Symmetry in bandlimited quantum field theory

Dominic Lewis

Supervisor: Nicolas C. Menicucci

Collaborator: Achim Kempf

RMIT UNIVERSITY

QuRMIT

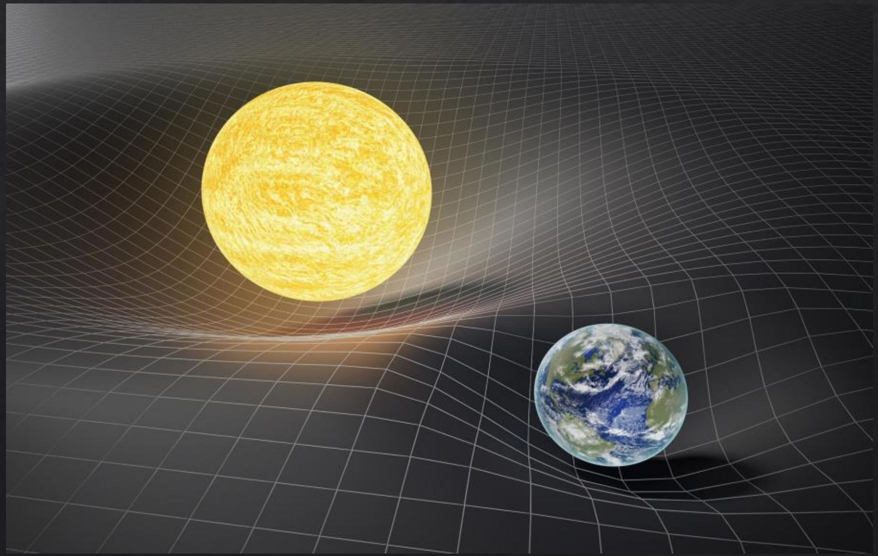
General relativity and QFT: like oil and water

QFT:
- Continuous theory at larger scales [1]
- Can break down at high energy [1, 2]
- Assumed discreteness at Planck scale [3]

https://www.dreamstime.com/royalty-free-stock-images-rough-fabric-texture-background-image18454809

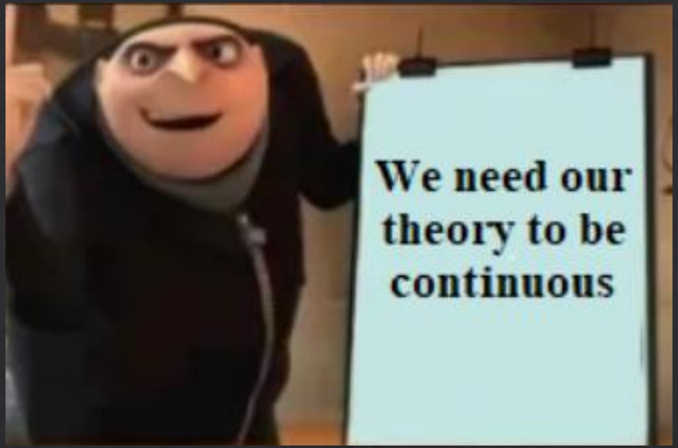
[1] M. E. Peskin, *An introduction to quantum field theory*. CRC press, 2018.
[2] J. Polchinski, "Renormalization and effective lagrangians," Nuclear Physics B, vol. 231, no. 2, pp. 269–295, 1984.
[3] J. Pye, W. Donnelly, and A. Kempf, "Locality and entanglement in bandlimited quantum field theory," Physical Review D, vol. 92, no. 10, p. 105022, 2015.

GR:
- Mathematics requires a 'smooth manifold' [3]
- Enforces continuity at ALL scales
- Enforces differentiability at ALL scales [3, 4]



https://scitechdaily.com/new-atomic-clocks-measure-time-dilation-of-einsteins-general-relativity-at-millimeter-scale/

[4] J. A. Wheeler, "On the nature of quantum geometrodynamics," Annals of Physics, vol. 2, no. 6, pp. 604–614, 1957


We need our theory to be continuous


We need our theory to be continuous
We need our theory to be discrete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This is a paradox


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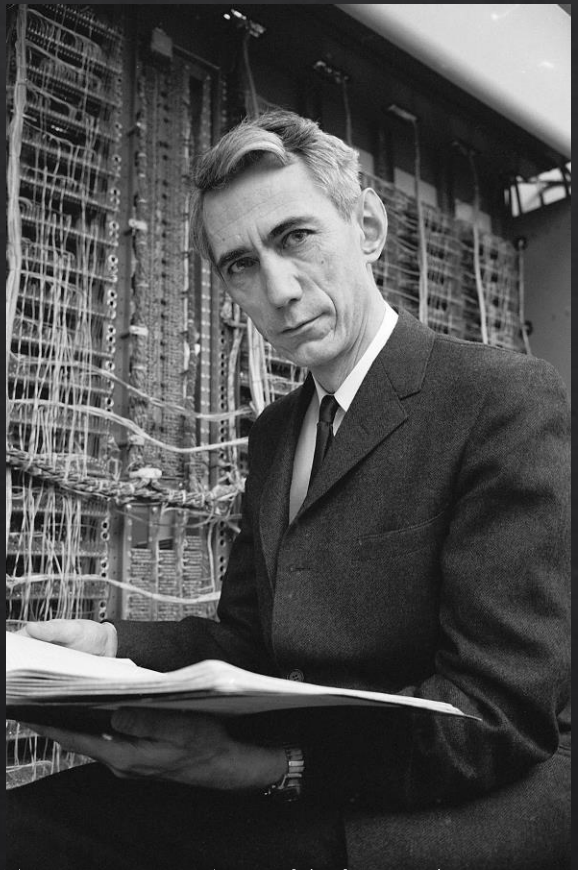
We need our theory to be continuous
We need our theory to be discrete
This is a paradox
This is a paradox
We can fix it with bandlimitation!

Bandlimitation, Shannon reconstruction, and bandlimited quantum fields



Claude Shannon – Father of information processing

Image: https://www.newyorker.com/tech/annals-of-technology/claude-shannon-the-father-of-the-information-age-turns-1100100

[5] C. E. Shannon, “A mathematical theory of communication,” The Bell system technical journal, vol. 27, no. 3, pp. 379–423, 1948.
[6] A. Kempf, “Fields over unsharp coordinates,” Phys. Rev. Lett., vol. 85, pp. 2873–2876, Oct 2000.

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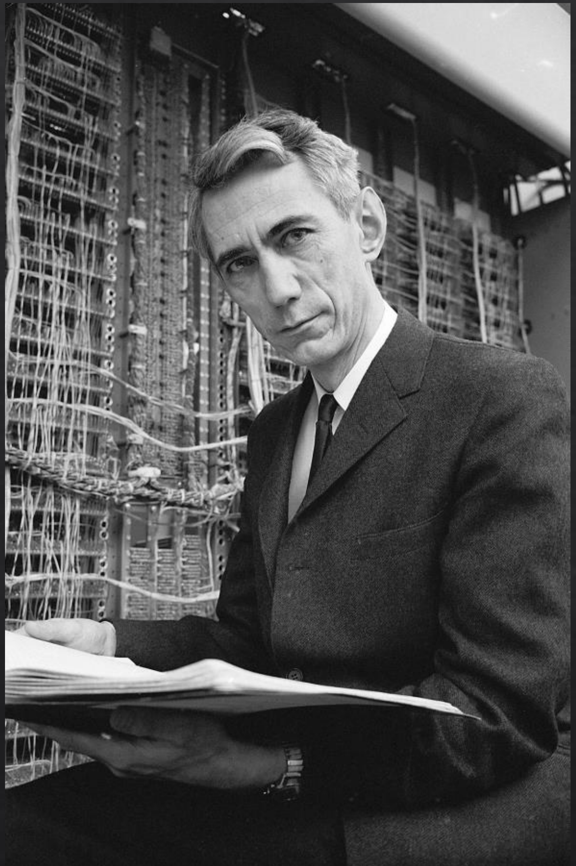

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Bandlimited function :

$$f(x) = \int_{-\Omega}^{\Omega} \frac{dk}{2\pi} \tilde{f}(k) e^{ikx}.$$

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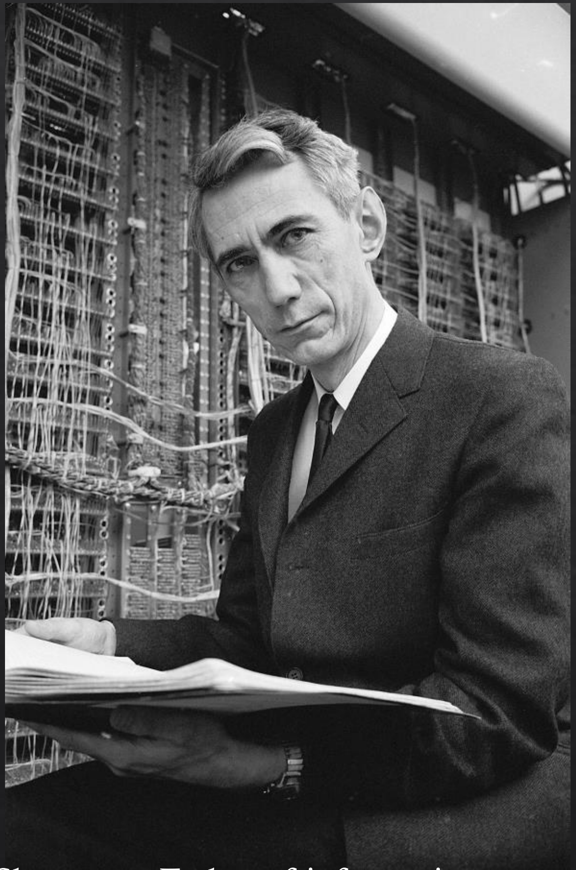

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Shannon reconstruction [5]:

$$f(x) = \sum_{j\in\mathbb{Z}} f(x_j) \mathrm{sinc}_{\pi}\left(\frac{x-x_j}{\Delta x}\right),$$

$\Delta x = \frac{\pi}{\Omega}$ ← Lattice spacing (Minimum length)

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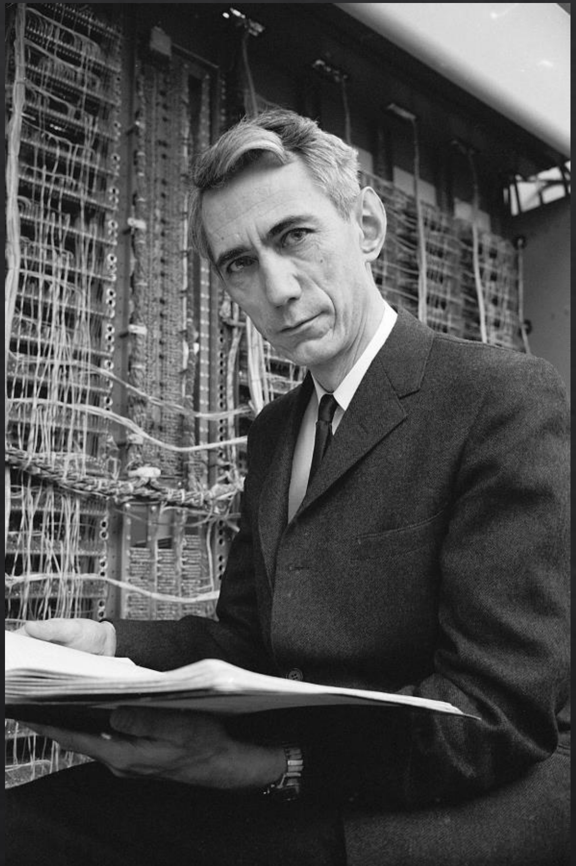

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Apply this to QFTs [6].

$$\hat{\phi}(x) = \int_{-\Omega}^{\Omega} \frac{dk}{2\pi} \frac{1}{\sqrt{2\omega_k}} \left(\hat{a}_k e^{ikx} + \hat{a}_k^{\dagger} e^{-ikx}\right) = \sum_{j\in\mathbb{Z}} \hat{\phi}(x_j) \mathrm{sinc}_{\pi}\left(\frac{x - x_j}{\Delta x}\right).$$

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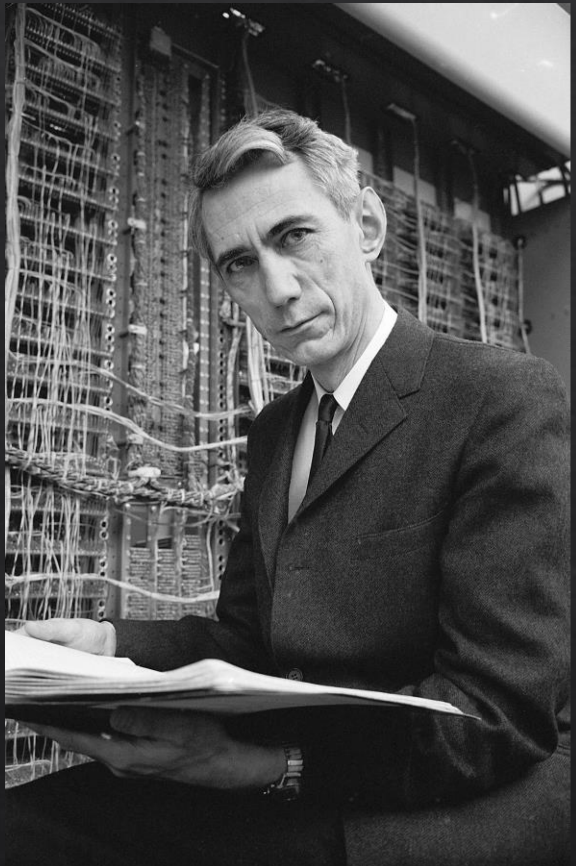

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Continuous field | Bandlimited in momentum | Equivalent discrete field

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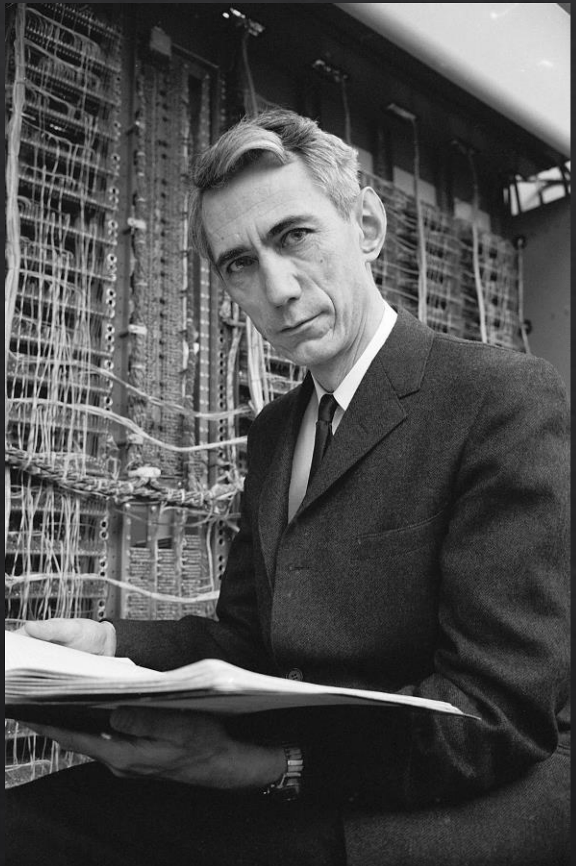
Bandlimitation, Shannon reconstruction, and bandlimited quantum fields



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Continuous field — Bandlimited in momentum — Equivalent discrete field

THIS IS A PERFECT EQUALITY!

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Bandlimited continuous theory
Discrete theory
Corporate needs you to find the difference between this picture and this picture
They're the same picture

Our work

Quantum lattice models that preserve continuous translation symmetry

Dominic G. Lewis,[1,*] Achim Kempf,[2,3] and Nicolas C. Menicucci[1,†]
[1] *Center for Quantum Computation and Communication Technology, School of Science, RMIT University, Melbourne, Victoria 3000, Australia*
[2] *Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada*
[3] *Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada*

RMIT UNIVERSITY

Symmetry, Interactions, and Correlations in Bandlimited Quantum Field Theory

A thesis submitted in fulfilment of the requirements for the degree of Bachelor of Science (Physics) (Honours)

Dominic Graham Lewis

Bachelor of Science (Physics) - RMIT University

- We introduce an ultraviolet cut-off to the momenta of a Klein-Gordon field
- We treat a discrete Harmonic chain field as samples of a continuous bandlimited field
- We investigate the continuous symmetry properties that lattice fields may possess
- We investigate the effect that a cut-off has on the nature of $\hat{\phi}^4$-interactions

Nicolas C. Menicucci

Achim Kempf

Key mathematics of bandlimited functions and fields

Shannon's sampling theorem:

$$f(x) = \sum_{j \in \mathbb{Z}} f(x_j)\,\mathrm{sinc}\left(\pi \frac{x - x_j}{\Delta x}\right)$$

Bandlimited first derivative:

$$\frac{\partial}{\partial x} f(x) = \frac{-1}{\Delta x} \sum_{m \neq 0} \frac{(-1)^m}{m} f(x + m\Delta x)$$

Bandlimited second derivative:

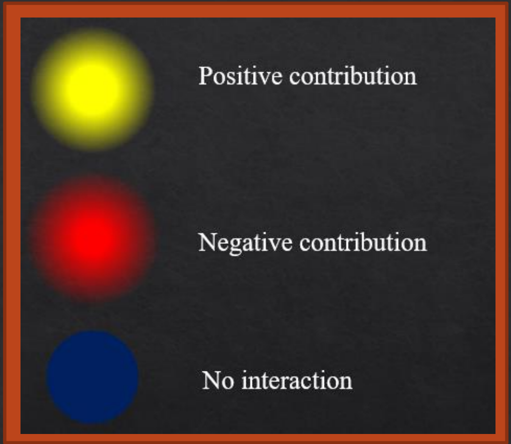
$$\frac{\partial^2}{\partial x^2} f(x) = -\left(\frac{\pi^3}{3} f(x) + \frac{2}{\Delta x^2} \sum_{m \neq 0} \frac{(-1)^m}{m^2} f(x + m\Delta x\right)$$

Outer product:

$$\int_{-\infty}^{\infty} f(x) g(x) dx = \Delta x \sum_{j \in \mathbb{Z}} f(x_j) g(x_j)$$

These are **perfect** equalities!

A look at field Hamiltonians

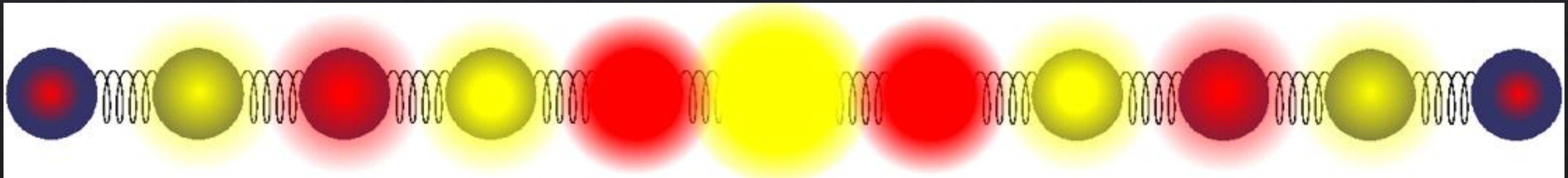


Bandlimited Continuous Klein-Gordon Hamiltonian:

$$\hat{H}_{KG} = \frac{1}{2}\int_{\mathbb{R}} dx\left[\hat{\pi}^2(x) + (\nabla\hat{\phi}(x))^2 + m^2\hat{\phi}^2(x)\right] = \frac{1}{2\Delta x}\sum_j \left[\hat{p}_j^2 + \left(\frac{\pi^2}{3} + \Delta x^2 m^2\right)\hat{q}_j^2 + \sum_{m\neq 0}\frac{2(-1)^m}{m^2}\hat{q}_j\hat{q}_{j+m}\right]$$

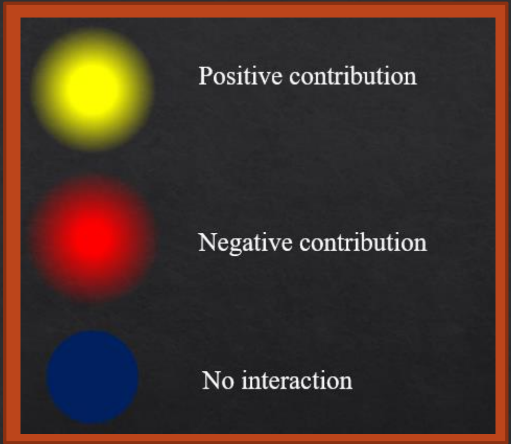
Continuous and Bandlimited

Discrete Equivalent



Position contribution: $(\nabla\hat{\phi}(x))^2 = \frac{\pi^2}{3}\hat{q}_j^2 + \sum_{m\neq 0}\frac{2(-1)^m}{m^2}\hat{q}_j\hat{q}_{j+m}$

A look at field Hamiltonians

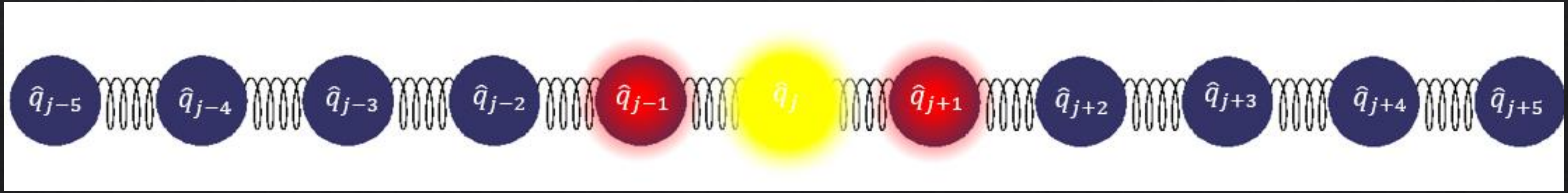


Bandlimited Discrete Harmonic Chain Hamiltonian:

$$\hat{H}_{HC} = \frac{1}{2}\sum_j \left[\frac{\hat{p}_j^2}{m} + K\left(\hat{q}_j^2 - \frac{1}{2}\hat{q}_{j+1}\hat{q}_j - \frac{1}{2}\hat{q}_j\hat{q}_{j-1}\right)\right] = \frac{1}{2}\int_{\mathbb{R}} dx \left[\frac{\Delta x}{m}\hat{\pi}^2(x) + \frac{K}{\Delta x}\left(\hat{\phi}(x+\Delta x) - \hat{\phi}(x)\right)^2\right]$$

Discrete

Continuous and Bandlimited



Bandlimited symmetry: translational invariance

Hamiltonian for a discrete harmonic chain:

$$\hat{H}_{HC} = \frac{1}{2}\sum_{j\in\mathbb{Z}}\left[\frac{\hat{p}_j^2}{m} + K\left(\hat{q}_{j+1}-\hat{q}_j\right)^2\right] = \frac{1}{2}\int_{\mathbb{R}}\left[\frac{\hat{\pi}^2(x)\Delta x}{m} + \frac{K}{\Delta x}(\hat{\phi}(x+\Delta x)-\hat{\phi}(x))^2\right]dx.$$

Discrete translational symmetry

Continuous translational symmetry

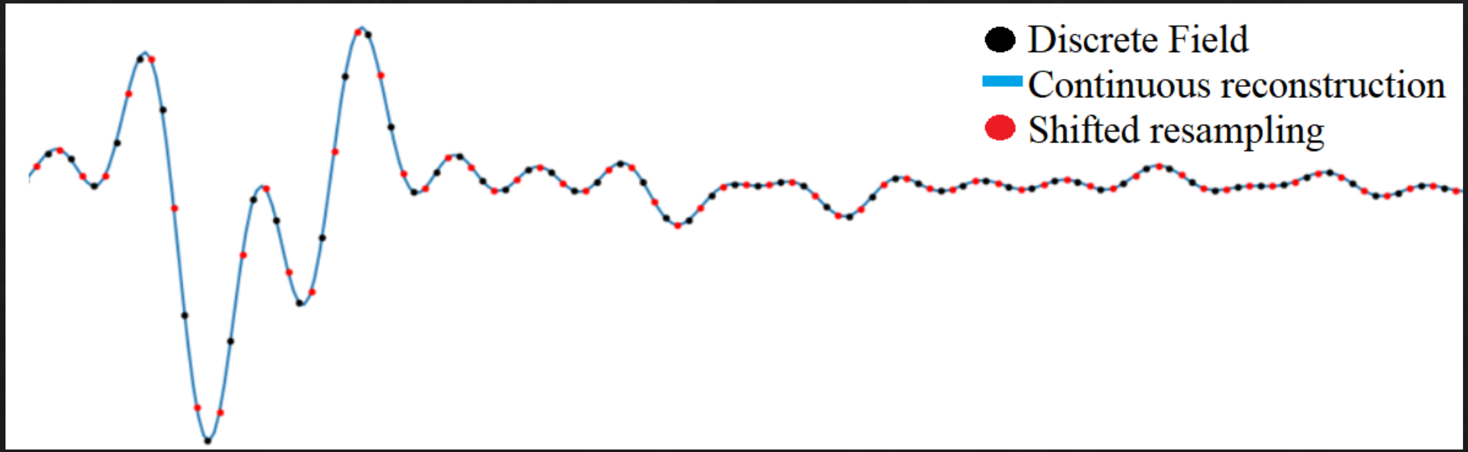
Shannon sampling:
$\hat{\phi}(x) = \sum_{j\in\mathbb{Z}}\hat{\phi}(x_j)\mathrm{sinc}_\pi\left(\frac{x-x_j}{\Delta x}\right)$

Through Shannon sampling and reconstruction the two forms are equivalent.
If one possesses fully continuous translational symmetry, so must the other!

Bandlimited symmetry: total momentum and translations

Total momentum operator: $\hat{P} = \int_{-\infty}^{\infty} \hat{\pi}(x) \frac{\partial}{\partial x} \hat{\phi}(x) dx$

Continuous operator in QFT



Bandlimited symmetry: total momentum and translations

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Continuous operator in QFT

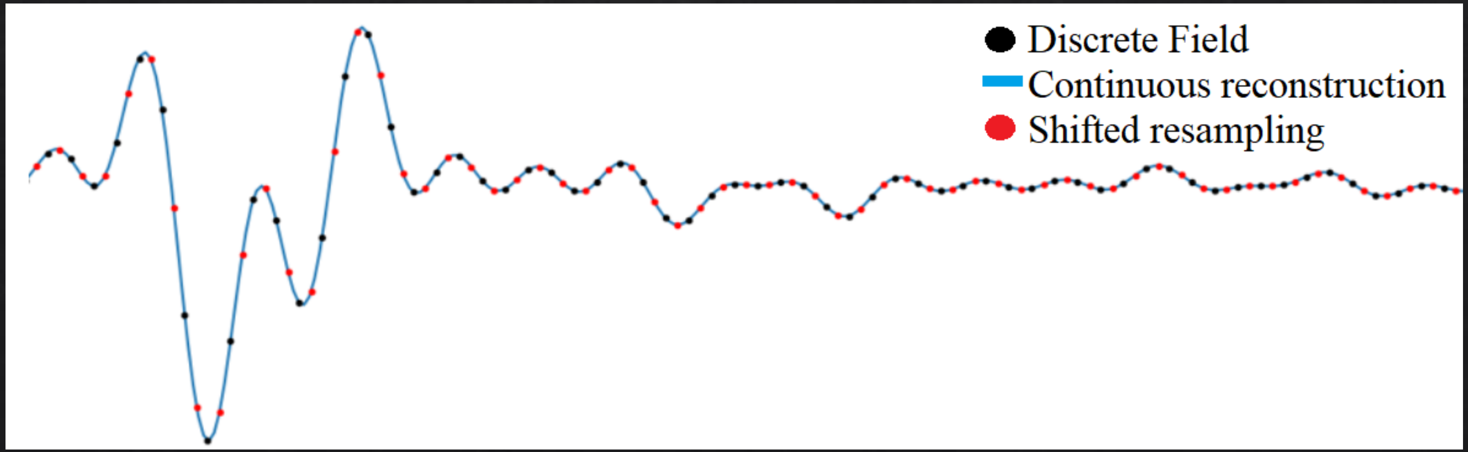
Shannon sampling:

$$\hat{\phi}(x) = \sum_{j \in \mathbb{Z}} \hat{\phi}(x_j) \mathrm{sinc}_{\pi}\left(\frac{x - x_j}{\Delta x}\right)$$

Bandlimited derivative:

$$\frac{d}{dx}\hat{\phi}(x) = \sum_{j \in \mathbb{Z}} \hat{\phi}(x_j) \frac{d}{dx} \mathrm{sinc}_{\pi}\left(\frac{x - x_j}{\Delta x}\right)$$

$$= \sum_{m \neq 0} \frac{-(-1)^m}{m \Delta x} \hat{\phi}(x_{j+m})$$



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Continuous operator
in QFT

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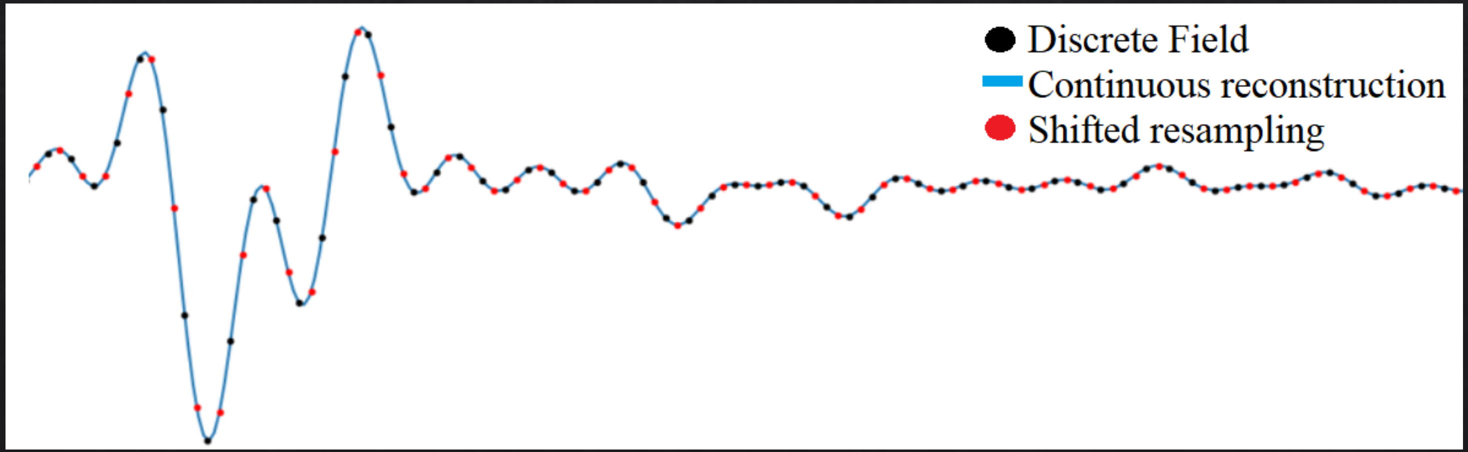
$= \sum_{m\neq 0} \frac{-(-1)^m}{m\Delta x} \hat{\phi}(x_{j+m})$



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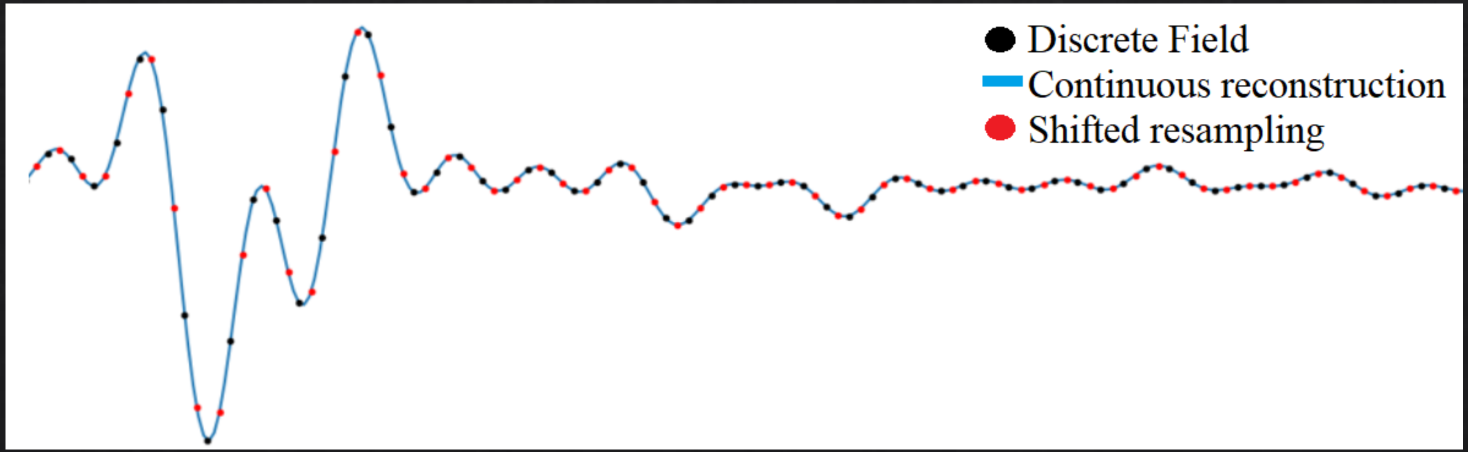

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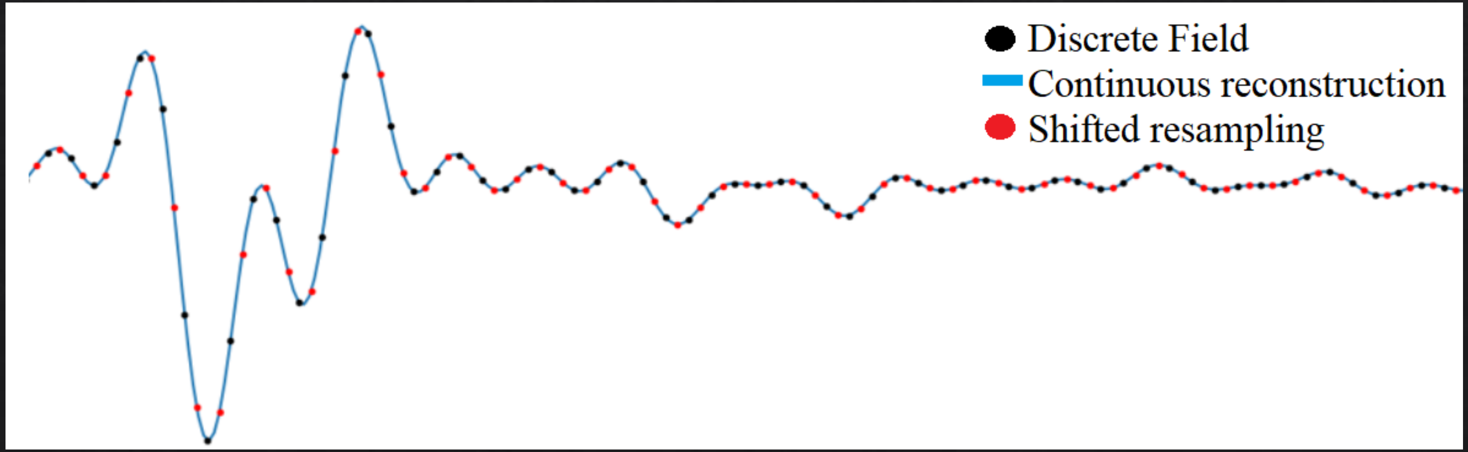
Continuous operator in QFT

Equivalent discrete operator

If a field is translationally symmetric, $\hat{P}$ produces continuous translations, even if the field is on a lattice!

$$e^{\frac{i}{\hbar}\hat{P}y} \hat{\phi}(x_j) e^{-\frac{i}{\hbar}\hat{P}y} = \hat{\phi}(x_j + y)$$



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Interpreting continuous translations

$\hat{\phi}(x_j)$

Dog image: https://auspost.com.au/about-us/corporate-responsibility/our-people/safety/dog-safety

Interpreting continuous translations

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Continuously shift the image behind the fence to the side by distance ‘a’

Dog image: https://auspost.com.au/about-us/corporate-responsibility/our-people/safety/dog-safety

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Continuously shift the image behind the fence to the side by distance ‘a’

$\hat{\phi}(x_j + a)$

Dog image: https://auspost.com.au/about-us/corporate-responsibility/our-people/safety/dog-safety

Interpreting continuous translations

$\hat{\phi}(x_j)$

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Continuously shift the image behind the fence to the side by distance ‘a’

$$\hat{\phi}(x + a) = \sum_{j\in\mathbb{Z}} \hat{\phi}(x_j + a)\mathrm{sinc}_\pi\left(\frac{x - x_j}{\Delta x}\right)$$

$\hat{\phi}(x_j + a)$

Dog image: https://auspost.com.au/about-us/corporate-responsibility/our-people/safety/dog-safety

Bandlimited interactions

$$\hat{H} = \frac{1}{2}\int_{\mathbb{R}} dx \left[\underbrace{\hat{\pi}^2(x) + \left(\nabla\hat{\phi}(x)\right)^2 + m^2\hat{\phi}^2(x)}_{\text{Free field}} + \underbrace{\frac{\lambda}{4!}\hat{\phi}^4(x)}_{\text{Interaction term}}\right]$$

Interaction term allows:

- Creation of new particles
- Destruction of existing particles
- Collisions/Scattering [7, 8, 9]

Shannon sampling:
$\hat{\phi}(x) = \sum_{j\in\mathbb{Z}} \hat{\phi}(x_j)\mathrm{sinc}_{\pi}\left(\frac{x-x_j}{\Delta x}\right)$

Q. What does this look like in a discrete representation?

$$\hat{H} = \frac{\Delta x}{2}\sum_{j\in\mathbb{Z}}\Bigg[\hat{\pi}^2(x_j) - \hat{\phi}(x_j)\Bigg(-\frac{\pi^2}{3\Delta x^2}\hat{\phi}(x_j) - \frac{2}{\Delta x^2}\sum_{m\neq 0}\frac{(-1)^m}{m^2}\hat{\phi}(x_{j+m})\Bigg) + m^2\hat{\phi}^2(x_j)$$

$$+ \frac{1}{\pi^2}\frac{\lambda}{4!}\hat{\phi}^2(x_j)\left[\frac{2\pi^2}{3}\hat{\phi}^2(x_j)\sum_{m\neq 0}\left(\frac{6}{m^2}\hat{\phi}^2(x_{j+m}) - \frac{2(-1)^m}{m^2}\hat{\phi}(x_j)\hat{\phi}(x_{j+m}) + \sum_{n\neq 0, n\neq m}\frac{6(-1)^{m-n}}{mn}\hat{\phi}(x_{j+m})\hat{\phi}(x_{j+n})\right)\right]\Bigg]$$

Free field (first line) — Interaction term (second line)

[7] K. Huang, *Quantum field theory: From operators to path integrals*. John Wiley & Sons, 2010.
[8] I. Montvay and G. M¨unster, *Quantum fields on a lattice*. Cambridge University Press, 1994.
[9] A. e. Izergin and V. Korepin, "Lattice versions of quantum field theory models in two dimensions," Nuclear Physics B, vol. 205, no. 3, pp. 401–413, 1982.

Bandlimited interactions: momentum representation

$$\hat{H} = \underbrace{\int_{|k|\leq\Omega} \frac{dk}{2\pi}\omega_k \hat{a}_k^\dagger \hat{a}_k}_{\text{Free field}} + \underbrace{\frac{1}{2}\frac{\lambda}{4!}\frac{1}{8\pi^3}\prod_{j=1}^{4}\left[\int_{|k_j|\leq\Omega}\frac{dk}{\sqrt{2\omega_{k_j}}}\left(\hat{a}_{k_j}+\hat{a}^\dagger_{-k_j}\right)\right]\delta(k_1+k_2+k_3+k_4)}_{\text{Interaction term}}$$

Free field

Interaction term

Bandlimit: $|k| < \Omega$

Specific interaction:

$$\int_{|k_1|\leq\Omega}\int_{|k_2|\leq\Omega}\int_{|k_3|\leq\Omega}\int_{|k_4|\leq\Omega}\frac{dk_1 dk_2 dk_3 dk_4}{4\sqrt{\omega_{k_j}}}\hat{a}^\dagger_{-k_1}\hat{a}_{k_2}\hat{a}_{k_3}\hat{a}_{k_4}\delta(k_1+k_2+k_3+k_4)$$

Two conditions:

- Momentum conservation: $k_1 = -(k_2+k_3+k_4)$
- UV cut-off bandlimit: $|k_1| = |-(k_2+k_3+k_4)| < \Omega$

Consequences:

- Particles with high momentum are unlikely to interact
- These particles become ‘transparent’ to interactions

If these conditions are not met then the interaction WILL NOT OCCUR!

Summary

- Through bandlimitation, discrete and continuous fields become equivalent.

- Well known continuous fields in continuous QFT are difficult to interpret in a lattice representation

- Discrete fields *gain* continuous symmetry!

- Interactions of particles with momentum well below the cut-off are unaffected by bandlimitation

- Particles with momentum close to or at the cut-off become transparent to interactions

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