

# Black holes, white holes, wormholes: geometry and physics

12.12.2022  
14:45

Daniel Terno

School of Mathematical & Physical Sciences



**MACQUARIE  
University**  
Research Centre for  
Astronomy, Astrophysics &  
Astrophotonics



AIP

**AUSTRALIAN INSTITUTE OF PHYSICS CONGRESS**

11-16 December 2022  
Adelaide Convention Centre

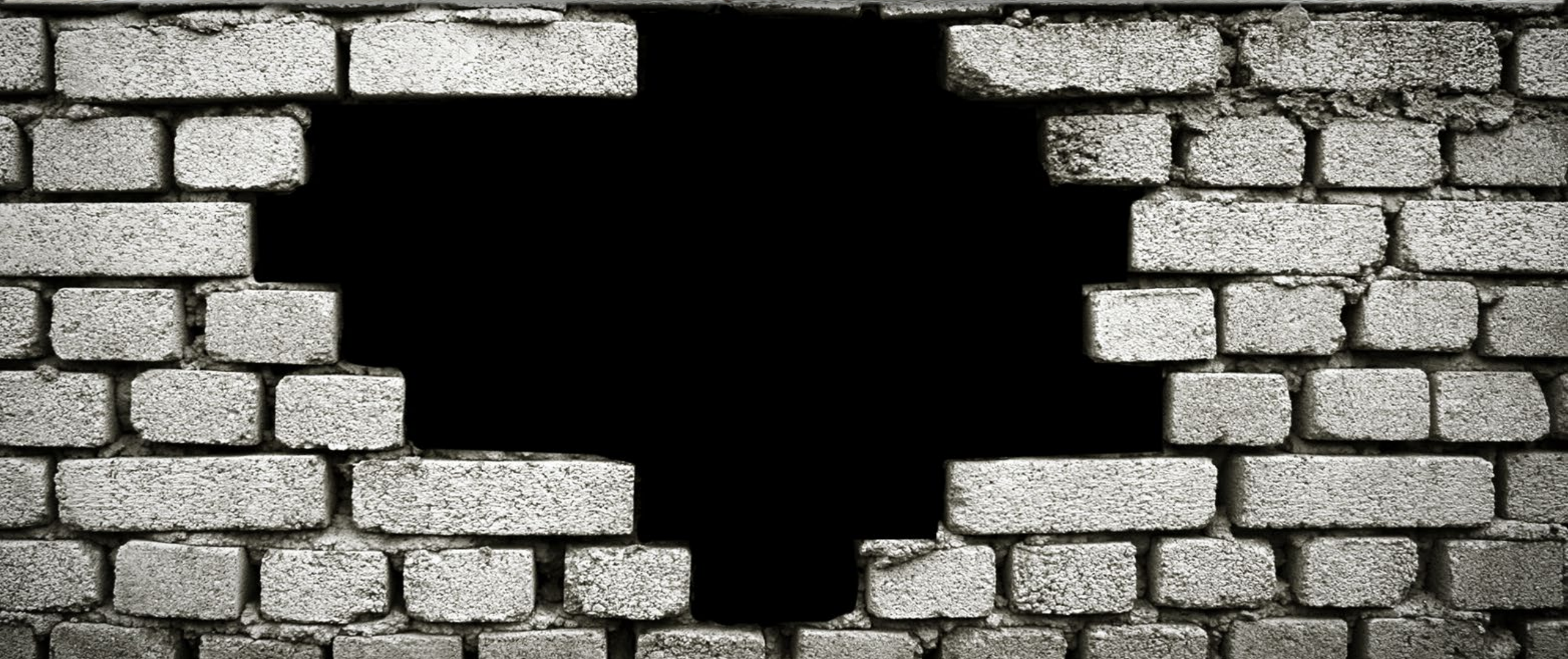




Mann, Murk, DRT,

*Black holes and their horizons in semiclassical  
and modified theories of gravity*

Int J Mod Phys D **31**, 2230015 (2022) [arXiv:2112.06515](https://arxiv.org/abs/2112.06515)





Mann, Murk, DRT,

*Black holes and their horizons in semiclassical  
and modified theories of gravity*

Int J Mod Phys D **31**, 2230015 (2022) [arXiv:2112.06515](https://arxiv.org/abs/2112.06515)

## Outline

- Existence of horizons as a math question
- Admissible solutions, their properties, and the holes they describe
  - Implications for black holes
    - Implications for wormholes

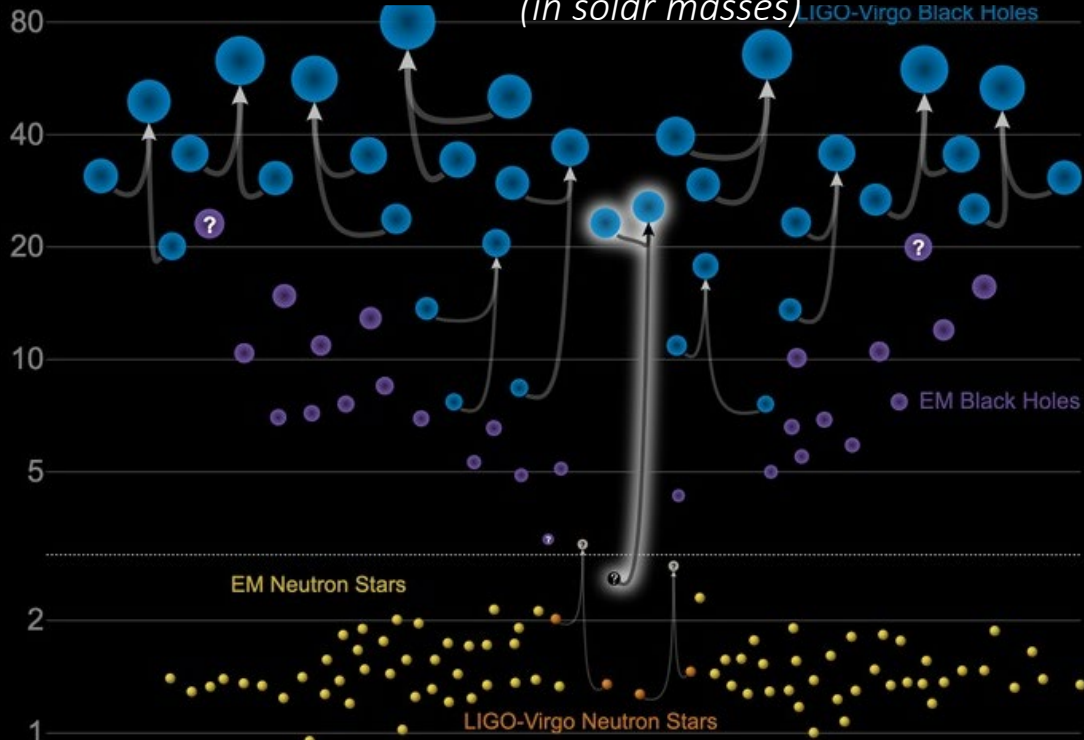
*What's next?*



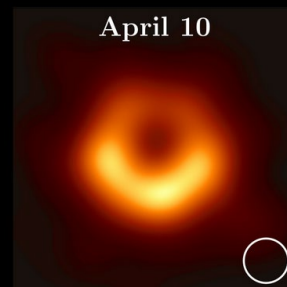
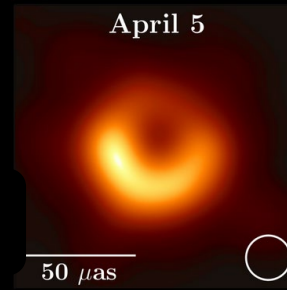
# astrophysical black holes

## known knowns

Masses in the stellar graveyard  
(in solar masses)



Updated 2020-05-16  
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern



EHT 2019

## known unknowns

IOP Publishing

Classical and Quantum Gravity

Class. Quantum Grav. 36 (2019) 143001 (178pp)

<https://doi.org/10.1088/1361-6382/ab0587>

Topical Review

### Black holes, gravitational waves and fundamental physics: a roadmap

Leor Barack<sup>1</sup>, Vitor Cardoso<sup>2,3,133</sup>, Samaya Nisanke<sup>4,5,6</sup>,  
Thomas P Sotiriou<sup>7,8</sup> (editors), Abbas Askar<sup>9,10</sup>,  
Chris Belczynski<sup>9</sup>, Gianfranco Bertone<sup>5</sup>, Edi Bon<sup>11,12</sup>,  
Diego Blas<sup>13</sup>, Richard Brito<sup>14</sup>, Tomasz Bulik<sup>15</sup>,  
Clare Burrage<sup>8</sup>, Christian T Byrnes<sup>16</sup>, Chiara Caprini<sup>17</sup>,

#### 4. The nature of compact objects

##### 4.1. No-hair theorems

##### 4.2. Non-Kerr black holes

4.2.1. Circumventing no-hair theorems.

4.2.2. Black holes with scalar hair.

4.2.3. Black hole scalarization.

4.2.4. Black holes in theories with vector fields and massive/bimetric theories.

4.2.5. Black holes in Lorentz-violating theories.

##### 4.3. Horizonless exotic compact objects

##### 4.4. Testing the nature of compact objects

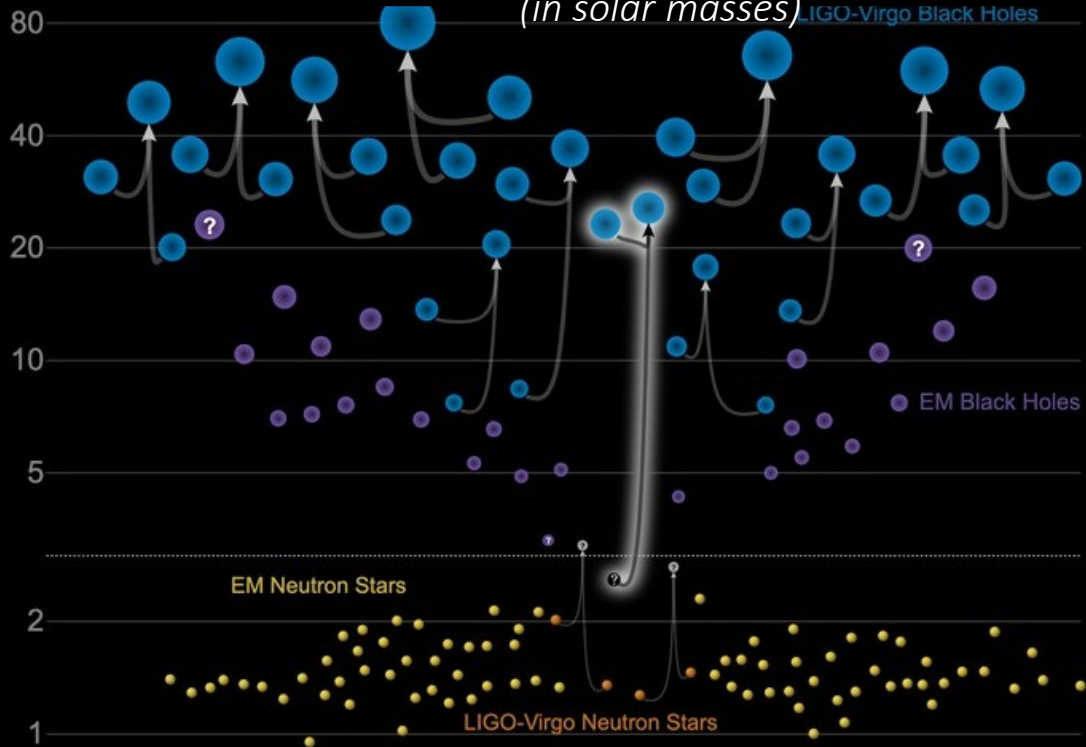
4.4.1. EM diagnostics.

4.4.2. GW diagnostics.

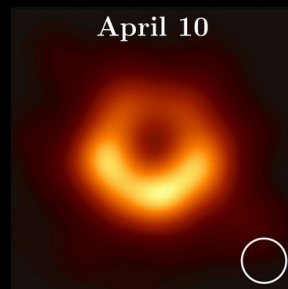
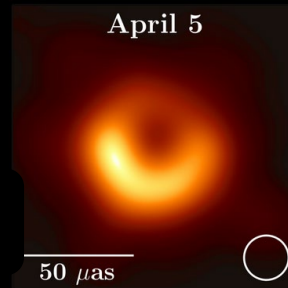
# astrophysical black holes

## known knowns

Masses in the stellar graveyard  
(in solar masses)



Updated 2020-05-16  
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern



## known unknowns

*Many definitions of a black hole:*

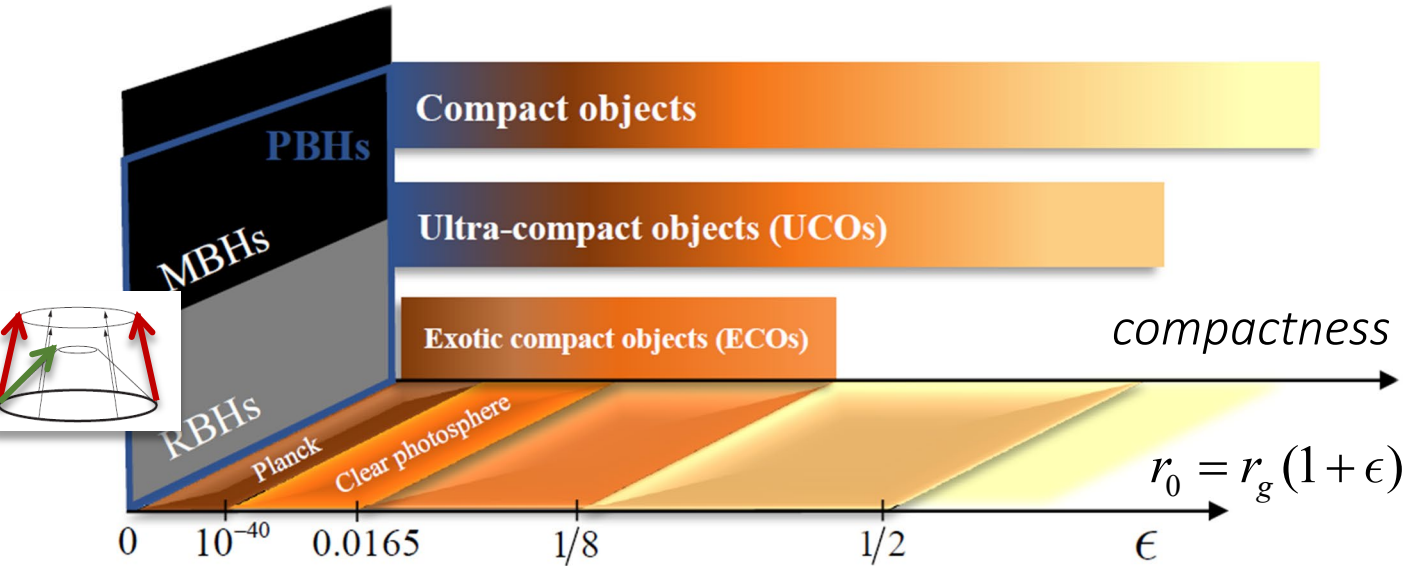
Curiel, Nature Astr. **3**, 27 (2019)

Frolov, arXiv:/gr-qc1411.6981

**physical black hole (PBH) =**  
a trapped spacetime region  
*[that has been formed at a finite time  
of a distant observer]*

# ultra-compact objects

## the zoo & physical black holes



UCO: has a photosphere

BH: has a horizon

MBH: has an event horizon

PBH: has a trapped region

ECO: non-BH UCO

*Why ECOs are called exotic?*

Buchdal's theorem  $\epsilon > 1/8$

~~$\rho > 0, p > 0$~~

~~GR~~

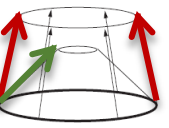
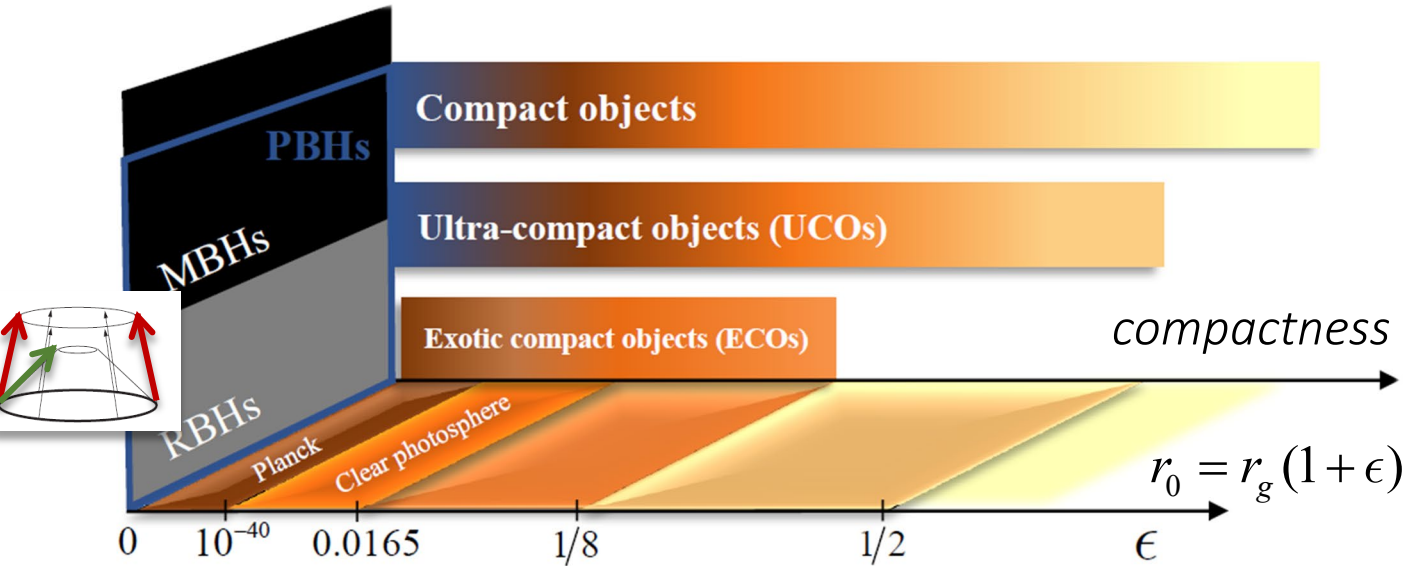
Exotic matter

- ◆ Modified
- ◆ Semiclassical
- ◆ Quantum gravity



# ultra-compact objects

## the zoo & physical black holes



- [1] Perpetual ongoing collapse, with an asymptotic horizon  $\epsilon \rightarrow 0$
- [2] Formation of a transient or an asymptotic object, where the compactness reaches a minimum at some finite asymptotic [=distant observer] time  $\epsilon \rightarrow \epsilon_{\min}$
- [3] Formation of an apparent horizon in finite asymptotic time  $\epsilon(t_f) = 0$

UCO: has a photosphere

BH: has a horizon

MBH: has an event horizon

PBH: has a trapped region

ECO: non-BH UCO

*Why ECOs are called exotic?*

Buchdal's theorem  
 $\epsilon > 1/8$

~~$\rho > 0, p > 0$~~

~~GR~~

Exotic matter


- ◆ Modified
- ◆ Semiclassical
- ◆ Quantum gravity

Cardoso and Pani, Nat. Astron. **1**, 586 (2017)

**assumptions**

1. The classical spacetime structure is still meaningful and is described by a metric  $g_{\mu\nu}$ .
2. Classical concepts, such as trajectory, event horizon or singularity can be used.
3. The metric is modified by quantum effects. The resulting curvature satisfies the semiclassical self-consistent equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \left\langle \hat{T}_{\mu\nu} \right\rangle_{\omega}$$

  $+E_{\mu\nu}$

4. Dynamics of the collapsing matter is still described classically using the self-consistent metric

**semiclassical physics**

**not assumed:** global structure, singularity, types of fields, quantum state, presence of Hawking radiation



## assumptions

1. The classical spacetime structure is still meaningful and is described by a metric  $g_{\mu\nu}$ .
2. Classical concepts, such as trajectory, event horizon or singularity can be used.
3. The metric is modified by quantum effects. The resulting curvature satisfies the semiclassical self-consistent equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \left\langle \hat{T}_{\mu\nu} \right\rangle_{\omega} + E_{\mu\nu}$$

4. Dynamics of the collapsing matter is still described classically using the self-consistent metric

## semiclassical physics

**not assumed:** global structure, singularity, types of fields, quantum state, presence of Hawking radiation

## physical black holes

- (i) a light-trapping region forms at a **finite** time of a distant observer
- (ii) curvature scalars [contractions of the Riemann tensor] are **finite** on the boundary of the trapped region

### *Worked-out consequences:*

- Spherical symmetry
- Kerr-Vaidya metrics

# spherical symmetry: recap + some results

---

## **PBH: the process**

- Use Schwarzschild coordinates to extract the info from divergencies
- Pick a nice form of the Einstein equations. Demand existence of real solutions
- Use null coordinates to help classification



# spherical symmetry: recap + some results

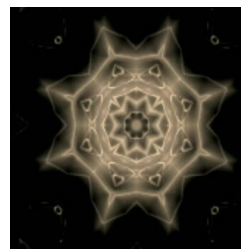
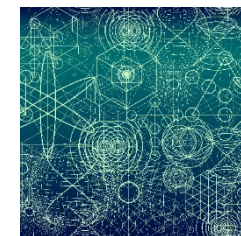
---

## PBH: the process

- Use Schwarzschild coordinates to extract the info from divergencies
- Pick a nice form of the Einstein equations. Demand existence of real solutions
- Use null coordinates to help classification

## PBH: the properties

- Finite infall time (according to a distant Bob)
- Collapse of a massive thin shell takes a finite time (according to Bob), but most of the mass remains [?]
- Outer apparent horizon is always **timelike**
- Null energy condition is violated [in the vicinity of the outer horizon]
- A **weak firewall**: energy density for escaping\* non-geodesic Alice diverges, but weakly.
- Inner apparent horizon is timelike or null.
- Some popular RBH models do not work.
- Usual proofs of instability of RBH do not apply



# spherical symmetry

---

## structure

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

□ circumference:  $2\pi r$

□ physical time at infinity:  $t$

$$f = 1 - 2M(t, r)/r$$

$$2M(t, r) \equiv C(t, r)$$

Misner-Sharp invariant mass

Schwarzschild radius

$$\max r_g = C(t, r_g)$$

$$C = r_g(t) + W(t, r) \blacktriangleright$$





# spherical symmetry

## structure

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

- circumference:  $2\pi r$
- physical time at infinity:  $t$

$$f = 1 - 2M(t, r)/r$$

$$2M(t, r) \equiv C(t, r)$$

Misner-Sharp invariant mass

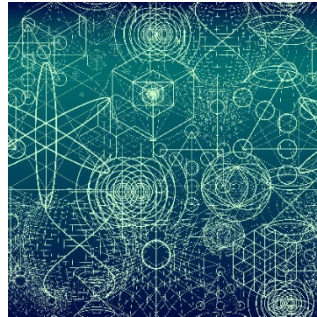
### Schwarzschild radius

$$\max r_g = C(t, r_g)$$

### Curvature scalars

$$T := T^\mu{}_\mu$$

$$\mathfrak{T} = T^{\mu\nu} T_{\mu\nu}$$



$$\mathfrak{T} := ((\tau^r)^2 + (\tau_t)^2 - 2(\tau_t^r)^2) / f^2$$

useful

$$\tau_t := e^{-2h} T_{tt}$$

$$\tau_t^r := e^{-h} T_t^r$$

$$\tau^r := T^{rr}$$

$$T := (\tau^r - \tau_t) / f$$

+regular terms\*

### Einstein equations

$$\partial_r C = 8\pi r^2 \tau_t / f,$$

$$\partial_t C = 8\pi r^2 e^h \tau_t^r,$$

$$\partial_r h = 4\pi r (\tau_t + \tau^r) / f^2$$

$$C = r_g(t) + W(t, r) \blacktriangleright$$



# spherical symmetry

## structure

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

- circumference:  $2\pi r$
- physical time at infinity:  $t$

$$f = 1 - 2M(t, r)/r$$

$$2M(t, r) \equiv C(t, r)$$

Misner-Sharp invariant mass

### Schwarzschild radius

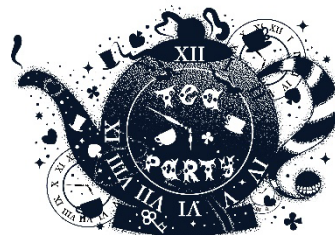
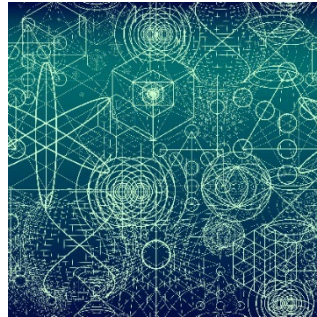
$$\max r_g = C(t, r_g)$$

$$C = r_g(t) + W(t, r) \blacktriangleright$$

### Curvature scalars

$$T := T^\mu{}_\mu$$

$$\mathfrak{T} = T^{\mu\nu} T_{\mu\nu}$$



$$\mathfrak{T} := ((\tau^r)^2 + (\tau_t)^2 - 2(\tau_t^r)^2) / f^2$$

useful

$$\tau_t := e^{-2h} T_{tt}$$

$$\tau_t^r := e^{-h} T_t^r$$

$$\tau^r := T^{rr}$$

$$T := (\tau^r - \tau_t) / f$$

+regular terms\*

All three components go to zero or diverge in the same way

### Einstein equations

$$\partial_r C = 8\pi r^2 \tau_t / f,$$

$$\partial_t C = 8\pi r^2 e^h \tau_t^r,$$

$$\partial_r h = 4\pi r (\tau_t + \tau^r) / f^2$$

$$\lim_{r \rightarrow r_g} \tau_a \sim \begin{cases} \pm \Upsilon^2 f^0 \\ \tau_a(t) f^k \end{cases}$$

$k=0,1^*$

$$\lim_{r \rightarrow r_g} \tau_t = \lim_{r \rightarrow r_g} \tau^r = -\Upsilon^2 \quad k=0$$



# spherical symmetry

## metrics

1. The limiting form (close apparent horizon) of dynamical metrics is almost uniquely defined (both  $k=0$  and  $k=1$ ).

$$x := r - r_g$$

$$C = r_g - 4\sqrt{\pi r_g^3} \Upsilon \sqrt{x} + \dots \quad h = -\frac{1}{2} \ln \frac{x}{\xi} + \dots \quad \leftarrow k=0$$

(dynamical BH/WoH; more static options)

$k=1 \rightarrow$

$$C = r - c_{32} x^{3/2} + \dots \quad h = -\frac{3}{2} \ln \frac{x}{\xi} + \dots$$

2. BH parameters are related via evaporation rate

$$\frac{dr_g}{dt} = -4\sqrt{\pi r_g \xi} \Upsilon$$

$$\frac{dr_g}{dt} = -\frac{c_{32} \xi^{3/2}}{r_g}$$

# spherical symmetry

## metrics

1. The limiting form (close apparent horizon) of dynamical metrics is almost uniquely defined (both  $k=0$  and  $k=1$ ).

$$x := r - r_g$$

$$C = r_g - 4\sqrt{\pi r_g^3} \Upsilon \sqrt{x} + \dots \quad h = -\frac{1}{2} \ln \frac{x}{\xi} + \dots \quad \leftarrow k=0$$

(dynamical BH/WoH; more static options)

$k=1 \rightarrow$

$$C = r - c_{32} x^{3/2} + \dots \quad h = -\frac{3}{2} \ln \frac{x}{\xi} + \dots$$

2. BH parameters are related via evaporation rate

$$\frac{dr_g}{dt} = -4\sqrt{\pi r_g \xi} \Upsilon$$

$$\frac{dr_g}{dt} = -\frac{c_{32} \xi^{3/2}}{r_g}$$

- No static  $k=0$  solutions
- Vaidya metrics are  $k=0$  solutions
- Reissner-Nordström, many static RBH are examples of  $k=1$  solutions:  $C = r_g + 8\pi r_g^2 \rho_g x + \dots$
- Popular dynamic RBH models are  $k=0$  solutions



# spherical symmetry

## metrics

3. Most convenient coordinates are retarded  $(u, r)$  for *white holes* and advanced  $(v, r)$  for *black holes*

$$dt = e^{-h} \left( e^{h_{\pm}} dv_{\pm} \mp f^{-1} dr \right)$$

$$ds^2 = -e^{2h_+} f dv^2 + e^{h_+} dv dr + r^2 d\Omega_2$$

$$= -e^{2h_-} f du^2 + e^{h_-} du dr + r^2 d\Omega_2$$

$$2M(t, r) \equiv C(t, r) \equiv C_-(u(t, r), r) \equiv \dots$$

E.g, in  $(v, r)$  the metric is regular at  $r_g \equiv r_+$  for  $r'_g < 0$  and singular for  $r'_g > 0$

$$e^{-h_+} \partial_v C_+ + f \partial_r C_+ = 8\pi r^2 \theta_v,$$

$$\partial_r C_+ = -8\pi r^2 \theta_{vr},$$

$$\partial_r h_+ = 4\pi r \theta_r.$$

$$\theta_v := e^{-2h_+} \Theta_{vv} = \tau_t,$$

$$\theta_{vr} := e^{-h_+} \Theta_{vr} = (\tau_t^r - \tau_t) / f,$$

$$\theta_r := \Theta_{rr} = (\tau^r + \tau_t - 2\tau_t^r) / f^2$$

# spherical symmetry

## metrics

3. Most convenient coordinates are retarded  $(u, r)$  for *white holes* and advanced  $(v, r)$  for *black holes*

$$dt = e^{-h} \left( e^{h_{\pm}} dv_{\pm} \mp f^{-1} dr \right)$$

$$ds^2 = -e^{2h_+} f dv^2 + e^{h_+} dv dr + r^2 d\Omega_2$$

$$= -e^{2h_-} f du^2 + e^{h_-} du dr + r^2 d\Omega_2$$

$$2M(t, r) \equiv C(t, r) \equiv C_-(u(t, r), r) \equiv \dots$$

E.g, in  $(v, r)$  the metric is regular at  $r_g \equiv r_+$  for  $r'_g < 0$  and singular for  $r'_g > 0$

$$e^{-h_+} \partial_v C_+ + f \partial_r C_+ = 8\pi r^2 \theta_v,$$

$$\theta_v := e^{-2h_+} \Theta_{vv} = \tau_t,$$

$$\partial_r C_+ = -8\pi r^2 \theta_{vr},$$

$$\theta_{vr} := e^{-h_+} \Theta_{vr} = (\tau_t^r - \tau_t) / f,$$

$$\partial_r h_+ = 4\pi r \theta_r.$$

$$\theta_r := \Theta_{rr} = (\tau^r + \tau_t - 2\tau_t^r) / f^2$$

$$r'_g < 0 \blacktriangleright (v, r)$$

$$\theta_{\text{in}} < 0, \theta_{\text{out}} < 0$$

BH solutions

$$r'_g > 0 \blacktriangleright (u, r)$$

$$\theta_{\text{in}} > 0, \theta_{\text{out}} > 0$$

WH solutions



# spherical symmetry

## metrics

3. Most convenient coordinates are retarded  $(u, r)$  for *white holes* and advanced  $(v, r)$  for *black holes*

$$dt = e^{-h} \left( e^{h_{\pm}} dv_{\pm} \mp f^{-1} dr \right)$$

$$ds^2 = -e^{2h_+} f dv^2 + e^{h_+} dv dr + r^2 d\Omega_2$$

$$= -e^{2h_-} f du^2 + e^{h_-} du dr + r^2 d\Omega_2$$

$$2M(t, r) \equiv C(t, r) \equiv C_-(u(t, r), r) \equiv \dots$$

E.g, in  $(v, r)$  the metric is regular at  $r_g \equiv r_+$  for  $r'_g < 0$  and singular for  $r'_g > 0$

$$e^{-h_+} \partial_v C_+ + f \partial_r C_+ = 8\pi r^2 \theta_v,$$

$$\theta_v := e^{-2h_+} \Theta_{vv} = \tau_t,$$

$$\partial_r C_+ = -8\pi r^2 \theta_{vr},$$

$$\theta_{vr} := e^{-h_+} \Theta_{vr} = (\tau_t^r - \tau_t) / f,$$

$$\partial_r h_+ = 4\pi r \theta_r.$$

$$\theta_r := \Theta_{rr} = (\tau^r + \tau_t - 2\tau_t^r) / f^2$$

$$r'_g < 0 \blacktriangleright (v, r)$$

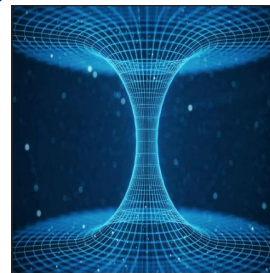
$$\theta_{in} < 0, \theta_{out} < 0$$

BH solutions

$$r'_g > 0 \blacktriangleright (u, r)$$

$$\theta_{in} > 0, \theta_{out} > 0$$

WH solutions



A wormhole throat: a marginal outer trapped surface +

Use all possible solution + impose wormhole requirements

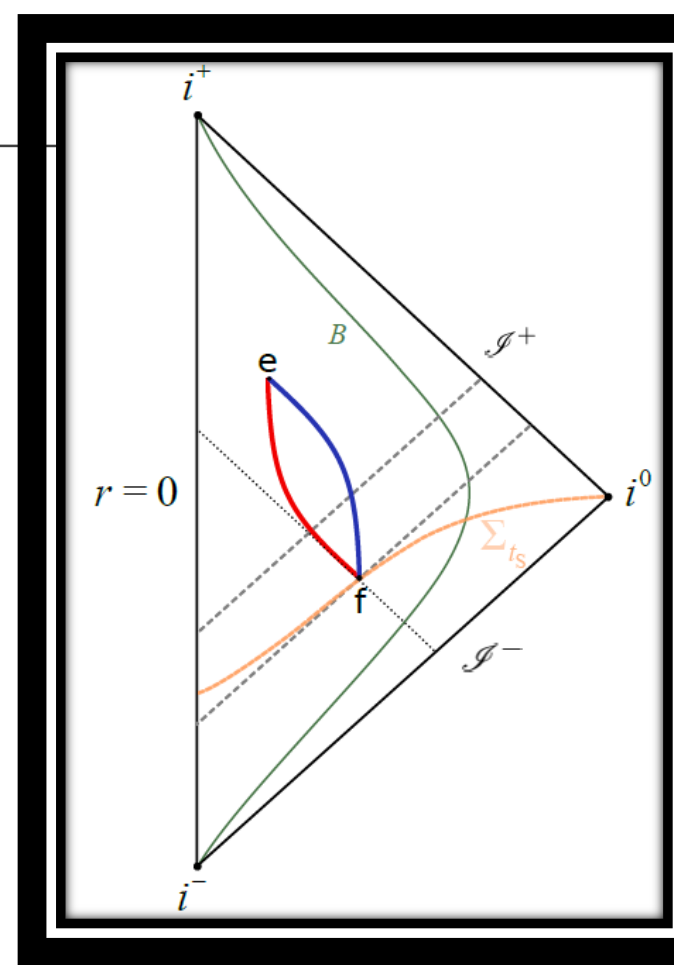
# spherical symmetry: recap + some results

## PBH: the process

- ❑ Use Schwarzschild coordinates to extract the info from divergencies
- ❑ Pick a nice form of the Einstein equations. Demand real solutions
- ❑ Use null coordinates to help classification

## PBH: the properties

- ❑ Finite infall/collapse time (according to a distant Bob)
- ❑ Outer apparent horizon is always **timelike**
- ❑ Null energy condition is violated [in the vicinity of the outer horizon]
- ❑ Unique formation scenario.
- ❑ A weak firewall: energy density for escaping\* non-geodesic Alice diverges, but weakly
- ❑ Some popular RBH models do not work.
- ❑ Generalized surface gravity: Kodama
- ❑ Interesting consistency/thermo implications
- ❑ If the 1<sup>st</sup> law+ thermality of evaporation work, then a short hair



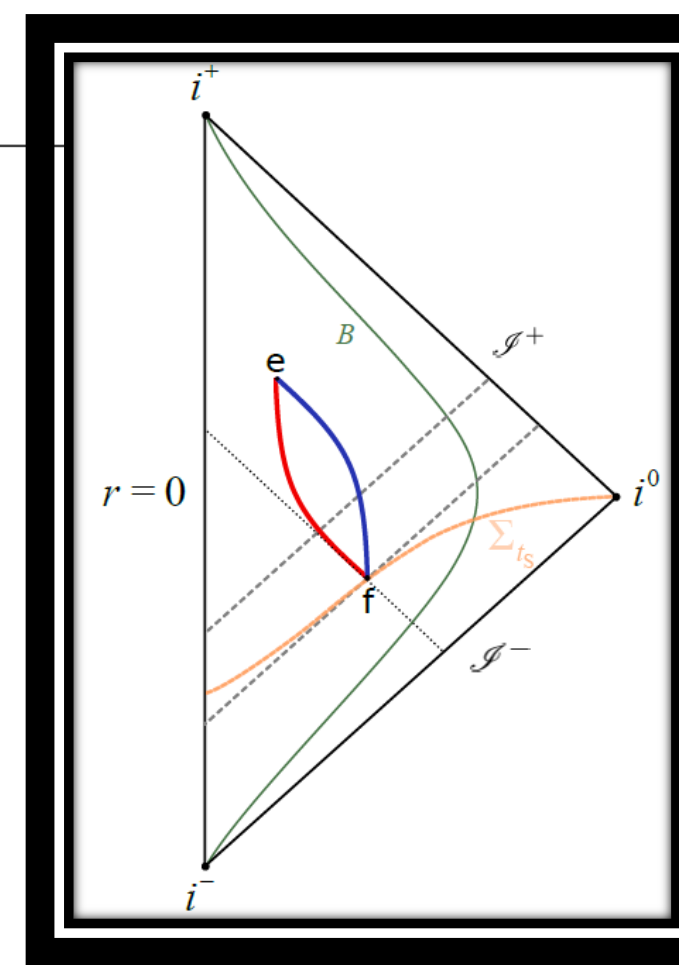
# spherical symmetry: recap + some results

## PBH: the process

- ❑ Use Schwarzschild coordinates to extract the info from divergencies
- ❑ Pick a nice form of the Einstein equations. Demand real solutions
- ❑ Use null coordinates to help classification

## PBH: the properties

- ❑ Finite infall/collapse time (according to a distant Bob)
- ❑ Outer apparent horizon is always **timelike**
- ❑ Null energy condition is violated [in the vicinity of the outer horizon]
- ❑ Unique formation scenario.
- ❑ A weak firewall: energy density for escaping\* non-geodesic Alice diverges, but weakly
- ❑ Some popular RBH models do not work.
- ❑ Generalized surface gravity: Kodama
- ❑ Interesting consistency/thermo implications
- ❑ If the 1<sup>st</sup> law+ thermality of evaporation work, then a short hair



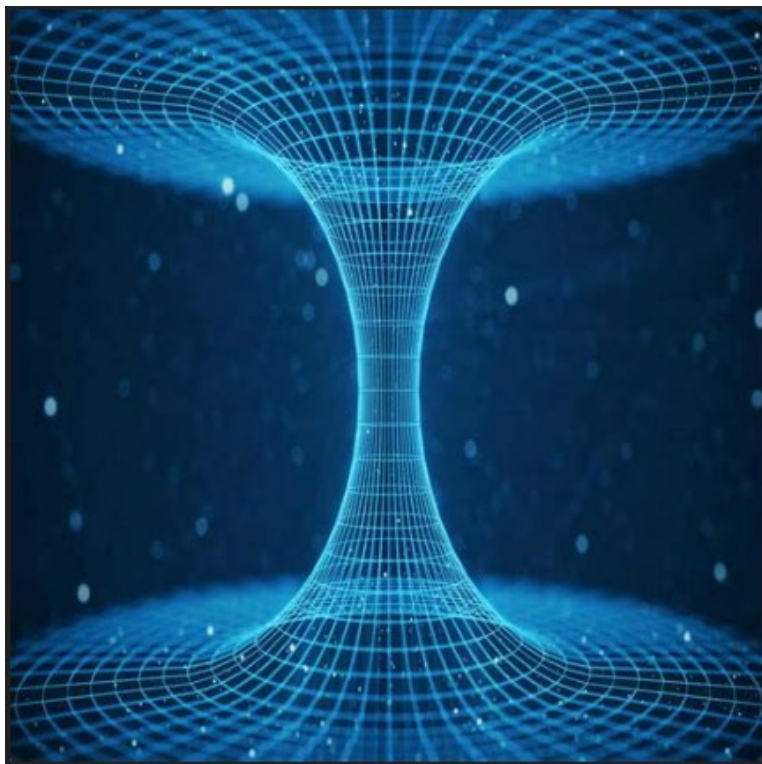
## Wormholes:

- ❑ A problem



# wormholes

## classic sci-fi



- ❑ A wormhole throat: a marginal outer trapped surface +
- ❑ Use all possible solution + impose wormhole requirements

DRT, *Inaccessibility of traversable wormholes*, Phys. Rev. D **106**, 044035 (2022)

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

$$ds^2 = -e^\Phi dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

Schwarzschild radius

$$\max r_g = C(t, r_g)$$

- ❑ Ellis-Morris-Thorne  $\Phi = 0, C = b_0^2/r$
- ❑ Simpson-Visser

$$ds^2 = -\left(1 - \frac{2m}{\sqrt{\eta^2 + a^2}}\right) dt^2 + \frac{d\eta^2}{1 - \frac{2m}{\sqrt{\eta^2 + a^2}}} + (\eta^2 + a^2) d\Omega_2,$$

# wormholes

---

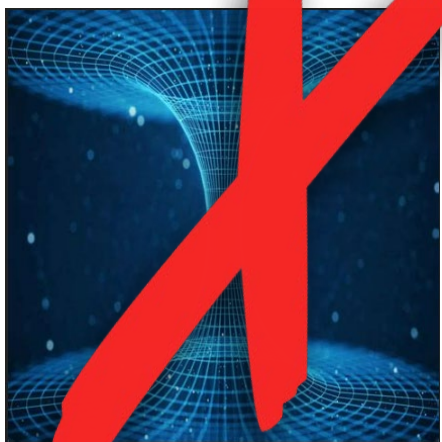
## consequences

- ❑ EMT and SV wormholes are  $k=1$
- ❑ More dynamical solutions are possible
- ❑ EMT and SV are not static limits of any of the admissible solutions
- ❑ Static limits of admissible solutions (if exist) have strong firewall and/or violate the QEI



$$\int_{\gamma} \dot{f}^2(\tau) \rho d\tau \geq -B(R, f, \gamma)$$

Kontou and Olum,  
PRD **91**, 104005 (2015).











*Horizon singularities & EMT*  
Mon 12/12, R2 @15:00

*BH thermo in dS*  
Wed 14/12, R2 @16:30

*PBH in modified gravity*  
Wed 14/12, R2 @16:45



**Fil Simovic**

**Ioannis Soranidis**

**PBH: Vaidya  
to Rindler  
& Hawking  
radiation**  
Thu 15/12,  
F&G, 1800+

**Pravin Dahal**



**Sebastian Murk**



# WORMHOLE 1

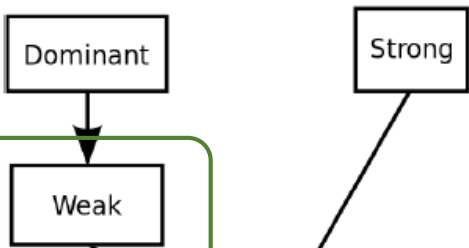




# spherical symmetry

## consequences

### Energy conditions



$$T_{\mu\nu} u^\mu u^\nu \geq 0$$

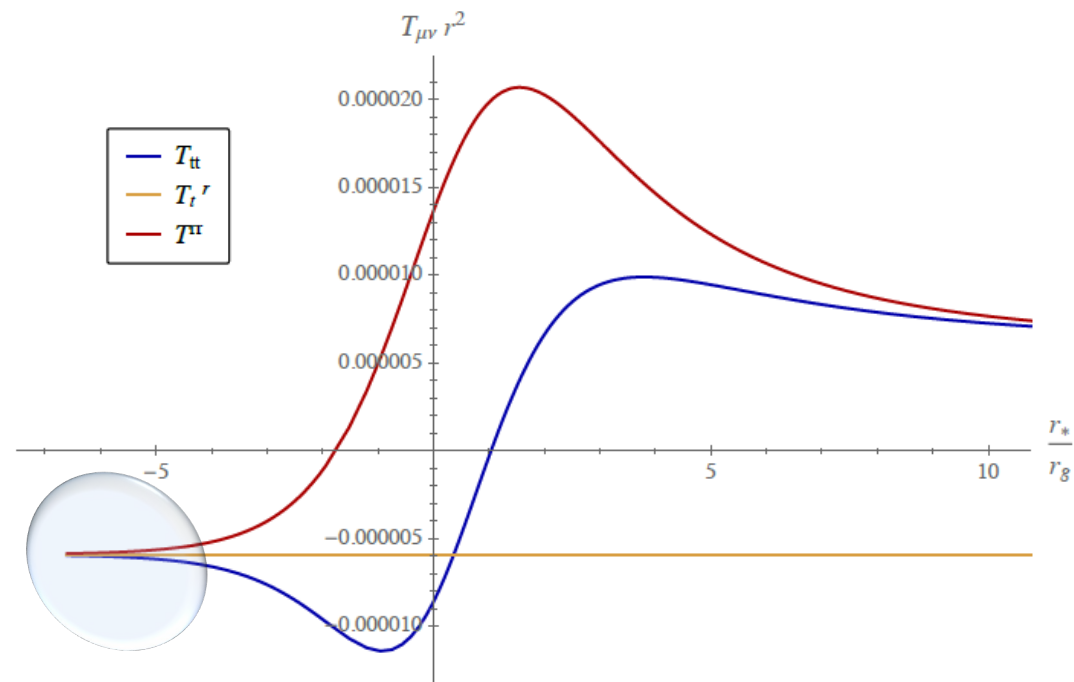
$$T_{\mu\nu} k^\mu k^\nu \geq 0 \Leftrightarrow R_{\mu\nu} k^\mu k^\nu$$

Solutions of the Einstein equations exist:  
the NEC **must** be violated

$\text{sgn}(T_{tt})$	$\text{sgn}(T_t^r)$	Time-evolution of Vaidya mass function	Black/White hole	NEC violation
-	-	$C'(v) < 0$	B	✓
-	+	$C'(u) > 0$	W	✓
				X
				X

Energy momentum ( $k=0$ )

$$T_{\hat{a}\hat{b}} = -\frac{\Upsilon^2(t)}{f} \begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$$



Levi and Ori, Phys. Rev. Lett. **117**, 231101 (2016)



# useful relations

**(v,r) & (t,r)**

$$\begin{aligned} x &:= r - r_g(t) \\ y &:= r - r_+(v) \end{aligned}$$

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

$$f = 4\sqrt{\pi r_g} \Upsilon \sqrt{x} + \ddot{h} = -\frac{1}{2} \ln \frac{x}{\xi} + \dots$$

$$ds^2 = -e^{2h_+} f(v,r) dv^2 + 2e^{h_+} dv dr + r^2 d\Omega_2$$

$$\begin{aligned} f &= 1 - \frac{C}{r} & C_+(v,r) &= r_+(v) + w_1(v)y + \mathcal{O}(y^2) \\ & & h_+(v,r) &= \chi_1(v)y + \mathcal{O}(y^2), \end{aligned}$$

$$x(r_+ + y, v) = r_+ + y - r_g(t(v, r_+ + y)) = -r_g'' y^2 / (2r_g'^2) + \mathcal{O}(y^3).$$

□ 1<sup>st</sup> step in the coordinate transformation ▲

□ 1<sup>st</sup> terms in the expansion ▼ (use invariance of the MS mass)

$$\text{(Dynamical } k=1 \text{ solution: } w_1=1) \quad w_1 = 1 - 2\sqrt{2\pi r_g^3 |r_g''|} \frac{\Upsilon}{|r_g'|}$$

## Parameter identification, 1.0

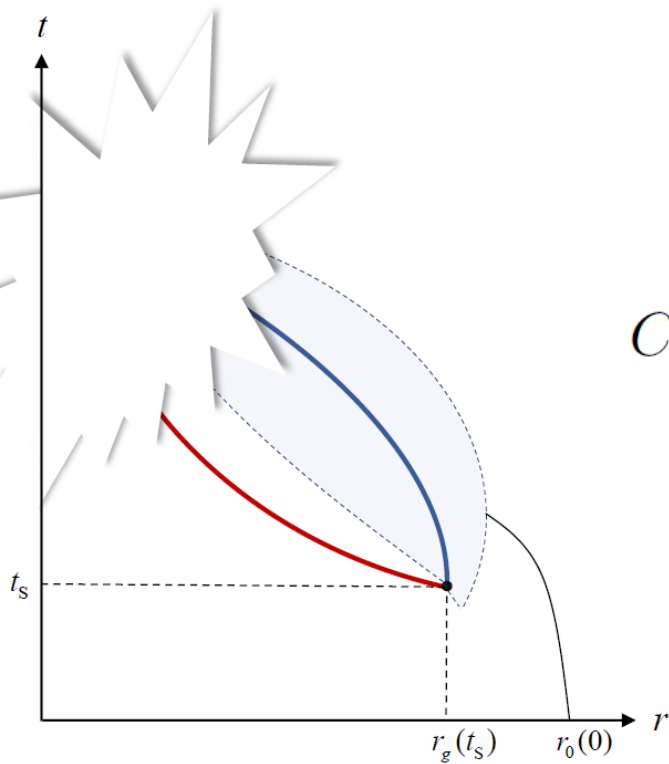
using the EMT transformation, near-horizon expansions:

(for the semiclassical evaporation law in a general form)

$$r_g'(t) = \Gamma(r_g) \quad r_+'(t) = \Gamma_+(r_+)$$

$$\begin{aligned} \Upsilon &= \frac{1}{2} \sqrt{\frac{|r_g''|}{2\pi r_g} \frac{|r_+'|}{|r_g'|}} \\ \xi &= \frac{r_g'^4}{2|r_g''| r_+'^2} \end{aligned}$$

# BH formation



□ A PBH forms as  $k=1$  solution and then evolves as (evaporating)  $k=0$  solution

$$C(v, r) = \Delta(v) + r_*(v) + \sum w_i(v)(r - r_*)^i$$



min gap  $C-r$

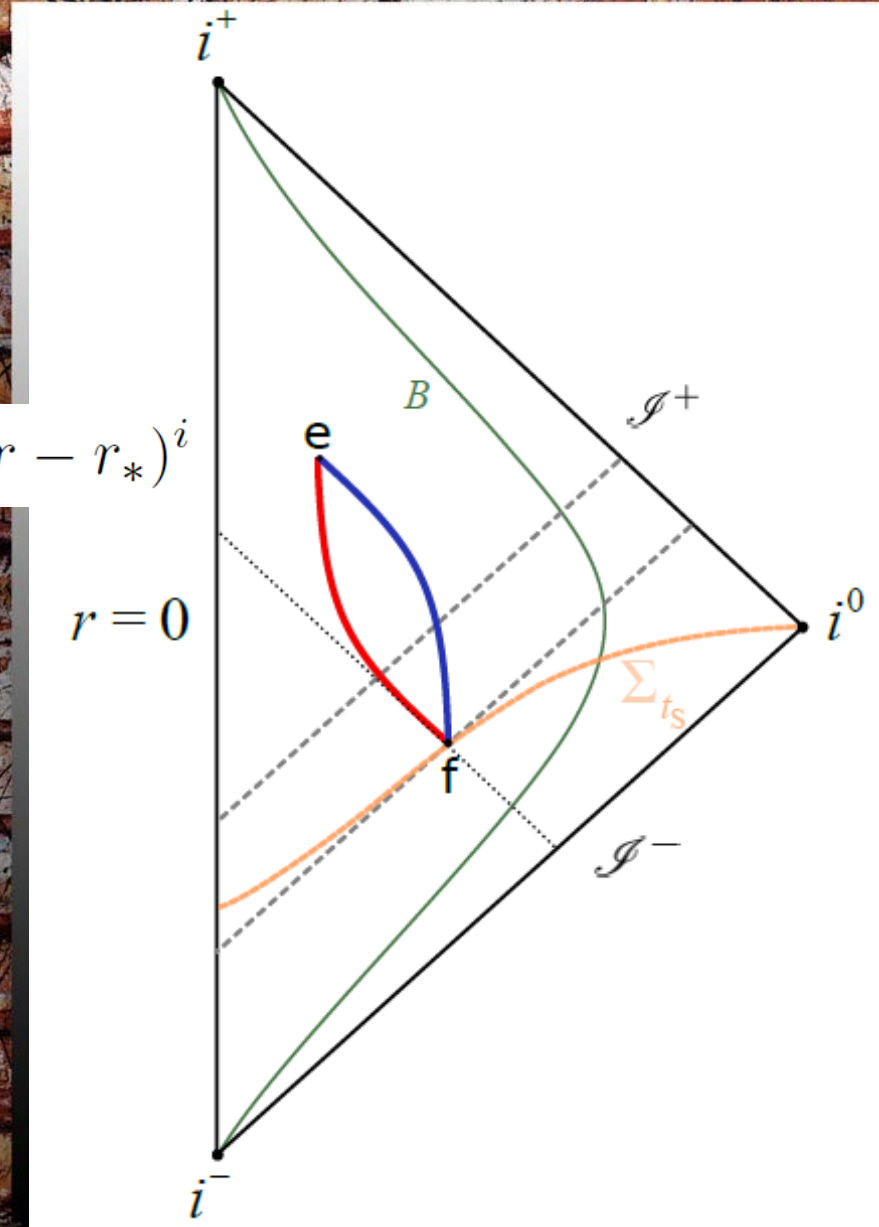


$r$  of min  $C-r$

Hence up to formation of the first marginally trapped surface  $w_1=1$

At the formation:  $\Delta(v_f)=0, r_+(v_f)=r_*(v_f)$

After formation:  $\Delta=0$ , but  $r_+(v)$  is not @ min



# Surface gravity

---

Surface gravity  $\kappa$  is:

- (a) inaffinity of null geodesics *on the horizon*
- (b) and the peeling off properties of null geodesics *near the horizon*

Interpretation: the force per unit mass as measured at infinity, to keep the observer stationary just outside the horizon (c) Stationary Killing horizon: (a)=(b)=(c)

Schwarzschild:  $\kappa = 1/4M = 1/2C$

 Surface gravity plays a key role in BH thermo NEC is true

0<sup>th</sup> law: surface gravity is constant on the horizon


1<sup>st</sup> law:

$$dM = \frac{\kappa}{8\pi} dA + \omega_H dJ$$

NEC is false

Temperature

$$T = \frac{\kappa}{2\pi} \frac{\hbar c^3}{G k_B}$$

 Surface gravity plays a key role in the Hawking radiation



# surface gravity

---

## @ outer apparent horizon

### *Peeling surface gravity*

$$\kappa_{\text{peel}} = \frac{e^{h(t,r)}}{r} (1 - \partial_r C(t,r)) \Big|_{r=r_g}$$

- Vanzo, Acquaviva, and Di Criscienzo, Class. Quant. Grav. **28**, 183001 (2011).
- Cropp, Liberati, Visser, Class. Quant. Grav. **30**, 125001 (2013)

$$\kappa_{\text{peel}} = 0, \infty^* \quad \text{both } k=0,1 \text{ solutions}$$

### *Kodama surface gravity*

$$\kappa_K = \frac{1}{2} \left( \frac{C(v,r)}{r^2} - \frac{\partial_r C(v,r)}{r} \right) \Big|_{r=r_g \equiv r_+}$$

- Hayward, Class. Quant. Grav. **15**, 3147 (1996).

$$C = r_+ + w_1(r - r_+) + \dots$$

$$\kappa_K = 0 \quad \text{for } k=1 \text{ solution}$$

$$\kappa_K \leq 1/2r_+ \quad \text{for } k=0 (w_1=1) \text{ solution}$$

# universality of BH dynamics

- If we want the 1<sup>st</sup> law with AH, then the metric is “close” to Vaidya

$$dM = \frac{\kappa}{8\pi} dA$$

$$\kappa_K = \frac{1}{2r_+} \Rightarrow w_1 = 0$$

- Use the relations between the coefficients

$$r_+'^2 = \frac{r_g'^2}{|r_g''| r_g}$$



$$w_1 = 1 - 2\sqrt{2\pi r_g^3 |r_g''|} \frac{\Upsilon}{|r_g'|}$$
$$\Upsilon = \frac{1}{2} \sqrt{\frac{|r_g''|}{2\pi r_g} \frac{|r_+'|}{|r_g'|}}$$

- Add Page's law

$$r_g'(t) = \Gamma(r_g) = -\frac{\alpha}{r_g^2}$$

then

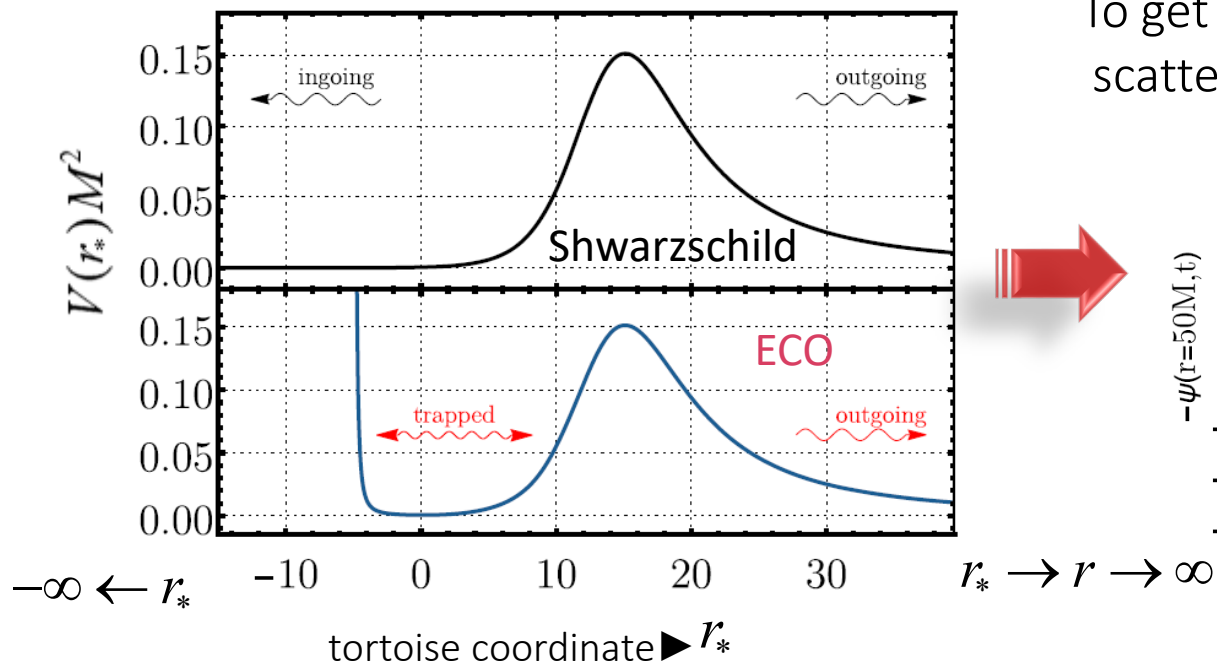
$$r_+' = -\frac{1}{2}$$

$$\Upsilon = \frac{1}{2\sqrt{2\pi} r_g}$$

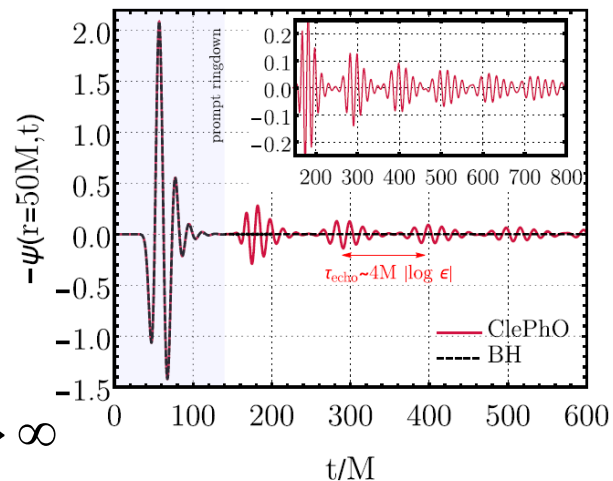
$$\xi = \frac{\alpha^2}{r_g^3}$$

Dahal, Simovic, Soranidis, DRT,  
soon(ish)

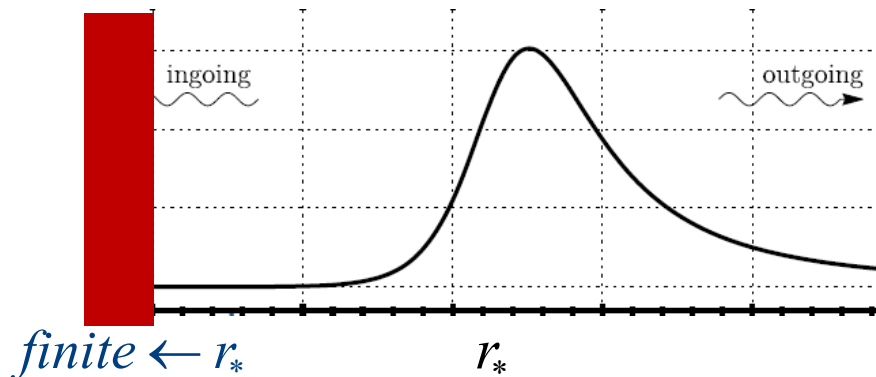
will be there a smoking gun?



To get to the QNM, waveforms:  
scattering in the effective potential



Cardoso and Pani,  
Liv. Rev. Rel. **22**, 4 (2019)



A missing piece

PBH give yet another potential:  
what are the QNM & waveforms?

