Unruh-DeWitt Detectors with Relativistic Centre of Mass

(arXiv:2211.10562 [quant-ph])

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Relativistic quantum mechanics

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  - Can one formulate a “first-quantised” relativistic quantum mechanics?
  - What can the transition from relativistic to non-relativistic quantum mechanics tell us?
  - Is it possible to define localised states in relativistic quantum mechanics?

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$$\hat{H}_I = \lambda \, \hat{\mu} \otimes \hat{\phi}(\boldsymbol{x})$$

  Originally proposed by Unruh [1] and later simplified by DeWitt [2]

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- Ultraviolet divergences with monopole model → introduce a spatial profile for detector [3]

$$\hat{H}_I = \lambda \int d^3x\, F(\boldsymbol{x} - \boldsymbol{x}_D)\, \hat{\mu} \otimes \hat{\phi}(\boldsymbol{x})$$

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[3] S. Schlicht, Class. Quantum Grav. **21**, 4647 (2004); J. Louko and A. Satz, Class. Quantum Grav. **23**, 6321 (2006)

Quantised centre of mass

- Traditional UDW model: field is quantised, but detector follows a classical worldline
- Model previously extended for quantised centre of mass in the non-relativistic regime [4, 5]

$$\hat{H}_D = \frac{\hat{\boldsymbol{p}}^2}{2M} + E|e\rangle\langle e| \,, \qquad \hat{H}_I = \lambda \int d^3x \, \hat{\mu} \otimes |\boldsymbol{x}\rangle\langle\boldsymbol{x}| \otimes \hat{\phi}(\boldsymbol{x})$$

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- Can calculate the transition rate perturbatively for some particular physical processes, such as spontaneous emission or absorption

$$\dot{P}[\psi_i] = \frac{\lambda^2}{2\pi} \int d^3p\, |\psi_i(\boldsymbol{p})|^2\, \mathcal{T}(\boldsymbol{p})$$

where the transition rate is a functional of the initial wavefunction ψ_i and a “template function” $\mathcal{T}$

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Relativistic quantised centre of mass

- Non-relativistic model should be extended:
  - Does not fully account for the relativistic dynamics of the detector
  - Mixing Lorentz and Galilean symmetries leads to spurious velocity-dependent effects [6, 7]

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- Choice between first- or second-quantised approach to modelling relativistic dynamics

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First- vs second-quantised models

- First-quantised model: $\hat{H}_I^{(1\text{st})} = \lambda_1 \int d^3x\, \hat{\mu} \otimes |\boldsymbol{x}\rangle\langle\boldsymbol{x}| \otimes \hat{\phi}(\boldsymbol{x})$
  - Free Hamiltonian: $\hat{H}_D = \sqrt{\hat{\boldsymbol{p}}^2 + \hat{M}^2}$
  - Mass-energy operator: $\hat{M} = M_g|g\rangle\langle g| + M_e|e\rangle\langle e| = m|g\rangle\langle g| + (m+E)|e\rangle\langle e|$

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- Second-quantised model, restrict detector to one-particle sector [9]:

$$\hat{H}_I^{(2\mathrm{nd})} = \lambda_2 \int d^3x \sum_{j\neq k} \hat{\psi}_j(\boldsymbol{x})\hat{\psi}_k(\boldsymbol{x}) \otimes \hat{\phi}(\boldsymbol{x}) \longrightarrow \hat{H}_I^{(2\mathrm{nd})}\Big|_{\mathcal{H}_1^D} = \lambda_2 \int d^3x \sum_{j\neq k} |\boldsymbol{x}_j\rangle\langle\boldsymbol{x}_k| \otimes \hat{\phi}(\boldsymbol{x})$$

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- Obtain different localisations [10]:

$$|\boldsymbol{x}\rangle_D{}^{(1\mathrm{st})} = \frac{1}{(2\pi)^{3/2}} \int d^3p \, e^{-i\,\boldsymbol{p}\cdot\boldsymbol{x}} \, |\boldsymbol{p}\rangle_D,$$

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Calculation of spontaneous emission rate

- Derive first-order perturbation in spontaneous emission rate

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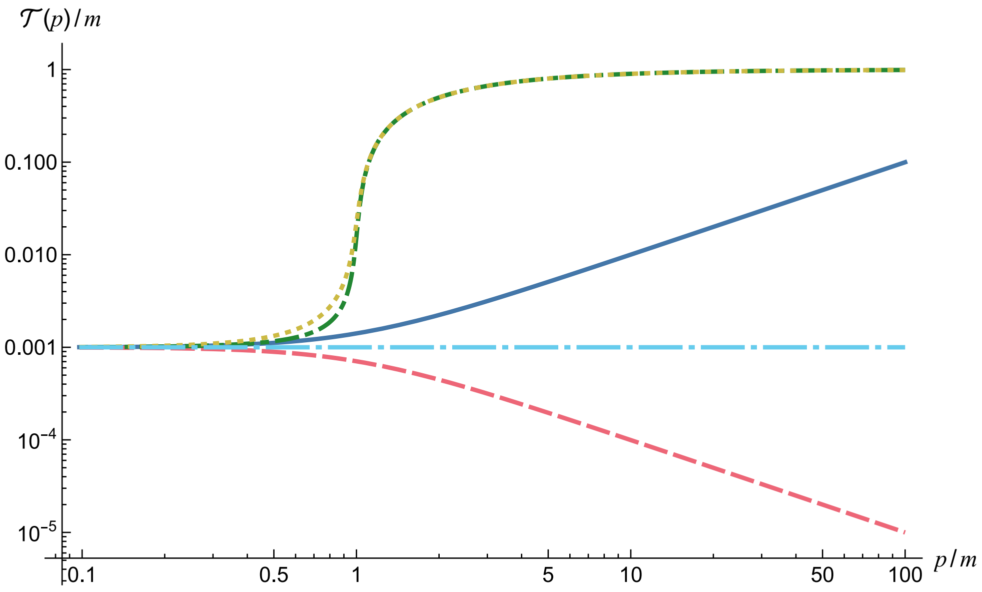
- In a vacuum, the template functions for the first- and second-quantised cases are

$$\mathcal{T}^{(1\text{st})}(\boldsymbol{p}) = \frac{1}{2}\left(1 - \frac{M_g^4}{M_e^4}\right)\sqrt{M_e^2 + \boldsymbol{p}^2}$$

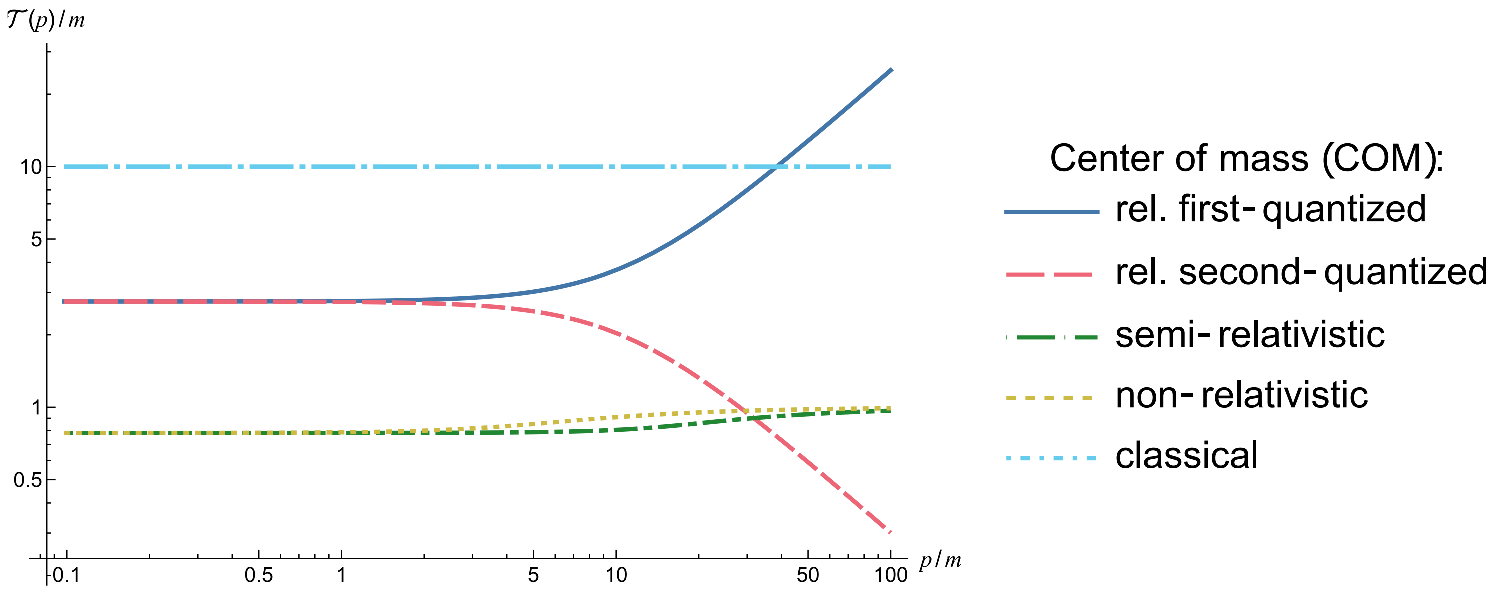
$$\mathcal{T}^{(2\text{nd})}(\boldsymbol{p}) = \frac{1}{2}\left(1 - \frac{M_g^2}{M_e^2}\right)\frac{1}{\sqrt{M_e^2 + \boldsymbol{p}^2}}$$

$$\lambda_1 = \sqrt{2\left(M_g^2 + M_e^2\right)}\,\lambda_2$$

Template functions for first- and second-quantised localisations in a vacuum



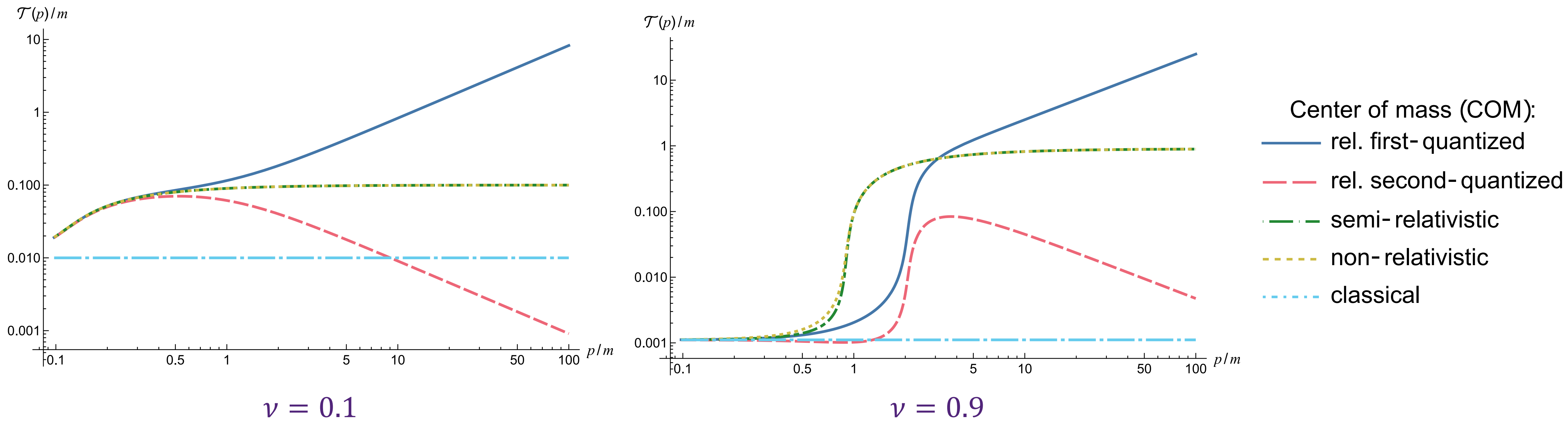
$E/m = 0.001$



$E/m = 10$

where m is the rest mass, i.e. $M_g \equiv m$ and $M_e \equiv m + E$.

Template functions for first- and second-quantised localisations in a medium



where ν is the propagation speed of the field $\hat{\phi}$ and $E/m = 0.001$.

Spontaneous emission rates for first- and second-quantised localisations in a vacuum

Consider detector initially in a Gaussian state

$$\dot{P}_{|e,0\rangle\to|g,1\rangle}[\psi_i] = \frac{\lambda^2}{2\pi}\int d^3p\,|\psi_i(\boldsymbol{p})|^2\mathcal{T}(\boldsymbol{p}) \quad \text{with} \quad \psi_i(\boldsymbol{p}) = \left(\frac{L^2}{2\pi}\right)^{3/4} e^{-\frac{L^2}{4}\boldsymbol{p}^2}$$

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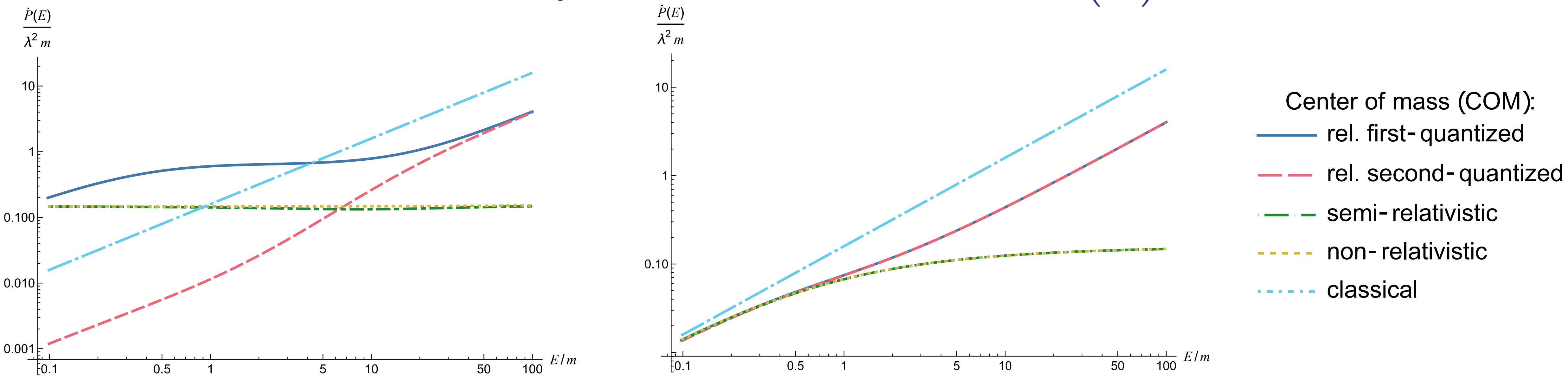
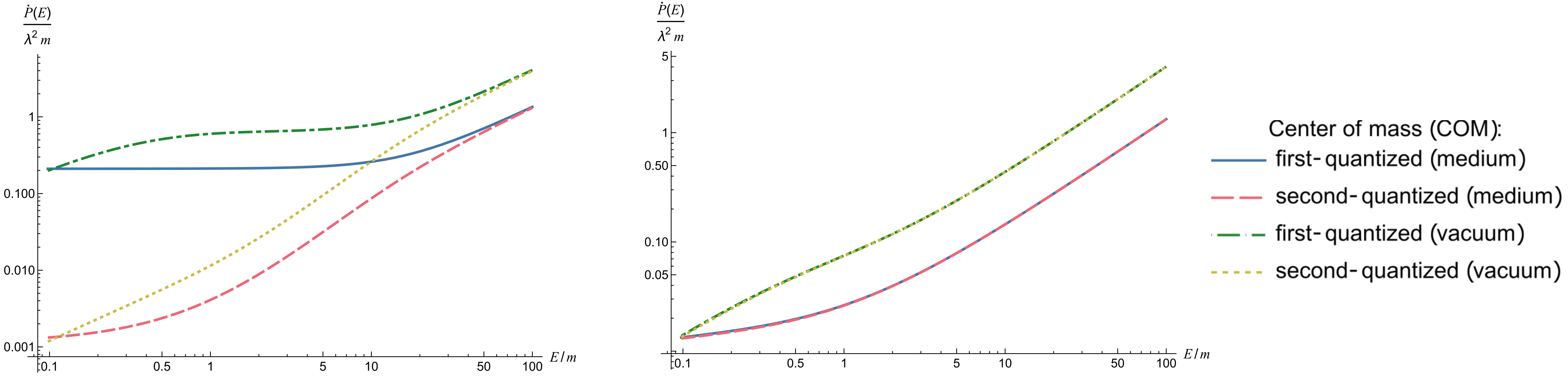


$L/\lambda_c = 0.1$ $\qquad$ $L/\lambda_c = 10$

where $\lambda_c \equiv m^{-1}$ is the Compton wavelength of the detector.

Spontaneous emission rates for a medium ($v = 0.1$) and vacuum ($v = c = 1$)



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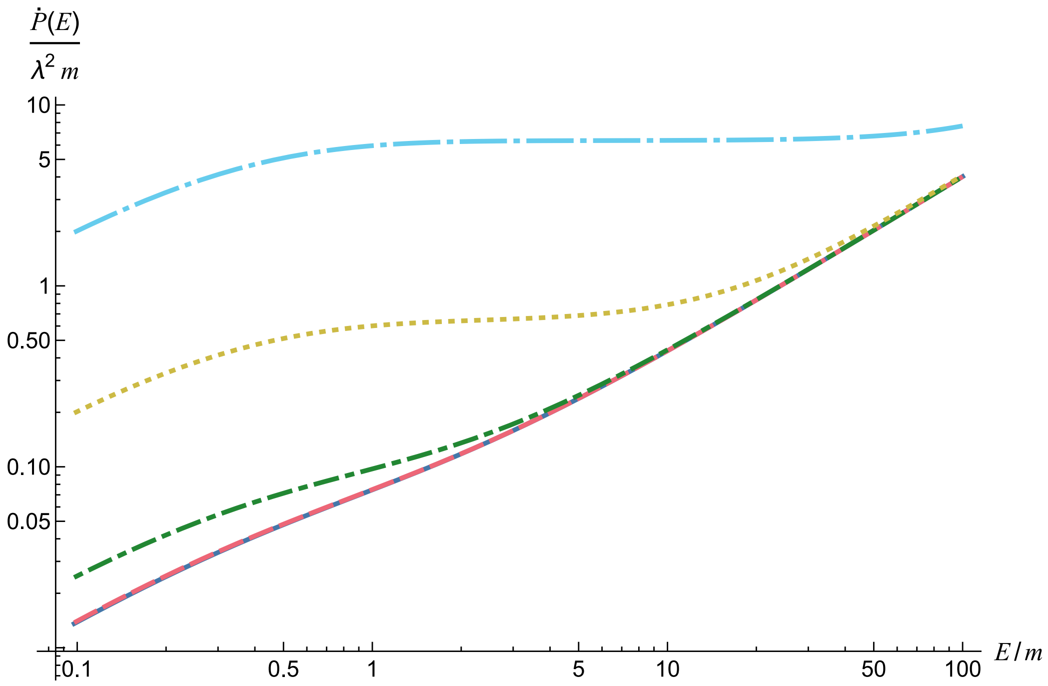
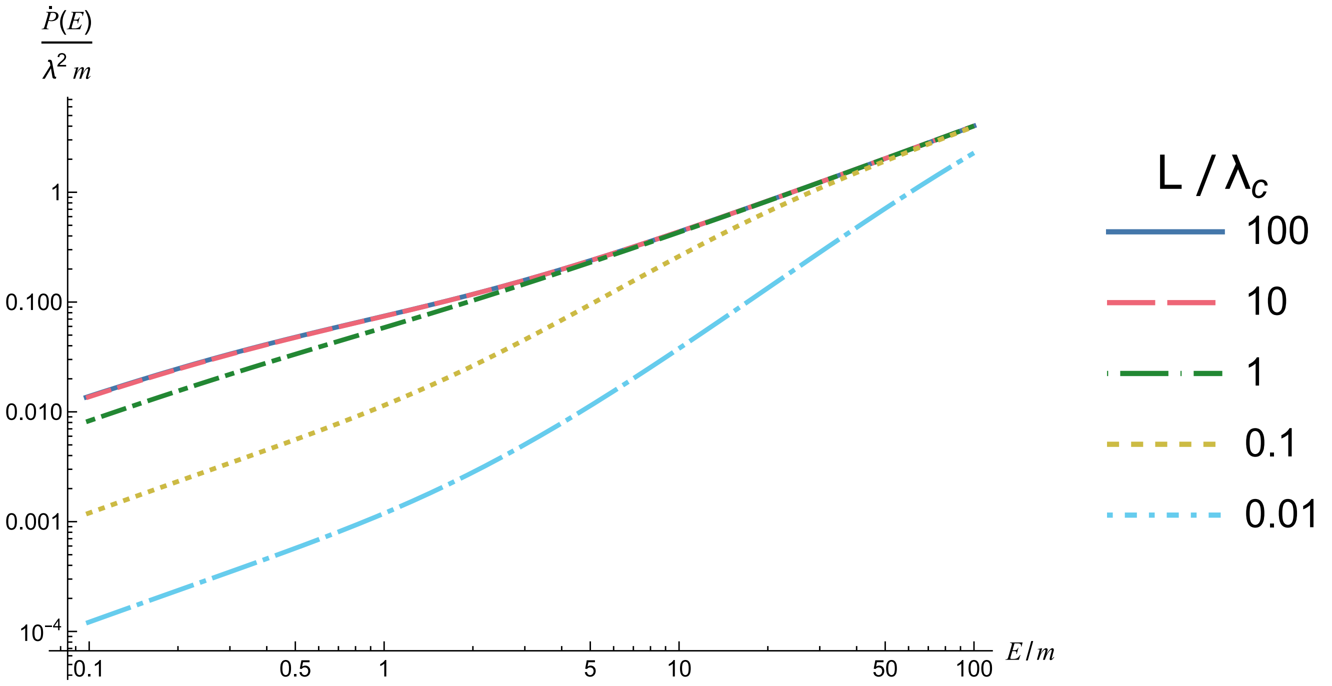
where $\lambda_c \equiv m^{-1}$ is the Compton wavelength of the detector.

Comparison of first- and second-quantised localisations in a vacuum



first-quantised



second-quantised

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  - Superpositions of Gaussians (position-momentum minimum uncertainty states)

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  - Consider alternate minimum uncertainty states, e.g. between position and velocity [11, 12]

$$(\Delta x)^2(\Delta v)^2 - (\Delta xv)^2 \geq \frac{1}{4}|\langle[\hat{x},\hat{v}]\rangle|^2, \quad \text{where} \quad \hat{v} = \frac{1}{i}\left[\hat{x},\hat{H}\right] = \hat{p}\hat{H}^{-1}$$

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- Extend formalism to include non-inertial detectors and curved spacetimes

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- Extend formalism to include non-inertial detectors and curved spacetimes
- Further investigate physical meaning of the first- and second-quantised localisations (alongside the Foldy-Wouthuysen transformation [13])

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[13] L. Foldy and S. Wouthuysen, Phys. Rev. **78**, 29-36 (1950)

Summary

- Relativistic quantum mechanical models of inertial UDW detectors
- Analytic results easily obtainable in a vacuum; numerical results in a medium
- Notable disagreement between first- and second-quantised models due to different detector localisations
- In principle, disagreement testable for detectors with large mean momenta

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