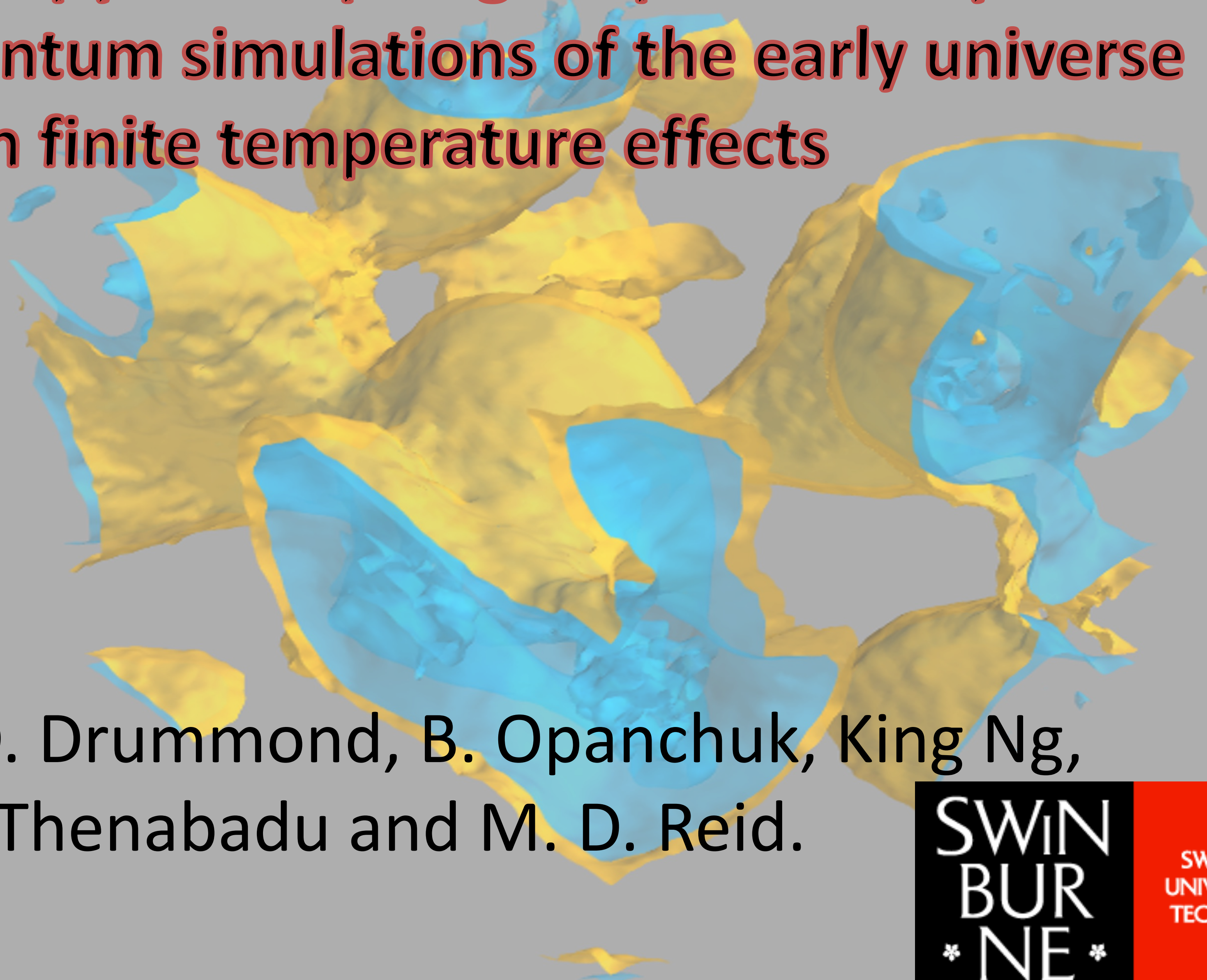
Entropy and topological phase analysis in quantum simulations of the early universe with finite temperature effects



P. D. Drummond, B. Opanchuk, King Ng, M. Thenabadu and M. D. Reid.

SWINBURNE UNIVERSITY OF TECHNOLOGY

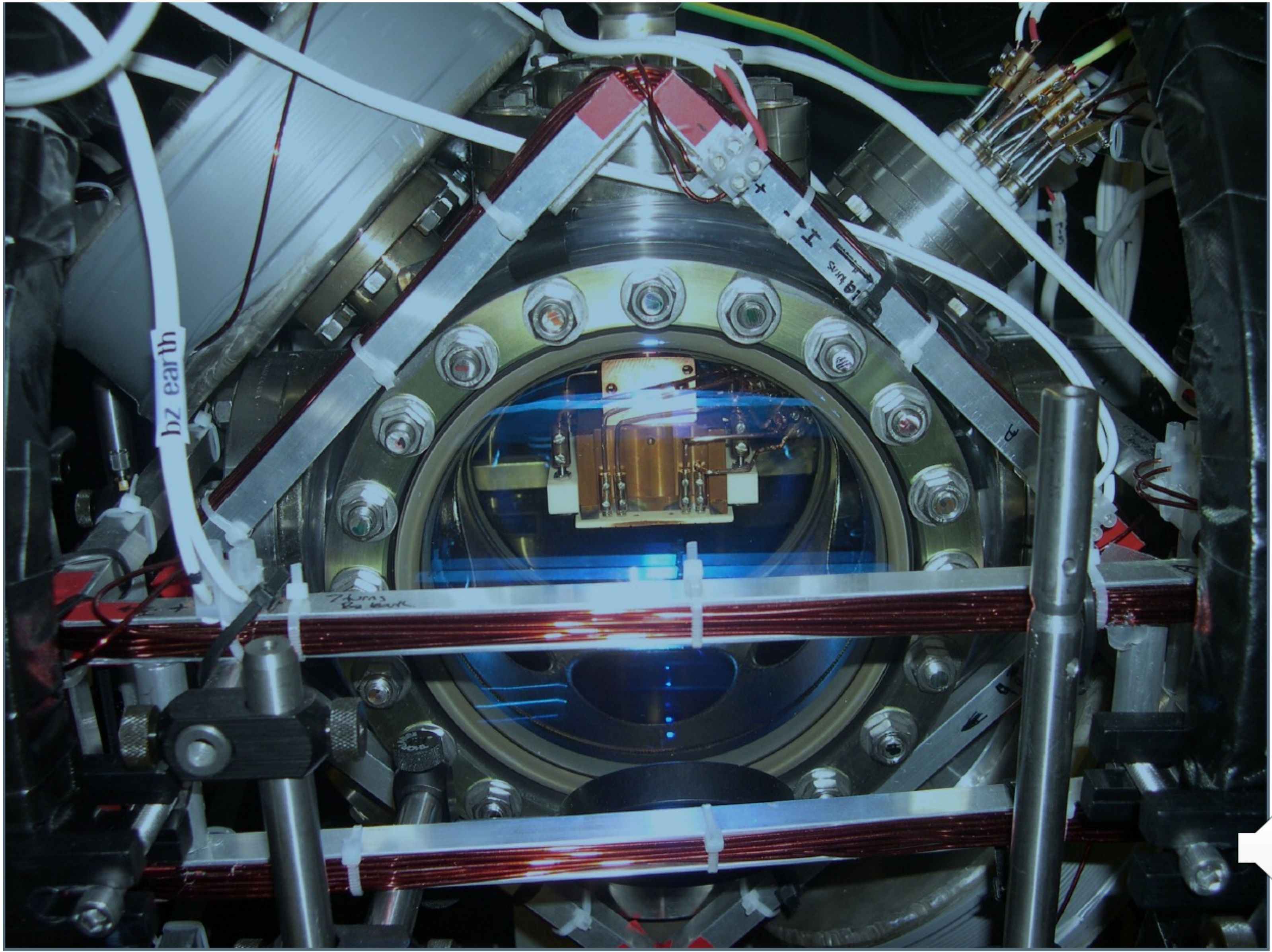
Early Universe emulation

- Quantum field theory: exponentially complex
  - Essential to current theories of inflation
  - Energies a trillion times larger than CERN
  - How can we compute what theory predicts?
- **Use ultracold BEC as relativistic simulator**
  - Check predictions with computer simulations
  - Need to include finite temperatures
  - **Possible model for matter-antimatter asymmetry**

Phase-space methods

- **Truncated Wigner (tW) technique**
  - ✓Well-tested in 3D BEC interferometry
  - ✓Uses a 1/N expansion, for N bosons/mode
  - ✓Can treat finite temperatures
- **Q-function**
  - ✓Almost identical to tW for macroscopic systems
  - ✓Always positive, so viable as reality model
  - ✓SEE: PDD&MD REID, PRR **2**, 033266 (2020).

Test case: Interferometry on an atom chip (Sidorov, Swinburne)



Bose gas master equation, finite temperature

A D-dimensional Bose gas has two spin components that are linearly coupled by an external microwave field.

$$\hat{H} = \hbar \int d^3\mathbf{x} \left[\frac{\hbar}{2m} \nabla \hat{\Psi}_i^\dagger \nabla \hat{\Psi}_i + V_i(\mathbf{x}) \hat{\Psi}_i^\dagger \hat{\Psi}_i + \frac{g_{ij}}{2} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i + \nu \hat{\Psi}_i^\dagger \hat{\Psi}_{3-i} \right]$$

Here, g_{ij} is the self- and cross-coupling in $D-$dimensions. Collisional damping follows a master equation,

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} \left[\hat{H}, \hat{\rho} \right] + \sum \kappa_\ell \int d^3\mathbf{x} \left[2\hat{O}_\ell \hat{\rho} \hat{O}_\ell^\dagger - \hat{O}_\ell^\dagger \hat{O}_\ell \hat{\rho} - \hat{\rho} \hat{O}_\ell^\dagger \hat{O}_\ell \right]$$

This includes self- and cross nonlinear damping, with

$$\hat{O}_\ell = \prod \hat{\Psi}_j^{\ell_j}$$

Stochastic time-evolution equations

M. J. Steel, M. K. Olsen, L. I. Plimak, P. D. Drummond, S. M. Tan, M. J. Collet, **D. F. Walls** and R. Graham, PRA**58**, 4824 (1998).

Result of Wigner operator mappings:

$$i\partial_\tau \psi_i = \left\{ -\frac{1}{2}\nabla_\zeta{}^2 + \gamma\psi_i^\dagger\psi_i + \gamma_c\psi_j^\dagger\psi_j \right\}\psi_i - \tilde{\nu}\psi_j,$$

$$-\sum \tilde{\kappa}_\ell \frac{\partial \tilde{O}_\ell^*}{\partial \psi_i^*}\tilde{O}_\ell + B_{ij}[\psi]\,\eta_j(t,x)$$

Scaling: $\tau = t/t_0$, $\zeta = x/x_0$,

$t_0 = \hbar/gn$; $x_0 = \hbar/\sqrt{gnm}$; $\langle \Delta\tilde{\psi}(\zeta)\Delta\tilde{\psi}^*(\zeta')\rangle = \frac{1}{2}\delta(\zeta-\zeta')$.

Finite temperature initial conditions

K. L. Ng, R. Polkinghorne, B. Opanchuk, P. D. Drummond, Journal of Physics A**52**, 035302 (2019).

- Finite temperatures are treated using the Bogoliubov theory..
- Divergences are removed with nonlinear chemical potential,

$$\mu_2 = g/L$$

$$\mu\left(\hat{N}\right) = \mu_1 \hat{N} + \frac{\mu_2}{2} : \hat{N}^2 : .$$

- **RESULT: Wigner initial distribution without divergence**

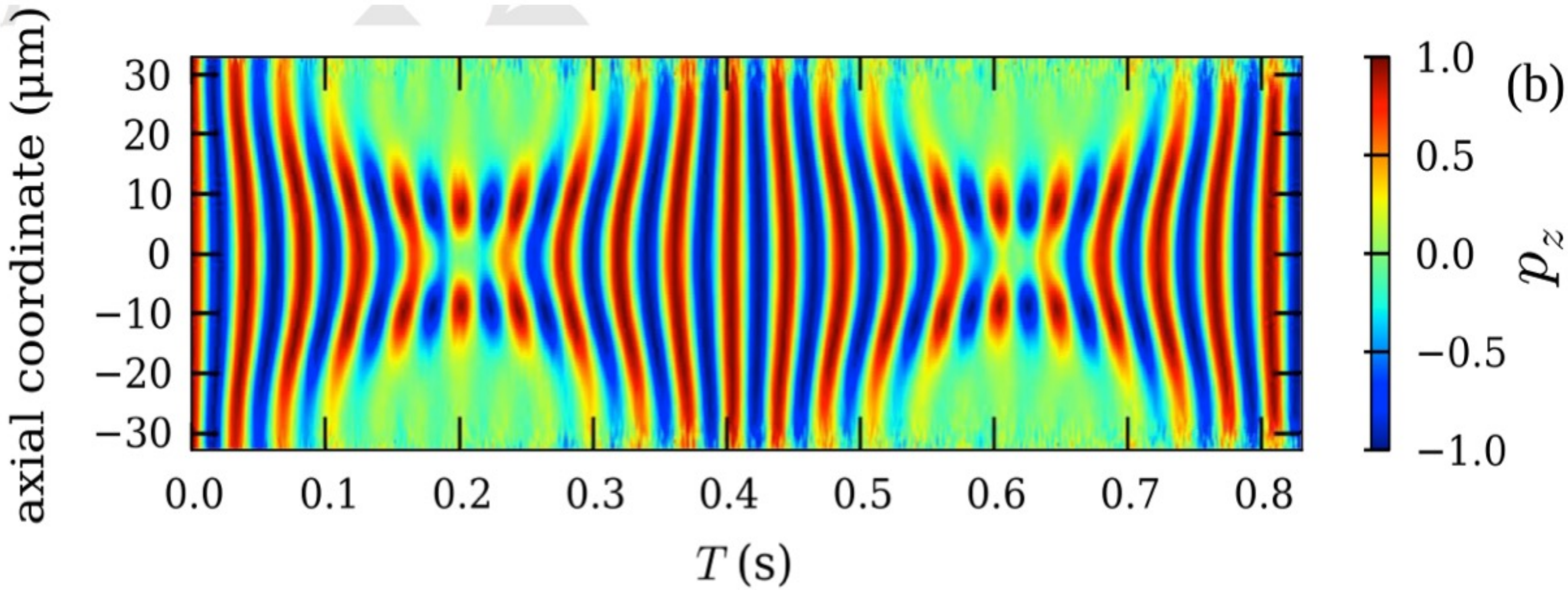
$$\psi_1 = \psi_c + \frac{1}{\sqrt{L}} \sum_k \left(u_k \beta_k e^{ikx} - v_k \beta_k^* e^{-ikx}\right)$$

$$\psi_2 = \frac{1}{\sqrt{L}} \sum_k \alpha_k e^{ikx},$$

$$\boxed{\begin{aligned} \langle |\alpha_k|^2 \rangle &= \frac{1}{2} \\ \langle |\beta_k|^2 \rangle &= n_k + \frac{1}{2}. \end{aligned}}$$

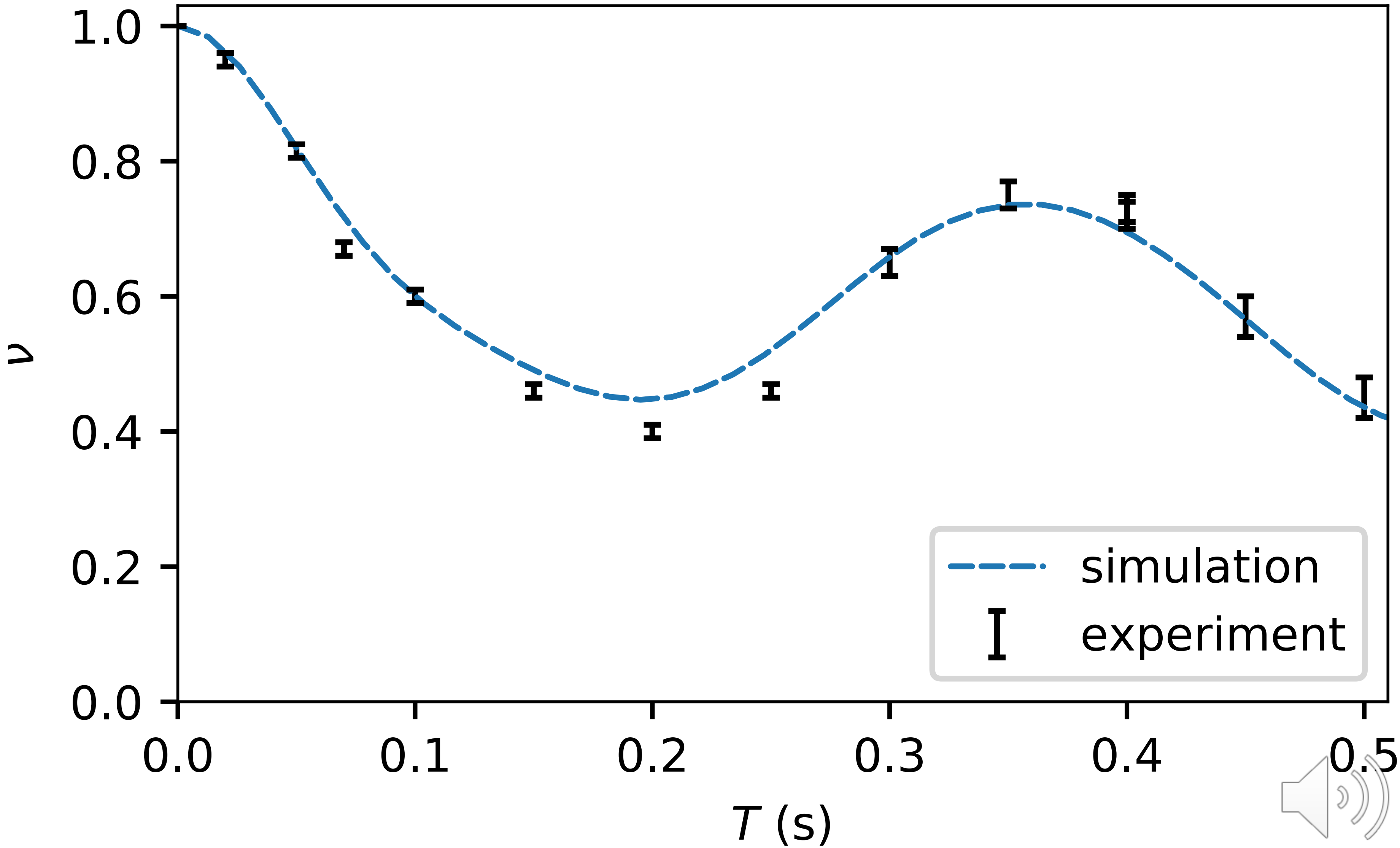
Oscillations in Rubidium experiment

A two-component, 4×10^4 atom ^{87}Rb BEC is in a harmonic trap with internal Zeeman states $|1, -1\rangle$ and $|2, 1\rangle$, which can be coupled via an RF field.

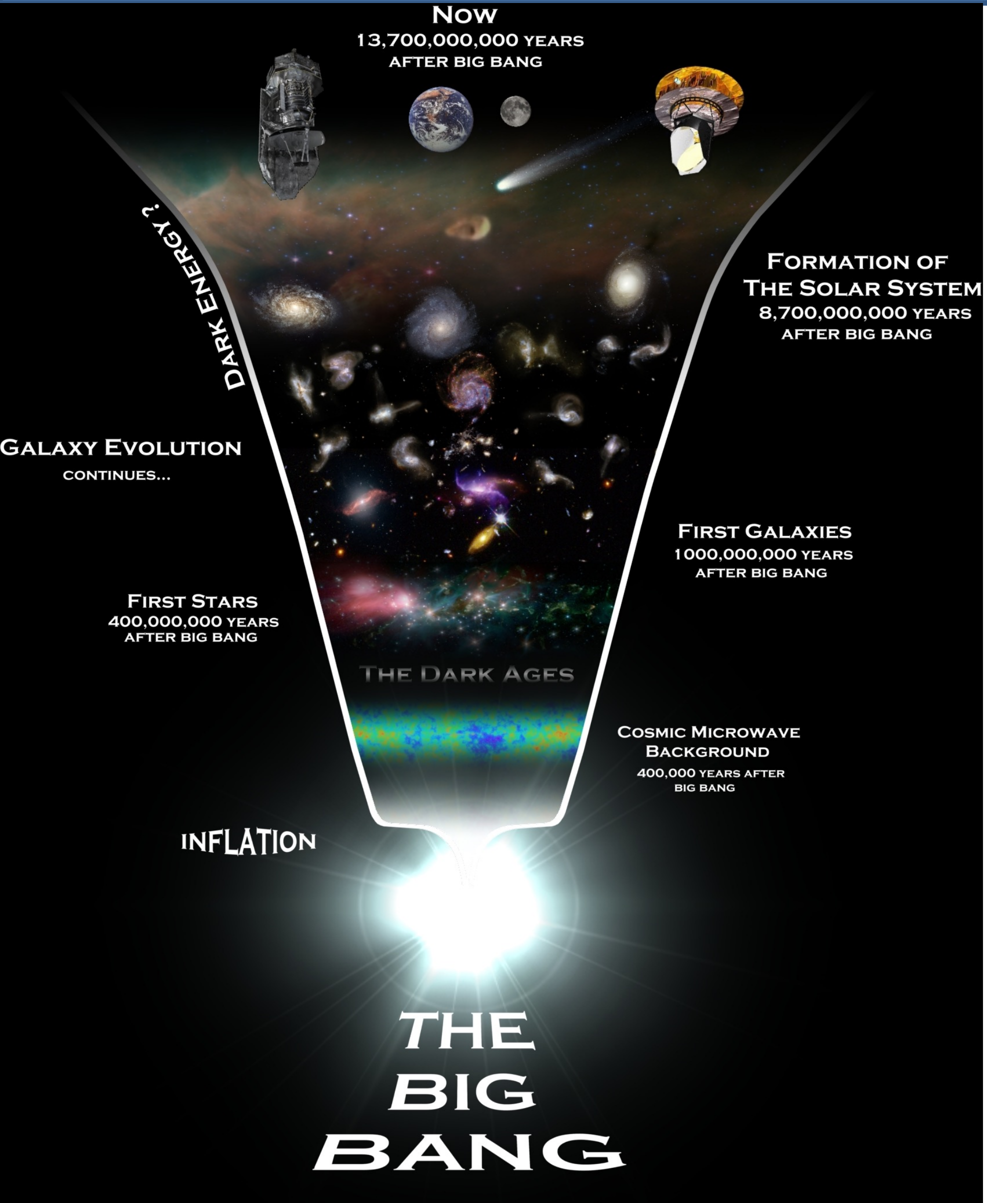


BEC interferometer fringe visibility, T=45nK

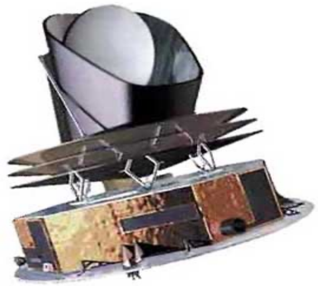
B. Opanchuk, et. al, Phys. Rev. A 100, 060102(R) (2019)



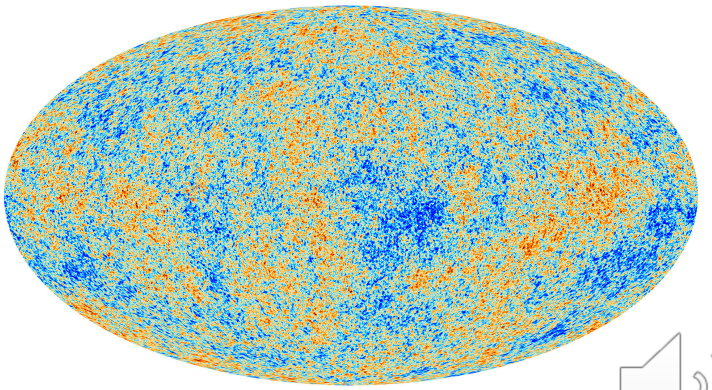
How can we test theories of the Big Bang?



Planck spacecraft was launched in May 2009. On 21 March 2013, the mission's all-sky map of the CMB was released



The CMB is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380,000 years old.



We can't see beyond that

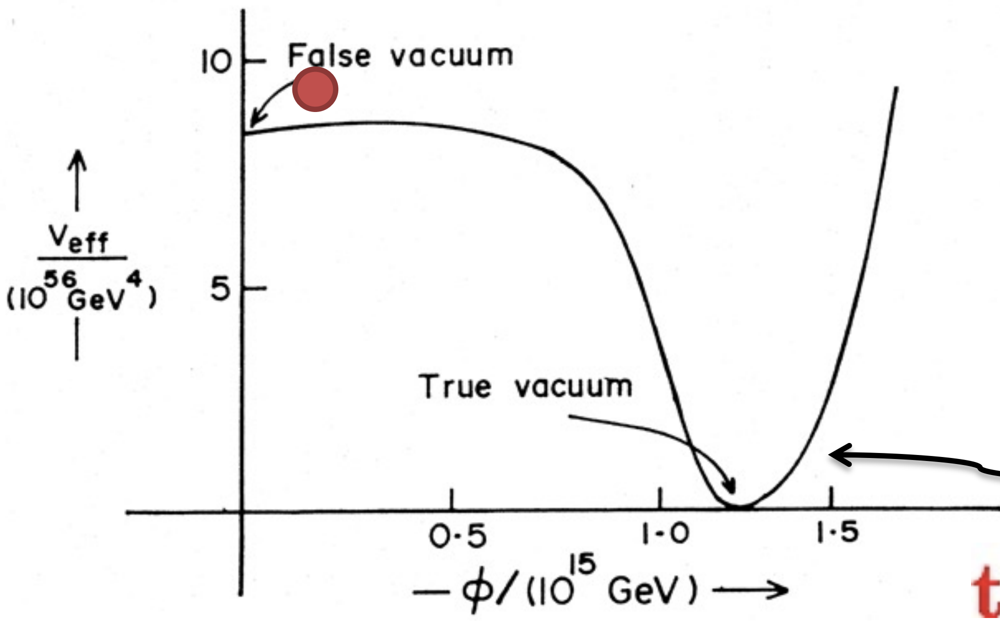
BGV theorem

Fate of the false vacuum: Semiclassical theory
Sidney Coleman
Phys. Rev. D 15, 2929 – 1977

Abstract

It is possible for a classical field theory to have two homogeneous stable equilibrium states with different energy densities. In the quantum version of the theory, the state of higher energy density becomes unstable through barrier penetration; it is a false vacuum….

Quantum models of the Big Bang



Potential energy on top of the hill is converted into kinetic energy of the rolling ball at the bottom of the hill.

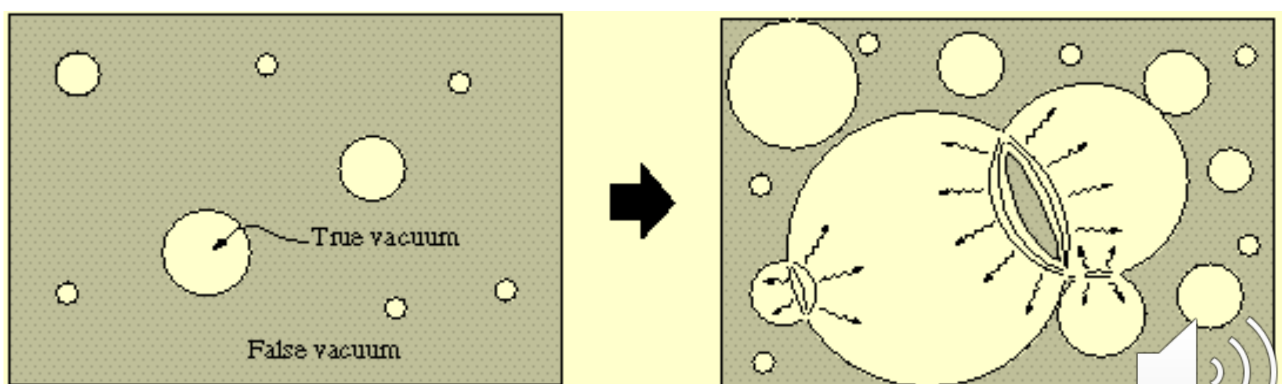
As a result a lot of energy was realised resulted in

BIG BANG

$t \sim 10^{-32}$sec

In reality the Universe has at least 3 dimensions. Bubbles appear during the transition to true vacuum.

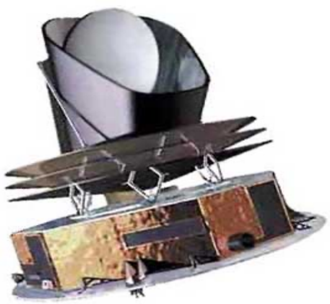
Are we in one of the bubbles….lonely….?

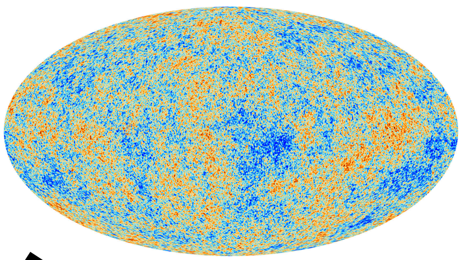


Similar to water boiling or bubbles in champagne

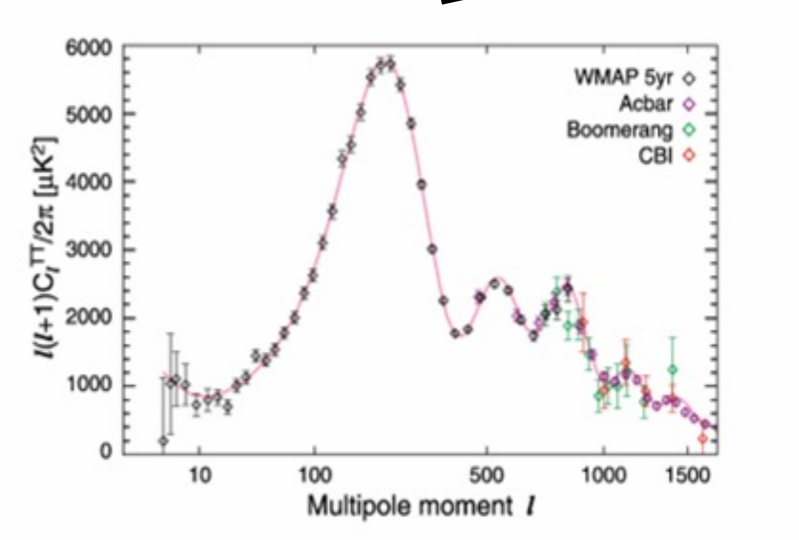
What is the observational evidence?

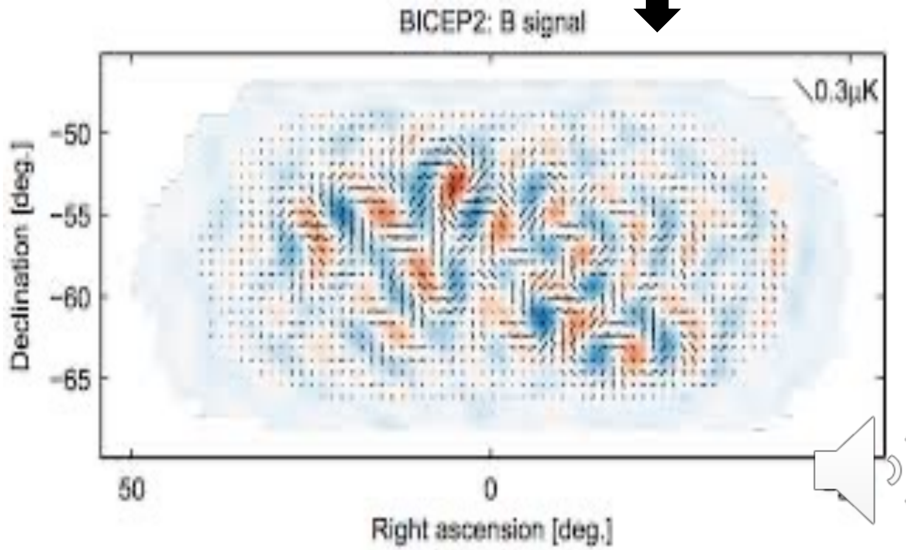
Planck spacecraft,
21 March 2013





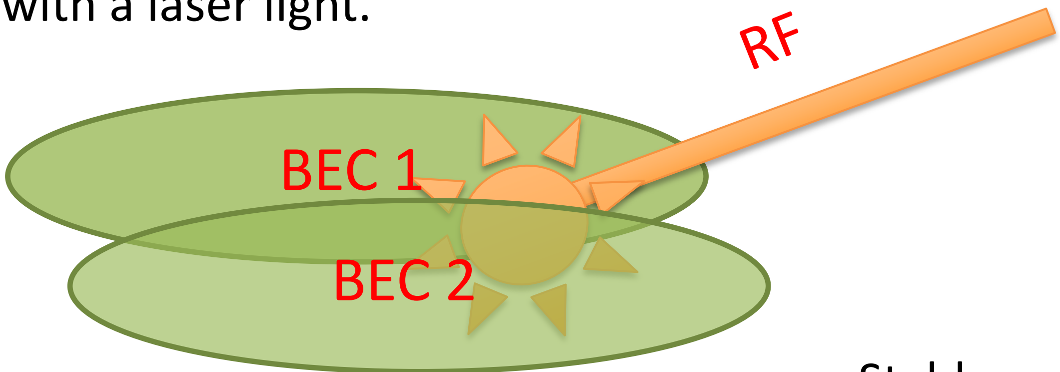
Bicep 2, Polar Bear,
South Pole Telescope



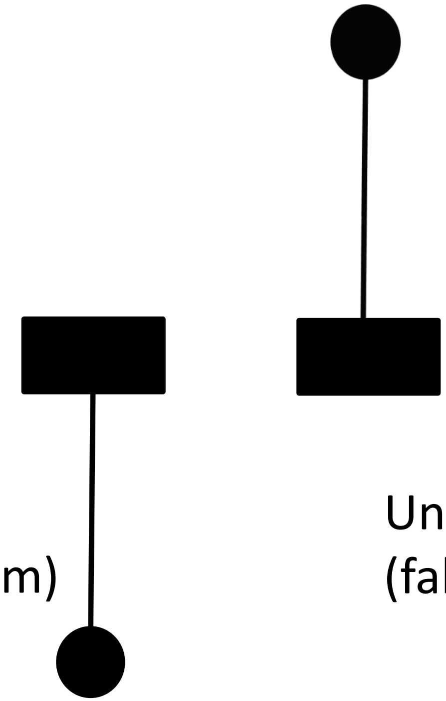


Analog quantum simulator

Take 2 BECs and couple them with a laser light.



$$\phi_1 - \phi_2 \approx \theta$$



Stable
(true vacuum)

$\theta = 0$

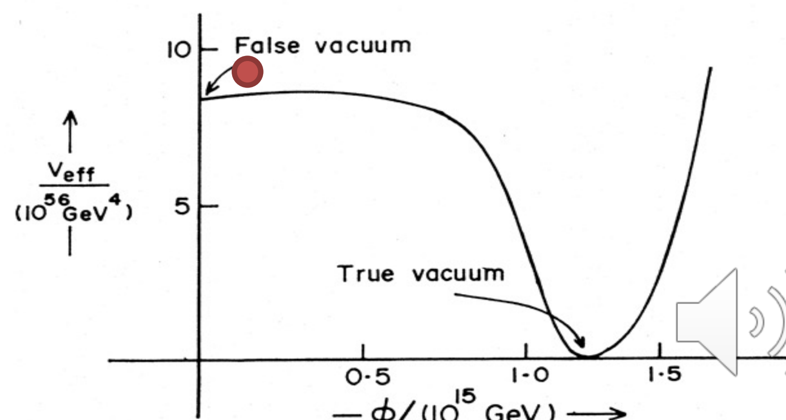
Unstable
(false vacuum)

$\theta = \pi$

Their phase difference behaves like a pendulum, which has stable and unstable points.

$$\partial_t^2\theta - c^2\nabla^2\theta = -\partial_\theta V(\theta)$$

-- relativistic field equation of the early Universe.



Early universe models

- The simplest model has a scalar inflaton field
- Relativistic, interacting quantum field dynamics
- $\phi(x)$ is described by the Lagrangian

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi),$$

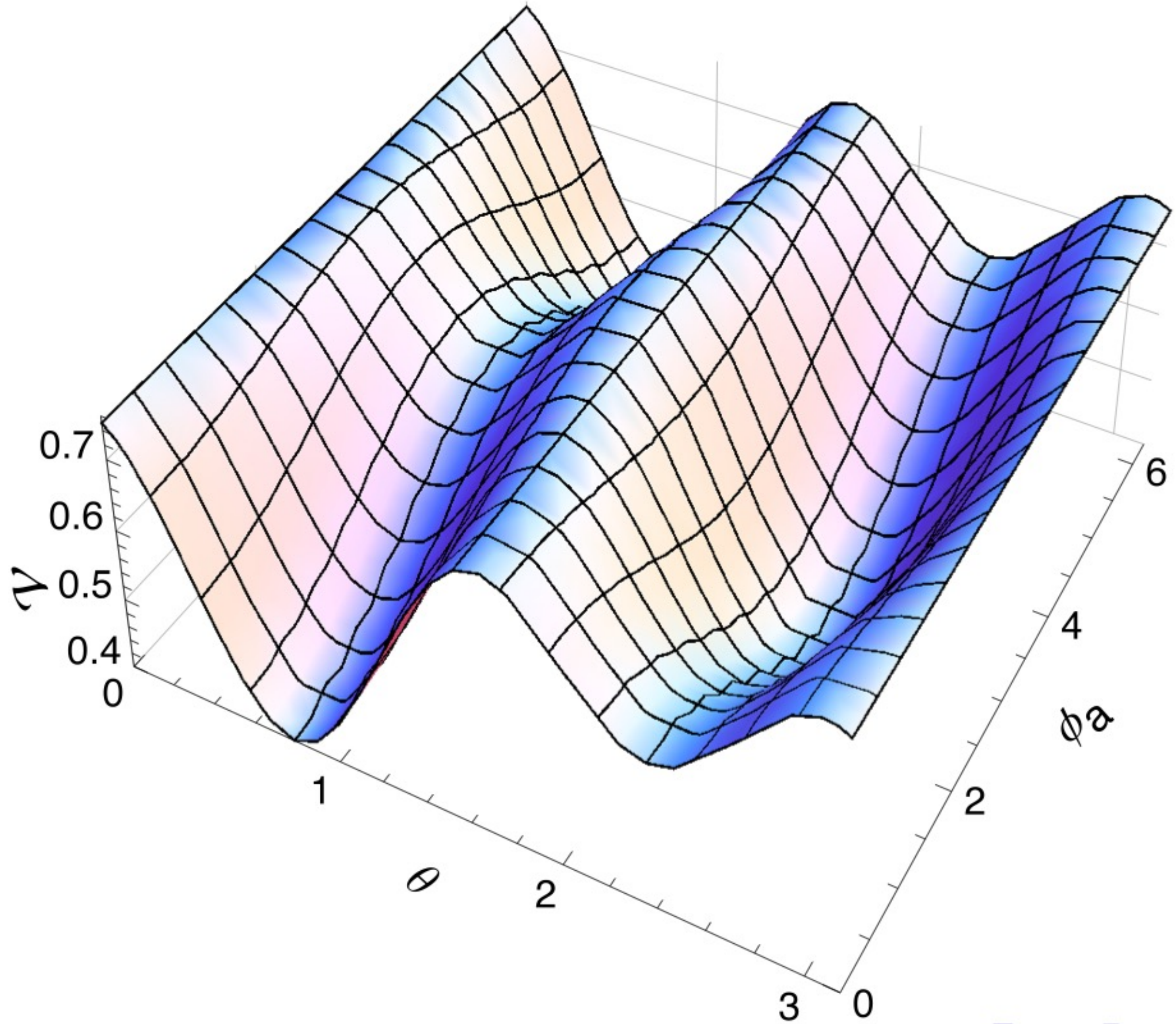
where $V(\phi)$ is the potential down which the scalar field rolls

Early universe quantum simulation

^{41}K Feshbach resonance

- zero inter- state scattering length at 685.7 G
  - nearly equal self-interactions,
  - unknown loss rates (can be estimated)
  - resonance not yet observed

Potential well with microwave coupling



Relativistic field theory at c=1 cm/s!

We introduce the quantum partition function
$\mathscr{Z} = \int \mathscr{D}(\psi^*, \psi) e^{-S[\psi^*, \psi]}$ where:

$$S[\psi^*, \psi] = \int d\mathbf{s} [\psi^*_\sigma \partial_\tau \psi_\sigma + H(\psi^*, \psi)]$$

.

- Imaginary time: $\mathbf{s} = (\tau, \mathbf{r})$, $\tau = it/\hbar \in [0, \beta]$
- $\psi_\sigma(\tau, \mathbf{r})$ is a complex field subject to the periodic boundary condition $\psi_\sigma(\beta, \mathbf{r}) = \psi_\sigma(0, \mathbf{r})$.
- First find static solution to identify vacua. This amounts to replacing $\psi_\sigma = \psi_0 = \mathrm{const}$ in the saddle-point approximation $\delta S / \delta \psi_\sigma = 0$.

Equivalent Sine-Gordon equation

$$\psi_1 = u e^{i(\phi_s+\phi_a)/2} \cos(\theta)$$
$$\psi_2 = u e^{i(\phi_s-\phi_a)/2} \sin(\theta),$$

- Canonical momentum: $\pi = \partial_\tau \phi_a / 4\gamma_{sa}$,
- Commutators: $\left[\phi_a(\zeta), \pi(\zeta')\right] = i\delta^D(\zeta-\zeta')$.
- Sine-Gordon equation:

$$\nabla^2\phi_a - \partial_{\zeta_0\zeta_0}\phi_a + \tilde{\alpha}\sin\phi_a = 0$$

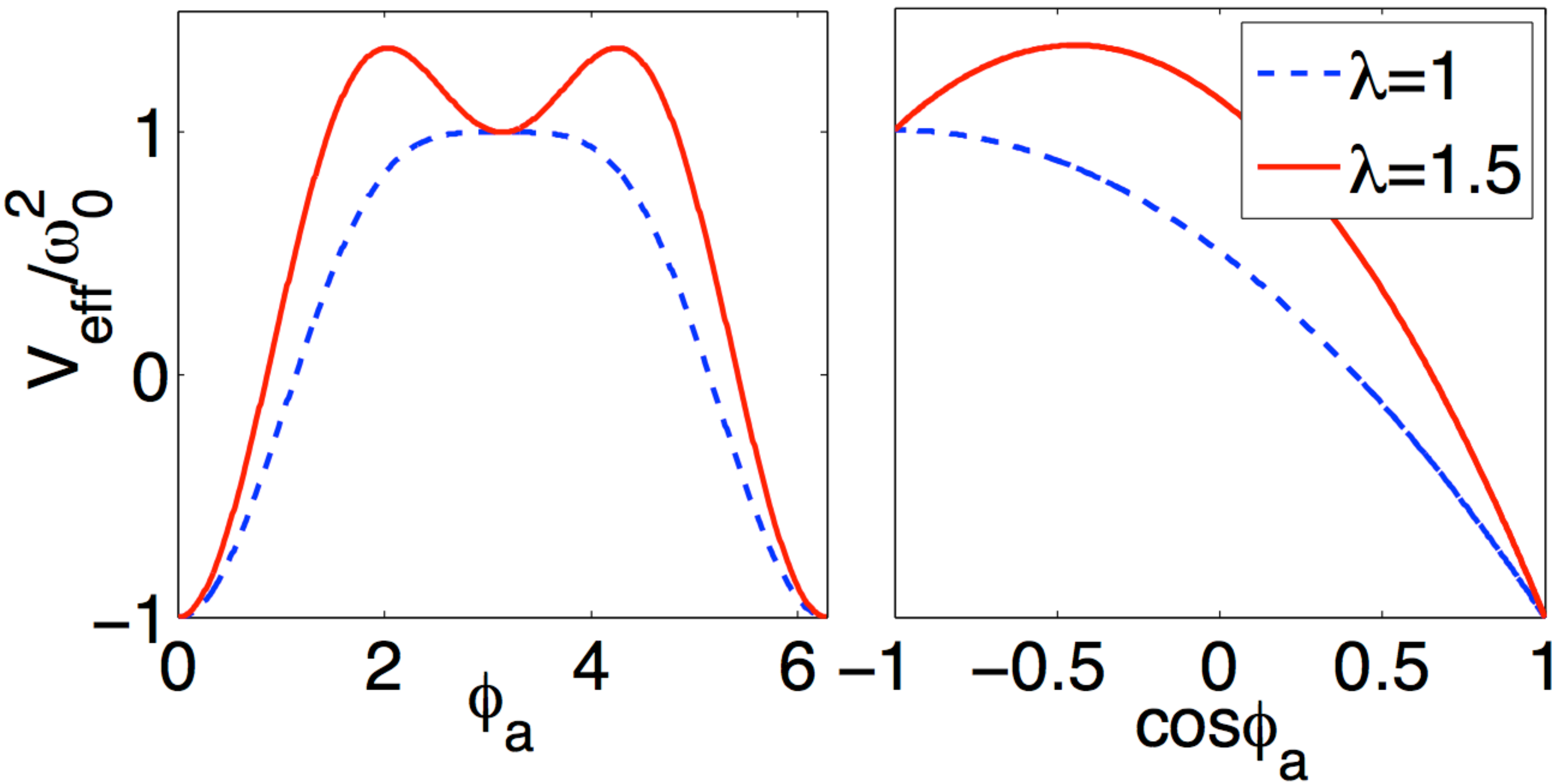
What about metastability: local minimum?

$$\frac{\partial \psi_j}{\partial t} = -\frac{i}{\hbar}\left[-\frac{\hbar^2 \nabla^2 \psi_j}{2m} + g\psi_j |\psi_j|^2 - \nu\left(t\right)\psi_{3-j}\right].$$

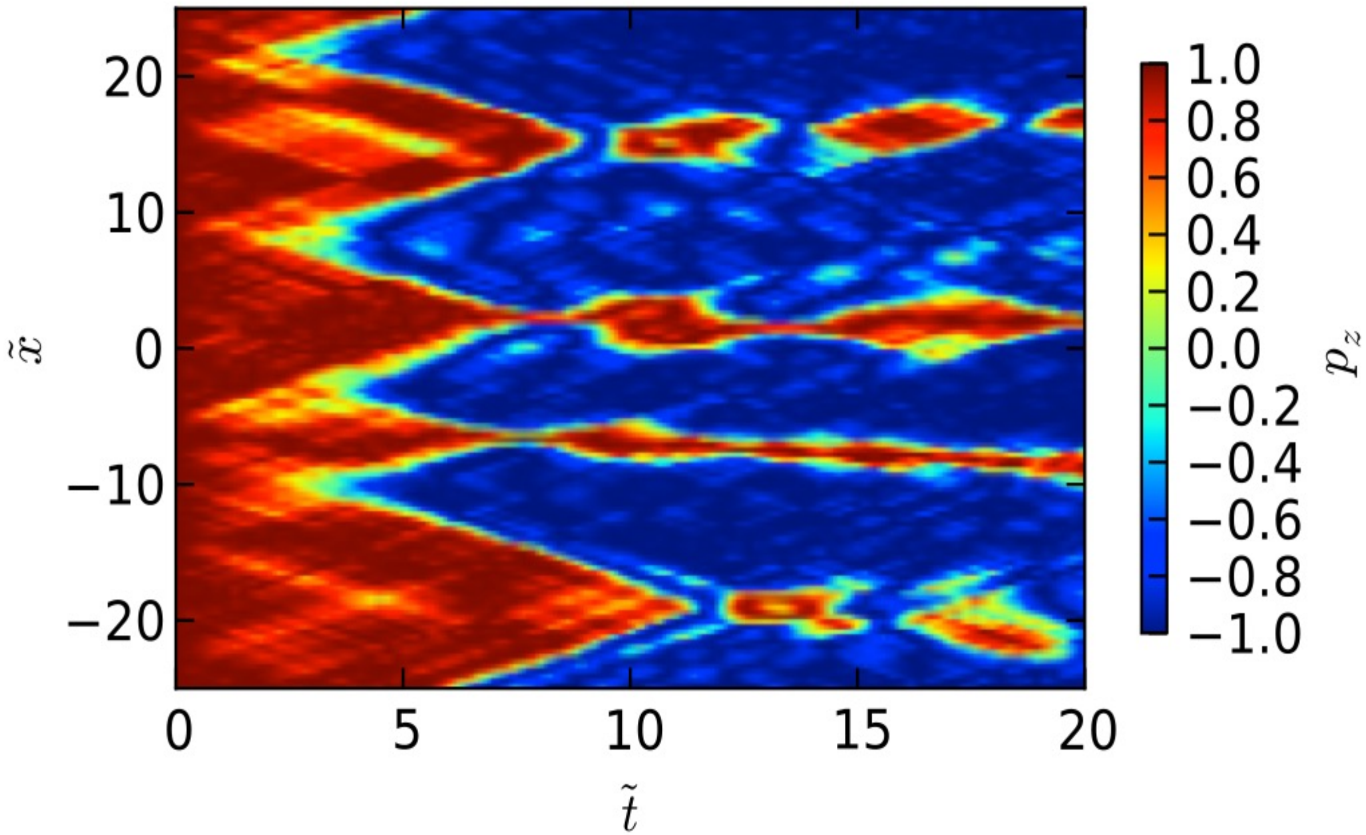
- By varying the tunnel coupling ν periodically in time, it is possible to establish the unstable vacuum at $\phi_a = \pi$
- Gives the Coleman model
- Microwave coupling $\nu_t = \nu + \delta\hbar\omega\cos(\omega t)$, where frequency of oscillations $\omega \gg \omega_0 \equiv 2\sqrt{\nu g \rho_0}/\hbar$
- $V(\phi) \to V_{\mathrm{eff}}(\phi_0) = -\omega_0^2\left[\cos\phi_0 - 0.5\lambda^2\sin^2\phi_0\right].$
- $\lambda = \delta\hbar\omega_0/\sqrt{2}\nu > 1$

O. Fialko et. al., Europhysics Letters **110**, 56001 (2015).

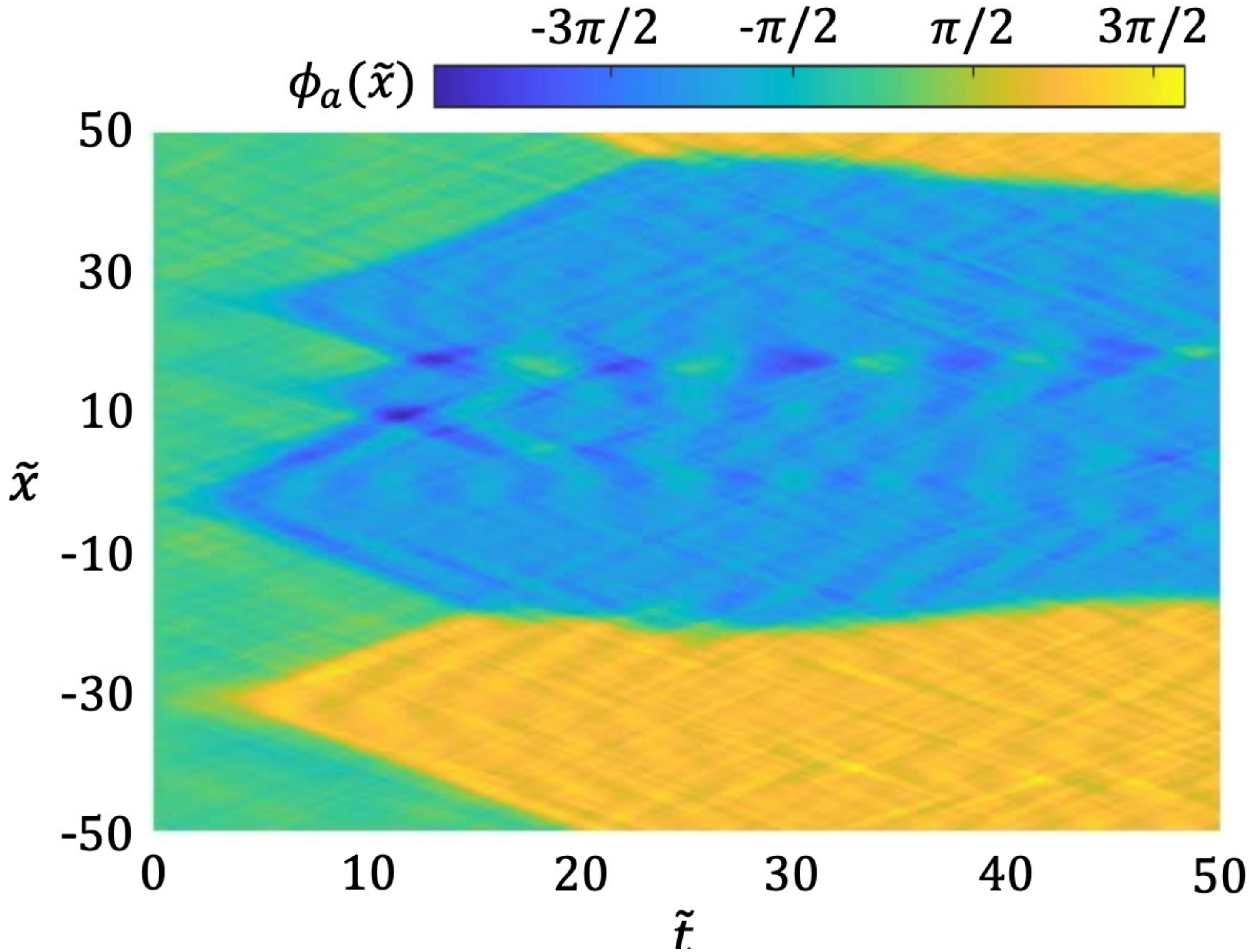
Effective potential



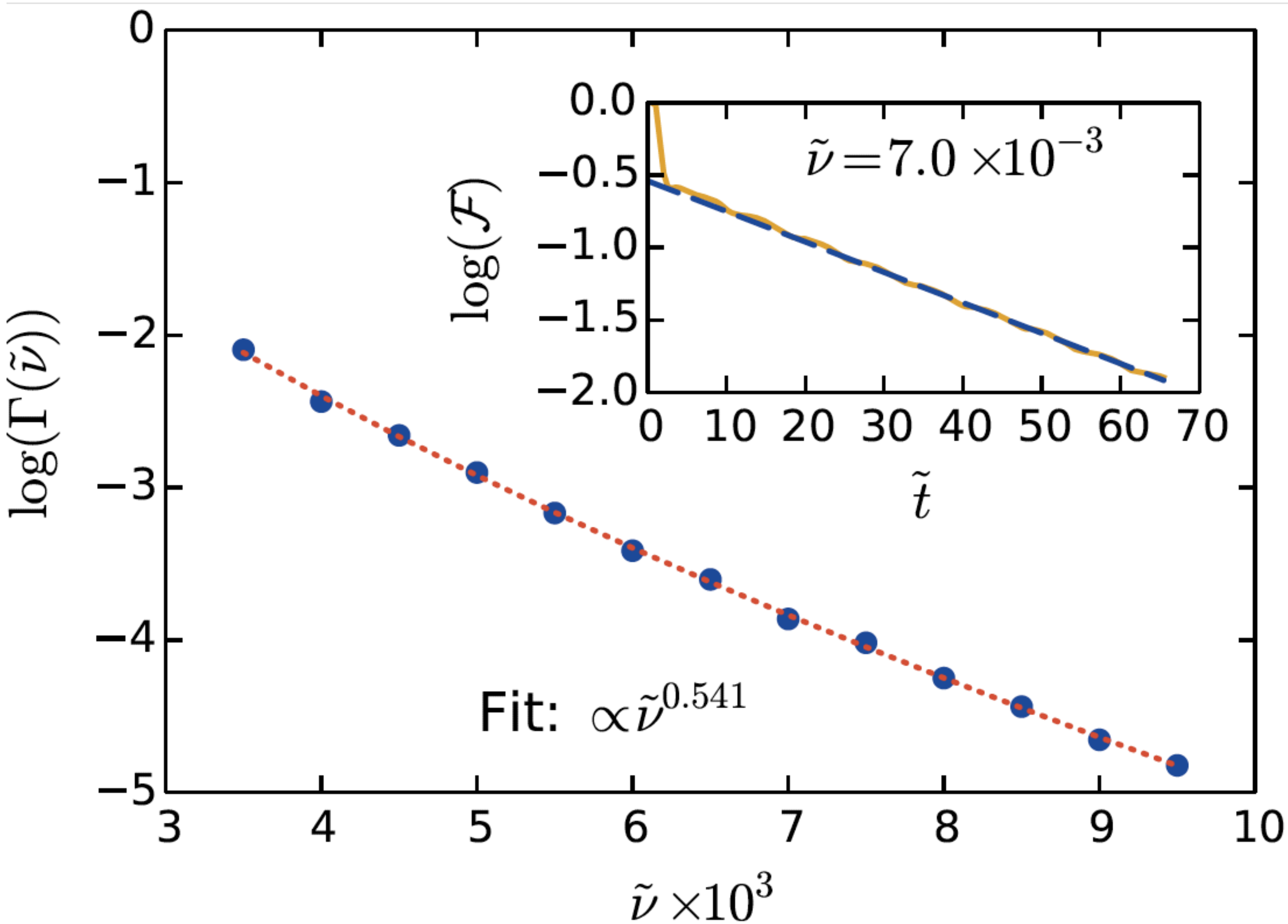
1D Vacuum bubbles expand at light-speed



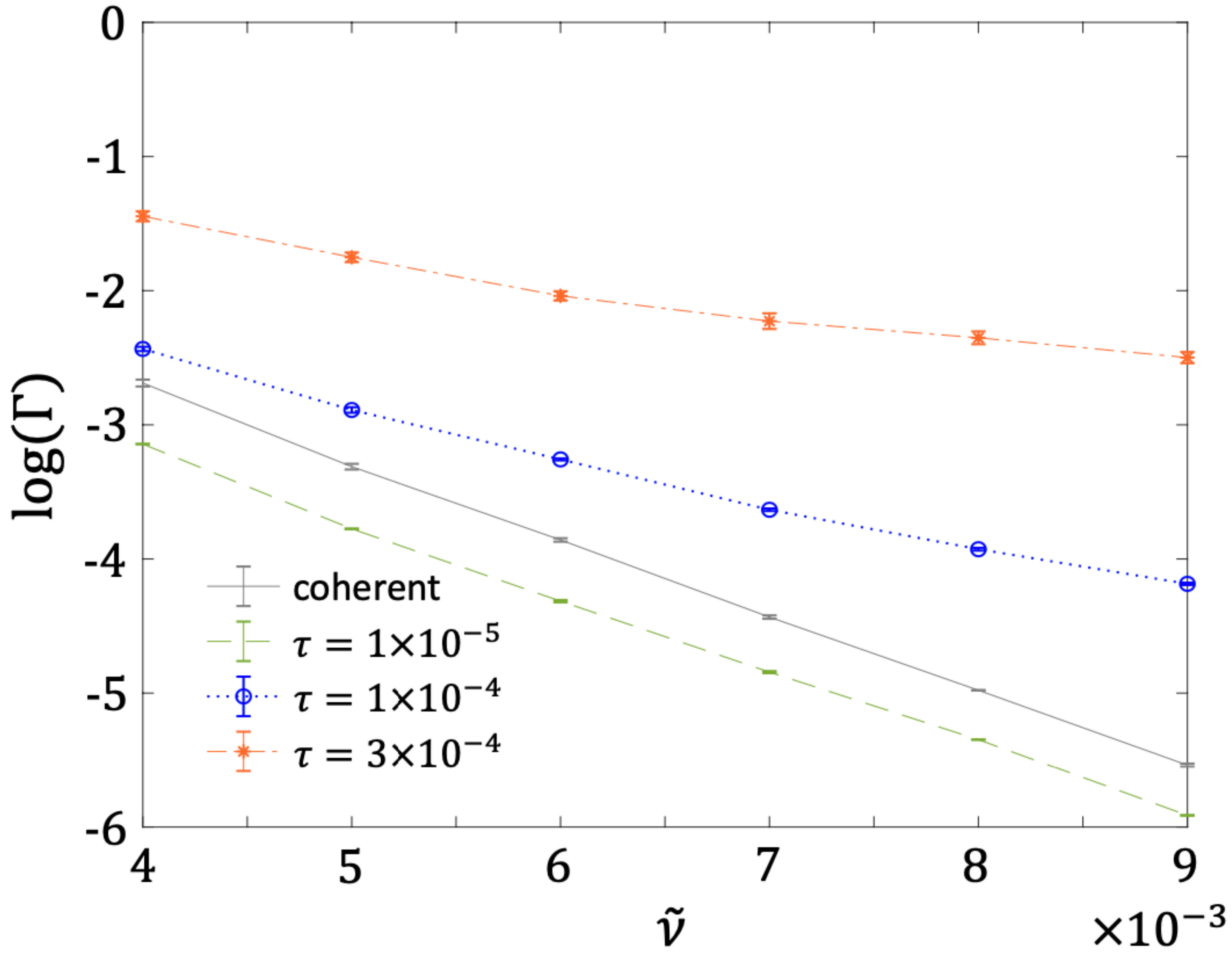
But there are two phase vacua: $\pm\pi$!



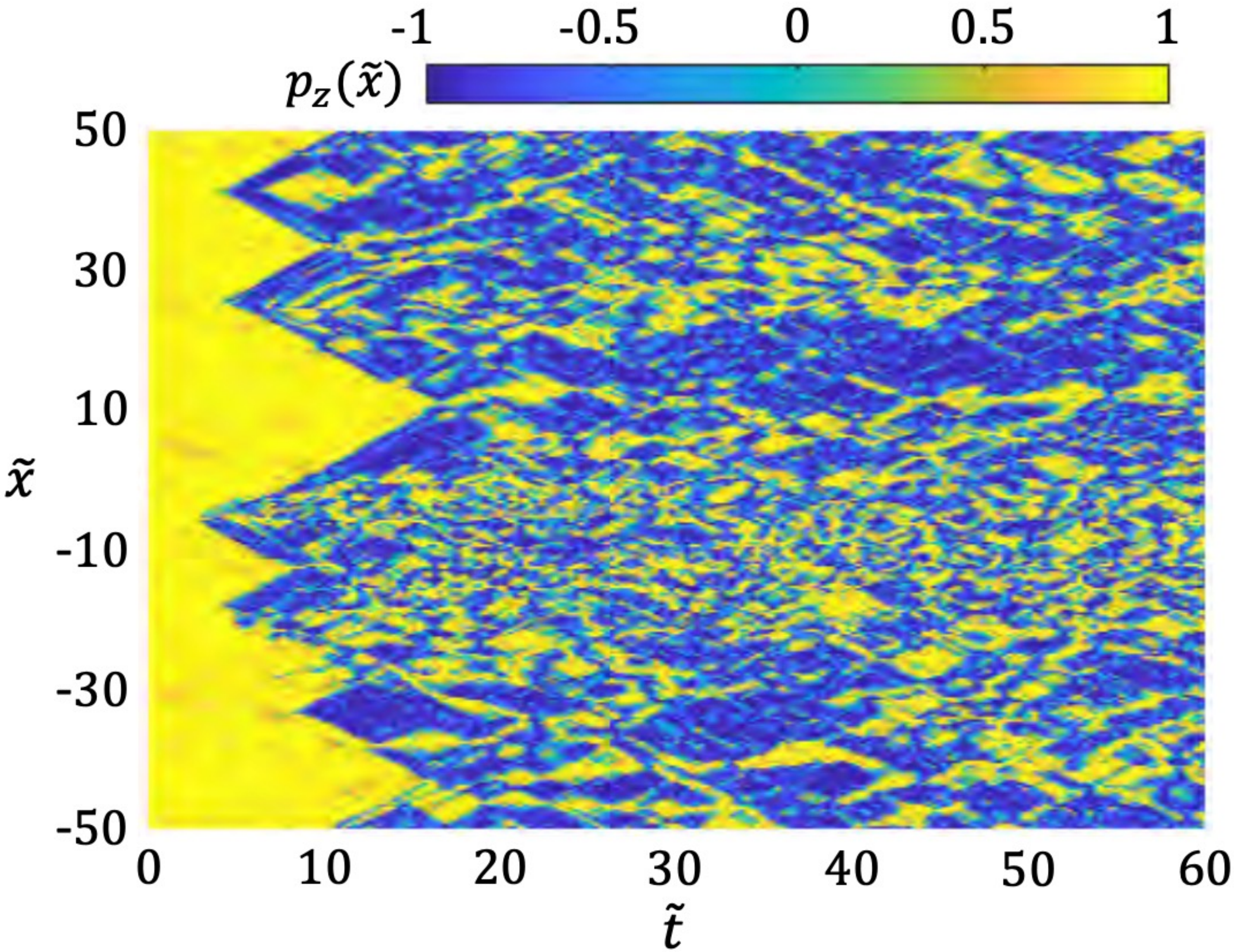
Tunneling rate depends on well-depth



Simulated tunneling rates at finite temperatures



Vacuum is more chaotic with temperature

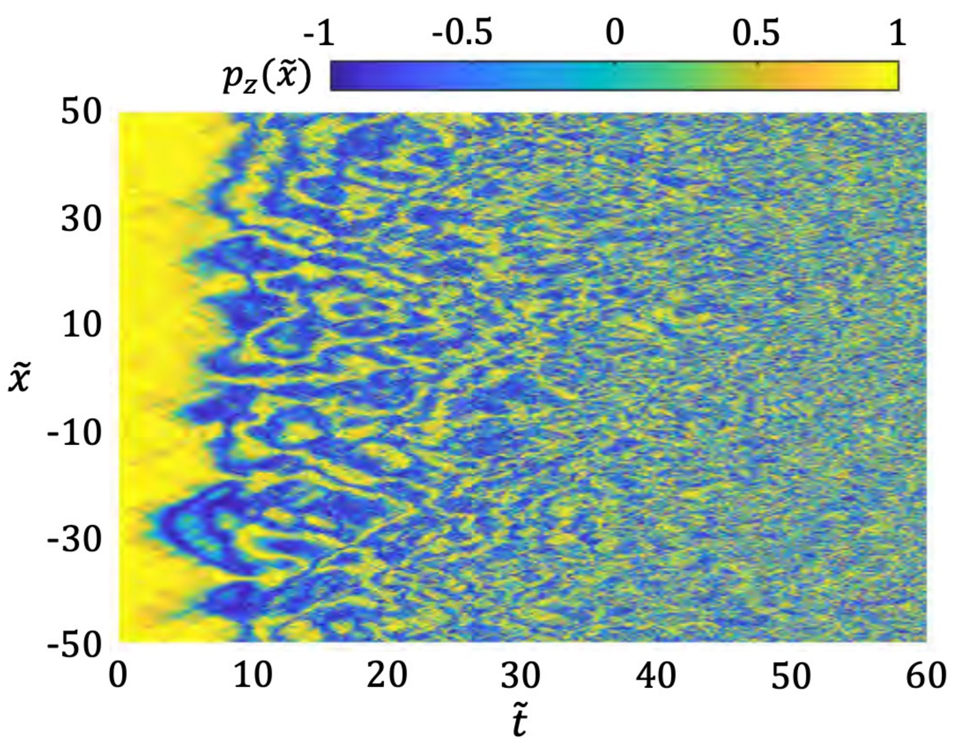


Momentum cutoff requirements with modulated coupling

There is a required momentum cutoff to prevent instabilities:

$$\widetilde{k}_c^2 \approx \frac{1}{2\widetilde{\nu}}\left(\sqrt{1+\widetilde{\omega}^2\widetilde{\nu}}-1\right)-\sigma,$$

Without a cutoff (eg a modulated trap potential) it is unstable:



Introduce a topological observational phase entropy

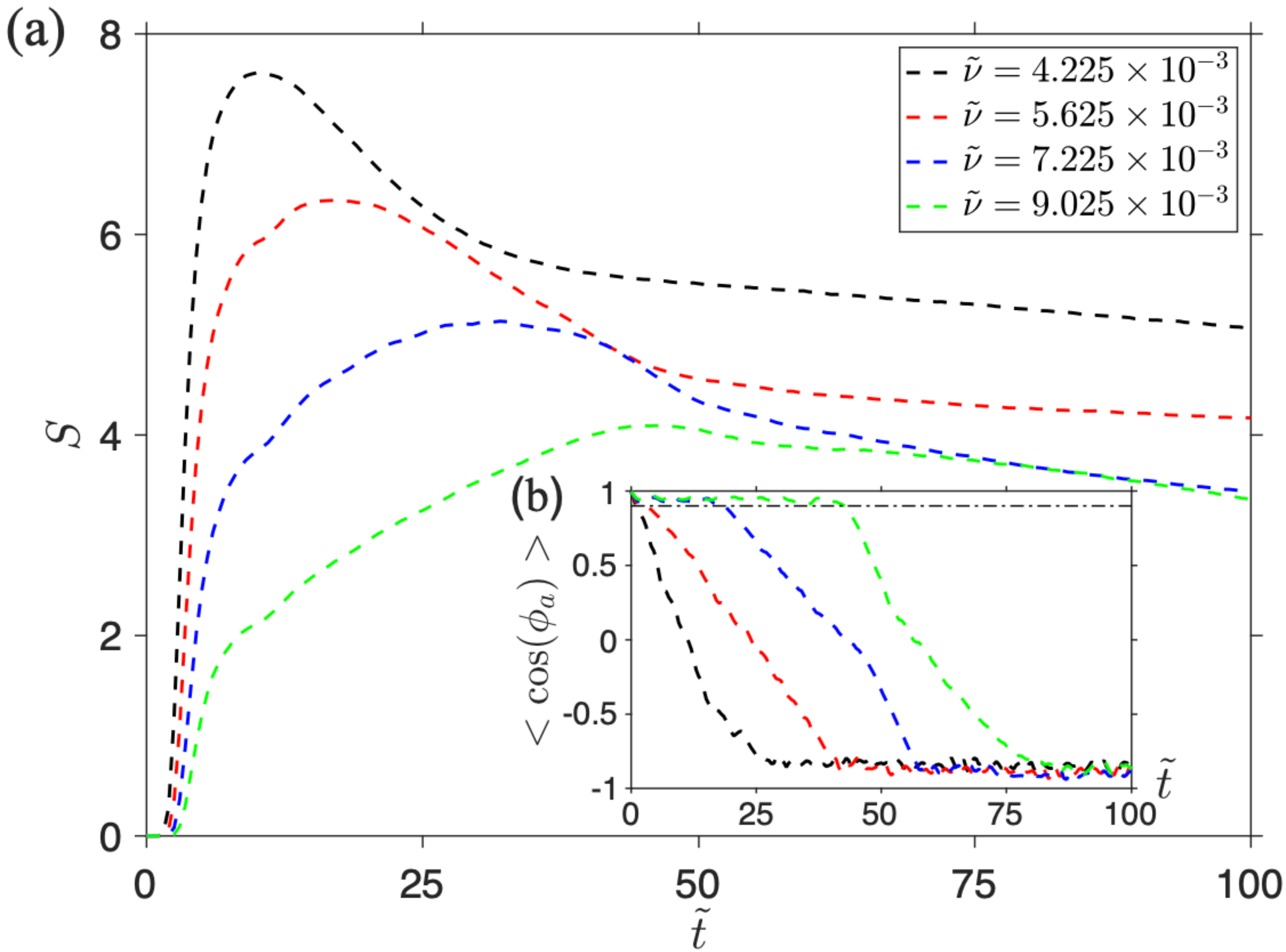
Based on the Wehrl entropy, defined for the Q-function

$$S_Q = -\int Q(\boldsymbol{\alpha}) \ln\left(Q(\boldsymbol{\alpha})\right) d^{2M}\boldsymbol{\alpha},$$

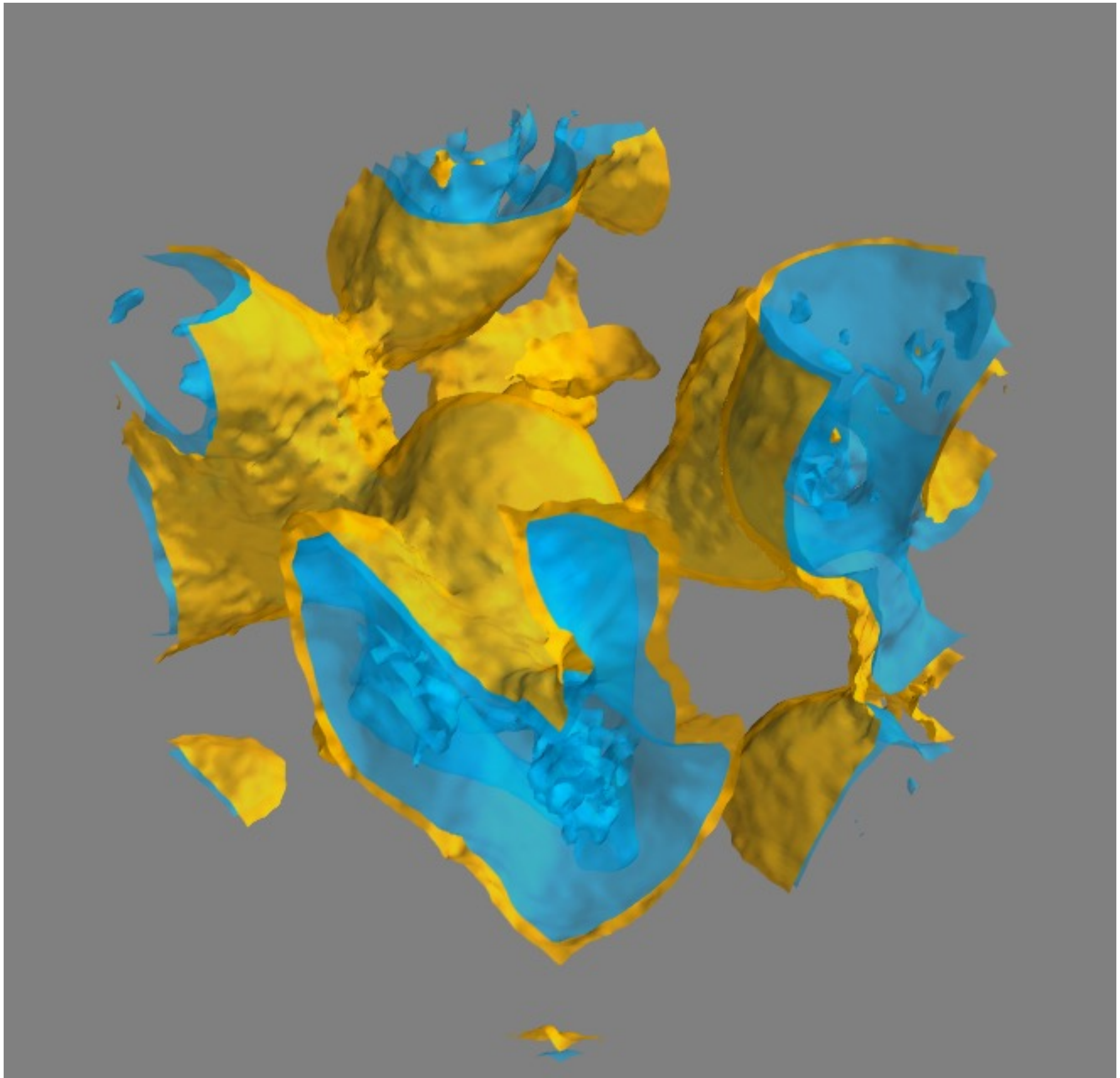
Given a topologically unwrapped phase measurement that distinguishes *p* different phases in l spatial regions, each possible phase sequence is binned, and the probability calculated from 10,000 simulations:

$$S_T = -\sum_i P_i \ln P_i \leq \ell \ln p.$$

Entropy is maximized during vacuum tunneling



3D bubbles



Summary: Topological effects due to two competing minima

- Topological entropy can decrease as well as increase!
- One topology dominates in any given “universe”
- Possible explanation for matter-antimatter asymmetry?
- INTERPRETATION: The Q-function can be used as an ontological model – is each simulation a REAL universe?