Interpretation of Dirac Fermions as a Four-Dimensional Gaussian

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The recently introduced Quantum Theory of Time (QTT) [1] describes the space-time wavefunction of a scalar particle as a 4-dimensional Gaussian, which is analogous to the s-orbital in atomic physics. This immediately suggests that a fermionic particle may be described in QTT by a wavefunction that is Gaussian in the radial component and periodic in the angular coordinates. To investigate this, we explore the possibility of deriving a solution to the Dirac equation [2] that is a 4-dimensional local wavefunction centred on the centre-of-mass coordinates. The overarching goal is to reinterpret the internal degrees of freedom of a Dirac fermion (particularly the electron) as a local wavefunction that is oriented in 4-dimensional space and time.

There has been some previous work exploring the local shape of an electron spinor, in terms of a generalised spherical-harmonic description of the wavefunction of a spin-1/2 particle. For example, Gatland [3] found that the angular momentum raising and lowering operators permit an odd number of states and thus forbade eigenstates with eigenvalues that are multiples of $\hbar$. Furthermore, Rubin [4] was able to compute eigenvalues of the Laplacian of symmetric transverse traceless tensor harmonics for dimensionality ≥ 3. He proved that the tensor harmonics occupy the space described by the original symmetric transverse traceless tensors, allowing easier linking of spherical harmonics to the electron spinor.

QTT attributes a localised Gaussian wavefunction in space and time to any particle, including a fermion, opening up further ways to explore the problem. In particular, we begin with a composite system of two 2-dimensional spin-1/2 particles to represent a single 4-dimensional spinor which we re-express in terms of a direct sum of L=0 and 1 angular momentum basis states. Finally, we use the corresponding spherical harmonic functions to show that a fermion is represented in QTT as a wavefunction that is localised both in time and the radial spatial coordinate like a scalar particle, but differs from a scalar particle in that it has additional angular variation in space.

[1] J. A. Vaccaro, *Proceedings of the Royal Society of London* **472**, 2185 (2016).

[2] P. A. M. Dirac, *Proceedings of the Royal Society of London A* **117**, 778 (1928)

[3] I. R. Gatland, *Am. J. Phys.* **74**, 3 (2006).

[4] M. A. Rubin, *J. Math. Phys.* **25**, 10 (1984).

See Also:

[1] J. A. Vaccaro, Quantum asymmetry between time and space, *Proc. R. Soc. A* **472**, 2185 (2016).

[2] F. Tanjia, A. Sadeghi, J. A. Vaccaro, Quantum asymmetry between space and time: phenomenological emergence of Lorentz invariance, submitted to *AIP Congress* (2022).

[3] K. Bordon, F. Tanjia, J. A. Vaccaro, Defining the Quantum Mechanical Time Observable, submitted to *AIP Congress* (2022).

[4] J. A. Vaccaro, Quantum asymmetry between time and space: emergence of time and the laws of motion from 4D space, submitted to *AIP Congress* (2021).