The prediction that black holes evaporate through the emission of Hawking radiation is widely regarded as one of the most impressive achievements of quantum field theory in curved spacetimes. Extensions of this result to various dynamical scenarios require generalisations of surface gravity. While different generalisations do in general not agree exactly, they are believed to be suitably close, particularly in the quasi-static limit. Nonetheless, we find that the two principal generalisations of surface gravity to dynamic spherically symmetric spacetimes are irreconcilable, and neither of them can describe the emission of nearly-thermal radiation. If semiclassical gravity is valid, this implies that it is impossible to simultaneously realise all of the necessary elements (event horizon, evaporation, thermal character of the radiation) that would be required for a self-consistent formulation of the information loss paradox. It also raises the question of which (if any) definition of surface gravity (or some closely related quantity) is related to the Hawking temperature of an evolving black hole.