Black Hole Thermodynamics

in de Sitter spacetimes

Fil Simovic

24th Australian Institute of Physics Congress

December 11-16th, 2022

MACQUARIE University
Research Centre for Astronomy, Astrophysics & Astrophotonics

PHYSICS: Launching our future

Black Hole Thermodynamics

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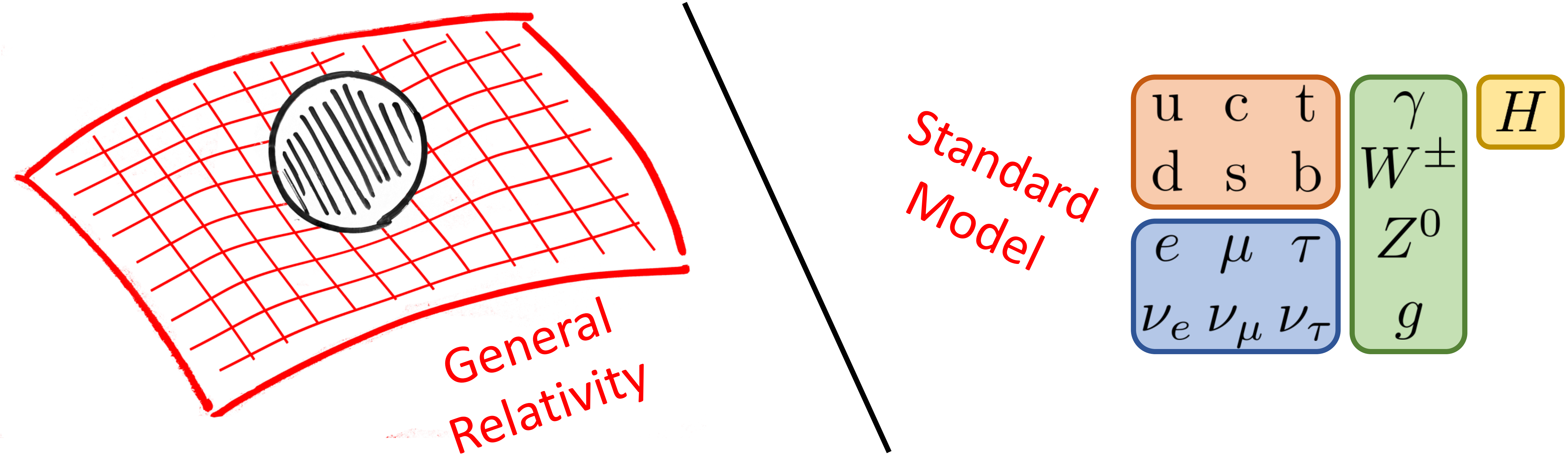
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

UNIVERSITY OF WATERLOO
CONCORDIA CUM VERITATE

Fundamental problems

- The entirety of the universe is described by two theories:



- There is a fundamental incompatibility between the two!

Why black holes?

- Black holes are one of the few objects in the universe where these issues manifest, due to the scales involved.

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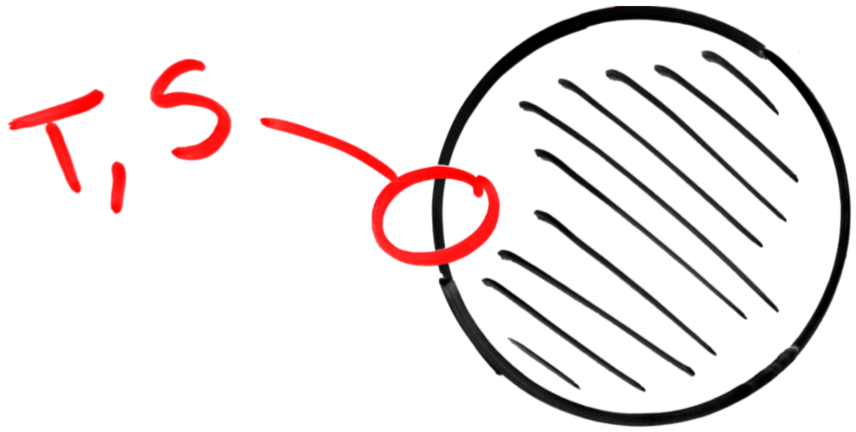

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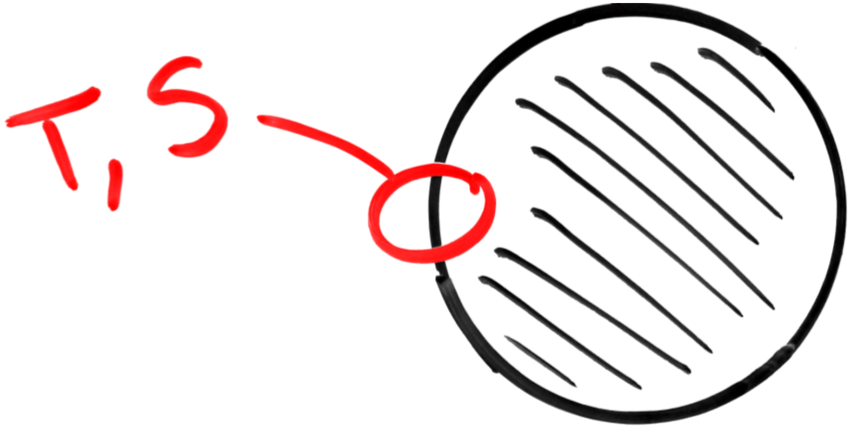

- Can we understand the thermodynamic nature of gravity?

Black hole thermodynamics

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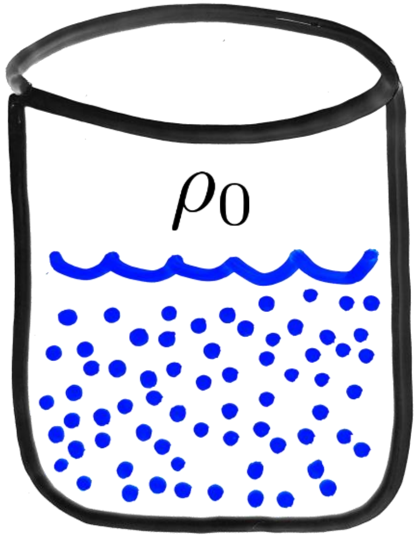
$$T = \frac{\kappa}{2\pi}\,, \quad S = \frac{A}{4} \qquad \rightarrow \qquad dM = T\, dS + \Phi\, dQ + \ldots + P dV?$$

$$P = -\Lambda/8\pi$$

Phase transitions

- Thermodynamic systems generically undergo phase transitions.
- Characterized by an 'order parameter' φ.



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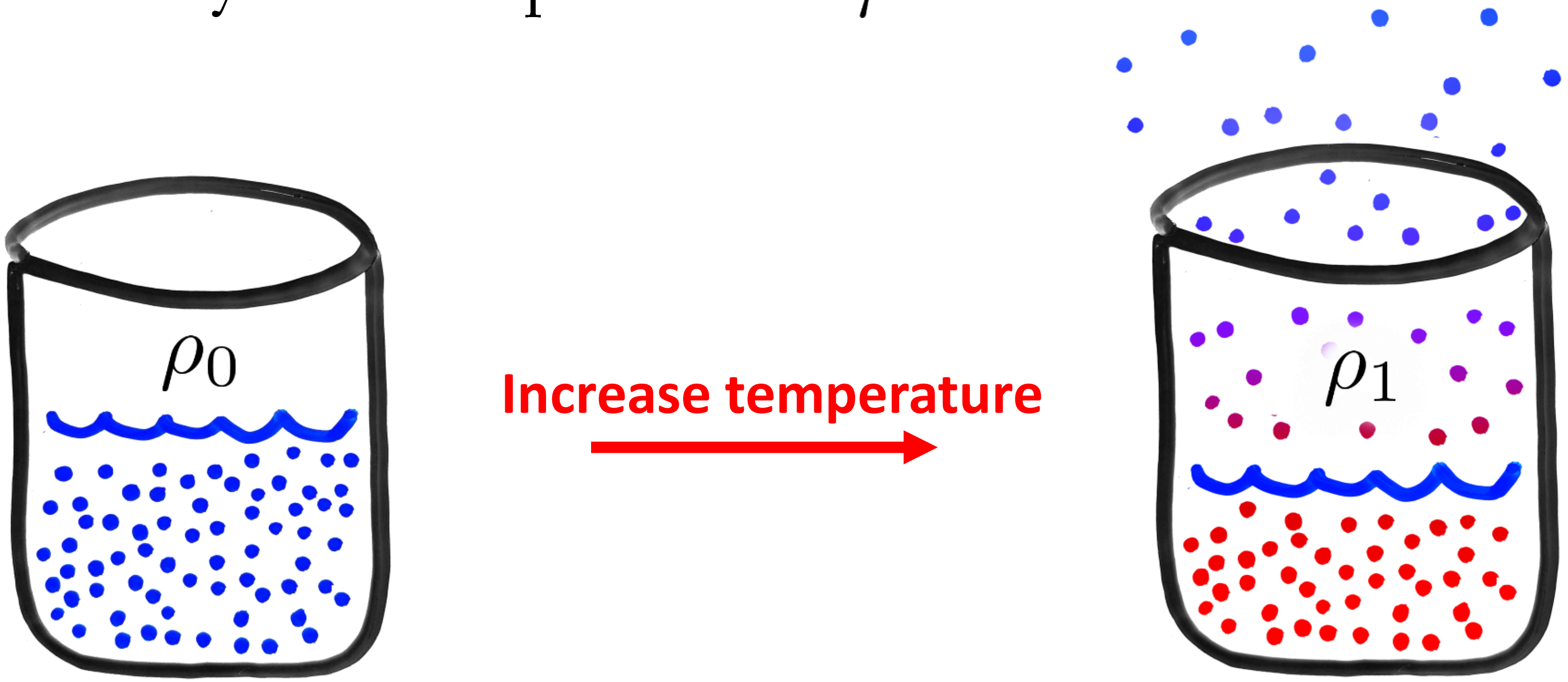

Phase transitions

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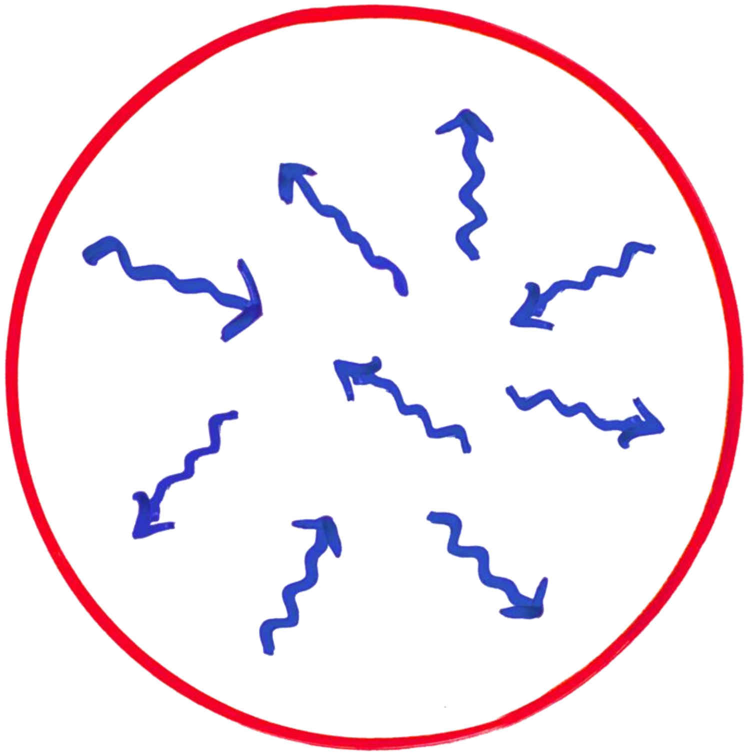

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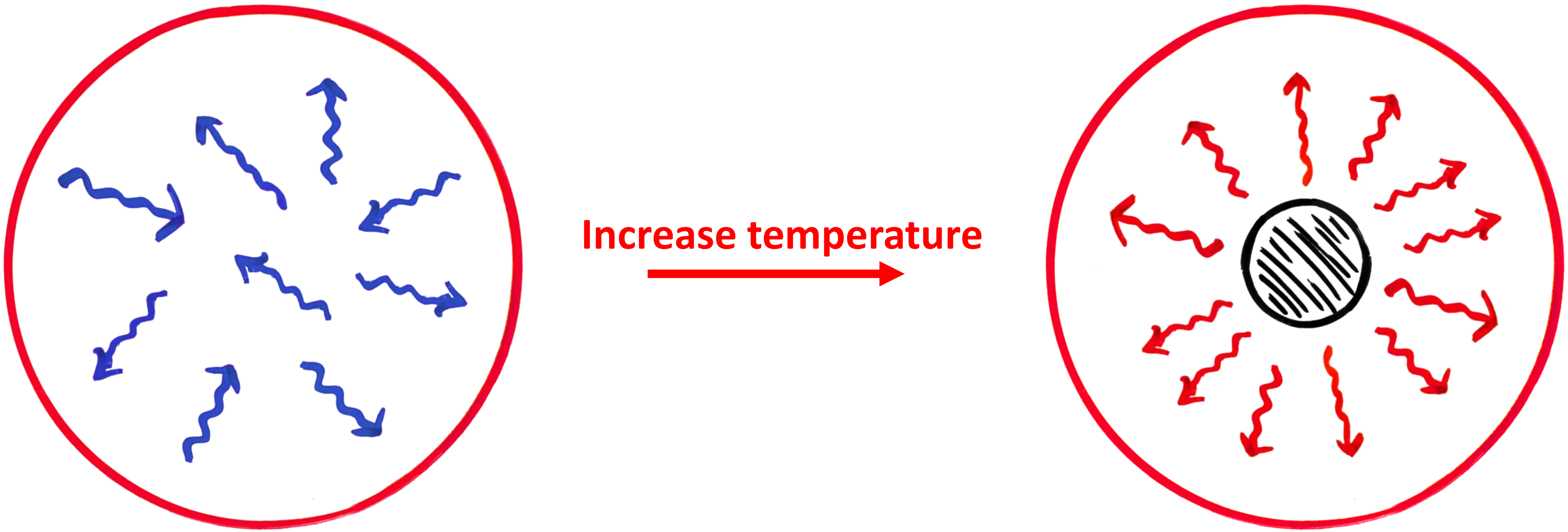

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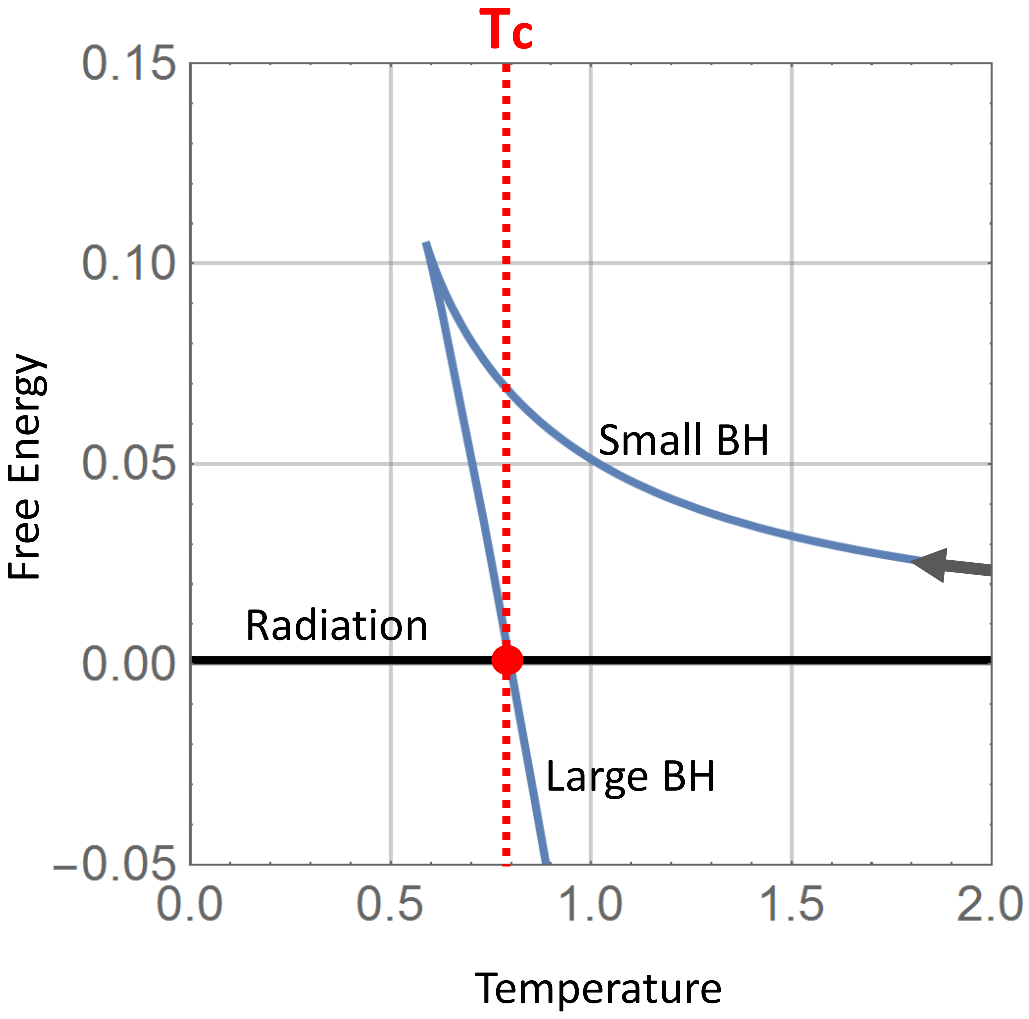
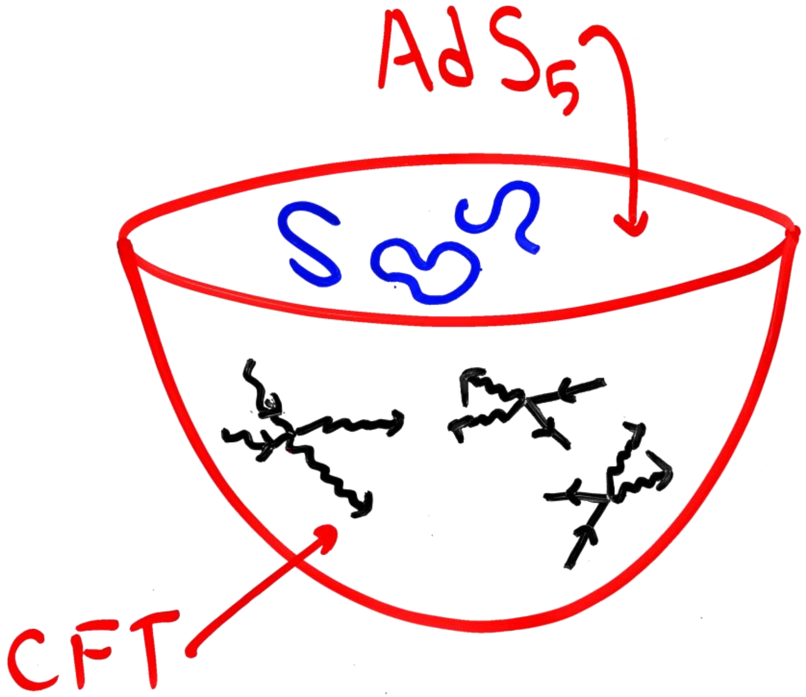
- When the temperature increases past T_c, a phase transition occurs.



The Hawking-Page transition

- The Hawking/Page transition is dual to a deconfinement transition in QGP through the AdS/CFT correspondence.

$$T_{\text{QCD}} \longleftrightarrow \frac{1}{r_{\text{horizon}}}$$



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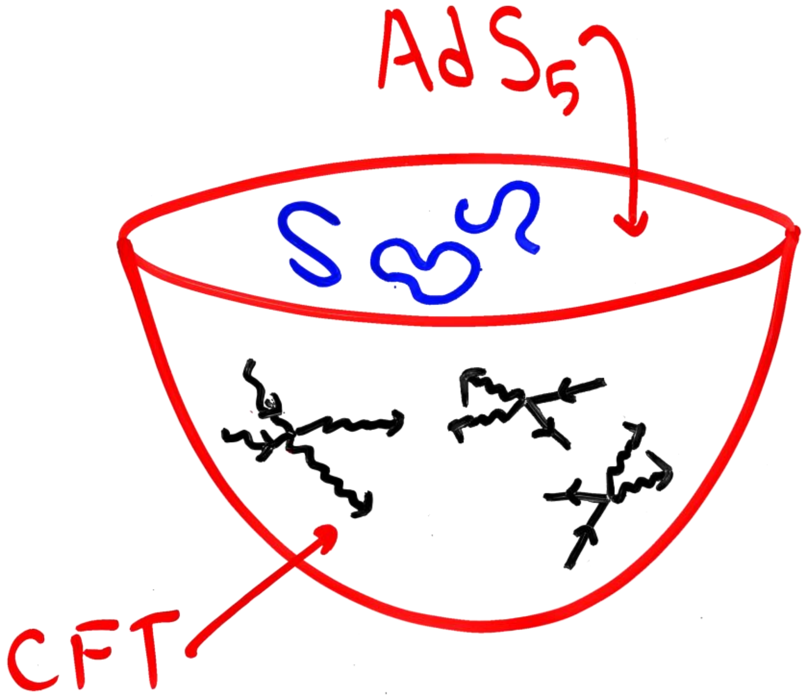

- Allows one to probe strongly coupled systems (where perturbation theory fails) through weakly coupled gravitational systems.

The Hawking-Page transition

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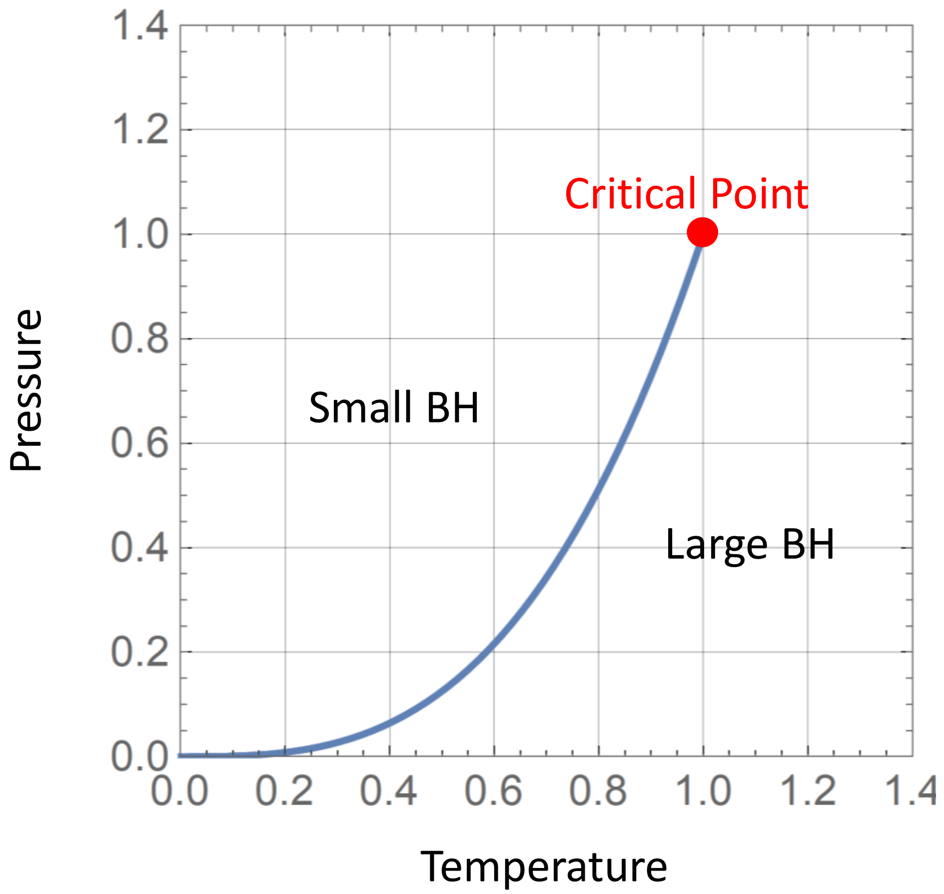

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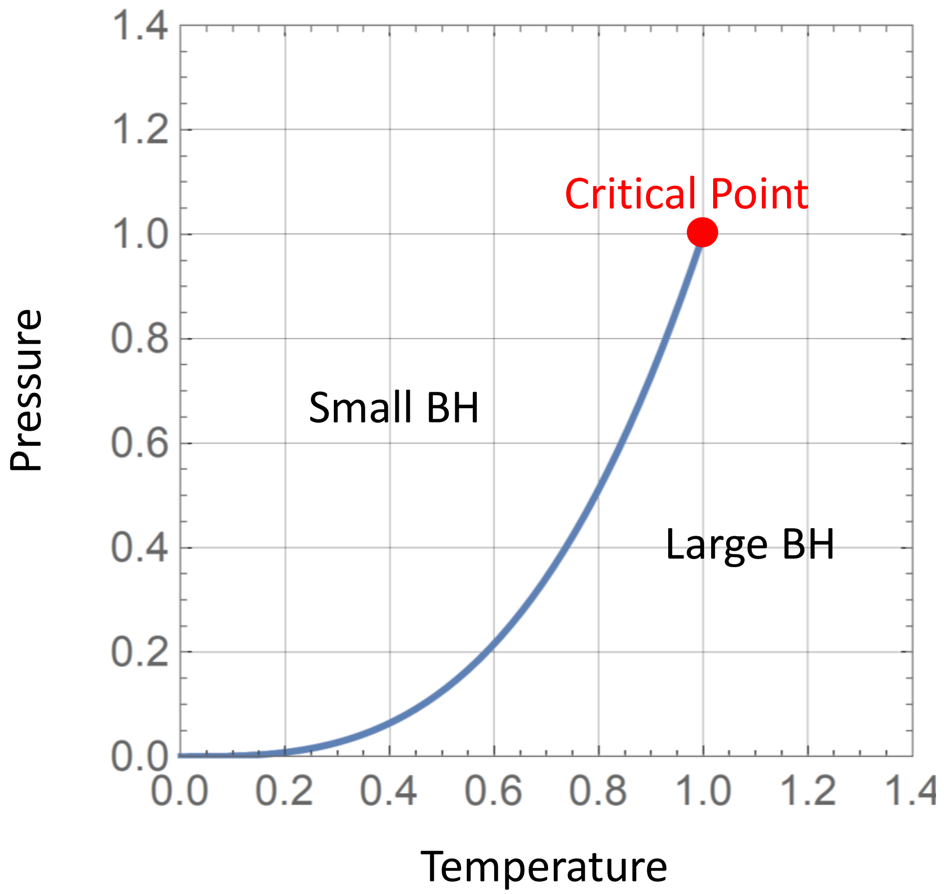
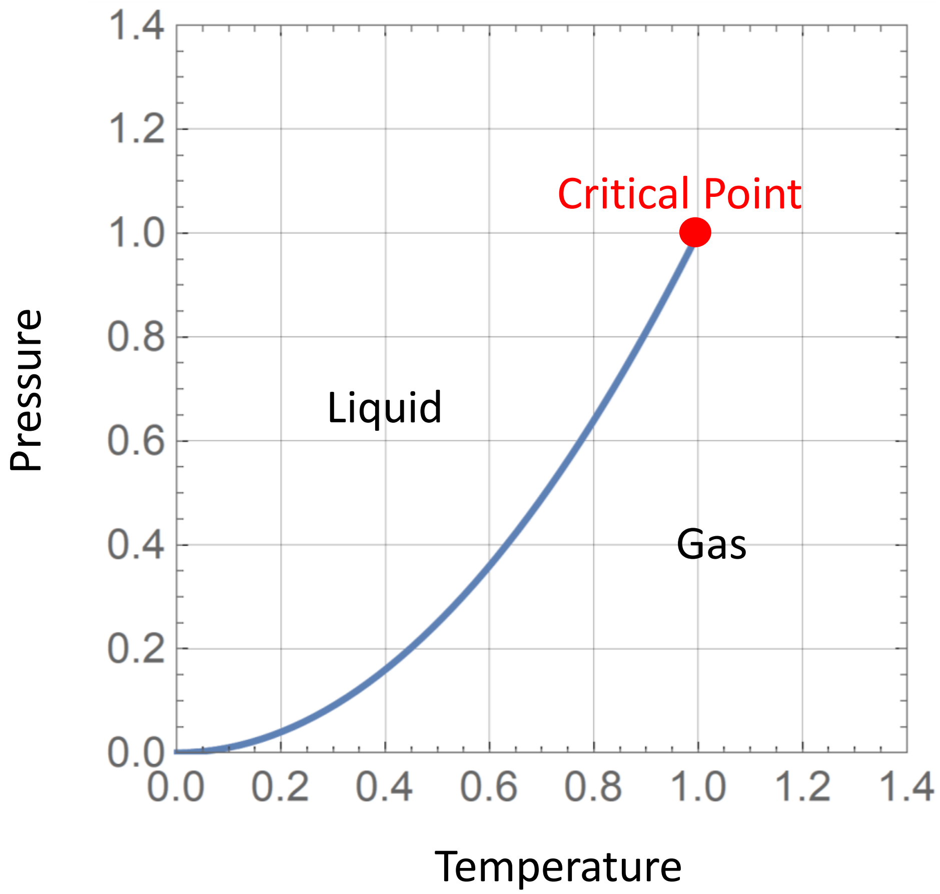
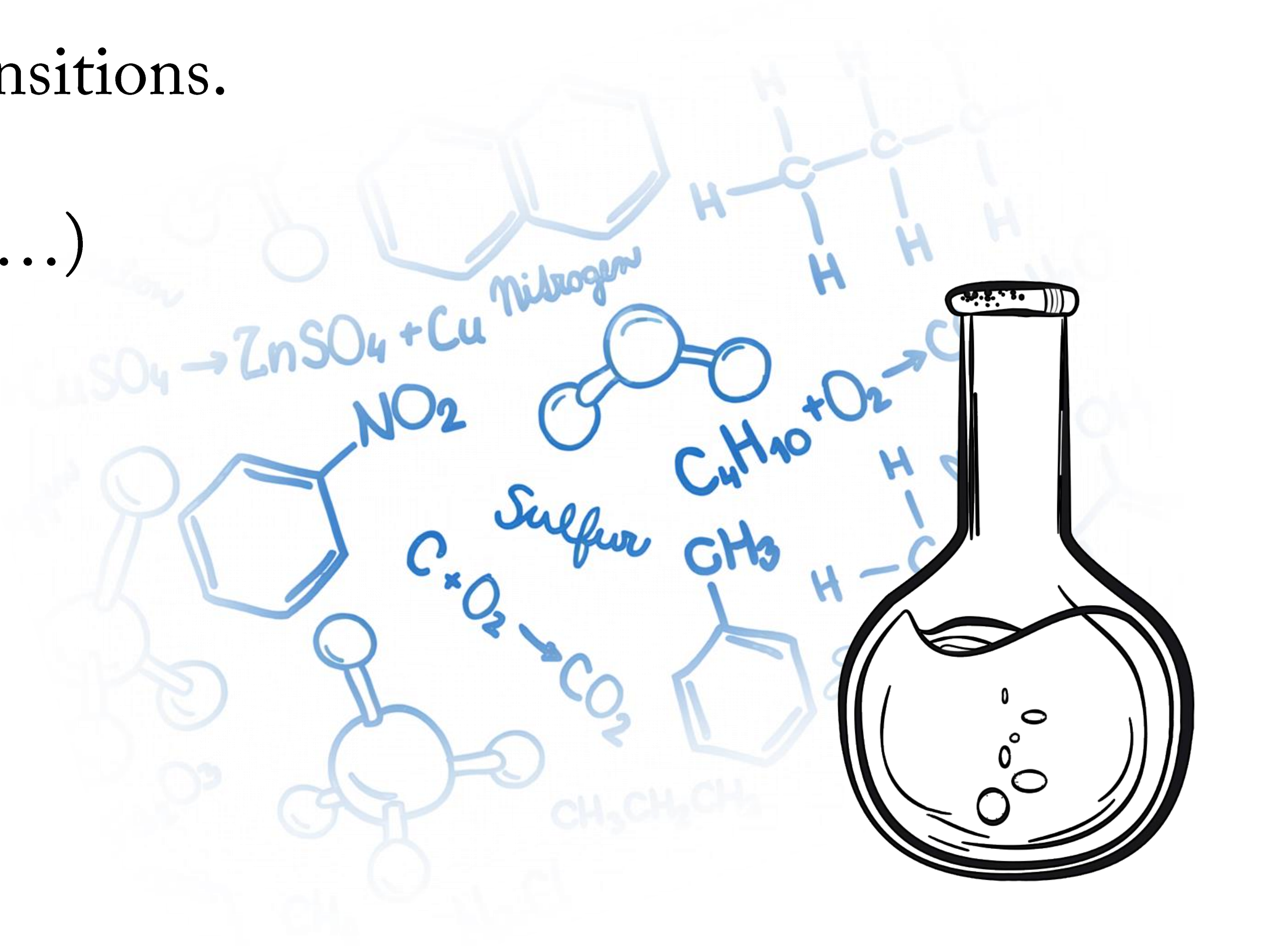




Rich phenomenology

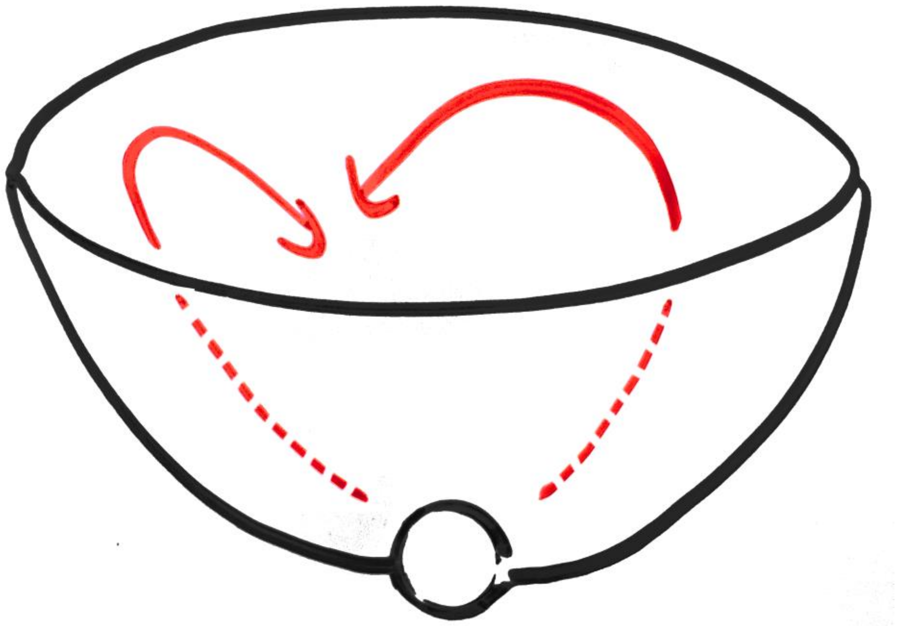
- Helium-like superfluid phase transitions.
- Triple points (water, mercury, etc…)
- Heat engines.
- Holographic superconductors.



David Kubizňák, Robert B. Mann, Mae Teo - ArXiv: 1608.06147

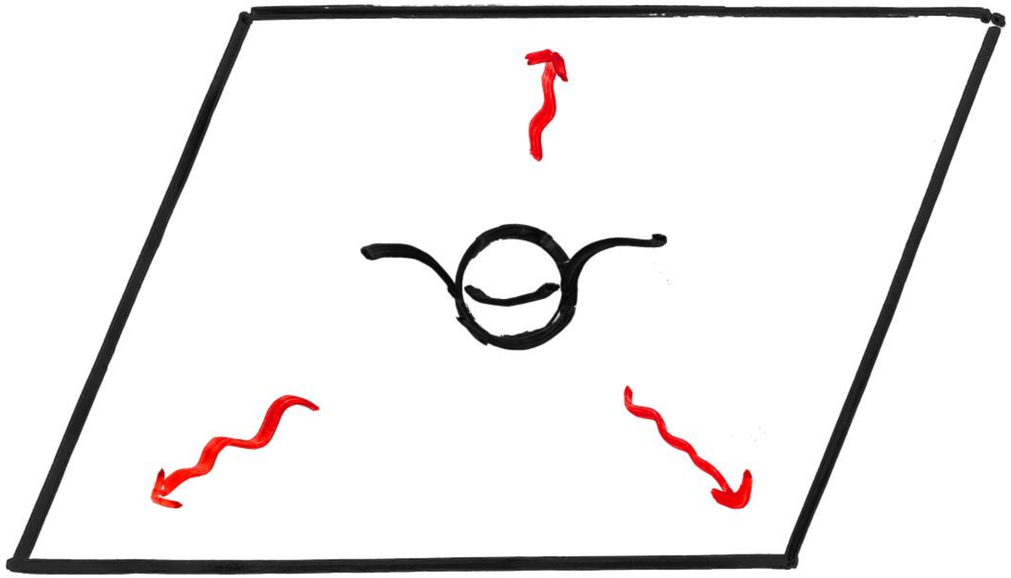
What about de Sitter?

- Anti-de Sitter space naturally confines radiation.



What about de Sitter?

- In asymptotically flat and de Sitter spacetimes, black holes evaporate and there is no thermal equilibrium.



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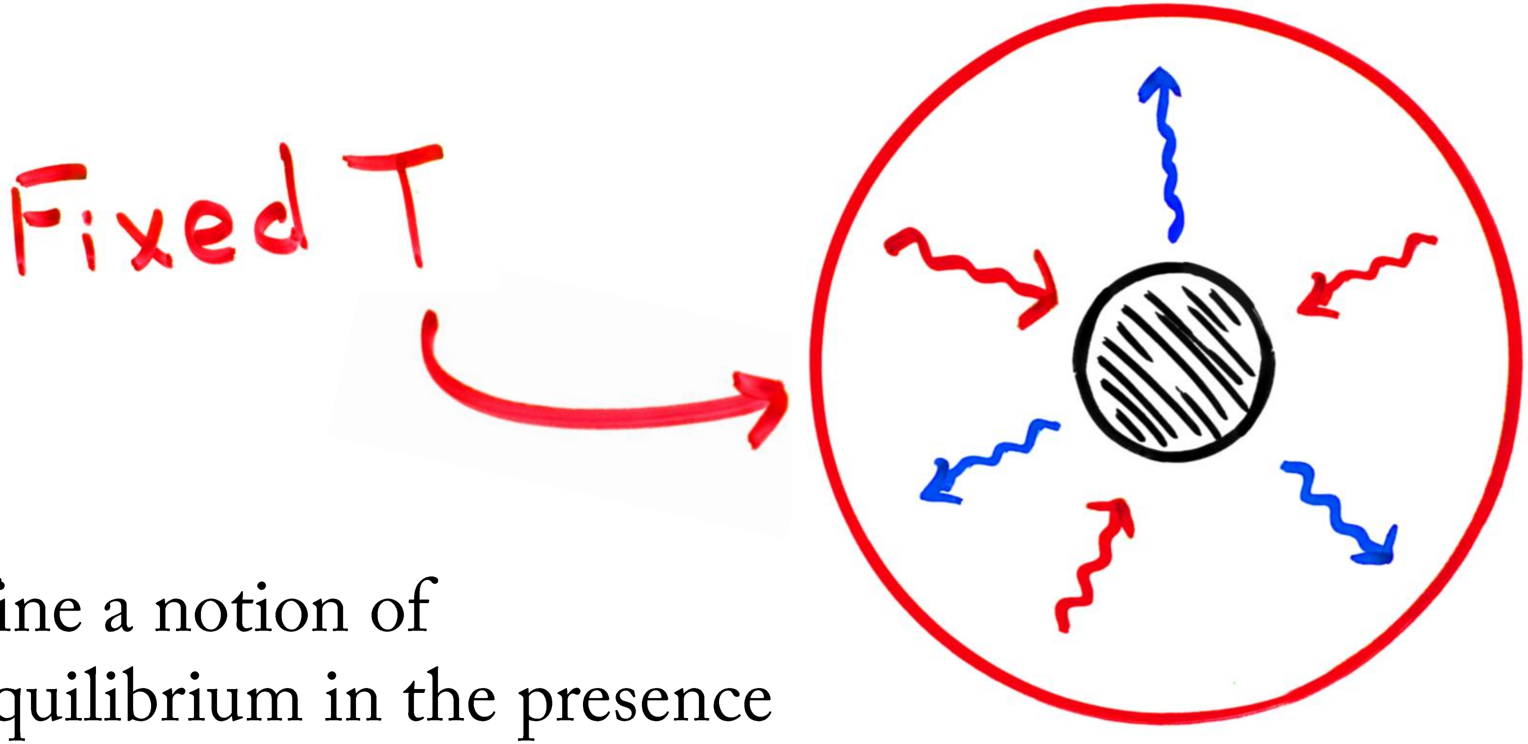
- de Sitter space is more relevant physically, but also more difficult.
- Difficulties with defining ‘mass’.
- Two horizons each with an independent temperature.
- Take a path integral approach with data specified on a boundary:

$$\mathcal{Z}(\beta) = \int \mathcal{D}[g]\, e^{iI_E/\hbar} \approx e^{-I_E[g_{cl}]/\hbar}$$

Black holes in cavities

- Embed the black hole in an isothermal cavity.



- Allows one to define a notion of thermodynamic equilibrium in the presence of a cosmological horizon.

The Euclidean action

1. Evaluate the on-shell Euclidean action, with appropriate boundary terms:

$$I_{\mathrm{E}} = -\frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{g}\,(R-2\Lambda) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3x\sqrt{k}\;K - I_0$$

Gibbons-Hawking-York

Background subtraction

The Euclidean action

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$$= \beta r_c\left[\sqrt{1-\frac{\Lambda r_c^2}{3}} - \sqrt{\left(1-\frac{r_h}{r_c}\right)\left(1-\frac{\Lambda}{3}(r_c^2+r_c r_h+r_h^2)\right)}\right] - \pi r_h^2$$

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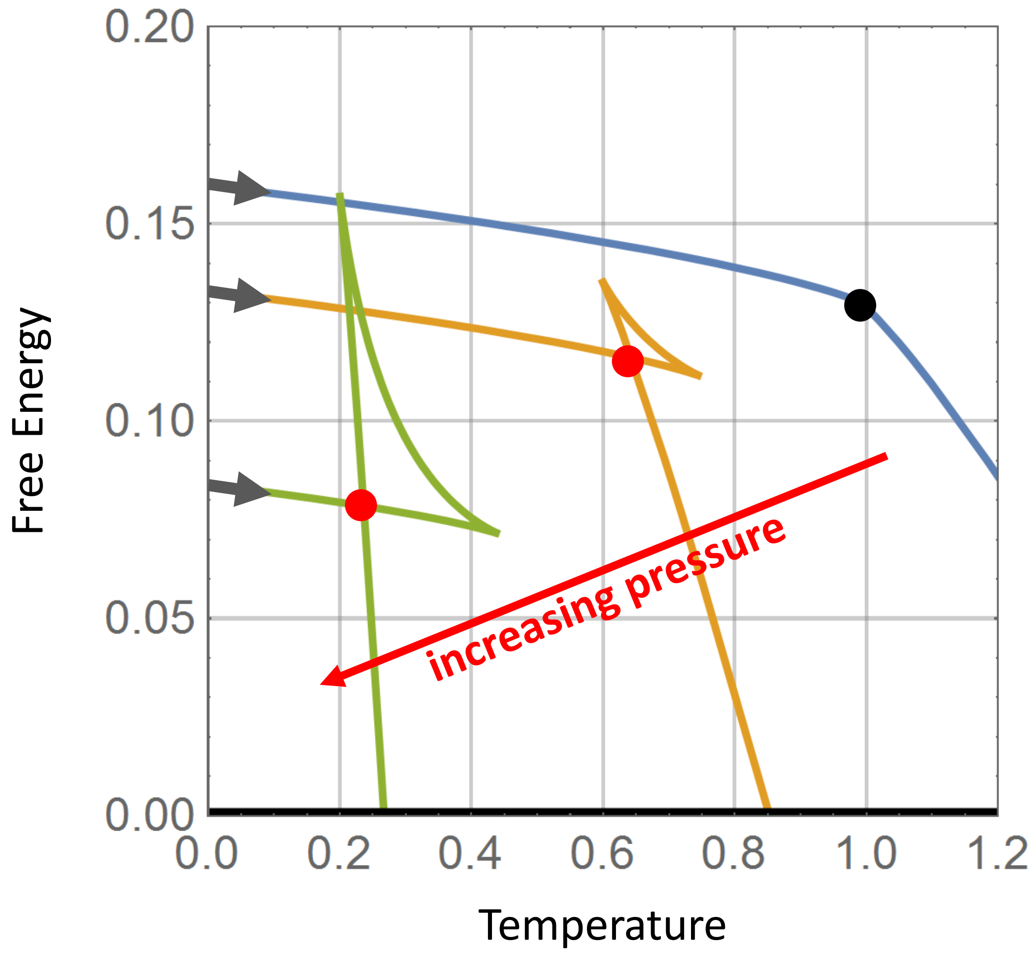
2. Determine thermodynamic quantities:

$$\langle E\rangle = \frac{\partial I_E}{\partial\beta}\,, \qquad S = \beta\left(\frac{\partial I_E}{\partial\beta}\right) - I_E\,, \qquad F = T I_E$$

Phase structure

Reissner–Nordstrom–de Sitter Black Holes

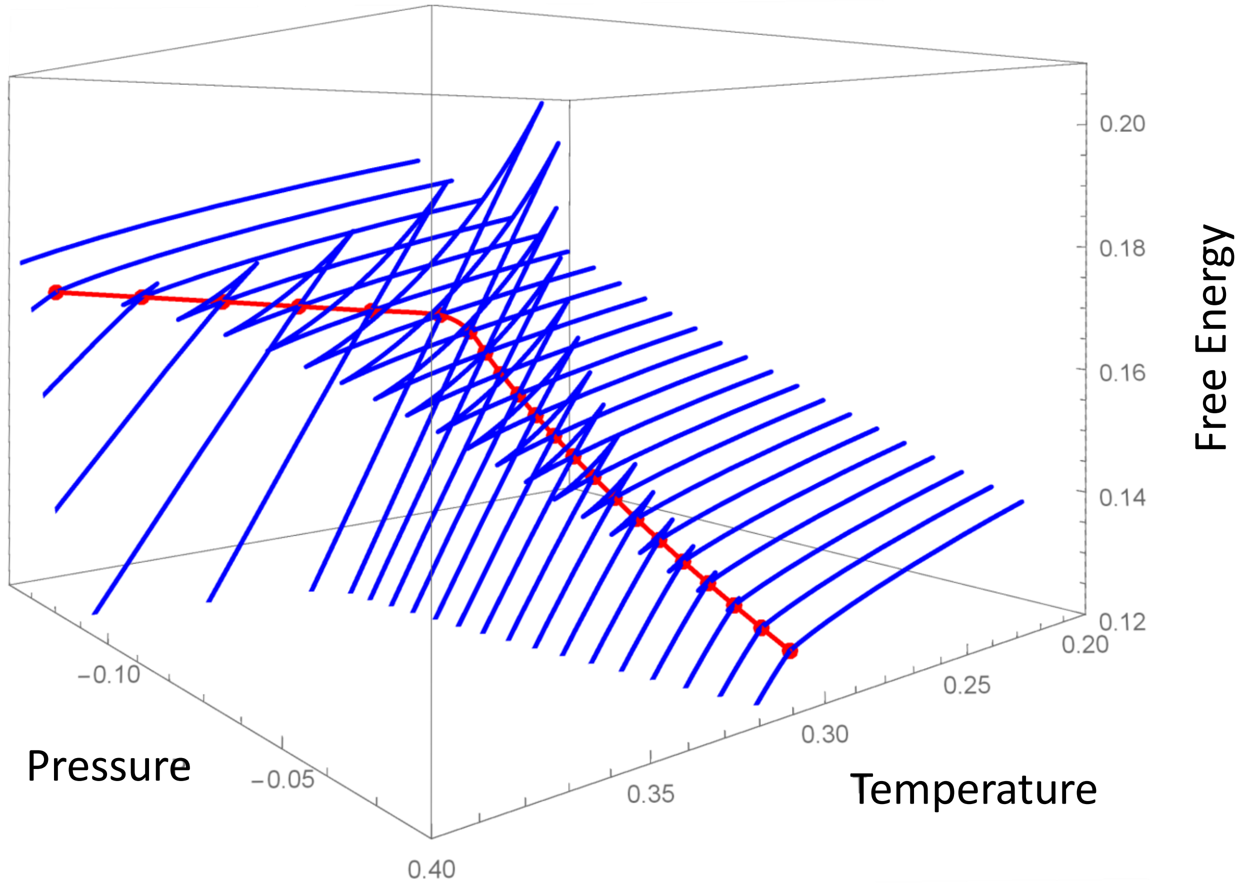


- When charge is present, a crossing in free energy occurs.
- Represents a phase transition between a large and small black hole.
- There is no transition to the radiation phase with fixed charge.

F. Simovic and R. B. Mann, Class. Quant. Grav. 36, 014002 (2019)

Phase structure

Reissner-Nordstrom-de Sitter Black Holes



- Free energy forms a “swallow-tube” in phase space.
- Phase transition exists only in a compact region in phase space, bounded by a minimum and maximum pressure.

Studied examples

- Schwarzschild and Reissner-Nordström-de Sitter black holes.
  ArXiv: 1807.11875
- Born-Infeld theory:
  ArXiv: 1904.04871
  $$\mathcal{L}_{BI} = 4b^2\left(1 - \sqrt{1 + \frac{F^{ab}F_{ab}}{2b^2}}\right)$$
- Gauss-Bonnet theory:
  ArXiv: 2002.01567
  $$\mathcal{L}_{GB} = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$
- Scalar fields:
  ArXiv: 2008.07593
  $$\mathcal{L}_{\phi} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{12}\phi^2 R - V(\phi)$$
- Exotic black holes, 4D Gauss-Bonnet, etc… ArXiv: 2107.11352, 2208.05500

Summary

- Black hole thermodynamics is most readily formulated in AdS, but can be done consistently in de Sitter spacetimes as well.
- Euclidean path integrals allow computation of partition functions.
- The presence of a cavity and cosmological horizon introduces interesting new phase structure.
- Applications through de Sitter space holography.

Thank you