad high energy behavior

MATRIX ELEMENTS AND CANCELLATIONS IN STABILIZED MODELS → ✔ Sum rules for the Stabilized model

Conclusions and perspective

Compactified theories of extra dimensions -> No low energy cut-off

Cancellations due to different diagrams reduce $O(s^5)$ growth to $O(s)$.

No low energy cut-off for consistent models of stabilization

Uncovered sum rules enforcing this cancellation

Can show -> Analysis extends to matter on brane or bulk

Consistent with literature on massive gravity.

Possible to double-copy a compactified gauge theory to compactified gravity for flat toroidal compactification

Pheno papers : Doing an unitarity analysis for DM models, ultralight radion as a candidate …

Theory papers : Spinor Helicity calculation ? More connections with massive gravity community …