Bandlimited quantum fields and their continuous and discrete properties

Dominic Lewis and Nicolas C. Menicucci

Center for Quantum Computation and Communication Technology, School of Science, RMIT University, Melbourne, Victoria 3001, Australia

One of the biggest paradoxes that comes from unifying quantum field theory with Einstein's theory of general relativity is one of continuity. The mathematics of general relativity requires a smooth manifold, a structure that is continuous and differentiable at all scales. Meanwhile, our most successful quantum field theories break down and give nonsensical results when treated at very small scales and thus are used under the assumption that these quantum fields are effectively discrete at the planck scale. In order to possibly unite these two theories effectively, it would be convenient to be able to write a quantum field theory as simultaneously continuous *and* discrete. Bandlimited quantum field theory allows us to do just that. By treating quantum fields as bandlimited and applying Shannon's sampling theorem [1], we are able to take continuous quantum field theories and express them as discrete objects without changing the physics of the field. Equivalently, we can take discrete quantum field theories on a lattice and describe them as samples of underlying continuous bandlimited fields [2, 3].

We apply Shannon's sampling theorem to continuous and discrete quantum fields to determine the continuous and discrete Hamiltonians of each field. Additionally, we show that discrete fields can possess continuous translational symmetry and all of the properties that come with such symmetry such as a generator of translations.

[1] C. E. Shannon, "A mathematical theory of communication," *The Bell system technical journal*, vol. 27, no. 3, pp. 379–423, 1948.

[2] A. Kempf, "Fields over unsharp coordinates," *Phys. Rev. Lett.*, vol. 85, pp. 2873–2876, Oct 2000. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.85.2873

[3] J. Pye, W. Donnelly, and A. Kempf, "Locality and entanglement in bandlimited quantum field theory," *Physical Review D*, vol. 92, no. 10, p. 105022, 2015.