

Measurement-based, Lorentz-covariant Bohmian trajectories of multiple photons

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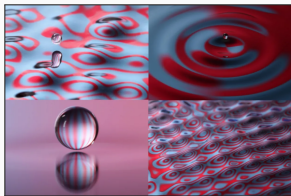


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QUANTUM THEORY AND BOHMIAN MECHANICS

Bohmian mechanics¹ (de Broglie-Bohm/pilot wave theory)²

- Deterministic, nonlocal, hidden-variable theory
 - $\psi(x, t) \Rightarrow$ guiding potential \Rightarrow particle dynamics \Leftrightarrow velocity field
- $\rho(x, t) = |\psi(x, t)|^2$ is the density of particle trajectories



¹David Bohm Phys. Rev. 85, 166 (1952)

²Most surveys report that $\sim 0 - 2\%$ of physicists prefer Bohmian mechanics over other interpretations

NONRELATIVISTIC BOHMIAN MECHANICS AND WEAK VALUES

Wiseman: Connection between weak values and Bohmian-type velocity field of nonrelativistic particles,³

$$V(x, t) = \frac{\langle x | \hat{p}_w | \psi(t) \rangle}{m} \Leftrightarrow \frac{j(x, t)}{\rho(x, t)}$$

Weakly measure \hat{p}_x , strong measurement reveals system to be in $|x\rangle$ i.e. particle at (x, t) .

Weak value definition:⁴ first-order term for weak interaction between system and pointer

$$\langle x | \hat{p}_w | \psi(t) \rangle = \text{Re} \frac{\langle x | \hat{p}_w | \psi(t) \rangle}{\langle x | \psi(t) \rangle}$$

³Wiseman, New J. Phys. **9** 165 (2007)

⁴Aharonov, Albert, and Vaidman, PRL **60** 1351 (1988)

NONRELATIVISTIC BOHMIAN TRAJECTORIES

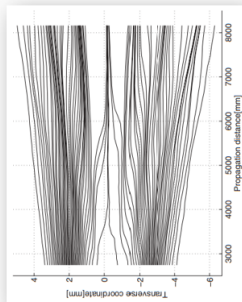
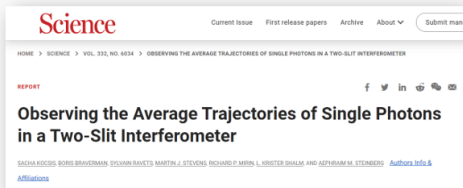


Figure 1: Nonrelativistic Bohmian trajectories in a double-slit interferometer, Kocsis et. al. Science, **332** 6034 (2011)

RELATIVISTIC BOHMIAN TRAJECTORIES OF PHOTONS



Lorentz covariant Bohmian mechanics

We extended Wiseman's formalism to relativistic regimes!

$$V(x, t) = \frac{dx}{d\tau} \frac{d\tau}{dt} = \frac{p}{E} \Rightarrow \frac{\langle x | \langle \hat{p}_w \rangle | \psi(t) \rangle}{\langle x | \langle \hat{H}_w \rangle | \psi(t) \rangle} \Leftrightarrow \frac{j_{\text{KG}}(x, t)}{\rho_{\text{KG}}(x, t)}$$

RELATIVISTIC BOHMIAN TRAJECTORIES OF PHOTONS

Trajectories satisfy **relativistic velocity addition** and **quantum-mechanical continuity**. Optics limit $\Rightarrow \rho_{\text{KG}}(x, t) \geq 0$.

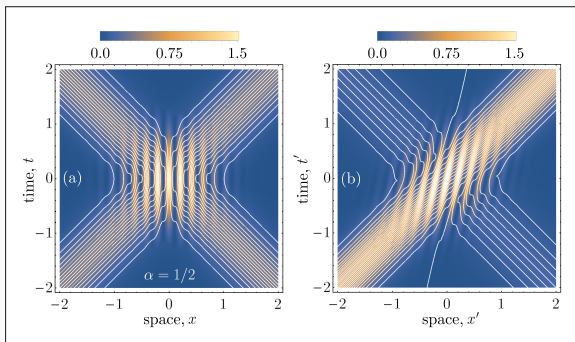


Figure 2: Photon trajectories. [J. Foo](#), E. Asmodelle, A.P. Lund, T.C. Ralph, *Nature Comms.* **13** 4002 (2022).

THE PHOTON METRIC

nonrelativistic dynamics \rightarrow relativistic dynamics

guiding potential \rightarrow guiding metric

ALCUBIERRE (PHOTON) METRIC

“Warp drive” solution in GR:

$$ds^2 = -(1 - v_s^2)dt^2 - 2v_s dx dt + dx^2$$

Define $v_s = (|V(x, t)| - 1) \operatorname{sgn}(V(x, t))$ then the speed of light according to faraway observers is

$$c = v_s + \operatorname{sgn}(V(x, t)) = V(x, t).$$

MULTIPLE PHOTONS?

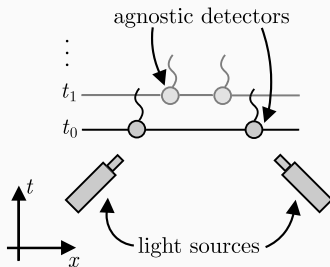
Weak value description of relativistic trajectories for multiple interacting photons?

$$\text{velocity} \Big|_{\text{WV}} \stackrel{???}{=} \text{velocity} \Big|_{\text{KG}}$$

CONSTRUCTING TWO-PHOTON VELOCITY FIELDS

Operational approach

- Initially separable state of the two photons
- Agnostic detectors
- Weak value velocities constructed from detector measurements



CONSTRUCTING TWO-PHOTON VELOCITY FIELDS

Multiparticle Klein-Gordon Approach

- Relativistic Klein-Gordon wavefunctions
- Symmetrise in spacetime

$$\psi(t_1, x_1, t_2, x_2) = \psi_1(t_1, x_1)\psi_2(t_2, x_2) + \psi_1(t_2, x_2)\psi_2(t_1, x_1)$$

- Evaluate velocity fields along a single timeslice

$$V_i(t_1, x_1, t_2, x_2) = \left. \frac{j_i(t_1, x_1, t_2, x_2)}{\rho_i(t_1, x_1, t_2, x_2)} \right|_{t_1=t_2=t}$$

where $i = 1, 2$ identifies the particle

TWO PHOTONS!

They yield the same result!

$$\text{velocity} \Big|_{\text{WV}} = \text{velocity} \Big|_{\text{KG}}$$

RELATIVISTIC TWO-PHOTON INTERACTION

Photons exhibit **nonclassical interference** due to their **indistinguishability** \Rightarrow photon-bunching.

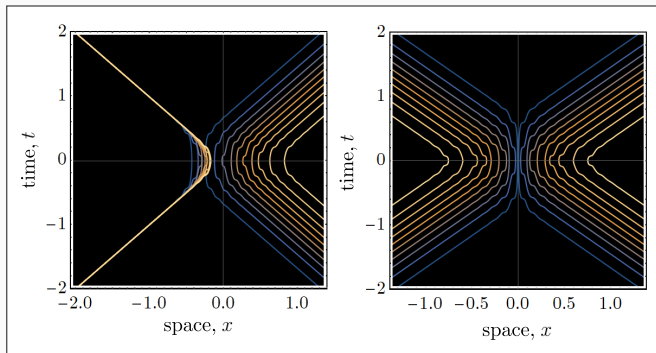


Figure 3: Bohmian trajectories for two indistinguishable photons. Colours indicate pairs of photons.

CONTINUITY

Quantum-mechanical continuity: density of trajectories matches quantum-mechanical density ρ_{KG} , holds for all time.

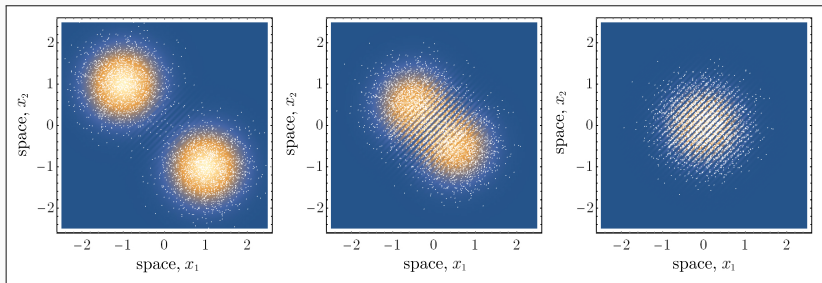


Figure 4: Randomly sampled initial conditions, demonstrating continuity of trajectories.

LORENTZ BOOSTS

Lorentz boost: $x \mapsto \gamma(x - vt)$ and $t \mapsto \gamma(t - vx)$.

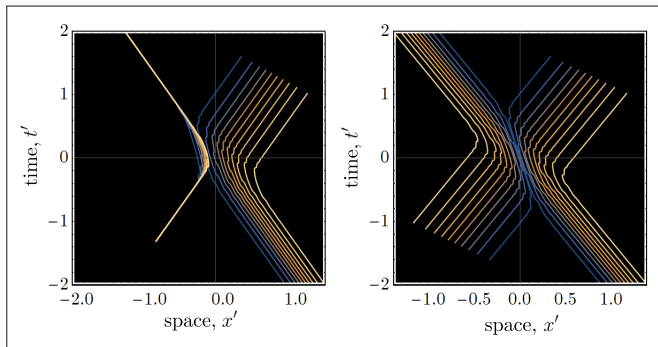


Figure 5: Boosted trajectories. [J. Foo](#), [A.P. Lund](#), [T.C. Ralph](#), [arXiv:23...](#)

CONCLUSIONS

Extended WV Bohmian framework to relativistic multiparticle interactions

- **Grounding** two-photon Bohmian trajectories in **weak values**

This gives us hope that our approach extends to:

- Entangling interactions
- Trajectories in curved space/gravitational potentials
- Particle production and annihilation processes

Email jfoobles@gmail.com for questions, suggestions, collaboration! Twitter: (at)jfoobles

LIFE IN A BOHMIAN UNIVERSE

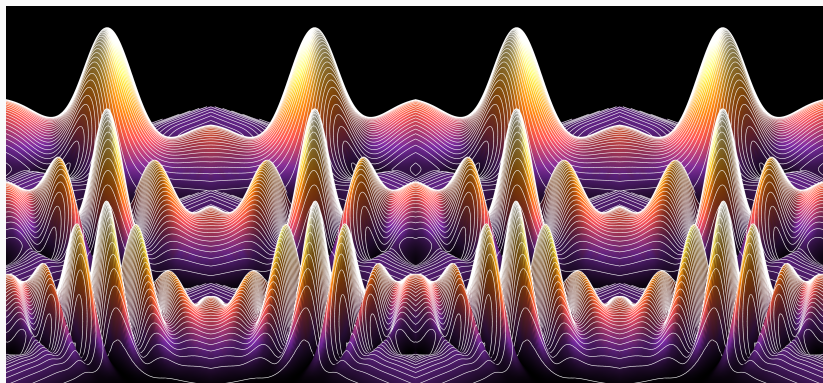


Figure 6: Front cover of the CQC2T annual report. *Life in a Bohmian universe*, J. Foo.

PHOTONS ARE LIKE BASKETBALLS

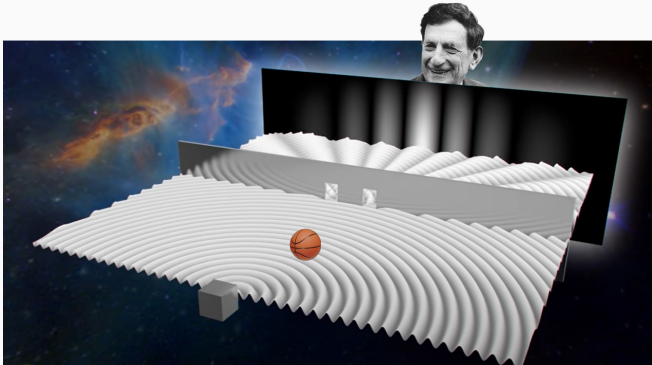


Figure 7: 3-Minute Thesis slide, J. Foo.