

Surface gravity

and information loss

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Surface gravity and the information loss problem

Robert B. Mann,^{1,2,*} Sebastian Murk^{3,4,†} and Daniel R. Terno^{3,‡}



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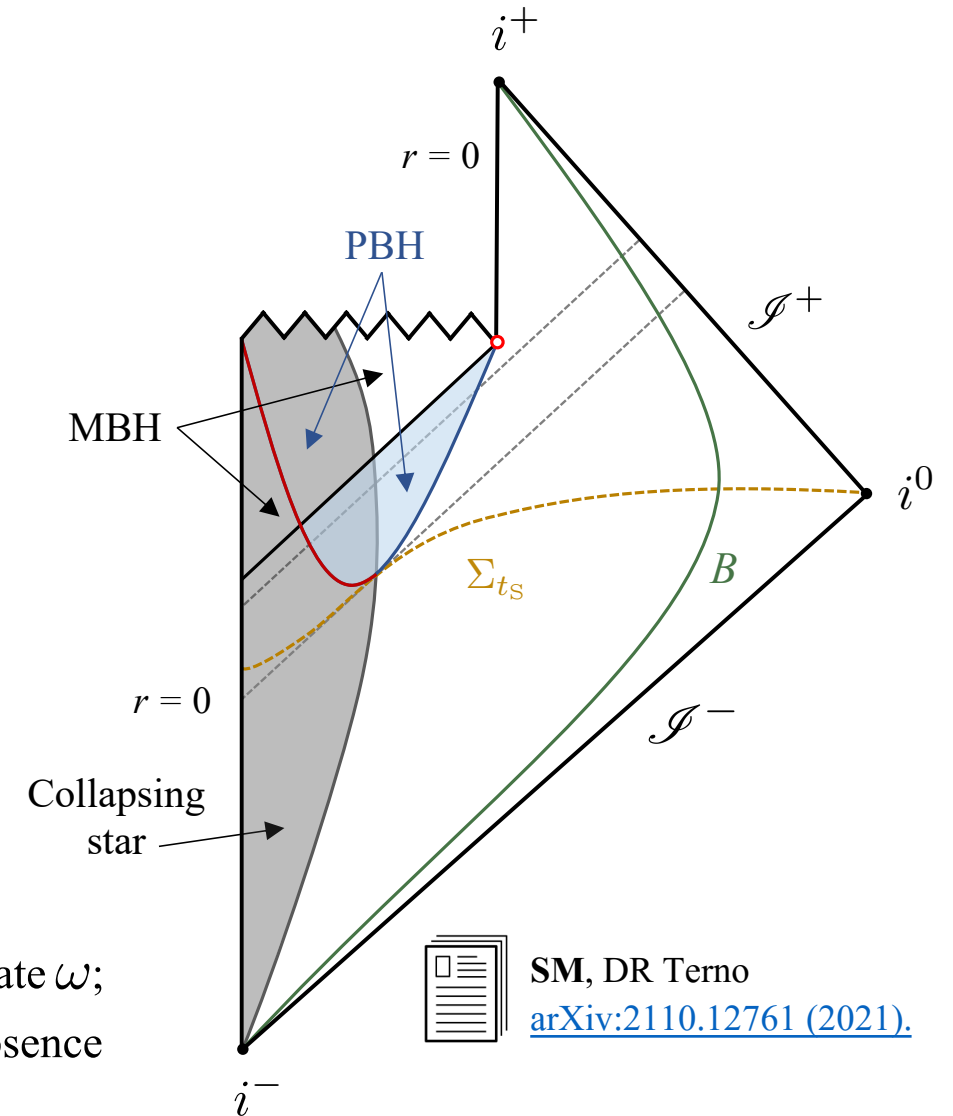
Semiclassical gravity: overview of our self-consistent approach

- i. Classical spacetime structure is still meaningful and described by metric; classical notions (e.g. horizons, trajectories) can be used.
- ii. Metric is modified by quantum effects. The resulting curvature satisfies semiclassical Einstein equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle_{\omega}$$

- iii. EMT describes total matter content, i.e. both the original collapsing matter and the produced excitations. Dynamics of collapsing matter is still described classically using metric.

No assumptions about: global/asymptotic structure of spacetime; quantum state ω ; status of energy conditions; presence or absence of singularity; presence or absence of Hawking radiation.



SM, DR Terno
[arXiv:2110.12761 \(2021\)](https://arxiv.org/abs/2110.12761).

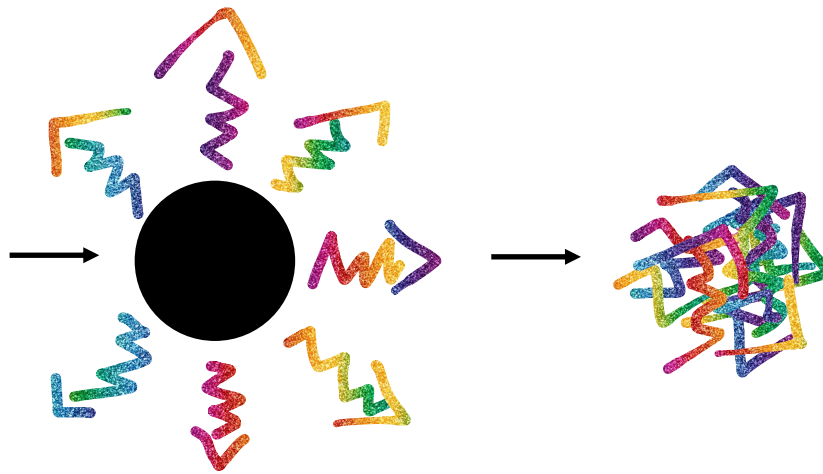
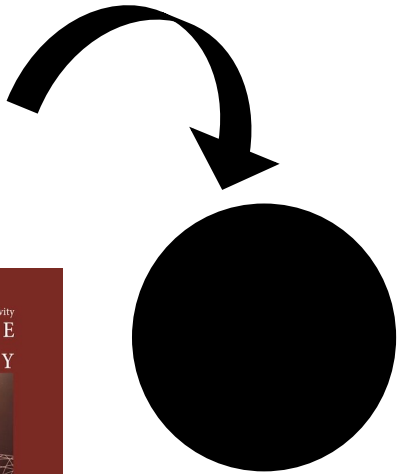
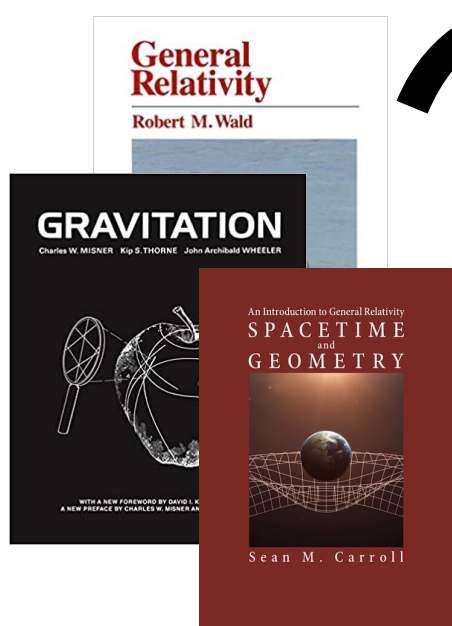


Information loss: a simplified view

$$\text{Semiclassical gravity: } R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} \equiv \langle \hat{T}_{\mu\nu} \rangle_{\omega}$$

Unitarity: information is preserved.



Thermal radiation:

uncorrelated; no information other than the **mass** and **temperature** of the BH.



Stephen W. Hawking, Cambridge (1993).



SW Hawking, [Nature](#) **248**, 30 (1974).



SW Hawking, [Commun. Math. Phys.](#) **43**, 199 (1975); **46**, 206(E) (1976).



Prerequisites for the paradox

The formulation of the information loss problem involves at least the following:

1) Formation of transient trapped region in finite time of distant observer

Provides the scattering-like setting to describe the states and their alleged information content “before” and “after”.

2) Formation of an event horizon

Needed to give an objective, observer-independent significance to tracing out of the black hole degrees of freedom.

3) Thermal or nearly-thermal character of the radiation

Responsible for disappearance of trapped region and high entropy of the reduced exterior density operator.



Prerequisites for the paradox: physical consequences

Note: event horizons are physically unobservable!



M Visser,
[Phys. Rev. D 90, 127502 \(2014\).](#)

→ work with apparent horizon

Note: for an evaporating black hole, existence of event horizon in finite time t implies formation of apparent horizon at some finite time t_s .

Details:



RB Mann, SM, DR Terno,
[Phys. Rev. D 105, 124032 \(2022\).](#)

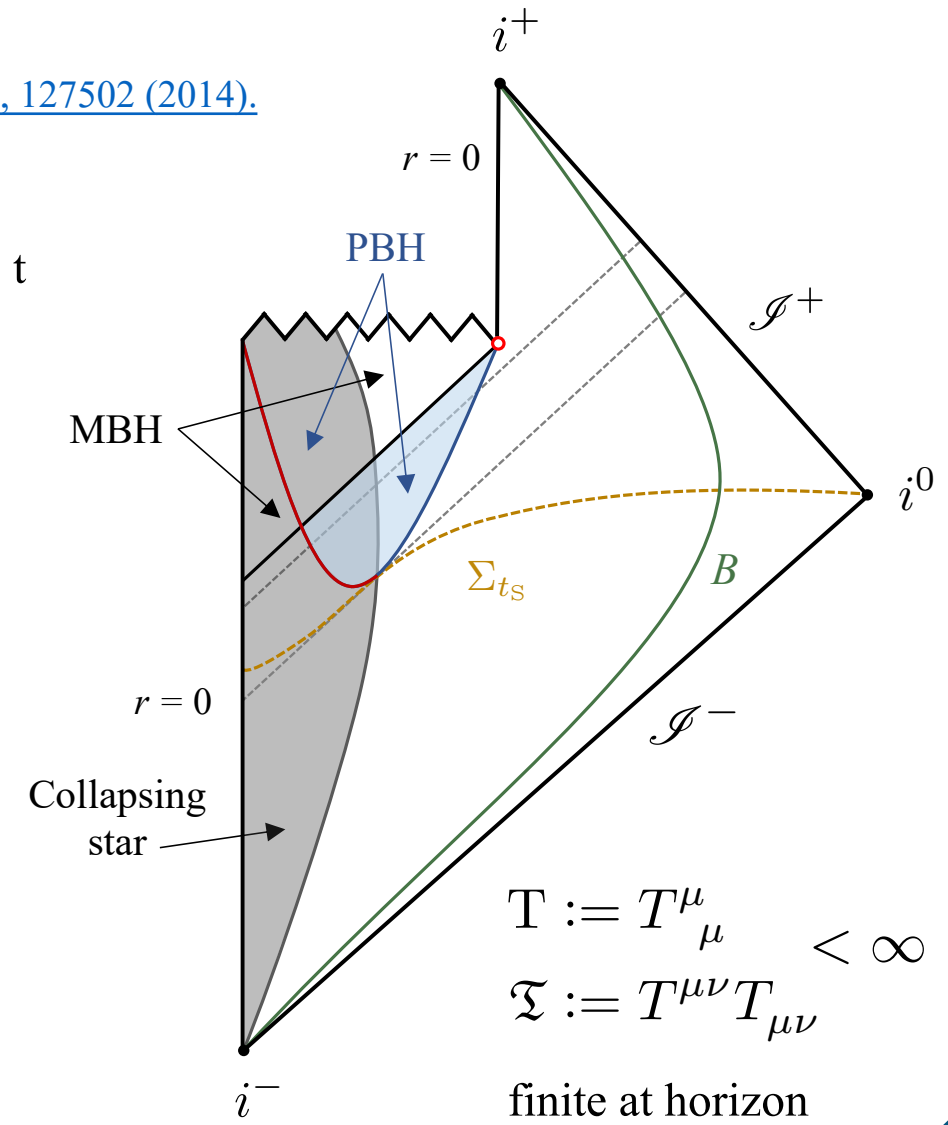
Necessary condition for the formulation of the paradox:

Formation of a regular apparent horizon in finite time as measured by the clock of a distant observer.

↳ **Unique scenario for black hole formation!**



SM, DR Terno,
[Phys. Rev. D 103, 064082 \(2021\).](#)





Semiclassical gravity: spherically symmetric setup

$$ds^2 = -e^{2h(t,r)} f(t,r) dt^2 + f(t,r)^{-1} dr^2 + r^2 d\Omega$$

where $f(t,r) := 1 - C/r := \partial_\mu r \partial^\mu r$

Misner-Sharp mass  CW Misner, DH Sharp, [Phys. Rev. 136, B571 \(1964\)](#).

Integrating factor in coordinate transformations, e.g.
 $dt = e^{-h} (e^{h+} dv - f^{-1} dr)$

Einstein equations
$\partial_r C = 8\pi r^2 \tau_t / f$
$\partial_t C = 8\pi r^2 e^h \tau_t^r$
$\partial_r h = 4\pi r (\tau_t + \tau^r) / f^2,$

Effective EMT components:

$$\tau_t := e^{-2h} T_{tt} \quad , \quad \tau_t^r := e^{-h} T_t^r \quad , \quad \tau^r := T^{rr}$$



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↑
Misner-Sharp mass



CW Misner, DH Sharp,
[Phys. Rev. 136, B571 \(1964\).](#)

Einstein equations

$$\partial_r C = 8\pi r^2 \tau_t / f$$

$$\partial_t C = 8\pi r^2 e^h \tau_t^r$$

$$\partial_r h = 4\pi r (\tau_t + \tau^r) / f^2,$$

Curvature scalars:

$$T := (\tau^r - \tau_t) / f$$

$$\mathfrak{T} := \left((\tau^r)^2 + (\tau_t)^2 - 2(\tau_t^r)^2 \right) / f^2$$

Effective EMT components:

$$\tau_t := e^{-2h} T_{tt}, \quad \tau_t^r := e^{-h} T_t^r, \quad \tau^r := T^{rr}$$

Solutions are characterised by scaling behaviour of EMT close to horizon: $\lim_{r \rightarrow r_g} \tau \sim \pm \Upsilon(t)^2 f(t,r)^k$



DR Terno, [Phys. Rev. D 101, 124053 \(2020\).](#)

SM, DR Terno, [Phys. Rev. D 103, 064082 \(2021\).](#)

Only two values of k are consistent: $k \in \{0, 1\}$

Both classes violate NEC near horizon.



Dynamic physical black hole solutions in spherical symmetry

SEBASTIAN MURK

PHYS. REV. D **105**, 044051 (2022)

	$k = 0$ solutions	$k = 1$ solution
Metric functions	$C = r_g - c_{12}\sqrt{x} + \sum_{j \geq 1}^{\infty} c_j x^j \quad (\text{k0.1})$ $h = -\frac{1}{2} \ln \frac{x}{\xi} + \sum_{j \geq \frac{1}{2}}^{\infty} h_j x^j \quad (\text{k0.2})$	$C = r_g + x - c_{32}x^{3/2} + \sum_{j \geq 2}^{\infty} c_j x^j \quad (\text{k1.1})$ $h = -\frac{3}{2} \ln \frac{x}{\xi} + \sum_{j \geq \frac{1}{2}}^{\infty} h_j x^j \quad (\text{k1.2})$
Leading coefficient	$c_{12} = 4\sqrt{\pi}r_g^{3/2}\Upsilon \quad (\text{k0.3})$	$c_{32} = 4r_g^{3/2}\sqrt{-\pi e_2/3} \quad (\text{k1.3})$
Horizon dynamics	$r'_g = \pm c_{12}\sqrt{\xi}/r_g \quad (\text{k0.4})$	$r'_g = \pm c_{32}\xi^{3/2}/r_g \quad (\text{k1.4})$

Describes black holes immediately after formation (and for the rest of their lifetime).

$$x := r - r_g$$

Describes formation of black holes.

Both violate the NEC near the horizon.

The formation of black holes follows a unique scenario that involves both classes of solutions!

The transition between them is continuous.

Details:



SM, DR Terno,
[Phys. Rev. D **103**, 064082 \(2021\).](#)



Surface gravity in stationary spacetimes

Hawking temperature: $T_H = \frac{\kappa}{2\pi}$ (for observer at infinity)

Several equivalent definitions, related to either

Inaffinity of null geodesics on the horizon:

$$\xi^\mu_{;\nu} \xi^\nu := \kappa \xi^\mu$$

Killing vector field

with norm $\sqrt{\xi^\mu \xi_\mu} = 0$

or

$$x := r - r_g$$

Peeling off properties of null geodesics near the horizon:

$$r \gtrsim r_g$$

$$\frac{dr}{dt} = \pm 2\kappa_{\text{peel}}(t)x + \mathcal{O}(x^2)$$



Surface gravity in dynamical spacetimes

In general dynamical spacetimes: no asymptotically timelike Killing vector.



H Kodama, [Prog. Theor. Phys. 63, 1217 \(1980\)](#).

G Abreu, M Visser, [Phys. Rev. D 82, 044027 \(2010\)](#).

F Kurpicz, N Pinamonti, R Verch, [Lett. Math. Phys. 111, 110 \(2021\)](#).

Role of Hawking temperature captured either by **peeling** or **Kodama surface gravity**.



C Barceló, S Liberati, S Sonego, M Visser,

[Phys. Rev. D 83, 041501\(R\) \(2011\)](#).

Indistinguishable for sufficiently slowly evolving horizons with properties close to their classical counterparts.

However: the similarity fails for dynamic spherically symmetric solutions!



RB Mann, SM, DR Terno,

[Phys. Rev. D 105, 124032 \(2022\)](#).



Surface gravity in dynamic spacetimes: peeling surface gravity



AB Nielsen, JH Yoon, [Class. Quantum Gravity 25, 085010 \(2008\)](#).

B Cropp, S Liberati, M Visser, [Class. Quantum Gravity 30, 125001 \(2013\)](#).

Consider **peeling surface gravity**:
$$\kappa_{\text{peel}} = \frac{e^{h(t,r_g)} (1 - C'(t, r_g))}{2r_g}$$

For example: k=0

$$C = r_g - c_{12}\sqrt{x} + \sum_{j \geq 1} c_j x^j$$

$$h = -\frac{1}{2} \ln \frac{x}{\xi} + \sum_{j \geq \frac{1}{2}} h_j x^j$$

With the metric functions C and h of the k=0 and k=1 solutions: $\kappa_{\text{peel}} \rightarrow \infty$ $\frac{dr}{dt} = \pm r'_g + a_{12}(t)\sqrt{x} + \mathcal{O}(x)$

Cf. stationary expression: $\frac{dr}{dt} = \pm 2\kappa_{\text{peel}}(t)x + \mathcal{O}(x^2)$



AB Nielsen, M Visser,

[Class. Quantum Gravity 23, 4637 \(2006\)](#).

Using Painlevé–Gullstrand coordinates (\bar{t}, r) : $\kappa_{\text{PG}_1} = \frac{1}{2r_g} (1 - \partial_r \bar{C}) \Big|_{r=r_g} \longrightarrow \kappa_{\text{PG}_1} = 0$



RB Mann, SM, DR Terno,

[Phys. Rev. D 105, 124032 \(2022\)](#).

$\kappa_{\text{PG}_2} = \frac{1}{2r_g} (1 - \partial_r \bar{C} + \partial_{\bar{t}} \bar{C}) \Big|_{r=r} \longrightarrow$ 3 possibilities (0, ∞ , finite) depending on behaviour of \bar{t}



Surface gravity in dynamic spacetimes: Kodama surface gravity

Defined via $\frac{1}{2} K^\mu (\nabla_\mu K_\nu - \nabla_\nu K_\mu) := \kappa_K K_\nu$ evaluated at horizon.

Kodama vector field: $K^\mu = (e^{-h_+}, 0, 0, 0)$ (v,r) coordinates

covariantly conserved: $\nabla_\mu K^\mu = 0,$

$\nabla_\mu J^\mu = 0, \quad J^\mu := G^{\mu\nu} K_\nu$

Result: $\kappa_K = \frac{1}{2} \left(\frac{C_+(v, r)}{r^2} - \frac{\partial_r C_+(v, r)}{r} \right) \Big|_{r=r_+} = \frac{(1 - w_1)}{2r_+}$

→ 0 at formation of black hole.

→ Approaches static value $\kappa = 1/(4M)$ only if metric is close to pure Vaidya metric.



RB Mann, SM, DR Terno, [Phys. Rev. D 105, 124032 \(2022\).](#)

→ **Contradicts semiclassical results.**



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1. Evaporation
2. Event horizon
3. Thermal character of the radiation

If semiclassical gravity is valid,
it is impossible to simultaneously realise all of the necessary elements
that are required for a self-consistent formulation of the information loss problem.

International Journal of Modern Physics D | Vol. 31, No. 09, 2230015 (2022) | Review Paper

Black holes and their horizons in semiclassical and modified theories of gravity

Robert B. Mann, Sebastian Murk and Daniel R. Terno

[arXiv:2112.06515 \[gr-qc\]](https://arxiv.org/abs/2112.06515)

Details:



RB Mann, SM, DR Terno, [Phys. Rev. D **105**, 124032 \(2022\)](https://doi.org/10.1103/PhysRevD.105.124032).

RB Mann, SM, DR Terno, [Int. J. Mod. Phys. D **31**, 2230015 \(2022\)](https://doi.org/10.1142/IntJModPhysD312230015).

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[Int. J. Mod. Phys. D 31, 2230015 \(2022\)](#)

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Abstract

For distant observers, black holes are trapped spacetime domains bounded by apparent horizons. We review properties of the near-horizon geometry emphasizing the consequences of two common implicit assumptions of semiclassical physics. The first is a consequence of the cosmic censorship conjecture, namely, that curvature scalars are finite at apparent horizons. The second is that horizons form in finite asymptotic time (i.e. according to distant observers), a property implicitly assumed in conventional descriptions of black hole formation and evaporation. Taking these as the only requirements within the semiclassical framework, we find that in spherical symmetry only two classes of dynamic solutions are admissible, both describing evaporating black holes and expanding white holes. We review their properties and present the implications. The null energy condition is violated in the vicinity of the outer horizon and satisfied in the vicinity of the inner apparent/anti-trapping horizon. Apparent and anti-trapping horizons are timelike surfaces of intermediately singular behavior, which manifests itself in negative energy density firewalls. These and other properties are also present in axially symmetric solutions. Different generalizations of surface gravity to dynamic spacetimes are discordant and do not match the semiclassical results. We conclude by discussing signatures of these models and implications for the identification of observed ultra-compact objects.

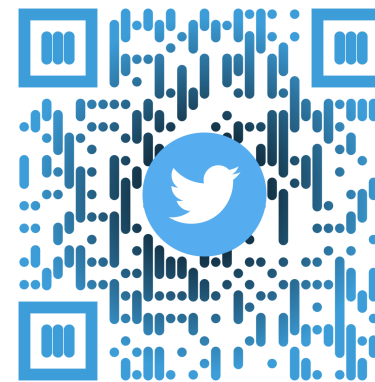
Keywords: Semiclassical gravity ■ modified gravity ■ black holes ■ apparent horizon ■ evaporation ■ white holes ■ energy conditions ■ thin shell collapse ■ surface gravity ■ information loss



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“an isolated black hole will evaporate completely via the Hawking process within a finite time. If the correlations between the inside and outside of the black hole are not restored during the evaporation process, then by the time that the black hole has evaporated completely, an initial pure state will have evolved to a mixed state, i.e., information will have been lost. In a semiclassical analysis of the evaporation process, such information loss does occur and is ascribable to the propagation of the quantum correlations into the singularity within the black hole.”



RM Wald,
[Living Rev. Relativ. 4, 6 \(2001\).](#)