Abstract

Discrete-variable Wigner function formalisms and the Weyl-Heisenberg displacements

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Continuous-variable Wigner functions (CVWFs) [1] are a useful tool in quantum information and computing. They provide a visual intuition of states over a phase-space by representing both position and momentum together; they can relate experimental results to the tomography of states (allowing one to reconstruct a WF from experiment); and when WFs go negative, they provide a resource for quantum computation. In quantum computing, we often deal with discrete systems and so it is natural to try and define a discrete-variable Wigner function (DVWF). DVWFs are an analog of the continuous case and they are defined over a until cell of phase space that are typically either $d \times d$ or $2d \times 2d$ sized unit cells.

Now, when one tries to define a DVWF [2–4], there are some caveats. Unlike the CVWFs, there is no agreed unique definition for a DVWF. As a result, DVWFs tend to differ in their definitions, phase-space size, and have caveats on dimensionality—they can sometimes be restricted to either even, odd, prime and prime-powered dimensions, or some combination of them.

In this work, I analytically defined a $2d \times 2d$ DVWF as well as a novel map I call a coarse grain map (CGM) that maps the phase space from one that is $2d \times 2d$ to one that is $d \times d$. I then compare three different DVWFs each defined through a CGM, Dirichlet kernel mapping (DKM) [5, 6], or William K. Wootters' definition [2]. The respective DVWFs, corresponding to a single Weyl-Heisenberg operator, were evaluated and compared using a so called 'division map'. What was found was that these functions are equivalent up to some non-trivial phase that is dependent on the displacement amounts.

References

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