$\mathcal{PT}$-Symmetric Quantum Rabi Model

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Abstract

In this work, we consider the $\mathcal{PT}$-symmetric quantum Rabi model (PTQRM), which describes a $\mathcal{PT}$-symmetric qubit coupled to a quantized light field. This model can be solved analytically in a reasonable parameter regime by using the adiabatic approximation (AA). With AA and numerical diagonalization, static and dynamic properties of the PTQRM are investigated. Particularly, a bunch of exceptional points (EPs) is found in the eigenspectrum and they turn out to be closely connected with the exactly solvable points in the Hermitian counterpart of the model. Interestingly, these EPs vanish and revive depending on the light-matter coupling strength. The time evolution of physical observables under the system Hamiltonian is also discussed. This work may shed some light on the research of non-Hermitian pure quantum systems.

Introduction

$\mathcal{PT}$-symmetric systems, which are invariant under the composited operation of parity $\mathcal{P}$ and time reversal $\mathcal{T}$, possess real eigenvalues without being Hermitian. The natural advantage of easy manipulation of gain and loss makes optical and photonic systems excellent platforms for realizing $\mathcal{PT}$symmetry.

Previous investigations on $\mathcal{PT}$-symmetric light-matter interaction models were focused on imaginary coupling [1] and periodically modulated systems [2]. In this work, we approach this problem by coupling a $\mathcal{PT}$-symmetric qubit to the light field.

The model

We consider a generalization of the quantum Rabi model (QRM) defined as

$$H = \omega a^\dagger a + \frac{\Delta}{2}\sigma_z + g\sigma_x\left(a^\dagger + a\right) + \frac{i\epsilon}{2}\sigma_x. \qquad (1)$$

Real parameters

ω: light field frequency,
Δ: level splitting,
g: coupling strength,
ϵ: imaginary bias,

with operators

$$\mathcal{P} = \sigma_z e^{i\pi a^\dagger a},$$

$\mathcal{T}$: take complex conjugate.

Features:

- the system is $\mathcal{PT}$-symmetric ($[H, \mathcal{PT}] = 0$), so we call it the PTQRM.
- there are an infinite number of exceptional points (EPs), the phase transition points from $\mathcal{PT}$-symmetric to $\mathcal{PT}$-broken.
- the key is competition between ϵ and Δ, so we can set them to be small to apply the Adiabatic Approximation (AA).

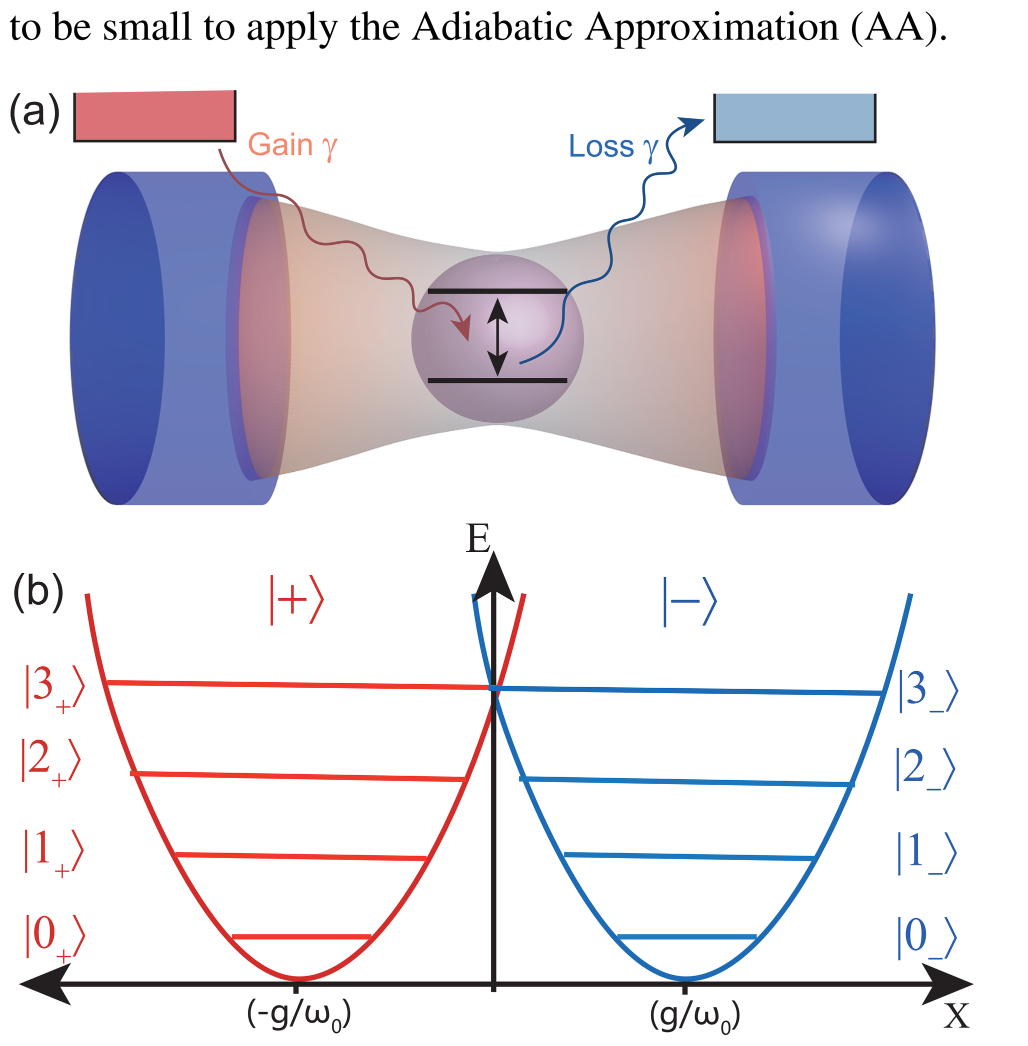

Figure 1: (a) Schematic of the system Hamiltonian (1) consisting of a $\mathcal{PT}$-symmetric qubit and a cavity. (b) Displaced oscillator interpretation of the model.

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References

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Spectrum and Eigenstates

We have calculated the spectra and eigenstates of the PTQRM. Key points:

- the amount and positions of EPs can be deduced from the results of the QRM.
- EPs vanish and revive.
- approximated locations can be derived by the AA, where $\epsilon = \Omega_n$.
- eigenstates are orthogonal at some points between the EPs (Judd points in the QRM independent of ϵ), which is absent in $\mathcal{PT}$-symmetric qubit.

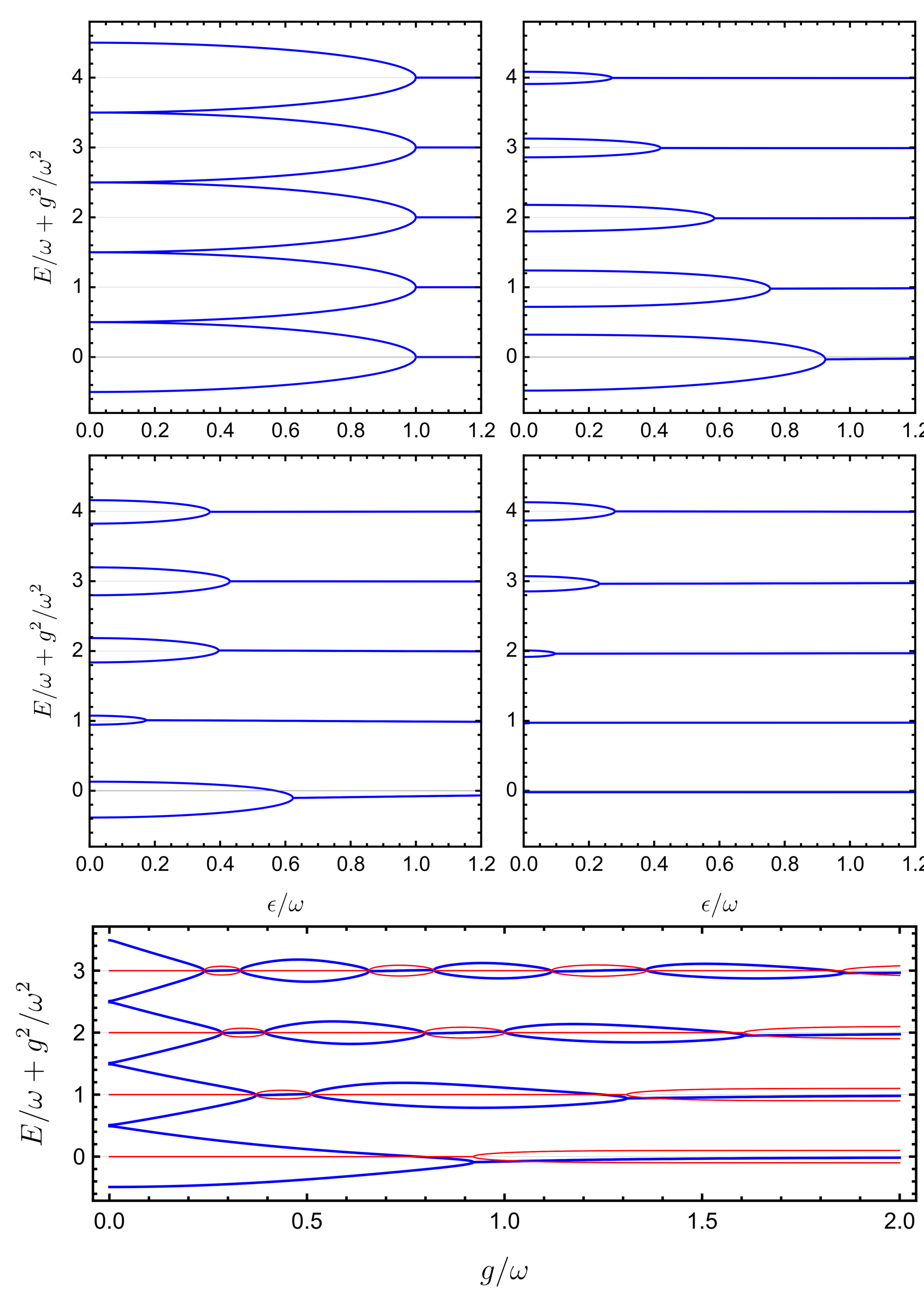

Figure 2: (a)-(d) Real energy spectra of the PTQRM against the qubit parameter ϵ, with the coupling strength $g/\omega = 0, 0.2, 0.5$ and 1.8. (e) Real (blue) and imaginary (red) energy spectra of the PTQRM against coupling strength g, with $\epsilon/\omega = 0.3$, $\Delta/\omega = 0.5$.

Under the AA the eigenvalues are given by

$$E_n^\pm = n\omega - g^2/\omega \pm \frac{1}{2}\sqrt{\Omega_n^2 - \epsilon^2} \qquad (2)$$

with non-normalized eigenstates

$$|\psi_n^\pm\rangle = \left(\frac{i\epsilon \pm \sqrt{\Omega_n^2 - \epsilon^2}}{\Omega_n}, 1\right)^T. \qquad (3)$$

In these expressions,

$$\Omega_n = \Delta e^{-2g^2/\omega^2} L_n\left(\frac{4g^2}{\omega^2}\right) \qquad (4)$$

with $L_n(x)$ being the nth Laguerre polynomial of x.

The locations of orthogonal points under AA are where the Laguerre polynomials vanish, where the normalized eigenvectors become

$$\psi_n^+ = \begin{pmatrix}1\\0\end{pmatrix}, \quad \psi_n^- = \begin{pmatrix}0\\1\end{pmatrix}, \qquad (5)$$

and are trivially orthogonal. The fidelity calculated with AA is shown in Fig. 3(b) in the dashed line, which agrees well with exact diagonalization.

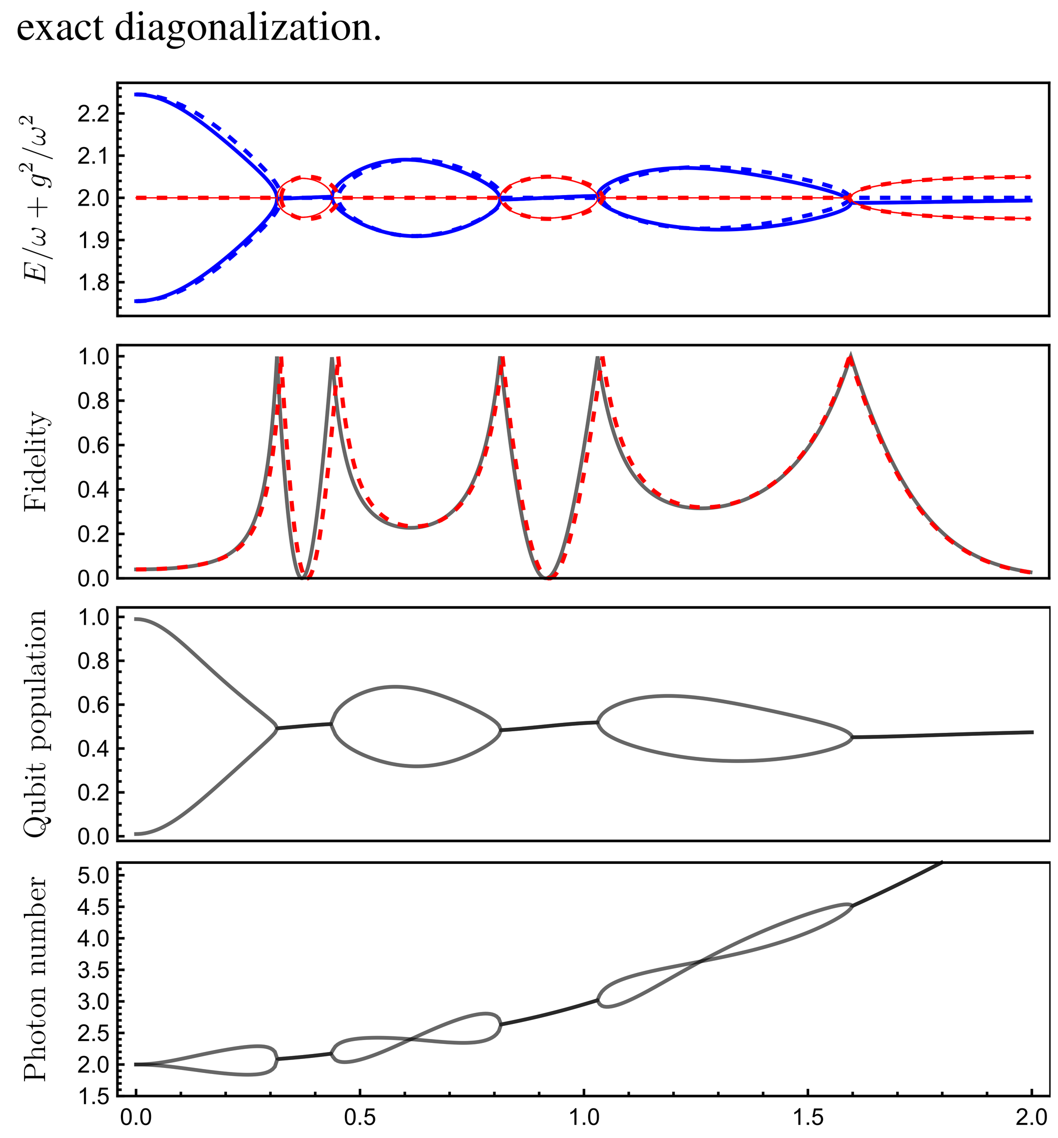

Figure 3: (a) Real part (thick blue) and imaginary part (thin red) of the energy spectrum against g for the 5th and 6th eigenstates. (b) The fidelity between these two eigenstates. (c) - (d) Qubit populations and mean photon number on these eigenstates. $\Delta/\omega = 0.5$ and $\epsilon/\omega = 0.1$ for all plots.

Dynamic behaviour

In this section, we investigate the dynamic behavior of the PTQRM by solving the time-dependent Schrodinger equation numerically.

The evolution of mean photon numbers can be divided into 3 stages:

- oscillation around the initial state at the beginning.
- convergence to eigenstate(s) with locally largest imaginary eigenvalue(s).
- divergence to infinite-photon states in large time scale.

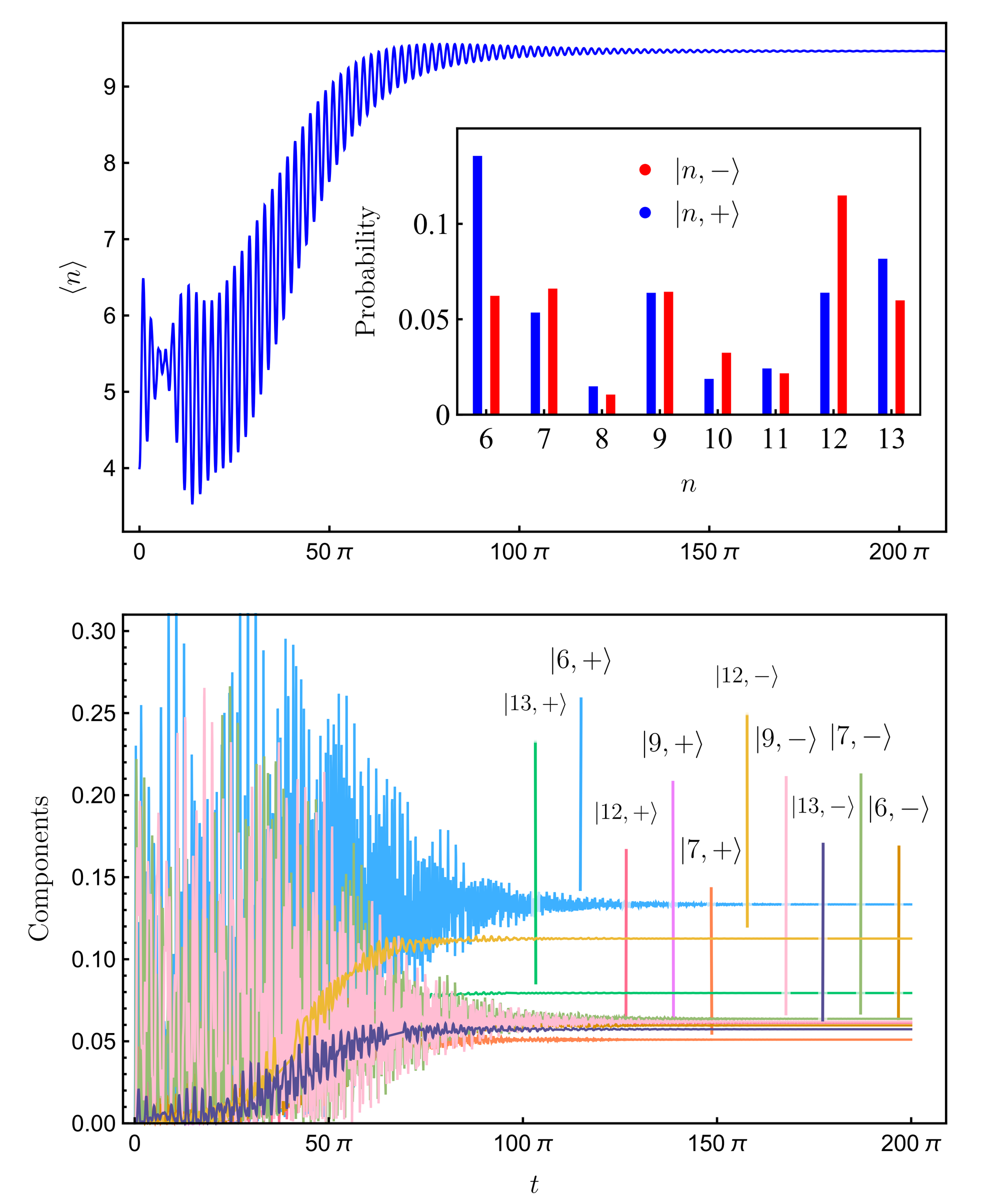

Figure 4: (a) Photon population evolution with initial state $|4,+\rangle$. Inset: The 19th (and 20th) eigenstate of the Hamiltonian. (b) States components evolution with initial state $|4,+\rangle$ at $g/\omega = 0.7$, $\Delta/\omega = 0.6$.

The stability increases with:

- smaller ϵ.
- larger displacement.

The dynamic behavior here is more difficult to fit in the AA picture: the effect of higher-order tunneling can be amplified by the growth factor through time in the $\mathcal{PT}$-broken phase.

The qubit population evolution is analyzed similarly and the numerical data for low-lying states agrees with recent experiments on $\mathcal{PT}$-symmetric qubits [3].

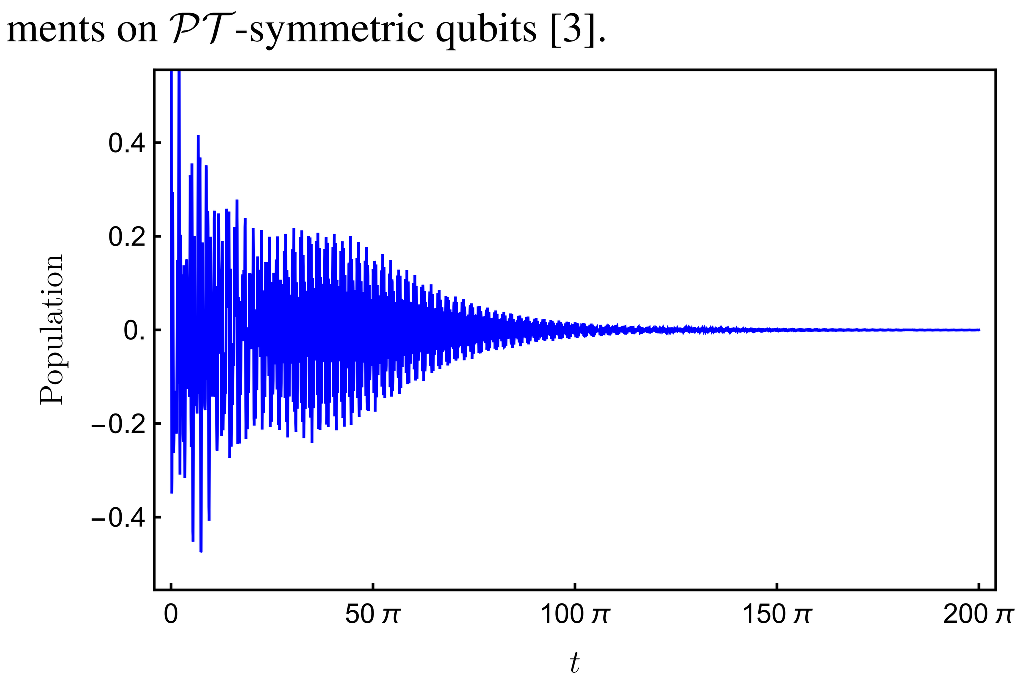

Figure 5: Qubit population evolution with initial state $|4,+\rangle$ at $g/\omega = 0.7$.

Summary

- An infinite number of exceptional points are observed in this PTQRM, which vanish and revive with the coupling strength.
- There are no overall $\mathcal{PT}$-symmetric and $\mathcal{PT}$-broken phases similar to $\mathcal{PT}$-symmetric bosonic systems [4].
- Any initial state grows to nearby $\mathcal{PT}$-broken eigenstates and diverges to infinite-photon states.
- The static part can be solved analytically using adiabatic approximation (AA), while the growth factors make AA less effective in the dynamic part.