Physical black holes

in modified theories of gravity

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References

SM, DR Terno
Phys. Rev. D **104**, 064048 (2021)

arXiv: [arXiv:2012.11209 [gr-qc]]

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Spherically symmetric black holes in metric gravity

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SM
Phys. Rev. D **105**, 044051 (2022)

arXiv: [arXiv:2110.14973 [gr-qc]]

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Physical black holes in fourth-order gravity

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arXiv:2111.00776 [gr-qc] (2021)

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Constraining modified gravity theories with physical black holes

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Einstein equations in GR and semiclassical/modified gravity

General relativity: $$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

JF Donoghue, Phys. Rev. Lett. **72**, 2996 (1994)
JF Donoghue, Phys. Rev. D **50**, 3874 (1994)
JF Donoghue, arXiv:gr-qc/9512024 (1995)

How can we account for quantum (gravitational) effects?

$T_{\mu\nu} := \langle \hat{T}_{\mu\nu} \rangle_\psi$

Expectation value of renormalized energy-momentum tensor in quantum state ψ.

[1] **Semiclassical gravity:** $$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

[2] **Modified gravity:** $$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda\mathcal{E}_{\mu\nu} = 8\pi T_{\mu\nu}$$

Accounts for deviations resulting from higher-order curvature corrections in the gravitational Lagrangian density.

$$\begin{aligned}\mathcal{L}_g &= \frac{M_{\rm Pl}^2}{16\pi}\left(R + \lambda\mathcal{F}(g^{\mu\nu}, R_{\mu\nu\rho\sigma})\right)\\ &= \frac{M_{\rm Pl}^2}{16\pi}R + a_1 R^2 + a_2 R_{\mu\nu}R^{\mu\nu} + a_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \ldots\end{aligned}$$

Semiclassical gravity: implicit assumptions

$$e.g.\ \mathrm{T} := T^{\mu}_{\mu}\ \textit{and}\ \mathfrak{T} := T^{\mu\nu}T_{\mu\nu}$$

1. **Regularity: curvature scalars are finite at apparent horizons**

Consequence of the cosmic censorship conjecture.

2. **Horizons form in finite asymptotic time (i.e. according to distant observers)**

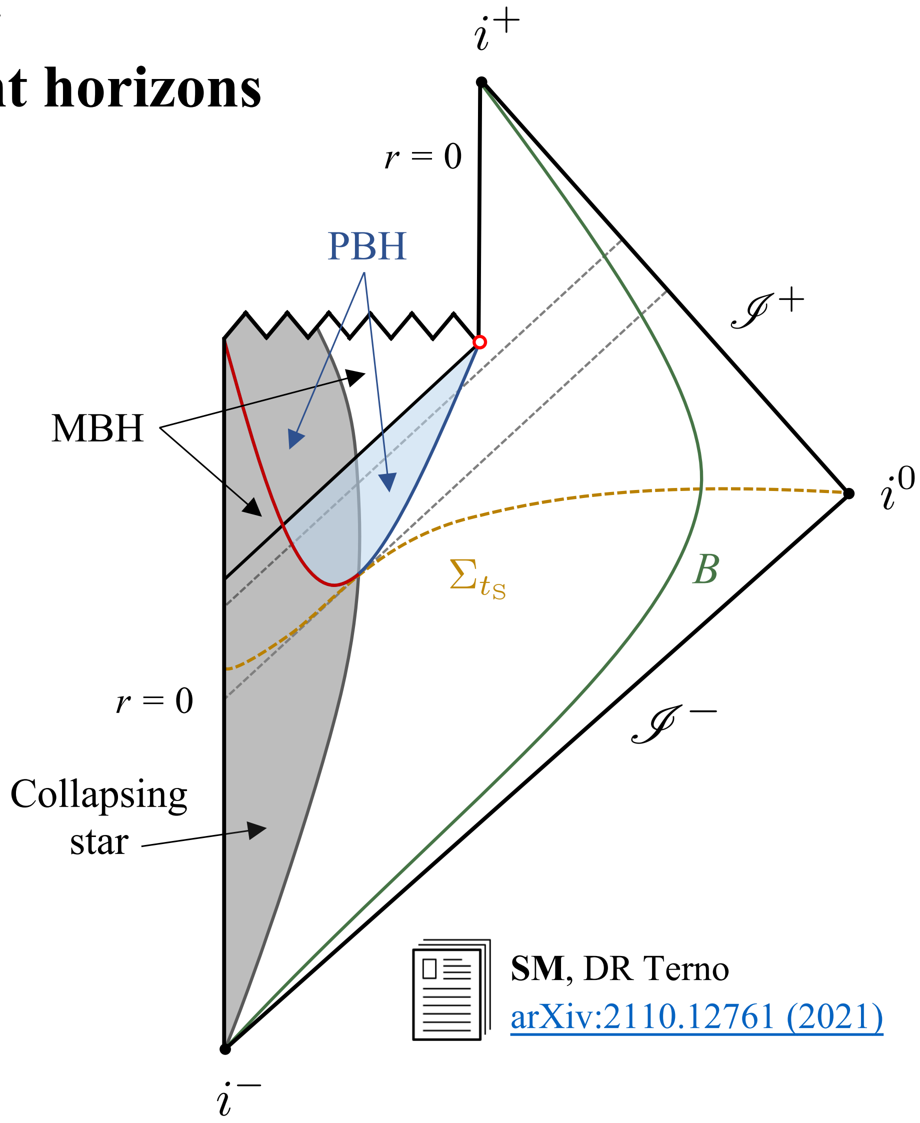
Implicitly assumed in conventional descriptions of black hole formation and evaporation.

Requires violation of NEC near outer apparent horizon

$$\boxed{T_{\mu\nu}\ell^{\mu}\ell^{\nu} < 0}$$

V Baccetti, RB Mann, **SM**, DR Terno
Phys. Rev D **99**, 124014 (2019)

RB Mann, **SM**, DR Terno
Int. J. Mod. Phys. D **31**, 2230015 (2022)



SM, DR Terno
arXiv:2110.12761 (2021)

Semiclassical gravity: only two dynamic solutions in spherical symmetry

$$ds^2 = \underbrace{-e^{2h(t,r)} f(t,r) dt^2}_{} + \underbrace{f(t,r)^{-1} dr^2}_{} + \underbrace{r^2 d\Omega^2}_{}$$

$$g_{\mu\nu} = \begin{bmatrix} \boxed{-e^{2h(t,r)} f(t,r)} & 0 & 0 & 0 \\ 0 & \boxed{f(t,r)^{-1}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{bmatrix}$$

Integrating factor in coordinate transformations, e.g.

$$dt = e^{-h}\left(e^{h_+} dv - f^{-1} dr\right)$$

$$f(t,r) := 1 - C/r := \partial_\mu r \partial^\mu r$$

Misner-Sharp mass

CW Misner, DH Sharp
Phys. Rev. **136**, B571 (1964)

Effective EMT components: $\tau_t := e^{-2h} T_{tt}$, $\tau_t{}^r := e^{-h} T_t{}^r$, $\tau^r := T^{rr}$

Semiclassical Einstein equations in spherical symmetry

$$\partial_r C = 8\pi r^2 \tau_t / f$$

$$\partial_t C = 8\pi r^2 e^h \tau_t{}^r$$

$$\partial_r h = 4\pi r \left(\tau_t + \tau^r\right) / f^2$$

Solutions are characterised by scaling behaviour of EMT close to horizon:

$$\lim_{r \to r_g} \tau \sim \pm \Upsilon(t)^2 f(t,r)^k$$

Only two values of k are consistent: $k \in \{0, 1\}$

DR Terno
Phys. Rev. D **101**, 124053 (2020)

SM, DR Terno
Phys. Rev. D **103**, 064082 (2021)

Dynamic physical black hole solutions in spherical symmetry

SEBASTIAN MURK PHYS. REV. D **105,** 044051 (2022)

	$k=0$ solutions		$k=1$ solution	
Metric functions	$C=r_g-c_{12}\sqrt{x}+\sum_{j\geq 1}^{\infty} c_j x^j$	(k0.1)	$C=r_g+x-c_{32}x^{3/2}+\sum_{j\geq 2}^{\infty} c_j x^j$	(k1.1)
	$h=-\frac{1}{2}\ln\frac{x}{\xi}+\sum_{j\geq\frac{1}{2}}^{\infty} h_j x^j$	(k0.2)	$h=-\frac{3}{2}\ln\frac{x}{\xi}+\sum_{j\geq\frac{1}{2}}^{\infty} h_j x^j$	(k1.2)
Leading coefficient	$c_{12}=4\sqrt{\pi}r_g^{3/2}\Upsilon$	(k0.3)	$c_{32}=4r_g^{3/2}\sqrt{-\pi e_2/3}$	(k1.3)
Horizon dynamics	$r_g'=\pm c_{12}\sqrt{\xi}/r_g$	(k0.4)	$r_g'=\pm c_{32}\xi^{3/2}/r_g$	(k1.4)

$$x := r - r_g$$

Describes black holes immediately after formation (and for the rest of their lifetime).

Describes formation of black holes.

Both violate the NEC near the horizon.

The formation of black holes follows a unique scenario that involves both classes of solutions!

The transition between them is continuous.

Details: **SM**, DR Terno
[Phys. Rev. D **103**, 064082 (2021)]

Black holes in modified gravity theories

Modified Einstein field equations: $$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \lambda \mathcal{E}_{\mu\nu} = 8\pi T_{\mu\nu}$$

Question: What are the constraints that any self-consistent **modified theory of gravity (MTG)** must satisfy to be compatible with semiclassical physical black holes?

Only assumption:
regular apparent horizon forms in finite time of distant observer.

So far: only vacuum solutions known explicitly in modified gravity.

SM, DR Terno
Phys. Rev. D **104**, 064048 (2021)

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Phys. Rev. D **105**, 044051 (2022)

Field equations in modified gravity theories

SM, DR Terno
Phys. Rev. D **104**, 064048 (2021)

SM
Phys. Rev. D **105**, 044051 (2022)

Modified Einstein field equations:

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \lambda \mathcal{E}_{\mu\nu} = 8\pi T_{\mu\nu}}$$

$$\mathcal{L}_g = \frac{M_{\mathrm{Pl}}^2}{16\pi}\left(R + \lambda \mathcal{F}(g^{\mu\nu}, R_{\mu\nu\rho\sigma})\right)$$

$$= \frac{M_{\mathrm{Pl}}^2}{16\pi} R + a_1 R^2 + a_2 R_{\mu\nu} R^{\mu\nu} + a_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ldots$$

No *a priori* assumptions are made about the modified term.

Perturbed metric:

$$g_\lambda := \bar{g}_{\mu\nu} + \lambda \tilde{g}_{\mu\nu}$$

Use physical value of horizon radius r_g to avoid artifactual divergences.

$$C_\lambda(t, r_g) = r_g$$

➡ **Results apply to almost all possible modifications of general relativity, irrespective of specific properties of a particular theoretical model!**

Qualitative overview of results

coordinate distance from the horizon

$> \mathcal{O}(\lambda)$

1. Treat higher-order terms as small. $\Rightarrow$ Expand in both $x := r - r_g$ and λ

Result: obtain constraints for the two classes of dynamic solutions.

2. Consider k=0 solutions where the leading terms in the effective EMT components are not determined by terms of order $\mathcal{O}(\lambda)$. $\Rightarrow$ Expand only in $x := r - r_g$

Result: obtain analogous constraint; no well-defined GR limit.

SM, DR Terno
Phys. Rev. D **104**, 064048 (2021)

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Phys. Rev. D **105**, 044051 (2022)

Non-perturbative (in λ) k=0 solutions

Consider solutions where the leading reduced components of the EMT are not determined by terms of order $\mathcal{O}(\lambda)$.

EMT expansion: $$\tau_a = \lambda\tilde{\Xi} + \underline{\lambda^2\tilde{\Xi}_{(2)}} + \sum_{j\geq\frac{1}{2}}^{\infty}\left[\left(\bar{\tau}_a\right)_j + \lambda\left(\tilde{\tau}_a\right)_j + \underline{\lambda^2\left(\tilde{\tau}_a^{(2)}\right)_j}\right]x^j$$

Metric functions: $$C = r_g - \lambda\sigma_{12}\sqrt{x} + \sum_{j\geq\frac{1}{2}}^{\infty}\left(\zeta_j + \lambda\sigma_j + \lambda^2\sigma_j^{(2)}\right)x^j,$$

$$h = -\frac{1}{2}\ln\frac{x}{\xi} + \sum_{j\geq\frac{1}{2}}^{\infty}\left(\eta_j + \lambda\omega_j + \lambda^2\omega_j^{(2)}\right)x^j$$

$\Rightarrow$ Obtain MTG constraints analogous to perturbative class of k=0 solutions

But:

It is impossible to determine the sign of $\tilde{\Xi}$ and $\tilde{\Xi}_{(2)}$ solely from the requirement of self-consistency of the modified Einstein equations.

$\Rightarrow$ **Unclear if violation of NEC is prerequisite for formation of PBH.**

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Phys. Rev. D **105**, 044051 (2022)

Overview of modified gravity constraints

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TABLE II. Necessary conditions for the existence of semiclassical PBHs in arbitrary metric MTG. To be compatible with semiclassical PBHs of the $k = 0$ ($k = 1$) type, the MTG terms of arbitrary metric MTG must conform to the structures prescribed by Eqs. (k0.I)–(k0.III) [Eqs. (k1.I)–(k1.III)] when expanded in terms of $x := r - r_g$. Additionally, their lowest-order coefficients must satisfy the three (two) identities given by Eqs. (k0.IV)–(k0.V) [Eqs. (k1.IV)–(k1.V)].

	$k = 0$ solutions	$k = 1$ solution
Decomposition of MTG terms	$\bar{\mathcal{E}}_{tt} = \frac{\text{æ}_{\overline{1}}}{x} + \frac{\text{æ}_{\overline{12}}}{\sqrt{x}} + \text{æ}_0 + \sum_{j\geq\frac{1}{2}}^{\infty} \text{æ}_j x^j$ (k0.I)	$\bar{\mathcal{E}}_{tt} = \frac{\text{æ}_{\overline{32}}}{x^{3/2}} + \frac{\text{æ}_{\overline{1}}}{x} + \frac{\text{æ}_{\overline{12}}}{\sqrt{x}} + \text{æ}_0 + \sum_{j\geq\frac{1}{2}}^{\infty} \text{æ}_j x^j$ (k1.I)
	$\bar{\mathcal{E}}_t{}^r = \frac{\text{œ}_{\overline{12}}}{\sqrt{x}} + \text{œ}_0 + \sum_{j\geq\frac{1}{2}}^{\infty} \text{œ}_j x^j$ (k0.II)	$\bar{\mathcal{E}}_t{}^r = \text{œ}_0 + \sum_{j\geq\frac{1}{2}}^{\infty} \text{œ}_j x^j$ (k1.II)
	$\bar{\mathcal{E}}^{rr} = \text{ø}_0 + \sum_{j\geq\frac{1}{2}}^{\infty} \text{ø}_j x^j$ (k0.III)	$\bar{\mathcal{E}}^{rr} = \sum_{j\geq\frac{3}{2}}^{\infty} \text{ø}_j x^j$ (k1.III)
Relations between MTG coefficients	$\text{æ}_{\overline{1}} = \sqrt{\bar{\xi}}\,\text{œ}_{\overline{12}} = \bar{\xi}\,\text{ø}_0$ (k0.IV)	$\text{æ}_{\overline{32}} = 2\bar{\xi}^{3/2}\text{œ}_0 - \bar{\xi}^3\text{ø}_{32}$ (k1.IV)
	$\text{æ}_{\overline{12}} = 2\sqrt{\bar{\xi}}\,\text{œ}_0 - \bar{\xi}\,\text{ø}_{12}$ (k0.V)	$\text{æ}_{\overline{1}} = 2\bar{\xi}^{3/2}(h_{12}\text{œ}_0 + \text{œ}_{12}) - \bar{\xi}^3(2h_{12}\text{ø}_{32} + \text{ø}_2)$ (k1.V)

Black holes in MTG: constraints for k=0 solutions

Structural decomposition of the MTG terms:

$$\bar{\mathcal{E}}_{tt} = \frac{æ_{\bar{1}}}{x} + \frac{æ_{\overline{12}}}{\sqrt{x}} + æ_0 x^0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} æ_j x^j$$

$$\bar{\mathcal{E}}_t{}^r = \frac{œ_{\overline{12}}}{\sqrt{x}} + œ_0 x^0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} œ_j x^j$$

$$\bar{\mathcal{E}}^{rr} = ø_0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} ø_j x^j$$

$$\bar{C} = r_g - c_{12}\sqrt{x} + \sum_{j\geqslant 1}^{\infty} c_j x^j$$

$$r_g' = -\frac{c_{12}\sqrt{\bar{\xi}}}{r_g}$$

Additional relations between coefficients:

$$\boxed{æ_{\bar{1}} = \sqrt{\bar{\xi}}\,œ_{\overline{12}} = \bar{\xi}ø_0}$$

$$æ_{\overline{12}} = 2\sqrt{\bar{\xi}}\,œ_0 - \bar{\xi}ø_{12}$$

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Black holes in MTG: constraints for k=0 solutions

Structural decomposition of the MTG terms:

$$\bar{\mathcal{E}}_{tt} = \frac{æ_{\bar{1}}}{x} + \frac{æ_{\overline{12}}}{\sqrt{x}} + æ_0 x^0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} æ_j x^j$$

$$\bar{\mathcal{E}}_t{}^r = \frac{œ_{\overline{12}}}{\sqrt{x}} + œ_0 x^0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} œ_j x^j$$

$$\bar{\mathcal{E}}^{rr} = ø_0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} ø_j x^j$$

$$\bar{C} = r_g - c_{12}\sqrt{x} + \sum_{j\geqslant 1}^{\infty} c_j x^j$$

$$r'_g = -\frac{c_{12}\sqrt{\bar{\xi}}}{r_g}$$

Additional relations between coefficients:

$$æ_{\bar{1}} = \sqrt{\bar{\xi}}œ_{\overline{12}} = \bar{\xi}ø_0$$

$$\boxed{æ_{\overline{12}} = 2\sqrt{\bar{\xi}}œ_0 - \bar{\xi}ø_{12}}$$

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Black holes in MTG: constraints for the k=1 solution

$$\bar{C} = r - c_{32}x^{3/2} + \sum_{j\geqslant 2}^{\infty} c_j x^j$$

$$r'_g = -\frac{c_{32}\bar{\xi}^{3/2}}{r_g}$$

Structural decomposition of the MTG terms:

$$\bar{\mathcal{E}}_{tt} = \frac{\text{æ}_{\overline{32}}}{x^{3/2}} + \frac{\text{æ}_{\bar{1}}}{x} + \frac{\text{æ}_{\overline{12}}}{\sqrt{x}} + \text{æ}_0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} \text{æ}_j x^j$$

$$\bar{\mathcal{E}}_t^{\ r} = \text{œ}_0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} \text{œ}_j x^j$$

$$\bar{\mathcal{E}}^{rr} = \sum_{j\geqslant\frac{3}{2}}^{\infty} \text{ø}_j x^j$$

Additional relations between coefficients:

$$\boxed{\text{æ}_{\overline{32}} = 2\bar{\xi}^{3/2}\text{œ}_0 - \bar{\xi}^3\text{ø}_{32}}$$

$$\text{æ}_{\bar{1}} = 2\bar{\xi}^{3/2}\left(h_{12}\text{œ}_0 + \text{œ}_{12}\right) - \bar{\xi}^3\left(2h_{12}\text{ø}_{32} + \text{ø}_2\right)$$

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Black holes in MTG: constraints for the k=1 solution

$$\bar{C} = r - c_{32}x^{3/2} + \sum_{j\geqslant 2}^{\infty} c_j x^j$$

$$r_g' = -\frac{c_{32}\bar{\xi}^{3/2}}{r_g}$$

Structural decomposition of the MTG terms:

$$\bar{\mathcal{E}}_{tt} = \frac{\ae_{\overline{32}}}{x^{3/2}} + \frac{\ae_{\bar{1}}}{x} + \frac{\ae_{\overline{12}}}{\sqrt{x}} + \ae_0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} \ae_j x^j$$

$$\bar{\mathcal{E}}_t^{\ r} = \oe_0 + \sum_{j\geqslant\frac{1}{2}}^{\infty} \oe_j x^j$$

$$\bar{\mathcal{E}}^{rr} = \sum_{j\geqslant\frac{3}{2}}^{\infty} \o_j x^j$$

Additional relations between coefficients:

$$\ae_{\overline{32}} = 2\bar{\xi}^{3/2}\oe_0 - \bar{\xi}^3\o_{32}$$

$$\boxed{\ae_{\bar{1}} = 2\bar{\xi}^{3/2}\left(h_{12}\oe_0 + \oe_{12}\right) - \bar{\xi}^3\left(2h_{12}\o_{32} + \o_2\right)}$$

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Do popular models satisfy the constraints?

Derive field equations from action: $\mathcal{S} = \int \sqrt{-g}\,\mathcal{L}_{\mathrm{g}}\, d^4x$

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Gravitational Lagrangian density: $\mathcal{L}_g = \frac{M_{\mathrm{Pl}}^2}{16\pi}\left(R + \lambda\mathcal{F}(g^{\mu\nu}, R_{\mu\nu\rho\sigma})\right)$

$$= \frac{M_{\mathrm{Pl}}^2}{16\pi} R + a_1 R^2 + a_2 R_{\mu\nu}R^{\mu\nu} + a_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \ldots$$

Tested so far:

[1] Starobinsky model $\mathfrak{F} = \varsigma R^2\,, \quad \varsigma = 16\pi a_1/M_{\mathrm{Pl}}^2$ ✓

$\mathfrak{f}(R) =: R + \lambda\mathfrak{F}(R)$ ✓

[2] Generic $f(R)$ theories $\mathfrak{F} = \varsigma R^q\,, \quad \mathfrak{F}' = \varsigma q R^{q-1}$

[3] Generic fourth-order gravity theories $\mathcal{S} = \int \sqrt{-g}\,\left(-\alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 + \gamma\kappa^{-2}R\right)\, d^4x$ ✓

What's next?

1. Test additional MTG: reformulations of Gauß-Bonnet gravity, higher-dimensional BH models;
2. Generalisation to non-spherically-symmetric spacetimes; consideration of angular momentum.

Review article

Stay in touch!

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Black holes and their horizons in semiclassical and modified theories of gravity

Robert B. Mann, Sebastian Murk and Daniel R. Terno

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Abstract

For distant observers, black holes are trapped spacetime domains bounded by apparent horizons. We review properties of the near-horizon geometry emphasizing the consequences of two common implicit assumptions of semiclassical physics. The first is a consequence of the cosmic censorship conjecture, namely, that curvature scalars are finite at apparent horizons. The second is that horizons form in finite asymptotic time (i.e. according to distant observers), a property implicitly assumed in conventional descriptions of black hole formation and evaporation. Taking these as the only requirements within the semiclassical framework, we find that in spherical symmetry only two classes of dynamic solutions are admissible, both describing evaporating black holes and expanding white holes. We review their properties and present the implications. The null energy condition is violated in the vicinity of the outer horizon and satisfied in the vicinity of the inner apparent/anti-trapping horizon. Apparent and anti-trapping horizons are timelike surfaces of intermediately singular behavior, which manifests itself in negative energy density firewalls. These and other properties are also present in axially symmetric solutions. Different generalizations of surface gravity to dynamic spacetimes are discordant and do not match the semiclassical results. We conclude by discussing signatures of these models and implications for the identification of observed ultra-compact objects.

Keywords: Semiclassical gravity ▪ modified gravity ▪ black holes ▪ apparent horizon ▪ evaporation ▪ white holes ▪ energy conditions ▪ thin shell collapse ▪ surface gravity ▪ information loss

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