Australian National University

Gravitational Waves from Young Neutron Stars

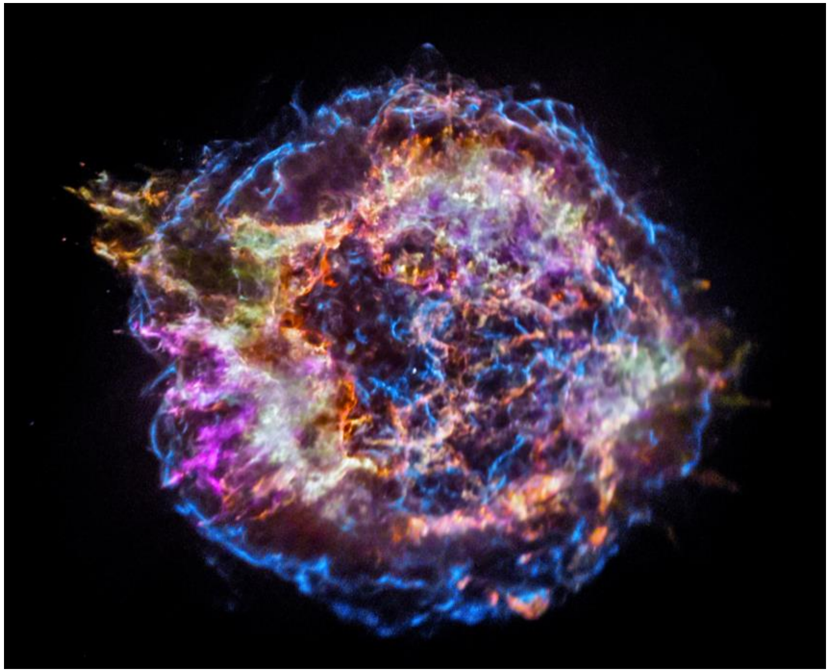
OzGrav
ARC Centre of Excellence for Gravitational Wave Discovery

ANU CENTRE FOR GRAVITATIONAL ASTROPHYSICS

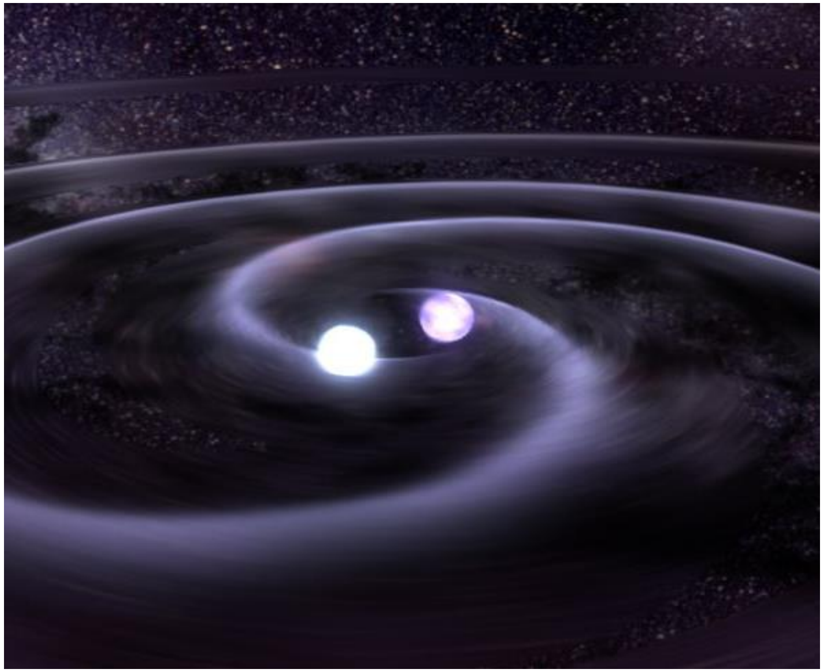
Ben Grace

Young Neutron Star Origins

Supernovae



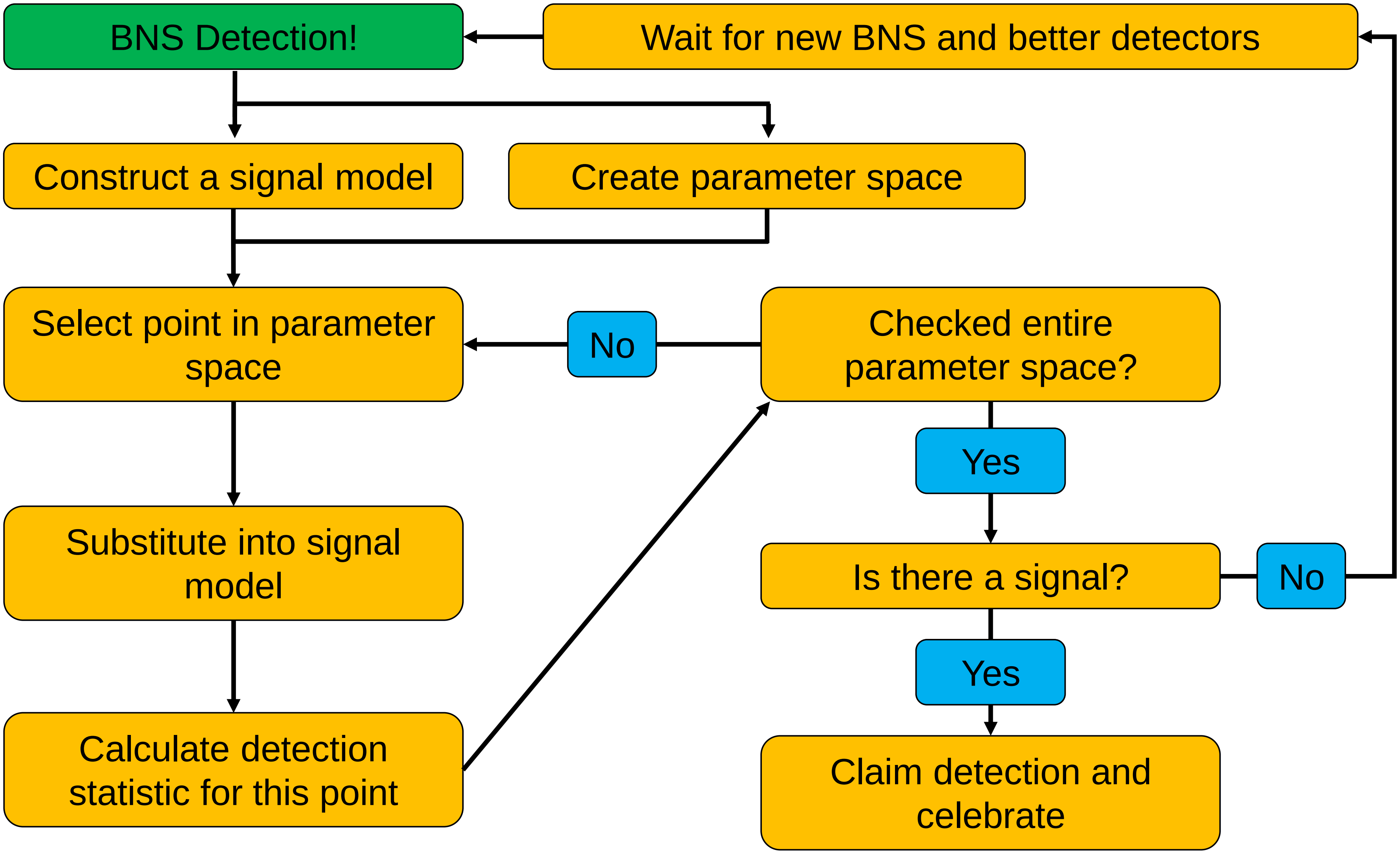
Neutron Star Coalescence

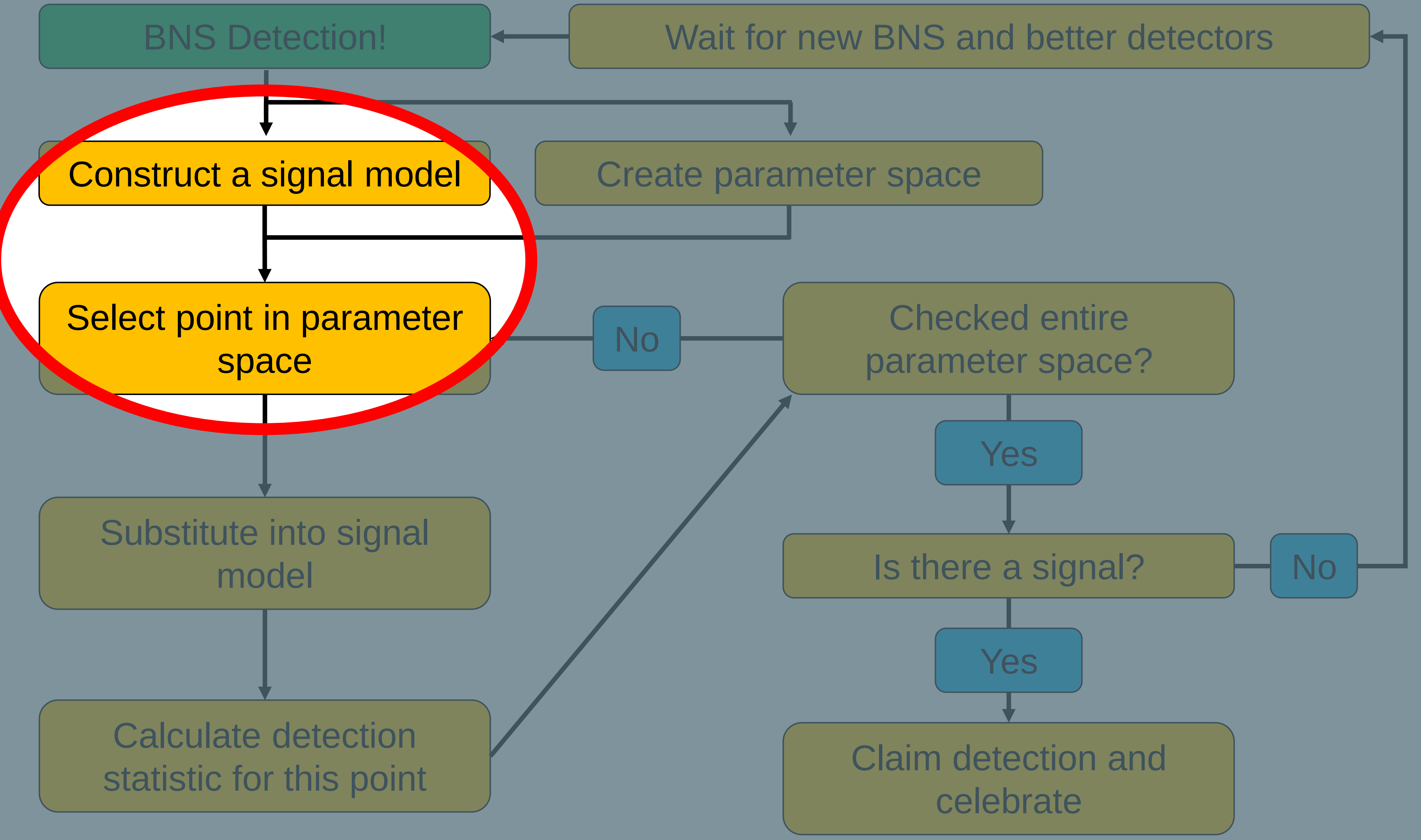


Credit: https://www.nasa.gov/mission_pages/chandra/images/index.html, https://svs.gsfc.nasa.gov/10543

Post Merger Scenarios

- Immediate black hole formation
- Hypermassive neutron star ($\leq$ 1s)
- Supramassive neutron star (~Hours)
- Stable, long lived neutron star


BNS Detection!
Wait for new BNS and better detectors
Construct a signal model
Create parameter space
Select point in parameter space
No
Checked entire parameter space?
Yes
Substitute into signal model
Is there a signal?
No
Yes
Calculate detection statistic for this point
Claim detection and celebrate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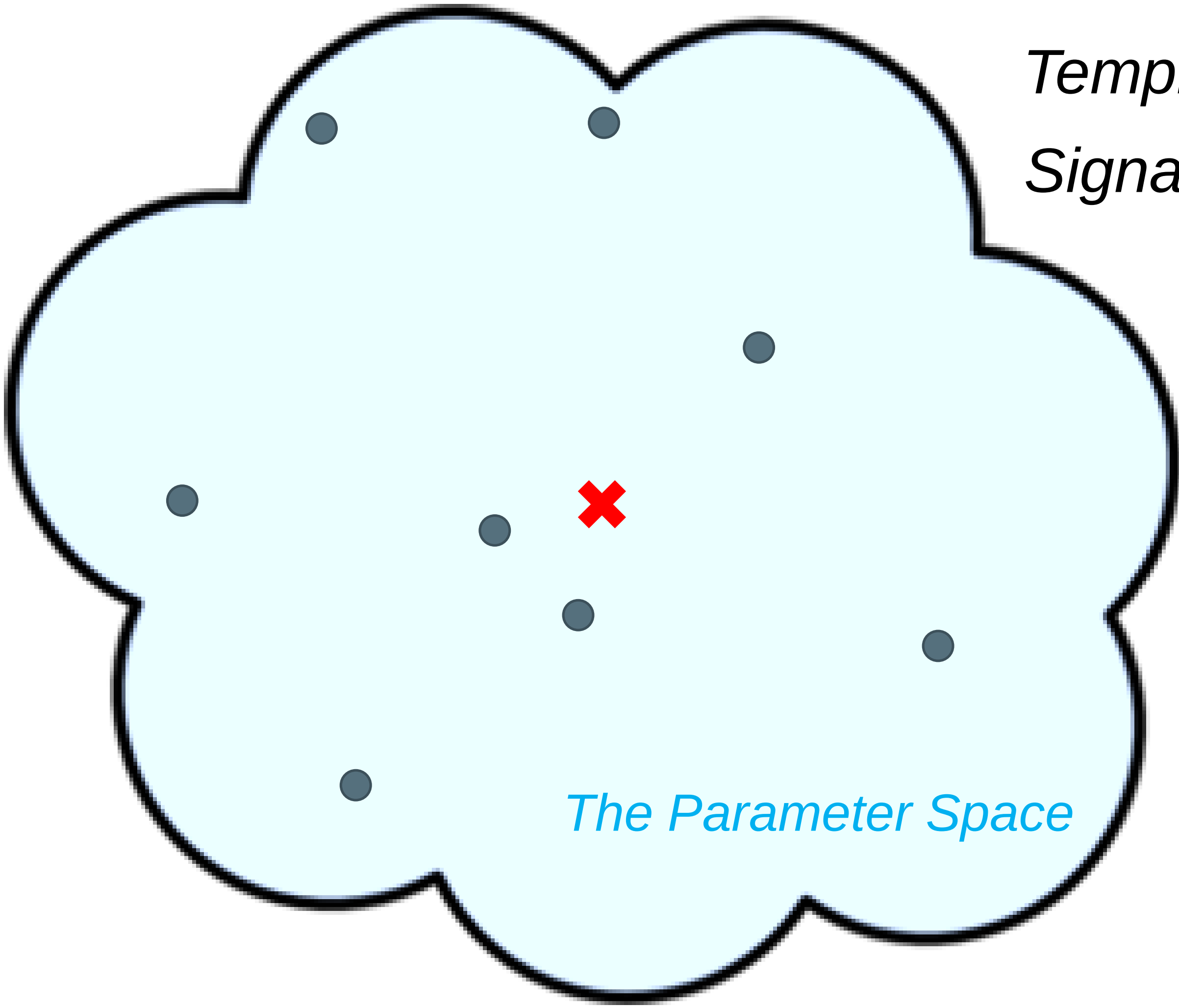


Template Bank = { ● , ● , ● , ... }

Signal Parameters = ✖

How close does ✖ need to be to a point ● for a detection to be made?

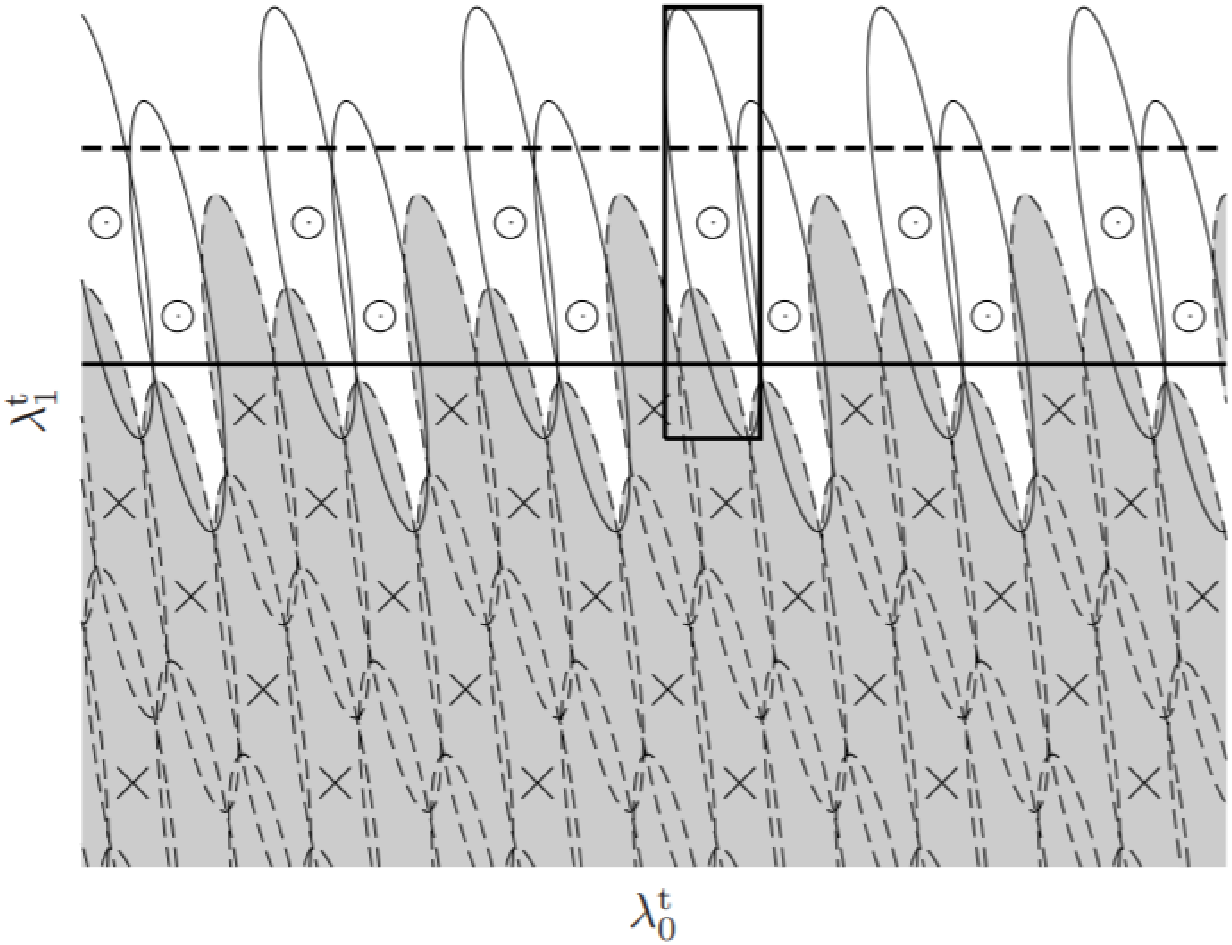
The Parameter Space Metric

$$g_{ij} = \langle \partial_i \phi \partial_j \phi \rangle - \langle \partial_i \phi \rangle \langle \partial_j \phi \rangle$$

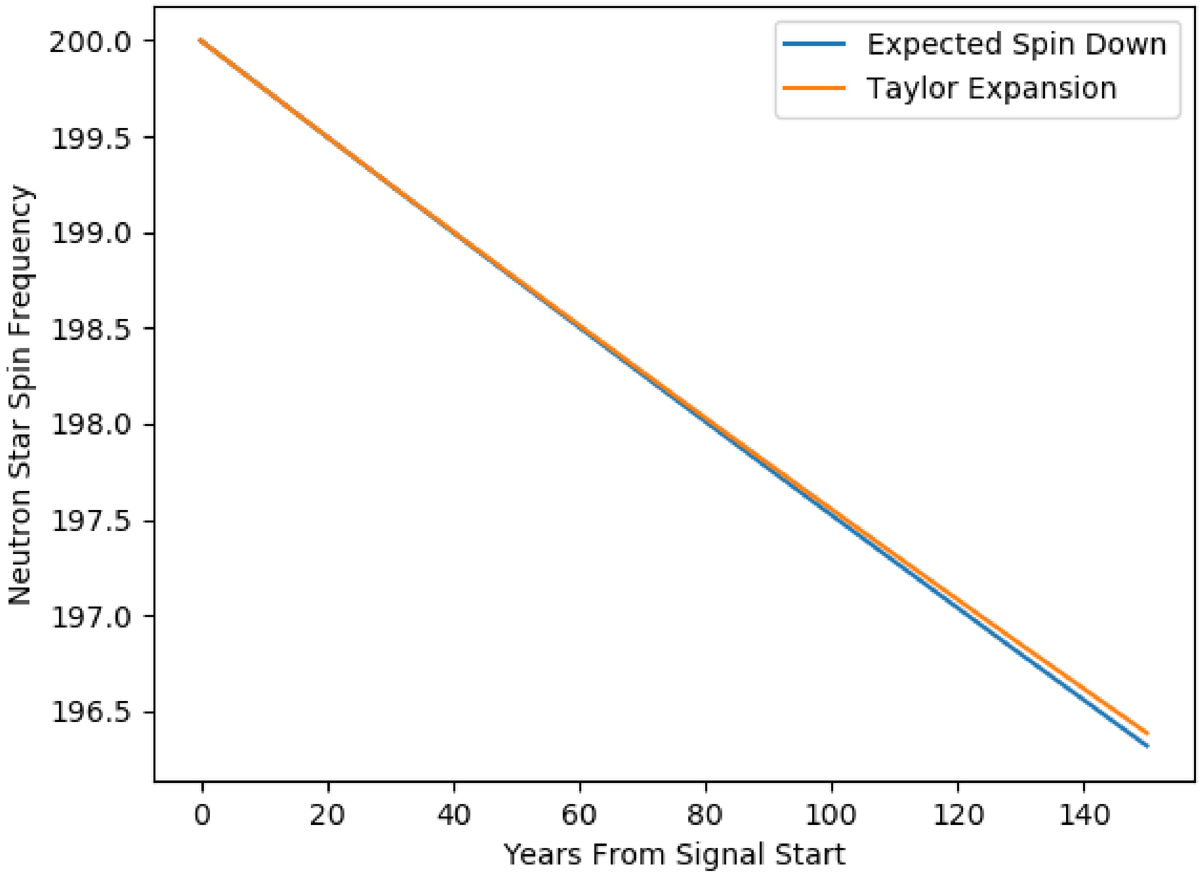
$$\phi(t) = 2\pi \int_0^t f_{GW}(t')\, dt'$$

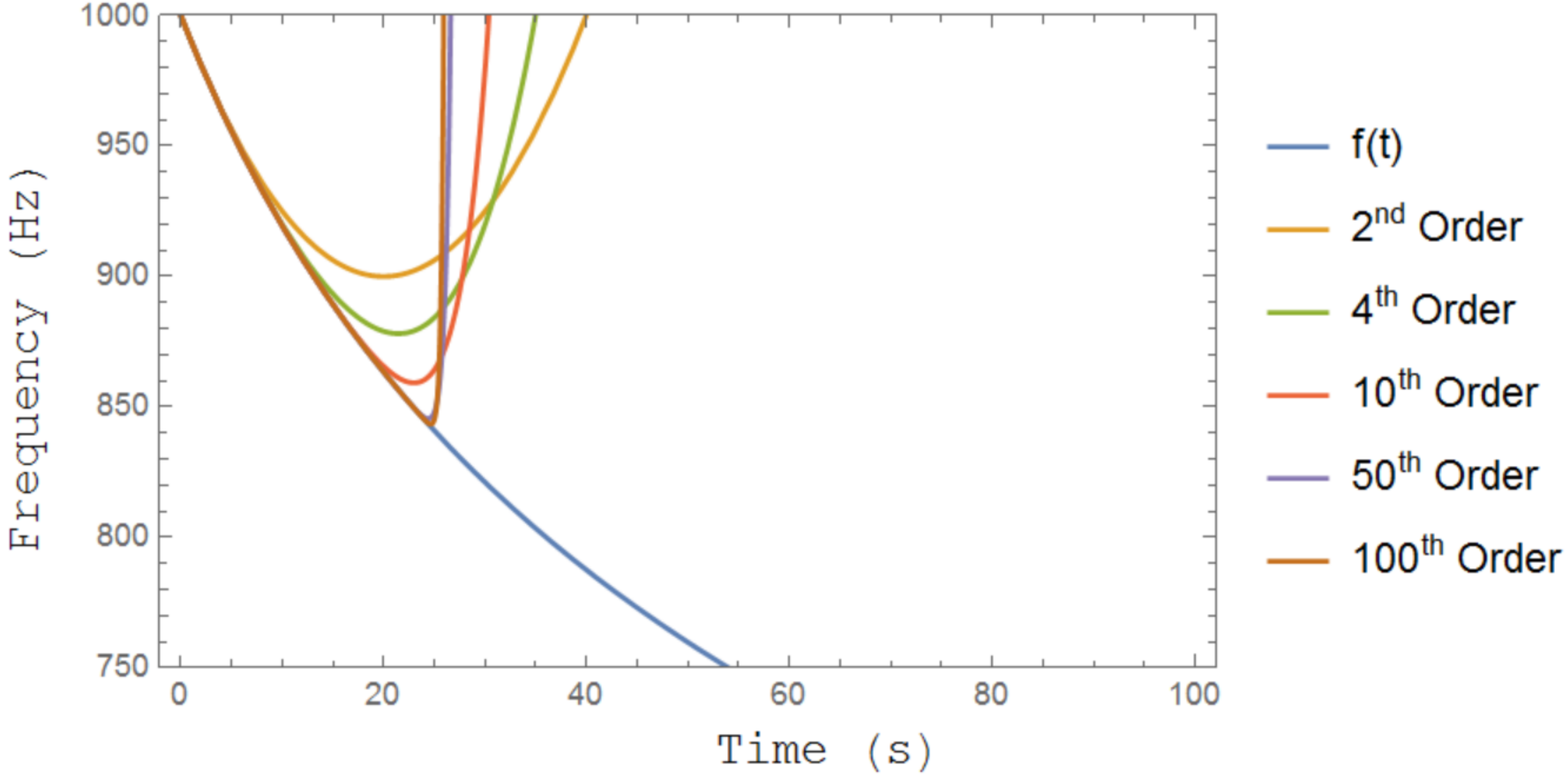
$$f(t) \approx f_0 + \dot{f}_0 t + \frac{1}{2} \ddot{f}_0 t^2 + \ldots$$

$$g_{ij} = \langle \partial_i \phi \partial_j \phi \rangle - \langle \partial_i \phi \rangle \langle \partial_j \phi \rangle$$

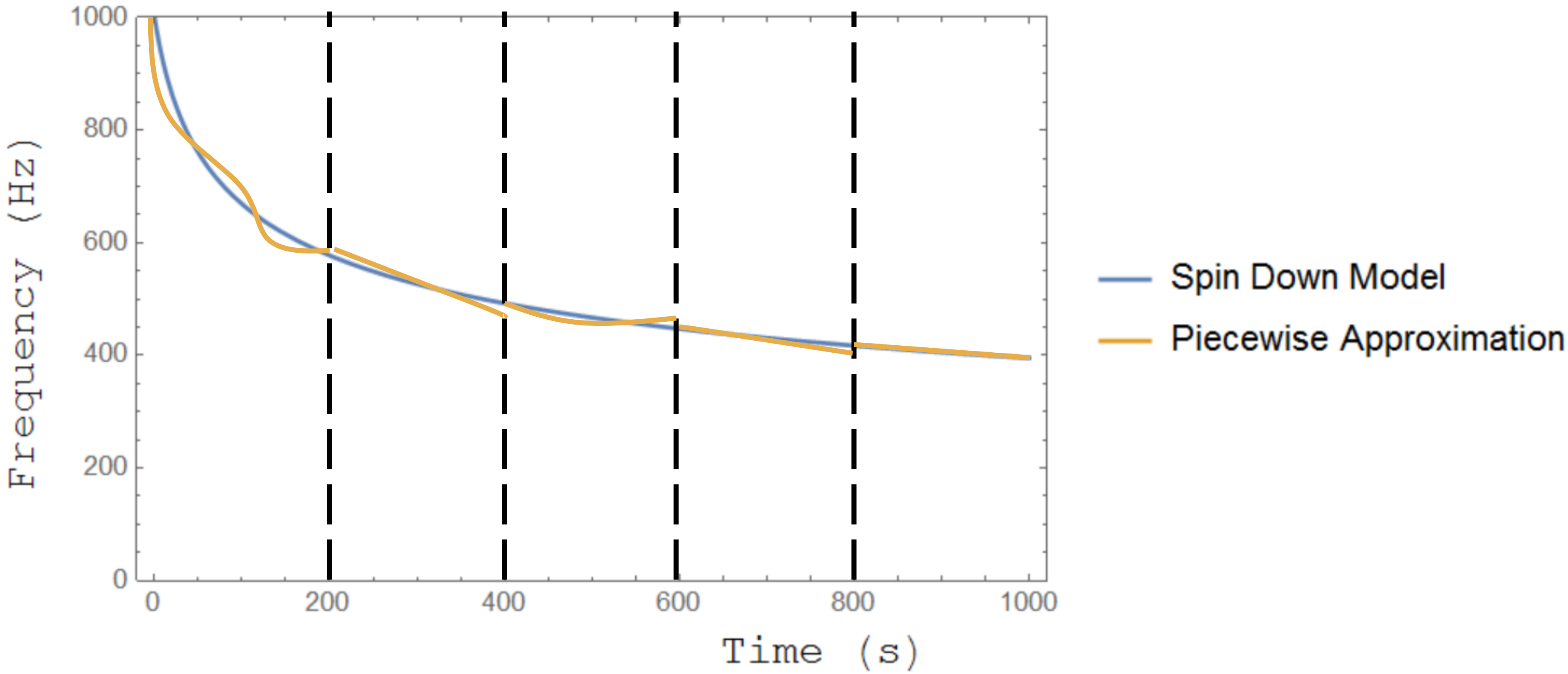


Continuous Waves from Long Lived Neutron Star




1000
950
900
850
800
750
0
20
40
60
80
100
Frequency (Hz)
Time (s)
f(t)
2nd Order
4th Order
10th Order
50th Order
100th Order

A Piecewise Approximation

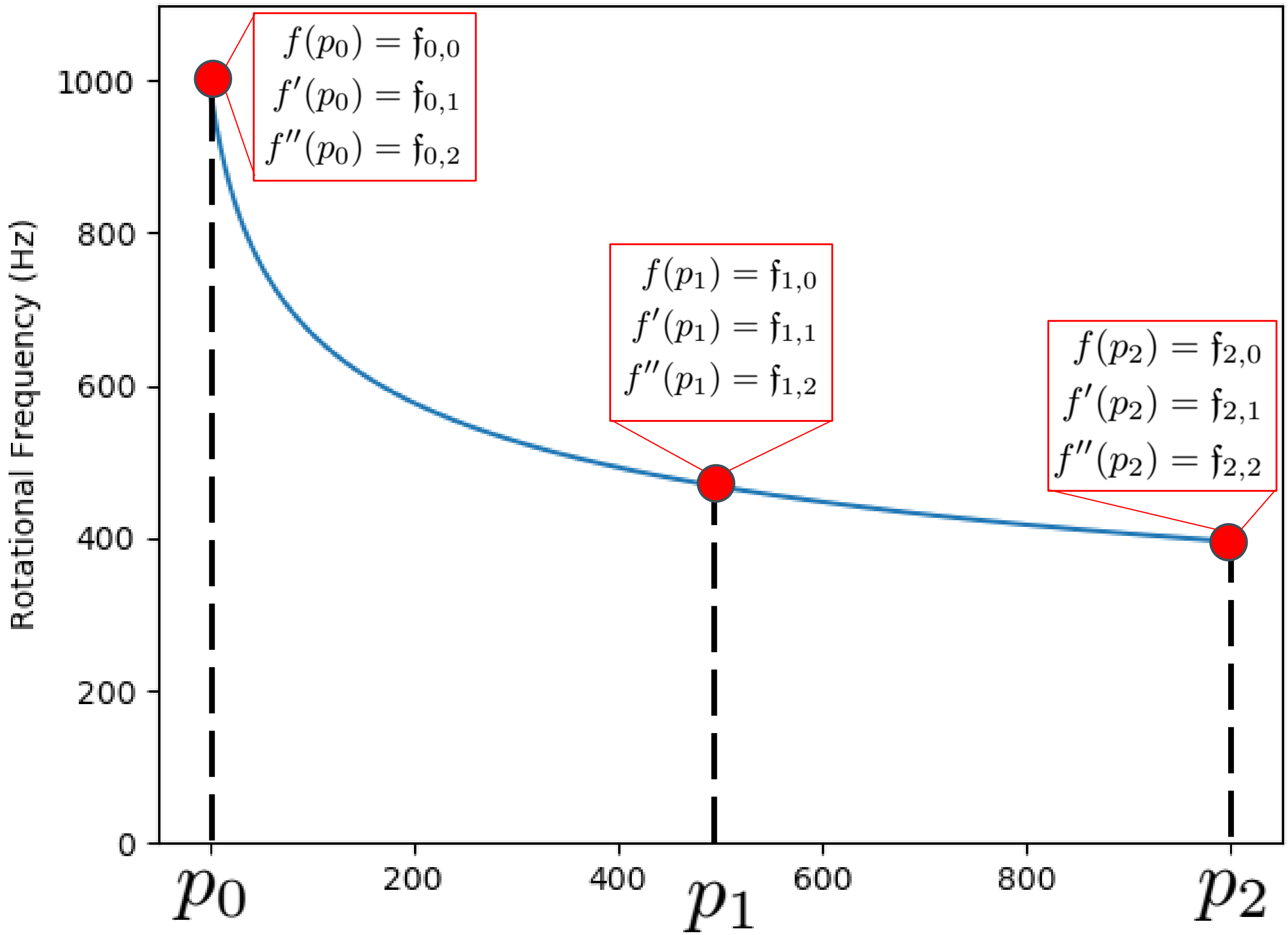


The Piecewise Model

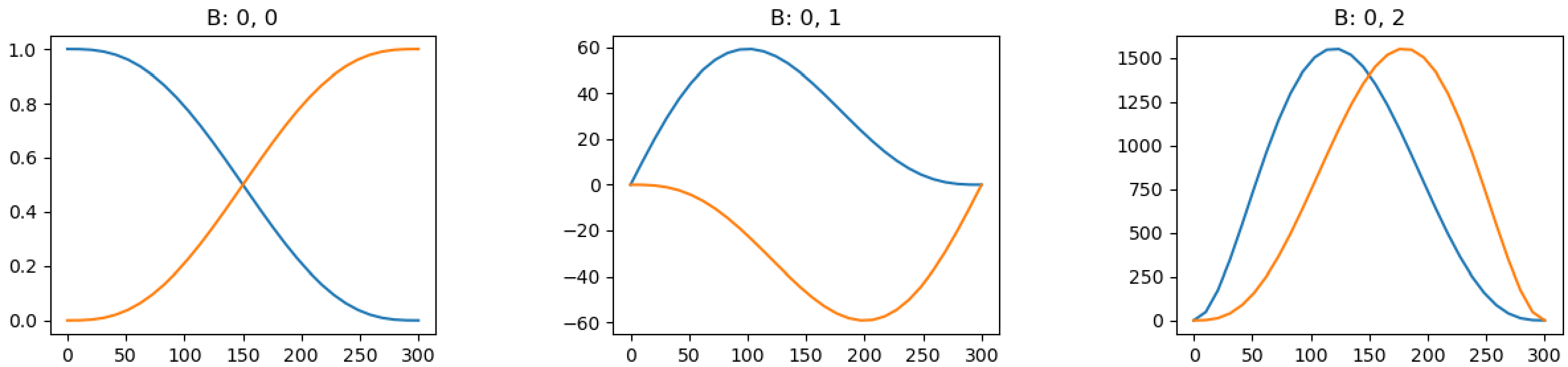
$$f(t) = \begin{cases} f_0 & p_0 \leq t < p_1 \\ f_1 & p_1 \leq t < p_2 \\ \ldots \end{cases}$$

$$f_j = \sum_{s=0}^{S-1} \mathfrak{f}_{j,s} B^0_{j,s}(t) + \mathfrak{f}_{j+1,s} B^1_{j,s}(t)$$

The Parameters of f(t):



Piecewise Basis Functions



X-axis: Seconds

Y-Axis: $f, \dot{f}, \ddot{f}$

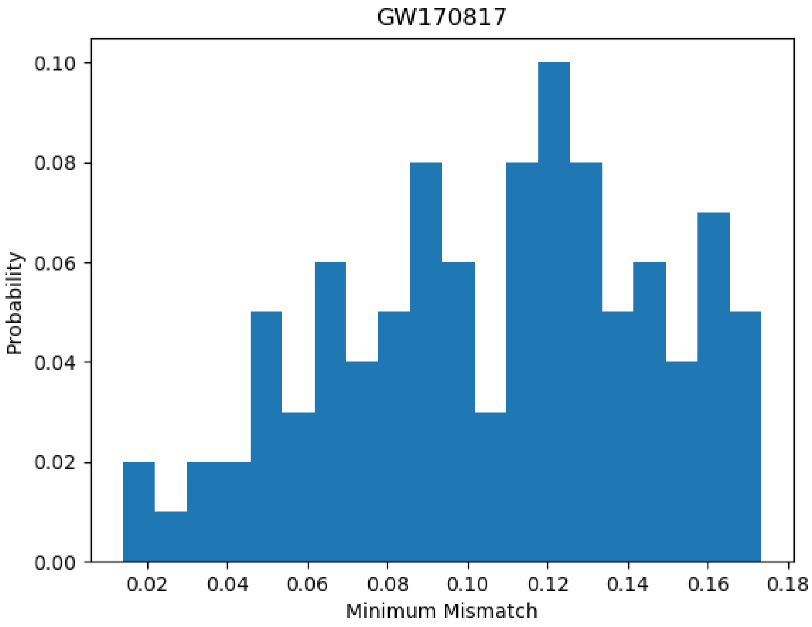
Yay! We have a new signal model!

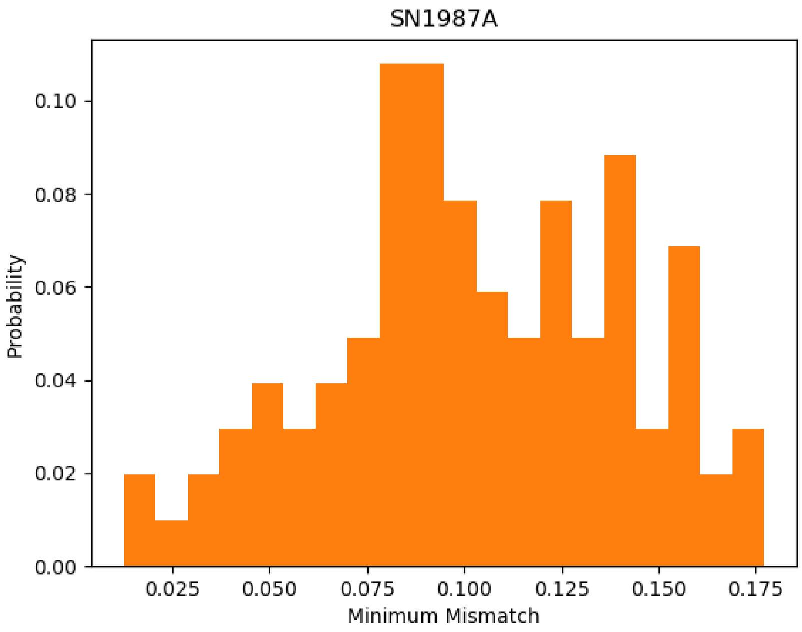
- How strongly does the method respond to a signal?
- And how sensitive is this method?

Strength of method response to a signal

- Can use a mismatch histogram
- The mismatch is ‘how far’ a template is from our signal parameters
- Fractional loss in signal to noise ratio of an exact match to a given template
- The lowest mismatch should always be below μ_{Max} (set before search)

Mismatch Histograms





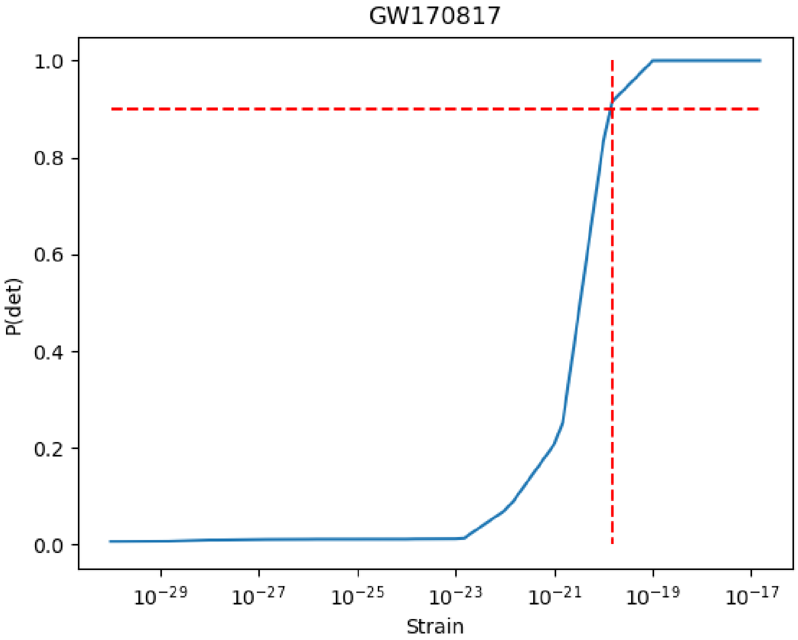
Average Lowest Mismatch: ~0.11

Average Lowest Mismatch: ~0.1

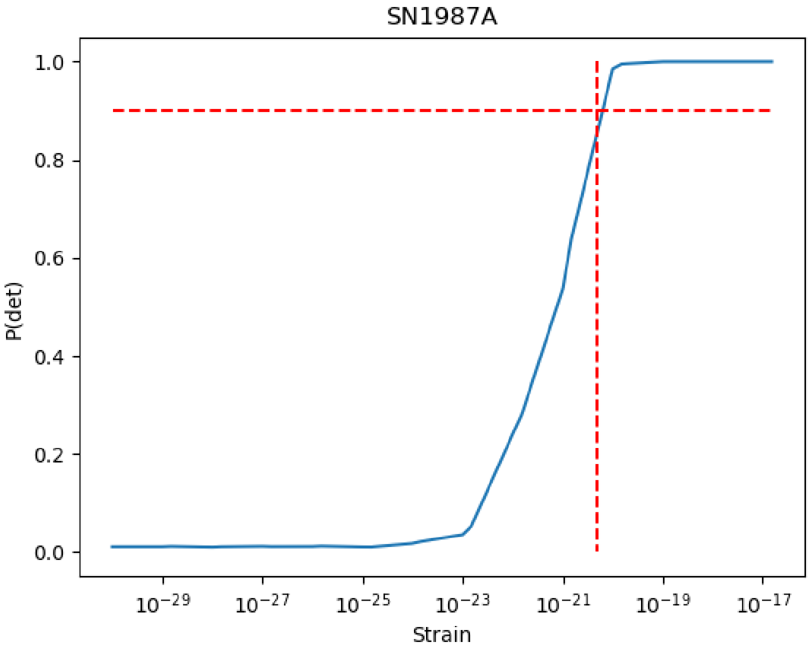
How sensitive is the method?

- Use detection probability
- Indicator of what signal strength we are sensitive to
- Quoted as the signal strength at which 90% of all templates have a detection statistic above a given threshold

Detection Probabilities



$$P(h_0 = 10^{-20}) = 90\%$$



$$P(h_0 = 5 \times 10^{-21}) = 90\%$$

At the moment

- Paper describing the method underway
- Simulations on data currently being tested
- Gravitational wave follow up of GW170817
- Gravitational wave search for 1987A supernova remnant

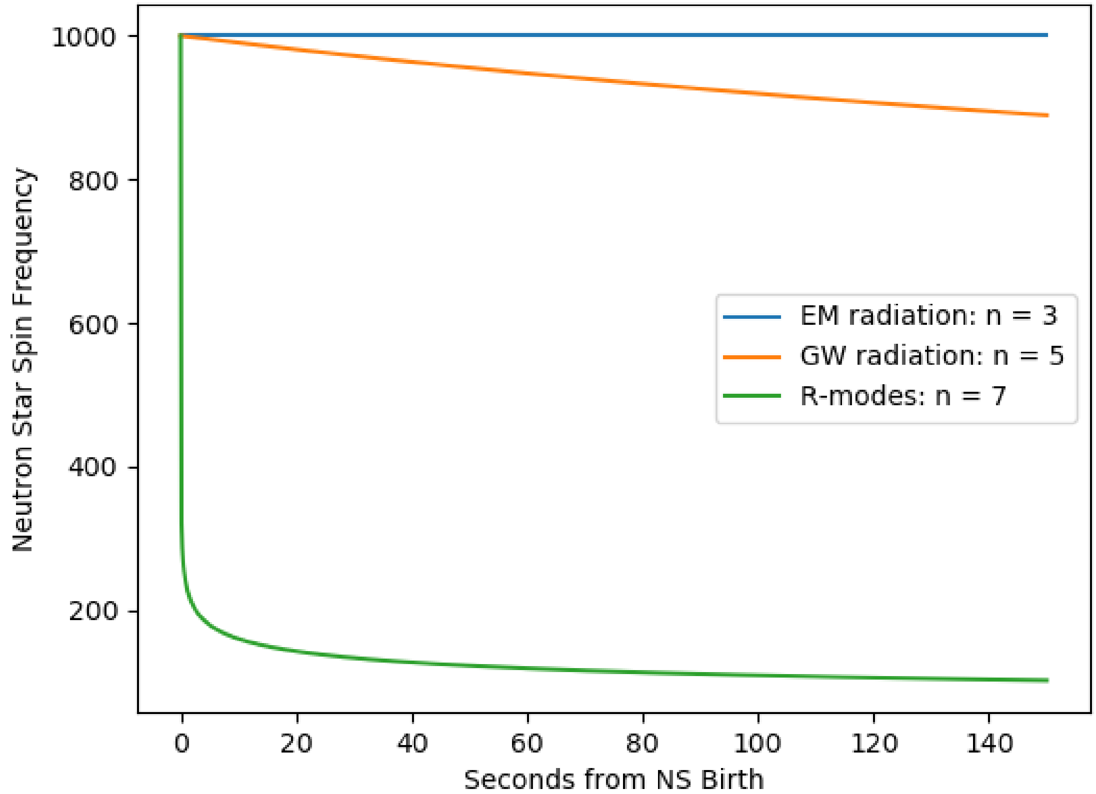
Extra Slides

Creating a Parameter Space

- What signals do we expect to see?
- What is physically interesting?
- Is it computationally feasible?

The General Torque Equation (GTE)

$$\dot{f} = -k f^n$$



What is Physically Interesting?

- Use the General Torque Equation to inform us

Parameter Constraints from the GTE

Parameters should follow the GTE

$$GTE\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1+n_{tol}), k_{i-1}(1+k_{tol}), \Delta p_i\right) \leq \mathfrak{f}_{i,0} \leq GTE\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1-n_{tol}), k_{i-1}(1-k_{tol}), \Delta p_i\right)$$
$$GTE'\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1+n_{tol}), k_{i-1}(1+k_{tol}), \Delta p_i\right) \leq \mathfrak{f}_{i,1} \leq GTE'\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1-n_{tol}), k_{i-1}(1-k_{tol}), \Delta p_i\right)$$
$$GTE''\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1-n_{tol}), k_{i-1}(1-k_{tol}), \Delta p_i\right) \leq \mathfrak{f}_{i,2} \leq GTE''\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1+n_{tol}), k_{i-1}(1+k_{tol}), \Delta p_i\right)$$

Braking index and k values should fall within a global range

$$n_{min}\frac{\mathfrak{f}_{i,1}^2}{\mathfrak{f}_{i,0}} \leq \mathfrak{f}_{i,2} \leq n_{max}\frac{\mathfrak{f}_{i,1}^2}{\mathfrak{f}_{i,0}}$$
$$\frac{\mathfrak{f}_{i,1}^2 \log\left(-\frac{\mathfrak{f}_{i,1}}{k_{i-1}(1+k_{tol})}\right)}{\mathfrak{f}_{i,0}\log\left(\mathfrak{f}_{i,0}\right)} \leq \mathfrak{f}_{i,2} \leq \frac{\mathfrak{f}_{i,1}^2 \log\left(-\frac{\mathfrak{f}_{i,1}}{k_{i-1}(1-k_{tol})}\right)}{\mathfrak{f}_{i,0}\log\left(\mathfrak{f}_{i,0}\right)}$$

Braking index and k values should be allowed to evolve over time

$$-k_{i-1}(1+k_{tol})\mathfrak{f}_{i,0}^{n_{i-1}(1+n_{tol})} \leq \mathfrak{f}_{i,1} \leq -k_{i-1}(1-k_{tol})\mathfrak{f}_{i,0}^{n_{i-1}(1-n_{tol})}$$
$$n_{i-1}(1-n_{tol})\frac{\mathfrak{f}_{i,1}^2}{\mathfrak{f}_{i,0}} \leq \mathfrak{f}_{i,2} \leq n_{i-1}(1+n_{tol})\frac{\mathfrak{f}_{i,1}^2}{\mathfrak{f}_{i,0}}$$

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Parameter Constraints from the GTE

Parameters should follow the GTE

$$GTE\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1+n_{tol}), k_{i-1}(1+k_{tol}), \Delta p_i\right) \le \mathfrak{f}_{i,0} \le GTE\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1-n_{tol}), k_{i-1}(1-k_{tol}), \Delta p_i\right)$$

$$GTE'\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1+n_{tol}), k_{i-1}(1+k_{tol}), \Delta p_i\right) \le \mathfrak{f}_{i,1} \le GTE'\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1-n_{tol}), k_{i-1}(1-k_{tol}), \Delta p_i\right)$$

$$GTE''\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1-n_{tol}), k_{i-1}(1-k_{tol}), \Delta p_i\right) \le \mathfrak{f}_{i,2} \le GTE''\left(\mathfrak{f}_{i-1,0}, n_{i-1}(1+n_{tol}), k_{i-1}(1+k_{tol}), \Delta p_i\right)$$

B[illegible]ing index and k values should fall within a glo[illegible] range

$$n_{min}\frac{\mathfrak{f}_{i,1}^2}{\mathfrak{f}_{i,0}} \le \mathfrak{f}_{i,2} \le n_{m}\text{[illegible]}$$

$$\frac{\mathfrak{f}_{i,1}^2 \log\left(-\frac{\text{[illegible]}}{k_{i-1}(1\text{[illegible]}}\right)}{\mathfrak{f}_{i,0}\text{[illegible]}} \text{[illegible]} \frac{\text{[illegible]}\log\left(-\frac{\mathfrak{f}_{i,1}}{k_{i-1}(1-k_{tol})}\right)}{\text{[illegible]}\log(\mathfrak{f}_{i,0})}$$

Braking i[illegible]k values should be all[illegible]lve over time

$$\text{[illegible]}k_{i-1}(1+k_{tol})\mathfrak{f}_{i,0}^{n_{i-1}(1+n_{tol})} \le \mathfrak{f}_{i,1} \le -k_{i-1}(1-k_{tol})\mathfrak{f}_{i,0}^{n_{i-1}\text{[illegible]}}$$

$$n_{i-1}(1-n_{tol})\frac{\mathfrak{f}_{i,1}^2}{\mathfrak{f}_{i,0}} \le \mathfrak{f}_{i,2} \le n_{i-1}(1+n_{tol})\frac{\mathfrak{f}_{i,1}^2}{\mathfrak{f}_{i,0}}$$

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Templates

	Cass A	1987A	GW170817
fmin	100	100	500
fmax	300	500	1000
nmin	2	2	2
nmax	7	5	7
Duration	8 days	30 days	6 hours
Templates	9.6×10^{11}	8.6×10^{10}	1.1×10^{10}

Templates

	Cass A	1987A	GW170817
fmin	100	100	500
fmax	300	500	1000
nmin	2	2	2
nmax	7	$5 \rightarrow 7$	$7 \rightarrow 5$
Duration	$8 \rightarrow 100$ days	$30 \rightarrow 100$ days	$6 \rightarrow 12$ hours
Templates	???	???	???

$$f_j = \sum_{k=1}^{K} \mathfrak{f}_{j,k} B_{j,k}^{s}(t) + \mathfrak{f}_{j+1,k} B_{j,k}^{e}(t)$$

$$f_{j+1} = \sum_{k=1}^{K} \mathfrak{f}_{j+1,k} B_{j+1,k}^{s}(t) + \mathfrak{f}_{j+2,k} B_{j+1,k}^{e}(t)$$

$$f_{j+2} = \sum_{k=1}^{K} \mathfrak{f}_{j+2,k} B_{j+2,k}^{s}(t) + \mathfrak{f}_{j+3,k} B_{j+2,k}^{e}(t)$$

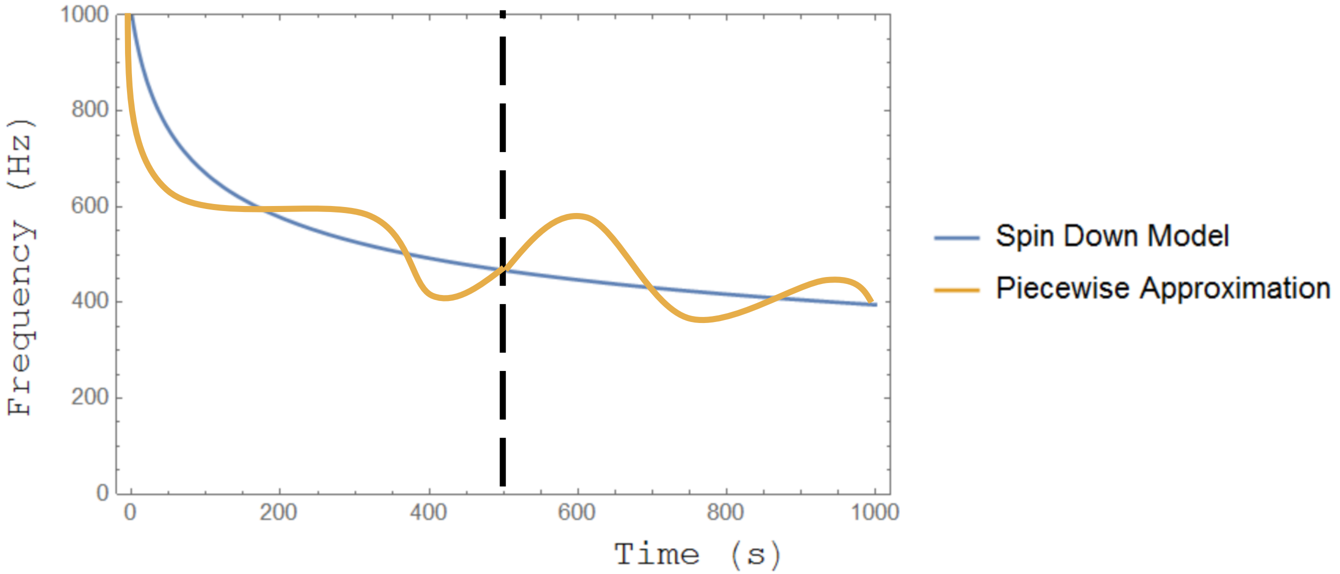
Requirements on our $B_{j,k}^{s/e}$

$$\delta_k^l = \frac{\partial^l}{\partial t^l} B_{j,k}^s(p_j)$$

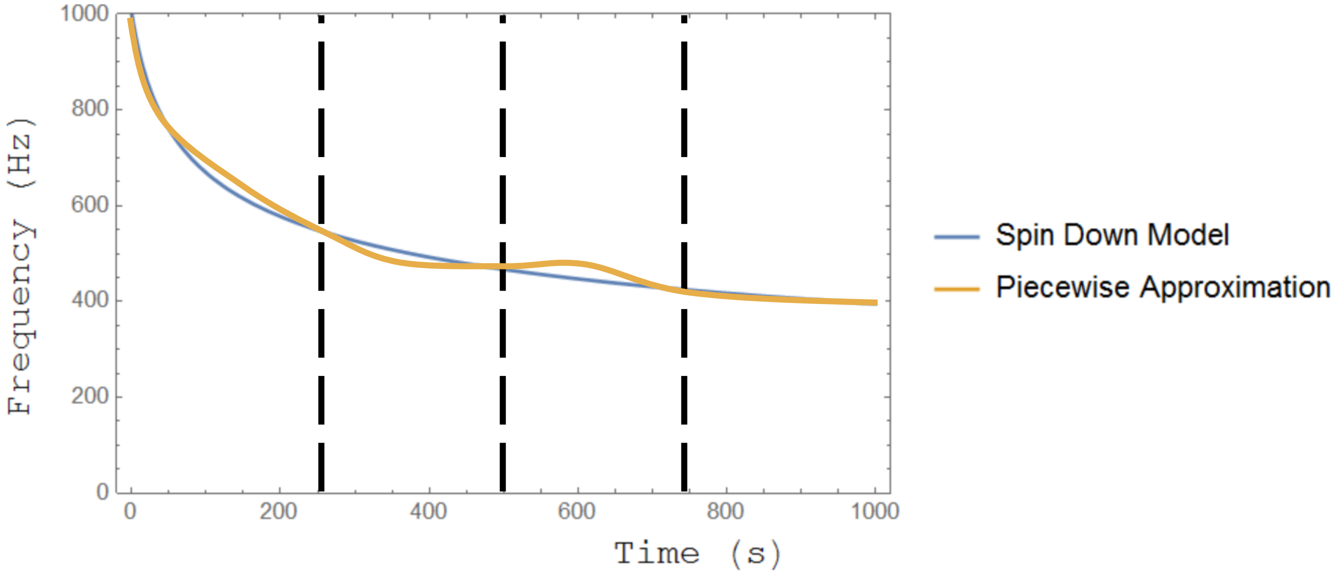
$$0 = \frac{\partial^l}{\partial t^l} B_{j,k}^s(p_{j+1})$$

$$0 = \frac{\partial^l}{\partial t^l} B_{j,k}^e(p_j)$$

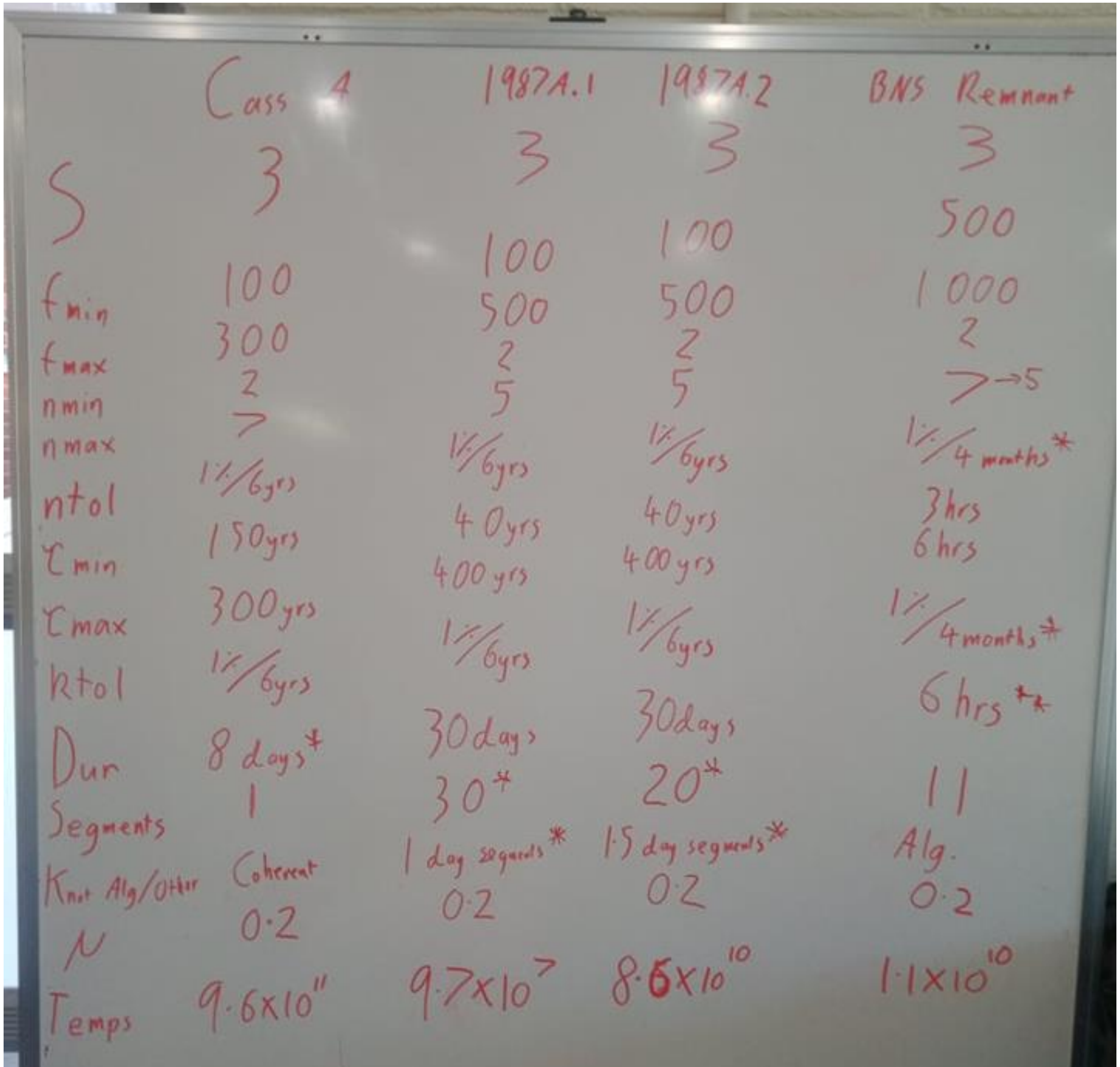
$$\delta_k^l = \frac{\partial^l}{\partial t^l} B_{j,k}^e(p_{j+1})$$



- Not accurate enough
- Small parameter space



- Accurate
- Large parameter space

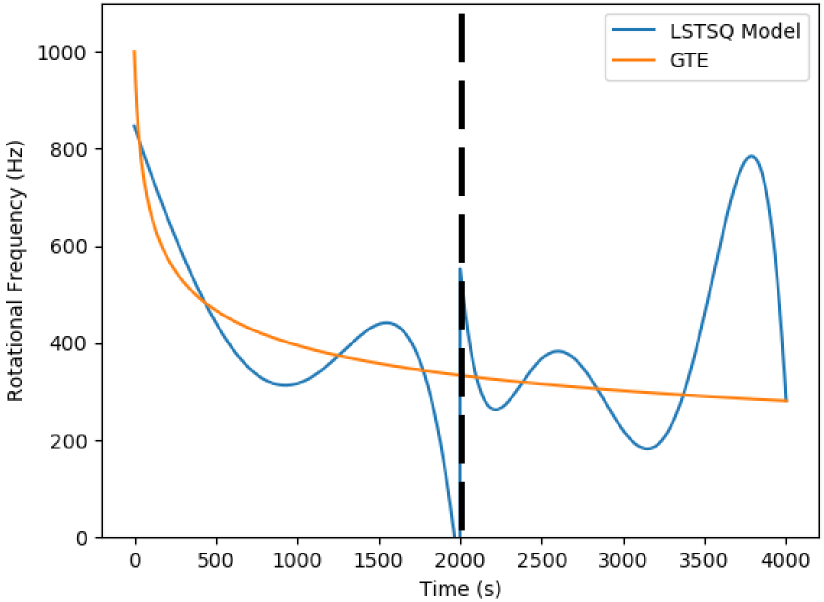


Cass 4
1987A.1
1987A.2
BNS Remnant
S
fmin
fmax
nmin
nmax
ntol
τmin
τmax
ktol
Dur
Segments
Knot Alg/Other
N
Temps
3
100
300
2
7
1%/6yrs
150yrs
300yrs
1%/6yrs
8 days*
1
Coherent
0·2
9·6x10¹¹
3
100
500
2
5
1%/6yrs
40yrs
400yrs
1%/6yrs
30days
30*
1 day segments*
0·2
9·7x10⁷
3
100
500
2
5
1%/6yrs
40yrs
400yrs
1%/6yrs
30days
20*
1·5 day segments*
0·2
8·6x10¹⁰
3
500
1000
2
7→5
1%/4 months*
3hrs
6hrs
1%/4 months*
6 hrs**
11
Alg.
0·2
1·1x10¹⁰

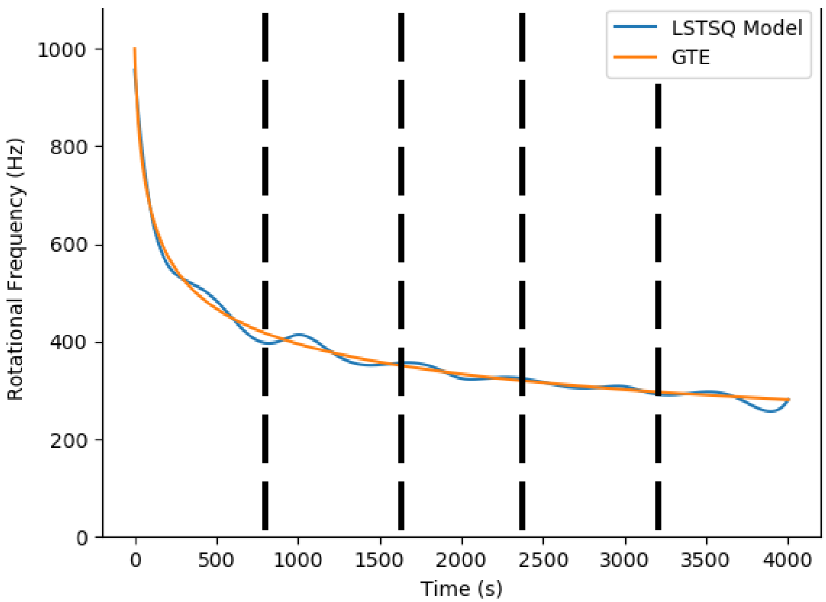
Piecewise Knots

$$f(t) = \begin{cases} f_0 & p_0 \leq t < p_1 \\ f_1 & p_1 \leq t < p_2 \\ \dots \end{cases}$$

- Longer piecewise segments means a smaller parameter space
- Shorter piecewise segments are more accurate



- Not accurate enough
- Smaller parameter space

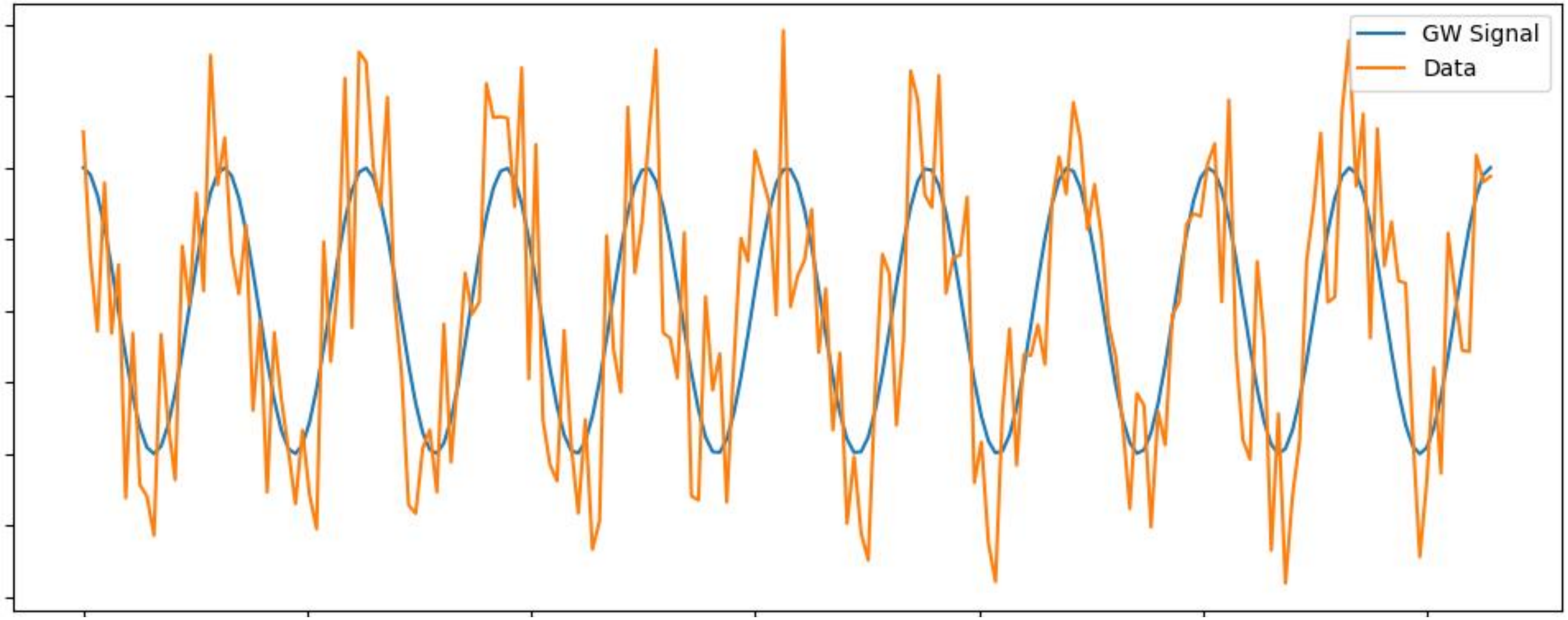


- Accurate
- Larger parameter space

Other knot considerations

- How accurate does our model need to be?
- Does knot choice change for different astrophysical sources?
- Each PW segment is a semi-coherent search segment

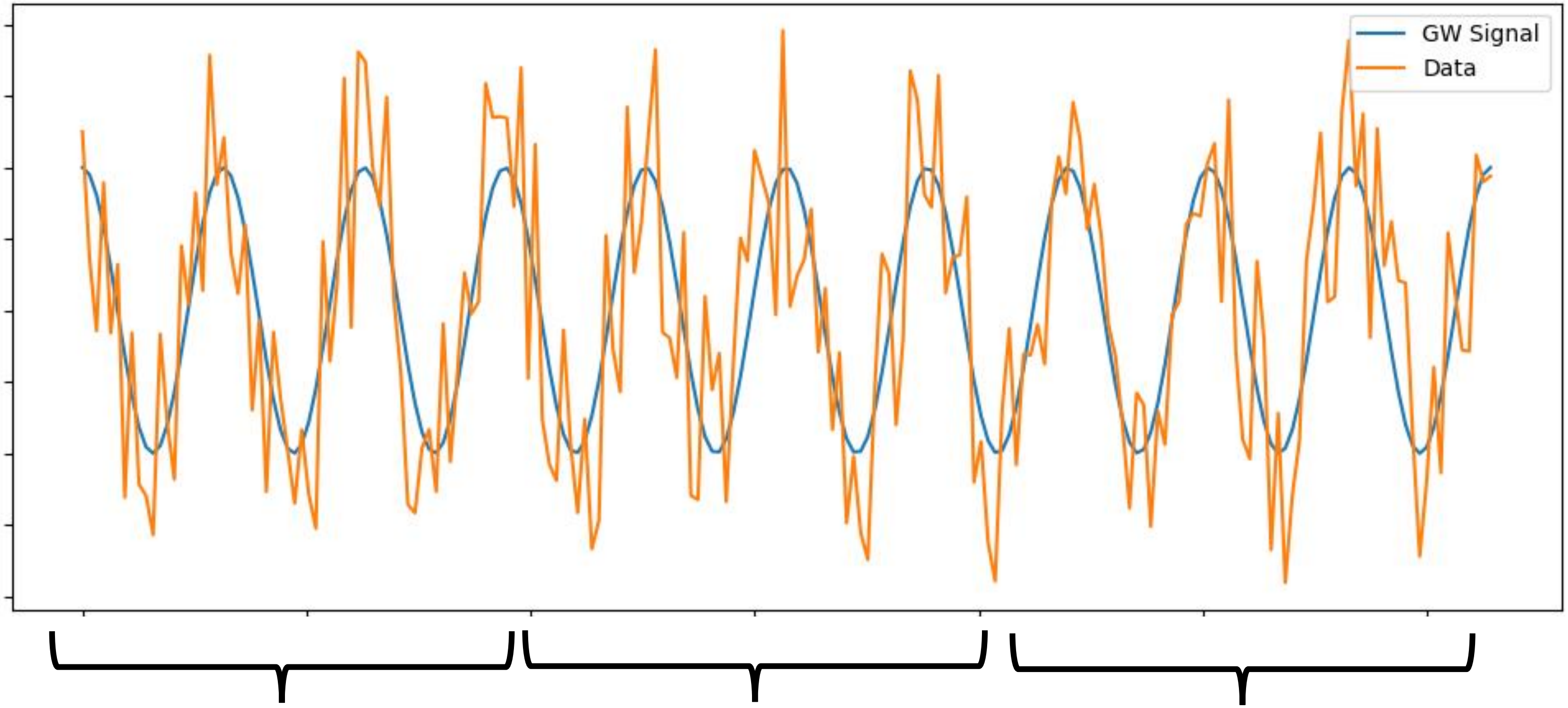
Coherent Searches



$$2\mathscr{F}$$

Calculate detection statistic using entire data segment

Semi-Coherent Searches



$$2\mathscr{F} = \frac{1}{3}\left(2\mathscr{F} + 2\mathscr{F} + 2\mathscr{F}\right)$$

Detection statistic is the average of detection statistics of each segment

Coherent

- More Sensitive
- More computationally expansive

Semi-Coherent

- Less Sensitive
- Less computationally expansive

For large data sets, semi-coherent searches are more sensitive for the same computational cost

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We will use the piecewise segments as individual semi-coherent segments

Model Accuracy

$$\max_{t \in [0,T]} |f_{GW}(t) - f_{Model}(t)| \leq \frac{1}{T}$$

For Our Piecewise Model

$$\max_{t \in [p_i, p_{i+1}]} |f_{GW}(t) - f_{PW}(t)| \leq \frac{1}{p_{i+1} - p_i}$$

Calculating knots

- The largest value a knot p_{i+1} can have occurs when

$$0 = \max_{t \in [p_i, p_{i+1}]} |f_{GW}(t) - f_{PW}(t)| - \frac{1}{p_{i+1} - p_i}$$