Horizon Singularities and Energy Momentum Tensor Classification

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Outline

Introduction

Framework: Semi-Classical gravity $\rightarrow$ Spacetime described by a metric and classical notions such as particles' trajectories and horizons are meaningful.

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Two Categories of Black Holes:

- **Mathematical Black Hole (MBH):** Solution of Einstein equations containing a singularity and an event horizon.
- **Physical Black Hole (PBH):** Trapped region of spacetime bounded by an apparent horizon. They can be Regular Black Holes (RBHs) without event horizon or singularity or may overlap, or be contained in an MBH.

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- **Event Horizon (EH):** Global Teleological entity which is even in principle physically unobservable! One needs to know the entire history of the universe![1]

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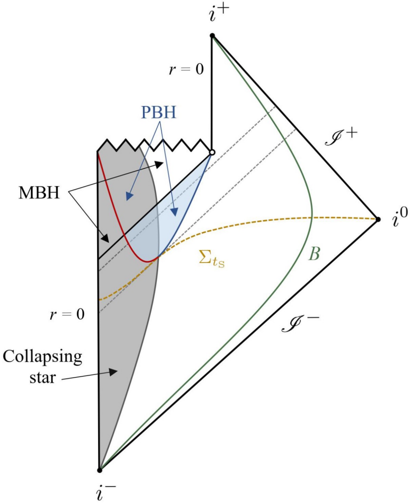
Both timelike and null particles can escape from the quantum ergosphere region!

↓

Useful to study their motion when it is possible to cross the AH.



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Self-Consistent Model

The model we are using for the description of spherically symmetric PBHs is based on two assumptions:[2]

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Semi-Classical Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

where $T_{\mu\nu} := \langle \hat{T}_{\mu\nu} \rangle_\omega$ expectation value of renormalized Energy Momentum Tensor in a quantum state ω describing **both** collapsing matter and produced excitations (**Joint Treatment**)

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Self-Consistent Model

The above assumptions lead to the main solution ($k = 0$) that describes evaporation but there are more, which play role at the formation stage.[3,4]

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The metric can be written in both Schwarzchild (t,r) coordinates and advanced coordinates (v,r).

$$ds^2 = -e^{2h(t,r)} f(t,r) dt^2 + f^{-1}(t,r) dr^2 + r^2 d\Omega^2$$

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with $x = r - r_g(t)$ and

$$f(t,r) = 1 - \frac{C(t,r)}{r}$$

$$C(t,r) = r_g(t) + c_{12}\sqrt{x} + c_1 x + \mathcal{O}\left(x^{3/2}\right), \; c_{12} = -4\sqrt{\pi} r^{3/2} Y < 0$$

$$h(t,r) = -\frac{1}{2} \ln \frac{x}{\xi} + \mathcal{O}(x)$$

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Important Features of the Model:

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Regularity Conditions

$$T_{00} = e^{2h(t,r)}\left(-\Upsilon^2(t) + e_{12}\sqrt{x} + e_1 x + \cdots\right)$$

$$T_{01} = f^{-1}(t,r)e^{h(t,r)}\left(-\Upsilon^2(t) + \varphi_{12}\sqrt{x} + \varphi_1 x + \cdots\right)$$

$$T_{11} = f^{-2}(t,r)\left(-\Upsilon^2(t) + p_{12}\sqrt{x} + p_1 x + \cdots\right)$$

Energy Momentum Tensor Classification

The EMT describing the self-consistent model in the orthonormal frame with the Schwarzchild coordinate description (t, r), has the following form,

$$T_{\hat{a}\hat{b}} = \begin{pmatrix} q+\mu_1 & q+\mu_2 & 0 & 0 \\ q+\mu_2 & q+\mu_3 & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

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where $x = r - r_g(t)$ and

- $q = \frac{r_g Y^2}{c_{12}\sqrt{x}} < 0$ (**Divergent Term!**)
- $\mu_1 = -\frac{r_g e_{12}}{c_{12}} + \frac{r_g^2(1-c_1)Y^2}{c_{12}^2} + k\sqrt{x} + \mathcal{O}(x)$
- $\mu_2 = -\frac{r_g \varphi_{12}}{c_{12}} + \frac{r_g^2(1-c_1)Y^2}{c_{12}^2} + \lambda\sqrt{x} + \mathcal{O}(x)$
- $\mu_3 = -\frac{r_g p_{12}}{c_{12}} + \frac{r_g^2(1-c_1)Y^2}{c_{12}^2} + \tilde{k}\sqrt{x} + \mathcal{O}(x)$

} **Finite Terms**

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The Hawking-Ellis Classification of the EMT is determined by the Lorentz invariant eigenvalues

$$Det(T_{\hat{a}\hat{b}} - \lambda \eta_{\hat{a}\hat{b}}) = 0$$

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We can directly determine that two of the eigenvalues are (P, P) and for the other two, it is necessary to solve the characteristic equation

$$\lambda^2 + (\mu_1 - \mu_3)\lambda + (\mu_2^2 - \mu_1\mu_3) + q(2\mu_2 - \mu_1 - \mu_3) = 0$$

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$\Delta(0) > 0 \rightarrow$ **Type I**

Only if the EMT behaves according to semi-classical analysis![5]

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NEC Violation$\rightarrow$ $T_{\hat{a}\hat{b}} l^{\hat{a}}_{out} l^{\hat{b}}_{out} = 4q + 4\mu_2 + \mathcal{O}\left(\sqrt{x}\right) < 0$

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First part can be written as $(q+\mu_2)k^{\hat{a}}k^{\hat{b}}$ where $k^{\hat{a}} = (1, -1, 0, 0)$ is the ingoing null vector. We have a negative energy density null dust or radiation going inside as seen by a static observer.$\rightarrow$ **Massless**

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Two fluid description!

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Two different observers:

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- Alice: The moving observer

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Eve, static at radial distance R, will observe energy density ρ_E given by the component $T_{\hat{0}\hat{0}}$ of the EMT in the orthonormal frame!

$$\rho_E = \frac{r_g Y^2}{c_{12}\sqrt{X}} + \mu_2 \text{ with } X = R - r_g$$

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$$\lim_{R \to r_g} \rho_E = -\infty$$

Eve observes divergent negative energy density on the apparent horizon!

Horizon Singularities

We study three different cases of Alice's motion, using the (v, r) coordinate form of the metric, with normalized 4-velocity given by $u_A^\mu = \left(\dot{V}, \dot{R}, 0, 0\right)$ and the observed energy density given by

$$\rho_A = T_{\mu\nu} u_A^\mu u_A^\nu$$

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What does Alice see when she tries to cross the apparent horizon?

For the **freely falling** case we have

$$\dot{V} = \frac{\dot{R} + \sqrt{\dot{R}^2 + f}}{e^{h_+} f} \simeq -\frac{e^{-h_+}}{2\dot{R}} + \mathcal{O}(f) \Rightarrow \rho_A : \textbf{Finite!}$$ [6]

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Horizon Singularities

For the case where Alice follows an **outgoing geodesic inside the quantum ergosphere of the PBH**, we have that the only way to escape is by making a transition to an ingoing trajectory and let the apparent horizon overtake[7]

$$\dot{V} = \frac{\dot{R} - \sqrt{\dot{R}^2 + f}}{e^h + f} \xrightarrow[\textit{Transition}]{} \dot{V} = \frac{\dot{R} + \sqrt{\dot{R}^2 + f}}{e^h + f}$$

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So Alice crosses the apparent horizon during the ingoing phase and thus observing

ρ_A : **Finite!**

Also the tidal forces experienced by her are finite too!

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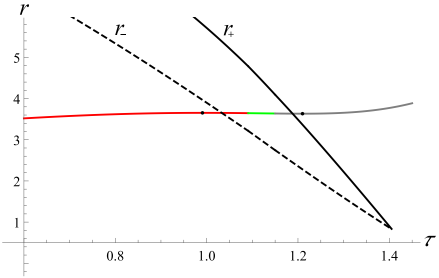

Figure: Escaping Particle's Trajectory from Regular Black Hole.

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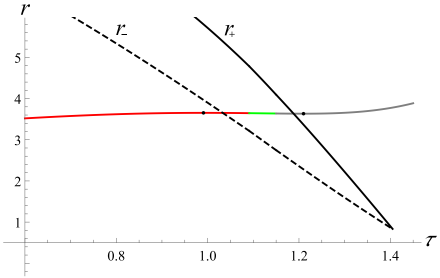

Figure: Escaping Particle's Trajectory from Regular Black Hole.

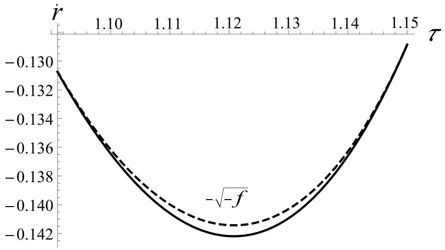

Figure: Radial velocity on the outgoing segment of the trajectory inside the trapped region.

Horizon Singularities

For the case where Alice follows an **outgoing non-geodesic** trajectory inside the quantum ergosphere of the PBH we consider the maximum allowed radial velocity $\dot{R} = -\sqrt{-f}$ which leads to

$$\dot{V} = \frac{e^{-h_+}}{\sqrt{-f}} \Rightarrow \rho_A \to -\infty \Rightarrow \textbf{Firewall}$$

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Also leads to divergent tidal forces, destroying any observer trying to go outside!

Summary

Main Results:

- NEC violating Energy Momentum Tensor, Type I, describing at least two fluids!
- Apparent Horizon has mildly singular features manifesting themselves as a firewall!

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Thank you!